Contents lists available at **SCCE**

Journal of Soft Computing in Civil Engineering

Journal homepage: http://www.jsoftcivil.com/

Refined Simplified Neutrosophic Similarity Measures Based on Trigonometric Function and Their Application in Construction Project Decision-Making

U. Solang [1] and J. Ye [1,2*]

1. Department of Civil Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing, Zhejiang Province 312000, P.R. China
2. Department of Electrical and Information Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing, Zhejiang Province 312000, P.R. China

Corresponding author: yehjun@aliyun.com, yejun@usx.edu.cn

http://dx.doi.org/10.22115/SCCE.2018.126129.1056

ARTICLE INFO

Article history:
Received: 03 April 2018
Revised: 07 May 2018
Accepted: 07 May 2018

Keywords:
Refined simplified neutrosophic set,
Tangent function,
Cotangent function,
Similarity measure,
Construction project,
Multiple attribute decision-making.

ABSTRACT

Refined simplified neutrosophic sets (RSNSs) are appropriately used in decision-making problems with sub-attributes considering their truth components, indeterminacy components and falsity components independently. This paper presents the similarity measures of RSNSs based on tangent and cotangent functions. When the weights of each element/attribute and each sub-element/sub-attribute in RSNSs are considered according to their importance, we propose the weighted similarity measures of RSNSs and their multiple attribute decision-making (MADM) method with RSNS information. In the MADM process, the developed method gives the ranking order and the best selection of alternatives by getting the weighted similarity measure values between alternatives and the ideal solution according to the given attribute weights and sub-attribute weights. Then, an illustrative MADM example in a construction project with RSNS information is presented to show the effectiveness and feasibility of the proposed MADM method under RSNS environments. This study extends existing methods and provides a new way for the refined simplified neutrosophic MADM problems containing both the attribute weight and the sub-attribute weights.

1. Introduction

The fuzzy set [1] is represented by the membership function for a fuzzy problem. However, the fuzzy set cannot be described by the non-membership function for a fuzzy problem. As the generalization of fuzzy sets, Atanassov [2] presented an intuitionistic fuzzy set (IFS), which is characterized by the membership and non-membership functions. However, IFS can only handle incomplete and uncertain information but not inconsistent and indeterminate information. Thus, a neutrosophic set (NS) was introduced by Smarandache [3], where the indeterminacy is quantified explicitly. In NS, the components of the truth, indeterminacy and falsity are denoted as T, I, F, and then they are expressed independently by the truth, falsity, indeterminacy membership functions defined in the real standard interval [0, 1] or non-standard interval $]^{-}0,1^{+}[$. After that, Simplified NSs introduced by Ye [4] are the subclasses of NSs and contain two concepts of single valued neutrosophic sets (SVNSs) and interval neutrosophic sets (INSs), then they were applied in decision making [5-17]. Even though they have been applied in real MADM problems, however, their decision-making methods cannot handle the problems of both the attributes and the sub-attributes. Smarandache [18] first defined n-value/refined neutrosophic set, which is composed of its n-sub-components represented by p truth sub-membership degrees, r indeterminacy sub-membership degrees, and s falsity sub-membership degrees satisfying $p + r + s = n$. Next, n-value/refined neutrosophic sets/multisets were applied to medical diagnoses and MADM [19-22]. Further, Ye and Smarandache [23] particularized the n-value/refined neutrosophic set to a refined single valued neutrosophic set (RSVNS), where its components T, I, F and the sub-components $T_1, T_2, ..., T_q$ and $I_1, I_2, ..., I_q$, and $F_1, F_2, ..., F_q$ are constructed as a RSVNS, and then they introduced the similarity measure using the union and the intersection of RSVNSs to deal with MADM problems with both attributes and sub-attributes. Then, the Dice, Jaccard and cosine similarity measures of refined simplified NSs (RSNS) have been proposed [24], along with their applications in MADM problems. Thereafter, cosine measures of refined interval NSs (RINS) were introduced by Fan and Ye [25] as an extension of RSVNS and used for MADM problems.

