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Regular Single Valued Neutrosophic Hypergraphs

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Abstract. In this paper we define the regular and totally regular single valued neutrosophic hypergraphs, and discuss the order and size along with properties of regular and totally regular single valued neutrosophic hypergraphs, we extended work on completeness of single valued neutrosophic hypergraphs.

Keywords: Single valued neutrosophic hypergraphs, regular single valued neutrosophic hypergraphs and totally regular single valued neutrosophic hypergraphs.

1 Introduction

The notion of neutrosophic sets (NSs) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (*t*), an indeterminacy-membership function (*i*) and a falsity membership function (*f*) independently, which are within the real standard or nonstandard unit interval $]^{-}0\ ,\ 1^{+}[$. In order to conveniently use NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices, hypergraphs can be applied to analyse architecture structures and to represent system partitions, Mordesen J.N and P.S Nasir gave the definitions for fuzzy hypergraphs. Parvathy. R and M. G. Karunambigai's paper introduced the concepts of Intuitionistic fuzzy hypergraphs and analyse its components, Nagoor Gani. A and Sajith Begum. S defined degree, order and size in intuitionistic fuzzy graphs and extend the properties. Nagoor Gani. A and Latha. R introduced irregular fuzzy graphs and discussed some of its properties.

Regular intuitionistic fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs are introduced by Pradeepa. I and Vimala. S in [0]. In this paper we extend regularity and totally regularity on single valued neutrosophic hypergraphs.

2 Preliminaries

Definition 2.1 Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x ∈ X; $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

Definition 2.2 Let A be a SVNS on X then support of A is denoted and defined by, $Supp(A) = \{ x : x \in X , T_A(x) > 0 , I_A(x) > 0 , F_A(x) > 0 \}$.

Definition 2.3 A hypergraph is an ordered pair *H = (X, E)*, where

(1) $X = \{x_1, x_2, \dots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ be a family of subsets of X.

(3) E_j for *j = 1,2,3,…,m* and $\bigcup_j (E_j) = X$.

The set X is called set of vertices and E is the set of edges(or hyper edges).

Definition 2.4 The single valued neutrosophic hypergraph is an ordered pair *H = (X, E),* where

(1) $X = \{x_1, x_2, \dots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ be a family of SVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for *j = 1, 2, 3, …, m* and $\bigcup_j Supp(E_j) = X$.

The set *X* is called set of vertices and *E* is the set of SVN-edges(or SVN-hyperedges).

Proposition 2.5 The single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

3 Regular and totally regular SVNHGs

Definition 3.1 The open neighbourrhood of a vertex *x* in single valued neutrosophic hypergraphs (SVNHGs) is the set of adjacent vertices of *x*, excluding that vertex and is denoted by *N(x)*.

Definition 3.2 The closed neighbourhood of a vertex *x* in single valued neutrosophic hypergraphs (SVNHGs) is the set of adjacent vertices of *x*, including that vertex and is denoted by *N[x].*

Example 3.3 Consider a single valued neutrosophic hyper graphs *H = (X, E),* where *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which are defined by

P = {(a, .1, .2, .3), (b, .4, .5, .6)}

Q = {(c, .1, .2, .3), (d, .4, .5, .6), (e, .7, .8, .9)}

R = {(b, .1, .2, .3), (c, .4, .5, .6)}

S = {(a, .1, .2, .3), (d, .4, .5, .6)}

Then the open neighbourhood of a vertex *a* contain *b* and *d*. The closed neighbourhood of a vertex *b* contain *b*, *a* and *c*.

Definition 3.4 Let *H = (X, E)* be a SVNHG, the open neighbourhood degree of a vertex *x,* which is denoted and defined by $deg(x) = (deg_T(x), deg_I(x), deg_F(x))$, where

$$deg_T(x) = \sum_{x \in X} T_E(x)$$

$$deg_I(x) = \sum_{x \in X} I_E(x)$$

$$deg_F(x) = \sum_{x \in X} F_E(x)$$

Example 3.5 Consider a single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which are defined by

P = {(a, .1, .2, .3), (b, .4, .5, .6)}

Q = {(c, .1, .2, .3), (d, .4, .5, .6), (e, .7, .8, .9)}

R = {(b, .1, .2, .3), (c, .4, .5, .6)}

S = {(a, .1, .2, .3), (d, .4, .5, .6)}

Then the open neighbourhood of a vertex *a* is *b* and *d*, and therefore the open neighbourhood degree degree of a vertex *a* is (.8, 1, 1.2).