It is well known that a similarity measure in decision-making theory is an important mathematical tool. So, Ye [12] put forward similarity measures of SVNS corresponding to cotangent function, then Mondal and Pramanik [22] presented the tangent function-based similarity measure of refined NSs (i.e. neutrosophic multi-sets) for the MADM problem without sub-attributes. However, their MADM methods [12, 19-22] cannot handle the MADM problems with both attributes and sub-attributes. In fact, there are no tangent and cotangent similarity measures for RSNSs in existing literature. Therefore, we introduce new similarity measures of RSNSs corresponding to tangent function and cotangent function to extend the existing decision-making methods of multiple attributes to MADM problems with attributes and sub-attributes, and then the developed method is applied in a MADM example on a construction project with both attributes and sub-attributes in RSNS (RSVNS and RINS) setting.

The rest of this article is constructed as the following. In the second section, we present the extended tangent function similarity measure and cotangent function similarity measure for SNSs in existing literature. The third section, the similarity measures of RSNSs were introduced corresponding to tangent and cotangent functions. In the fourth section we present the MADM method using the tangent and cotangent similarity measures of RSNSs. In the fifth section, an illustrative example on decision-making of a construction project with attributes and sub-attributes is given in RSNS (RSVNS and RINS) setting. Lastly, this article is concluded in the sixth section.

2. Trigonometric function-based similarity measures

NS [3] is described by the three components *T*, *I*, *F*, which are defined independently as the membership degrees of the truth, indeterminacy and falsity within a real standard interval [0,1] or a nonstandard interval $]^-0,1^+[$. For its application in real science and engineering, this can been constrained in the real standard interval [0,1]. Thereby, as a simplified form or a subclass of a NS Ye [4] presented the concept of simplified NS. The simplified NS contains SVNS and INS. A simplified NS *P* in a universe of discourse *X* with the element *x* is denoted as $P = \{<x, T_P(x), I_P(x), F_P(x)>|x \in X\}$ where each membership function is considered as a singleton or a sub-interval in the real standard [0,1], such that $T_P(x), I_P(x), F_P(x) \in [0,1]$ for SVNS or $T_P(x), I_P(x), F_P(x) \subseteq [0,1]$ for INS. An element $<x, T_P(x), I_P(x), F_P(x)>$ in the simplified NS *P* is called a simplified neutrosophic number (SNN), simply denoted as $p = \langle t_p, i_p, f_p \rangle$, which contains single valued and interval neutrosophic numbers.

Similarity measures mainly describe the similarity degree between different objects. Assume two simplified NSs in the universe of discourse *X* are $P = \{p_1, p_2, \ldots, p_n\}$ for $p_j \in P$ $\{j = 1, 2, \ldots, n\}$ and $Q = \{q_1, q_2, \ldots, q_n\}$ for $q_j \in Q$ $(j = 1, 2, \ldots, n)$ such that $p_j = <t_{pj}, i_{pj}, f_{pj}>$ and $q_j = <t_{qj}, i_{qj}, f_{qj}>$. Then, the tangent function and cotangent function-based similarity measures between two SVNSs *P* and *Q* are expressed below [12, 22]:

$$S_1(P,Q) = \frac{1}{n}\sum_{j=1}^{n} 1 - \tan[(|t_{pj} - t_{qj}| + |i_{pj} - i_{qj}| + |f_{pj} - f_{qj}|)\frac{\pi}{3\times 4}], \quad (1)$$

$$S_2(P,Q)=\frac{1}{n}\sum_{j=1}^{n} \cot[\frac{\pi}{4} + (|t_{pj} - t_{qj}| + |i_{pj} - i_{qj}| + |f_{pj} - f_{qj}|)\frac{\pi}{3\times 4}]. \quad (2)$$

For INSs, the tangent function and cotangent function-based similarity measures between two INSs *P* and *Q* are presented by

$$S_3(P,Q) = \frac{1}{n}\sum_{j=1}^{n} 1 - \tan[(|\inf t_{pj} - \inf t_{qj}| + |\sup t_{pj} - \sup t_{qj}| + |\inf i_{pj} - \inf i_{qj}| + |\sup i_{pj} - \sup i_{qj}| + |\inf f_{pj} - \inf f_{qj}| + |\sup f_{pj} - \sup f_{qj}|)\frac{\pi}{6\times 4}], \quad (3)$$

$$S_4(P,Q) = \frac{1}{n}\sum_{j=1}^{n} \cot[\frac{\pi}{4} + (|\inf t_{pj} - \inf t_{qj}| + |\sup t_{pj} - \sup t_{qj}| + |\inf i_{pj} - \inf i_{qj}| + |\sup i_{pj} - \sup i_{qj}| + |\inf f_{pj} - \inf f_{qj}| + |\sup f_{pj} - \sup f_{qj}|)\frac{\pi}{6\times 4}]. \quad (4)$$

According to the tangent and cotangent similarity measure properties in [12, 22], the cotangent and tangent similarity measures $S_k(P, Q)$ (k = 1, 2, 3, 4) between two simplified NSs P and Q also have the following properties:

(R1) $0 \leq S_k(P, Q) \leq 1$;

(R2) $S_k(P, Q) = S_k(Q, P)$;

(R3) $S_k(P, Q) = 1$ if and only if $P = Q$;

(R4) Suppose M is also a simplified NS in the universe X, If $P \subseteq Q \subseteq M$, then $S_k(P, M) \leq S_k(P, Q)$ and $S_k(P, M) \leq S_k(Q, M)$.

3. Tangent and cotangent similarity measures of RSNSs

This section presents the tangent and cotangent similarity measures between RSNSs and the weighted tangent and cotangent similarity measures of RSNSs containing both the weights of their elements and the weights of their sub-elements, which are more suitable for solving MADM problems with sub-attributes.

If a simplified NS (SVNS or INS) $P = \{p_1, p_2, p_3, \ldots, p_n\}$ for $p_j \in P$ (j = 1, 2, …, n) is refined, $p_j = \langle t_j, i_j, f_j \rangle$ consists of the sub-components such as $p_{ji} = \langle t_{j1}, t_{j2}, \ldots, i_{j1}, i_{j2}, \ldots, f_{j1}, f_{j2}, \ldots \rangle$. Then RSNS contains RSVNS with the components $t_{j1}, t_{j2}, \ldots \in [0,1]$, $i_{j1}, i_{j2}, \ldots \in [0,1]$, $f_{j1}, f_{j2}, \ldots \in [0,1]$ and $0 \leq t_{ji} + i_{ji} + f_{ji} \leq 3$ and RINS with $t_{j1}, t_{j2}, \ldots \subseteq [0,1]$, $i_{j1}, i_{j2}, \ldots \subseteq [0,1]$, $f_{j1}, f_{j2}, \ldots \subseteq [0,1]$ and $0 \leq \sup t_{ji} + \sup i_{ji} + \sup f_{ji} \leq 3$.

Then, we introduce the tangent and cotangent functions to similarity measures of RSNSs. Assume we consider two RSNS $P = \{p_1, p_2, \ldots, p_n\}$ and $Q = \{q_1, q_2, \ldots, q_n\}$ for $p_j \in P$ and $q_j \in Q$ (j = 1, 2, …, n), where $p_{ji} = \langle (t_{pj1}, t_{pj2}, \ldots, t_{pjr(j)}), (i_{pj1}, i_{pj2}, \ldots, i_{pjr(j)}), (f_{pj1}, f_{pj2}, \ldots, f_{pjr(j)}) \rangle$ and $q_{ji} = \langle (t_{qj1}, t_{qj2}, \ldots, t_{qjr(j)}), (i_{qj1}, i_{qj2}, \ldots, i_{qjr(j)}), (f_{qj1}, f_{qj2}, \ldots, f_{qjr(j)}) \rangle$ for $p_{ji} \in p_j$ and $q_{ji} \in q_j$ (i = 1, 2,..., $r(j)$; j = 1, 2, …, n). Thus, the similarity measures between two RSVNSs and between two RINSs based on the trigonometric functions of tangent and cotangent are given as follows:

(a) Similarity measures of RSVNSs

$$T_1(P,Q) = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{r(j)}\sum_{i=1}^{r(j)}\{1 - \tan[(|t_{pjr(j)} - t_{qjr(j)}| + |i_{pjr(j)} - i_{qjr(j)}| + |f_{pjr(j)} - f_{qjr(j)}|)\frac{\pi}{3\times 4}]\}, \tag{5}$$

$$T_2(P,Q) = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{r(j)}\sum_{i=1}^{r(j)}\cot[\frac{\pi}{4} + (|t_{pjr(j)} - t_{qjr(j)}| + |i_{pjr(j)} - i_{qjr(j)}| + |f_{pjr(j)} - f_{qjr(j)}|)\frac{\pi}{3\times 4}]\ . \tag{6}$$

(b) Similarity measures of RINSs

$$T_3(P,Q) = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{r(j)}\sum_{i=1}^{r(j)}\{1-\tan[(|inf\ t_{pjr(j)}-inf\ t_{qjr(j)}|+|sup\ t_{pjr(j)}-sup\ t_{qjr(j)}|+|inf\ i_{pjr(j)}-inf\ i_{qjr(j)}|+|sup\ i_{pjr(j)}-sup\ i_{qjr(j)}|+|inf\ f_{pjr(j)}-inf\ f_{qjr(j)}|+|sup\ f_{pjr(j)}-sup\ f_{qjr(j)}|)\frac{\pi}{6\times 4}]\}, \quad (7)$$

$$T_4(P,Q) = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{r(j)}\sum_{i=1}^{r(j)}\cot[\frac{\pi}{4}+(|inf\ t_{pjr(j)}-inf\ t_{qjr(j)}|+|sup\ t_{pjr(j)}-sup\ t_{qjr(j)}|+|inf\ i_{pjr(j)}-inf\ i_{qjr(j)}|+|sup\ i_{pjr(j)}-sup\ i_{qjr(j)}|+|inf\ f_{pjr(j)}-inf\ f_{qjr(j)}|+|sup\ f_{pjr(j)}-sup\ f_{qjr(j)}|)\frac{\pi}{6\times 4}]. \quad (8)$$

Similar to the properties (R1)-(R4) of the similarity measures discussed above, the simplified neutrosophic similarity measures $T_k(P, Q)$ (k = 1, 2, 3, 4) based on cotangent and tangent functions also contain the following properties:

(R1) $0 \le T_k(P, Q) \le 1$;

(R2) $T_k(P, Q) = T_k(Q, P)$;

(R3) $T_k(P, Q) = 1$ if and only if $P = Q$;

(R4) Suppose M is also a RSNS in the universe X, If $P \subseteq Q \subseteq M$, then $T_k(P, M) \le T_k(P, Q)$ and $T_k(P, M) \le T_k(Q, M)$.

To apply them efficiently in decision-making, we need to consider the weights of elements in RSNS as $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ and the weights of sub-elements in RSNS as $\boldsymbol{w_j} = (w_{11}, w_{22}, \ldots, w_{nr(j)})$ (i = 1, 2, …, $r(j)$; j =1, 2, …, n). Thus, the weighted similarity measures are presented as follows:

(a) The weighted similarity measures between RSVNSs

$$W_1(P,Q) = \sum_{j=1}^{n} w_j \sum_{i=1}^{r(j)} w_{ji}\{1-\tan[(|t_{pjr(j)}-t_{qjr(j)}|+|i_{pjr(j)}-i_{qjr(j)}|+|f_{pjr(j)}-f_{qjr(j)}|)\frac{\pi}{3\times 4}]\}, \quad (9)$$

$$W_2(P,Q) = \sum_{j=1}^{n} w_j \sum_{i=1}^{r(j)} w_{ji}\cot[\frac{\pi}{4}+(|t_{pjr(j)}-t_{qjr(j)}|+|i_{pjr(j)}-i_{qjr(j)}|+|f_{pjr(j)}-f_{qjr(j)}|)\frac{\pi}{3\times 4}]\ ; \quad (10)$$