Definition 3.6 Let *H = (X, E)* be a SVNHG, the closed neighbourhood degree of a vertex *x,* which is denoted and defined by

$$deg[x] = (deg_T[x], deg_I[x], deg_F[x])$$

where

$$deg_T[x] = deg_T(x) + T_E(x)$$

$$deg_I[x] = deg_I(x) + I_E(x)$$

$$deg_F[x] = deg_F(x) + F_E(x)$$

Example 3.7 Consider a single valued neutrosophic hypergraphs *H = (X, E)*, where *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which is defined by

P = {(a, .1, .2, .3), (b, .4, .5, .6)}

Q = {(c, .1, .2, .3), (d, .4, .5, .6), (e, .7, .8, .9)}

R = {(b, .1, .2, .3), (c, .4, .5, .6)}

S = {(a, .1, .2, .3), (d, .4, .5, .6)}

The closed neighbourhood of a vertex *b* contain *b*, *a* and *c,* hence the closed neighbourhood degree of a vetex *a* is (.9, .1.2, 1.5).

Definition 3.8 Let *H = (X, E)* be a SVNHG, then *H* is said to be an *n*-regular SVNHG if all the vertices have the same open neighbourhood degree $n = (n_1, n_2, n_3)$.

Definition 3.9 Let *H = (X, E)* be a SVNHG, then *H* is said to be an *m*-totally regular SVNHG if all the vertices have the same closed neighbourhood degree $m = (m_1, m_2, m_3)$.

Proposition 3.10 A regular SVNHG is the generalization of regular fuzzy hypergraphs and regular intuitionistic fuzzy hypergraphs.

Proposition 3.11 A totally regular SVNHG is the generalization of totally regular fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs.

Example 3.12 Consider a single valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R, S},* which are defined by

P = {(a, .8, .2, .3), (b, .8, .2, .3)}

Q = {(b, .8, .2, .3), (c, .8, .2, .3)}

R = {(c, .8, .2, .3), (d, .8, .2, .3)}

S = {(d, .8, .2, .3), (a, .8, .2, .3)}

Here the open neighbourhood degree of every vertex is (1.6, .4, .6), hence *H* is regular SVNHG and closed neighbourhood degree of every vertex is (2.4, .6, .9). Hence *H* is both regular and totally regular SVNHG.

Theorem 3.13 Let *H = (X, E)* be a SVNHG which is both regular and totally regular SVNHG then *E* is constant.

Proof: Suppose *H* is an *n*-regular and an *m*-totally regular SVNHG. Then

$$deg(x) = n = (n_1, n_2, n_3)$$

$$deg[x] = m = (m_1, m_2, m_3)$$

for all $x \in E_i$. Consider the *deg[x] = m,* hence by definition $deg(x) + E_i(x) = m$ this implies that $E_i(x) = m - n$ for all *x* in E_i. Therefore *E* is constant.

Remark 3.14 The converse of above theorem need not to be true in general.

Example 3.15 Consider a SVNHG *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R, S}, which is* defined by

P = {(a, .8, .2, .3), (b, .8, .2, .3)}

Q = {(b, .8, .2, .3), (d, .8, .2, .3)}

R = {(c, .8, .2, .3), (d, .8, .2, .3)}

S = {(d, .8, .2, .3), (a, .8, .2, .3)}

Here *E* is constant but *deg(a) = (1.6, .4, .6)* and *deg(d) = (2.4, .6, .9).* Therefore *deg(a)* and *deg(d)* are not equal, hence *H* is not regular SVNHG. Also *deg[a] = (2.4, .6, .9)* and *deg[d] = (3.2, .8, 1.2),* thus *deg[a]* and *deg[d]* are not equal, hence *H* is not totally regular SVNHG, we conclude that *H* is neither regular and nor totally regular SVNHG.

Theorem 3.16 Let *H = (X, E)* be a SVNHG, then *E* is costant on *X* if and only if following are equivalent

(1) *H* is regular SVNHG.

(2) *H* is totally regular SVNHG.

Proof: Suppose *H = (X, E)* be a SVNHG and *E* is constant in *H*, then $E_i(x) = c = (c_1, c_2, c_3)$ for all $x \in E_i$. Suppose *H* is an n-regular SVNHG, then $deg(x) = n = (n_1, n_2, n_3)$ for all $x \in E_i$. Consider $deg[x] = deg(x) + E_i(x) = n + c$ for all $x \in E_i$. Hence *H* is totally regular SVNHG. Next suppose that *H* is an *m*-totally regular SVNHG, then $deg[x] = m = (m_1, m_2, m_3)$ for all $x \in E_i$, that is $deg(x) + E_i(x) = m$ for all $x \in E_i$. This implies that $deg(x) = m - c$ for all $x \in E_i$. Thus *H* is regular SVNHG.

Conversely: Suppose contrary *E* is not constant, that is E_i(x) and E_i(y) not equal for some *x* and *y* in *X.* Let *H = (X, E)* be an n-regular SVNHG, then $deg(x) = n = (n_1, n_2, n_3)$ for all $x \in E_i$. Consider

$$deg[x] = deg(x) + E_i(x) = n + E_i(x)$$

$$deg[y] = deg(y) + E_i(y) = n + E_i(y)$$

since E_i(x) and E_i(y) are not equal for some *x* and *y* in *X,* hence *deg[x]* and *deg[y]* are not equal, thus *H* is not totally regular SVNHG, which contradict to our assumption. Next let *H* be totally regular SVNHG, then *deg[x] = deg[y],* that is

$$deg(x) + E_i(x) = deg(y) + E_i(y)$$

$$deg(x) - deg(y) = E_i(y) - E_i(x)$$

since RHS of above equation is nonzero, hence LHS of above equation is also nonzero, thus *deg(x)* and *deg(y)* are not equals, so *H* is not regular SVNHG, which is again contradict to our assumption, thus our supposition was wrong, hence *E* must be constant, this completes the proof.