(b) The weighted similarity measures between RINSs

$$W_3(P,Q) = \sum_{j=1}^{n} w_j \sum_{i=1}^{r(j)} w_{ji}\{1-\tan[(|inf\ t_{pjr(j)}-inf\ t_{qjr(j)}|+|sup\ t_{pjr(j)}-sup\ t_{qjr(j)}|+|inf\ i_{pjr(j)}-inf\ i_{qjr(j)}|+|sup\ i_{pjr(j)}-sup\ i_{qjr(j)}|+|inf\ f_{pjr(j)}-inf\ f_{qjr(j)}|+|sup\ f_{pjr(j)}-sup\ f_{qjr(j)}|)\frac{\pi}{6\times 4}]\}, \quad (11)$$

$$W_4(P,Q)=\sum_{j=1}^{n} w_j \sum_{i=1}^{r(j)} w_{ji}\cot[\frac{\pi}{4}+(\left|inf\ t_{pjr(j)}-inf\ t_{qjr(j)}\right|+\ \left|sup\ t_{pjr(j)}-sup\ t_{qjr(j)}\right|+\left|inf\ i_{pjr(j)}-inf\ i_{qjr(j)}\right|+\left|sup\ i_{pjr(j)}-sup\ i_{qjr(j)}\right|+\left|inf\ f_{pjr(j)}-inf\ f_{qjr(j)}\right|+\left|sup\ f_{pjr(j)}-sup\ f_{qjr(j)}\right|)\frac{\pi}{6\times 4}]. \quad (12)$$

The above weighted similarity measures $W_k(P, Q)$ (k = 1, 2, 3, 4) based on tangent and cotangent obviously also contain the following properties:

(R1) $0 \le W_k(P, Q) \le 1$;

(R2) $W_k(P, Q) = W_k(Q, P)$;

(R3) $W_k(P, Q) = 1$ if and only if $P = Q$;

(R4) Suppose M is also a simplified NS in the universe X, If $P \subseteq Q \subseteq M$, then $W_k(P, M) \le W_k(P, Q)$ and $W_k(P, M) \le W_k(Q, M)$.

4. MADM method based on the proposed similarity measures of RSNSs

In a MADM problem that has multiple attributes with their sub-attributes, this section proposes a MADM method using the proposed similarity measures of RSNSs.

Let's consider a set of m alternatives $P = \{P_1, P_2, \ldots, P_m\}$ to be judged under attributes $Z = \{z_1, z_2, \ldots, z_n\}$ with their sub-attributes $z_j = \{z_{j1}, z_{j2}, \ldots, z_{jr(j)}\}$ for j = 1, 2, …, n. Then, we give the evaluation of the alternatives over the attributes and sub-attributes by RSNSs (RSVNSs and RINSs). Table 1 shows the relative evaluation values between alternatives and the attributes and sub-attributes, known as the RSNS decision matrix $D=(p_{sji})_{m\times n}$, where p_{sji} (i = 1, 2, …, $r(j)$; j = 1, 2, …, n; s = 1, 2, …, m) represents the evaluation value of P_j regarding each sub-attribute $z_{jr(j)}$.

Table 1.
The RSNS decision matrix $D=(p_{sji})_{m\times n}$.

	z_1	z_2	…	z_n
	$z_{11}, z_{12}, \ldots, z_{1r(1)}$	$z_{21}, z_{22}, \ldots, z_{2r(2)}$	…	$z_{n1}, z_{n2}, \ldots, z_{nr(n)}$
P_1	$p_{11r(1)}$	$p_{12r(2)}$	…	$p_{1nr(n)}$
P_2	$p_{21r(1)}$	$p_{22r(2)}$	…	$p_{2nr(n)}$
P_3	$p_{31r(1)}$	$p_{32r(2)}$	…	$p_{3nr(n)}$
…	…	…	…	…
P_m	$p_{m1r(1)}$	$p_{m2r(2)}$	…	$p_{mnr(n)}$