Definition 3.17 Let *H = (X, E)* be a regular SVNHG, then the order of SVNHG *H,* which is denoted and defined by *O(H) = (p, q, r)*, where

$$p = \sum_{x \in X} T_{E_i}(x)$$

$$q = \sum_{x \in X} I_{E_i}(x)$$

$$r = \sum_{x \in X} F_{E_i}(x)$$

for every x ∈ X and the size of regular SVNHG, which is denoted and defined by $S(H) = \sum_{i=1}^{n}(S_{E_i})$, where S(E$_i$) = (a, b, c), which is defined by

$$a = \sum_{x \in E_i} T_{E_i}(x)$$

$$b = \sum_{x \in E_i} I_{E_i}(x)$$

$$c = \sum_{x \in E_i} F_{E_i}(x).$$

Example 3.18 Consider the SVNHG *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R, S},* which is defined by

P = {(a, .8, .2, .3), (b, .8, .2, .3)}

Q = {(b, .8, .2, .3), (c, .8, .2, .3)}

R = {(c, .8, .2, .3), (d, .8, .2, .3)}

S = {(d, .8, .2, .3), (a, .8, .2, .3)}

Here the order and the size of *H* are given (3.2, .8, 1.2) and (6.4, 1.6, 2.4), respectively.

Proposition 3.19 The size of an *n*-regular SVNHG *H = (X, E)* is nk/2, where |X|= k.

Proposition 3.20 Let *H = (X, E)* be an m-totally regular SVNHG, then *2S(H) + O(H) = mk*, where *|X|= k.*

Corollary 3.21 Let *H = (X, E)* be an n-regular and an m-totally regular SVNHG, then *O(H) = k(m - n)*, where *|X|= k.*

Proposition 3.22 The dual of an *n*-regular and an *m*-totally regular SVNHG *H= (X, E)* is again an n-regular and an m-totally regular SVNHG.

Definition 3.23 The SVNHG is said to be complete SVNHG if for every *x* in *X, N(x) = {x: x in X-{x}},* that is *N(x)* contains all remaining vertices of *X* except *x.*

Example 3.24 Consider a single valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R},* which is defined by

P = {(a, .4, .6, .3), (c, .8, .2, .3)}

Q = {(a, .8, .8, .3), (b, .8, .2, .1), (d, .8, .2, .1)}

R = {(c, .4, .9, .9), (d, .7, .2, .1), (b, .4, .2, .1)}

Here *N(a) = {b, c, d} , N(b) = {a, c, d}, N(c) = {a, b, d}, N(d) = {a, b, c}.* Hence *H* is complete SVNHG.

Remark 3.25 In a complete SVNHG *H = (X, E)* the cardinality of *N(x)* is same for every vertex.

Theorem 3.26 Every complete SVNHG *H = (X, E)* is both regular and totally regular if is constant in *H.*

Proof: Let *H = (X, E)* be a complete SVNHG, suppose *E* is constant in *H,* so ∀$x \in E_i$, $E_i(x) = c = (c_1, c_2, c_3)$, since SVNHG is complete, then by definition for every vertex *x* in *X*, *N(x) = { x : x in X-{x}}*, the open neighbourhood degree of every vertex is same. Hence $deg(x) = n = (n_1, n_2, n_3)$ for all x ∈ E_i. Hence complete SVNHG is regular SVNHG. Also $deg[x] = deg(x) + E_i(x) = n + c$ for all x ∈ E_i. Thus *H* is totally regular SVNHG.

Remark 3.27 Every complete SVNHG is totally regular even if *E* is not constant.

Definition 3.28 The SVNHG is said to be *k*-uniform if all the hyperedges have same cardinality.

Example 3.29 Consider a SVNHG *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R},* which is defined by

P = {(a, .8, .2, .3), (b, .7, .5, .3)}

Q = {(b, .8, .1, .8), (c, .8, .4, .2)}

R = {(c, .8, .1, .4), (d, .8, .9, .5)}

4 Conclusion

Theoretical concepts of graphs and hypergraphs are highely utilized by computer science applications. The SVNHG are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of SVNHGs can be applied in various areas of engineering and computer science. In this paper we defined the concept of regular and totally regular SVNHGs. We plan to extend our research work to regular and totally regular on Bipolar SVNHGs, regular and totally regular on interval valued neutrosophic hypergraphs, irregular and totally irregular on SVNHGs, irregular and totally irregular on bipolar SVNHGs.

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