In Table 1, each alternative P_s in the set $P = \{P_1, P_2, \ldots, P_m\}$ is evaluated under all attributes $Z = \{z_1, z_2, \ldots, z_n\}$ and sub-attributes $z_j = \{z_{j1}, z_{j2}, \ldots, z_{jr(j)}\}$ by RSNN $p_{sji} = \langle (t_{sj1}, t_{sj2}, \ldots, t_{sjr(j)}), (i_{sj1}, i_{sj2}, \ldots, i_{sjr(j)}), (f_{sj1}, f_{sj2}, \ldots, f_{sjr(j)}) \rangle$ (i = 1, 2, …, $r(j)$; j = 1, 2, …, n; s = 1, 2, …, m). The importance of attributes and sub-attributes is presented as the weight vectors $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ for the set $Z = \{z_1, z_2, \ldots, z_n\}$ and $\boldsymbol{w_j} = (w_{j1}, w_{j2}, \ldots, w_{jr(j)})$ for the set of sub-attributes such that $\sum_{j=1}^{n} w_j=1$ and $\sum_{i=1}^{r(j)} w_{ji}=1$ with $w_j, w_{ji} \in [0,1]$.

The ideal RSNN or the ideal solution is given from the RSNS decision matrix $D=(p_{sji})_{m\times n}$ as follows:

p_{pji} = <(t_{pj1}, t_{pj2}, …,$t_{pjr(j)}$), (i_{pj1}, i_{pj2},…,$i_{pjr(j)}$), (f_{pj1}, f_{pj2},…, $f_{pjr(j)}$)> = <($\max_s(t_{sj1})$, $\max_s(t_{sj2})$, …,$\max_s(t_{sjr(j)})$), ($\min_s(i_{sj1})$, $\min_s(i_{sj2})$, …,$\min_s(i_{sjr(j)})$), ($\min_s(f_{sj1})$, $\min_s(f_{sj2})$,…, $\min_s(f_{sjr(j)})$)> for RSVNNs (13)

or p_{pji} = <(t_{pj1}, t_{pj2}, …,$t_{pjr(j)}$),(i_{pj1}, i_{pj2},…,$i_{pjr(j)}$),(f_{pj1}, f_{pj2},…, $f_{pjr(j)}$)> = <([$\max_s(\inf t_{sj1})$, $\max_s(\sup t_{sj1})$], [$\max_s(\inf t_{sj2})$, $\max_s(\sup t_{sj2})$], …, [$\max_s(\inf t_{sjr(j)})$, $\max_s(\sup t_{sjr(j)})$]), ([$\min_s(\inf i_{sj1})$, $\min_s(\sup i_{sj1})$], [$\min_s(\inf i_{sj2})$, $\min_s(\sup i_{sj2})$], …, [$\min_s(\inf i_{sjr(j)})$, $\min_s(\sup i_{sjr(j)})$]), ([$\min_s(\inf f_{sj1})$, $\min_s(\sup f_{sj1})$], [$\min_s(\inf f_{sj2})$, $\min_s(\sup f_{sj2})$], …, [$\min_s(\inf f_{sjr(j)})$, $\min_s(\sup f_{sjr(j)})$])> for RINNs. (14)

Then, the ideal solution/alternative is presented as $P^* = \{p_1^*, p_2^*, \ldots, p_n^*\}$, where $p_j^* = (p_{pj1}, p_{pj1}, \ldots, p_{pjr(j)})$ for $j = 1, 2, \ldots, n$.

Thus, we use the equations (9) and (11) or (10) and (12) to get the values of $W_k(P_s, P^*)$ (k = 1, 3 or 2, 4; $s = 1, 2, \ldots, m$). By the similarity measure values between the ideal solution P^* and each alternative set P_s, all the alternatives are ranked and the best one is determined based on the one with biggest weighted similarity measure value given by $W_k(P_s, P^*)$ among the alternatives.

5. Illustrative example

A successful project can be achieved by many interacted factors as presented in previous literatures [14-16], which mainly depends on the decision-making method. Hence, the manager has to effectively make accurate and reliable decision according to the presented requirements or objective attributes with their highly subjective judgmental factors to select the best alternative for some project.

In a construction project, the manager has to select the best alternative in the decision set of the alternatives $P = \{P_1, P_2, P_3, P_4\}$ suggested by different personalities or departments like administration department, technical department, finance department, etc. to meet the requirements and the objectives of the project from the contractor company, as well as the contracting company. The following two cases composed of the suggested alternatives with their attributes set $Z = \{z_1, z_2, z_3\}$ and sub-attributes set $z_j = \{z_{j1}, z_{j2}, z_{jr(j)}\}$ (j = 1, 2, 3) in a construction project are presented to describe the applicability of the proposed method. Here, the attributes and sub-attributes of alternatives are shown in Table 2.

Table 2.
The attributes and sub-attributes.

z_1: Budget	z_2: Quality	z_3: Delivery
z_{11}: Human resource cost	z_{21}: Experience or performance	z_{31}: Schedule
z_{12}: Materials and equipment cost	z_{22}: Technology	z_{32}: Communication
z_{13}: Facilities		z_{33}: Risk and uncertainties

Case1. Under RSVNS environment, the evaluation values of the decision set of the alternatives $P = \{P_1, P_2, P_3, P_4\}$ over the attributes and sub-attributes must belong to the interval [0,1]. The weight vector of the given attribute set $Z = \{z_1, z_2, z_3\}$ is $\boldsymbol{w} = (0.3, 0.4, 0.3)$ and the weight vectors of the sub-attribute sets $\{z_{11}, z_{12}, z_{13}\}$, $\{z_{21}, z_{22}\}$ and $\{z_{31}, z_{32}, z_{33}\}$ are given respectively as $\boldsymbol{w}_1 = (0.5, 0.3, 0.2)$, $\boldsymbol{w}_2 = (0.6, 0.4)$ and $\boldsymbol{w}_3 = (0.4, 0.2, 0.4)$. Thus, the RSVNS decision matrix $D=(p_{sji})_{4\times 3}$ corresponding to the alternatives with respect to the three attributes with their given sub-attributes is given in Table 3.

Table 3.
The RSVNS decision matrix $D=(p_{sji})_{4\times 3}$.

	z_1	z_2	z_3
	z_{11}, z_{12}, z_{13}	z_{21}, z_{22}	z_{31}, z_{32}, z_{33}
P_1	$\langle(0.5,0.5,0.6),(0.3,0.4,0.2),(0.2,0.1,0.2)\rangle$	$\langle(0.7,0.8),(0.1,0.2),(0.1,0.2)\rangle$	$\langle(0.9,0.8,0.5),(0.1,0.1,0.3),(0,0.1,0.2)\rangle$
P_2	$\langle(0.7,0.6,0.5),(0.2,0.2,0.3),(0.1,0.2,0.2)\rangle$	$\langle(0.9,0.5),(0.1,0.3),(0.2,0.2)\rangle$	$\langle(0.7,0.6,0.8),(0.1,0.3,0.1),(0.2,0.1,0.1)\rangle$
P_3	$\langle(0.8,0.6,0.8),(0,0.3,0.1),(0.2,0.1,0.1)\rangle$	$\langle(0.7,0.6),(0.2,0.1),(0.3.0.1)\rangle$	$\langle(0.5,0.6,0.6),(0.2,0.2,0.3),(0.3,0.2,0.1)\rangle$
P_4	$\langle(0.6,0.7,0.7),(0.2,0.2,0.1),(0.2,0.1,0.2)\rangle$	$\langle(0.5,0.8),(0.3,0.1),(0.1,0)\rangle$	$\langle(0.8,0.8,0.6),(0.1,0.2,0.2),(0.1,0,0.2)\rangle$

To get the value of RSVNS for P^* we apply the formula (13) to obtain the following ideal solution:

$$P^* = \{\langle(0.8,0.7,0.8),(0,0.2,0.1),(0.1,0.1,0.1)\rangle, \langle(0.9,0.8),(0.1,0.1).(0.1,0)\rangle,$$
$$\langle(0.9,0.8,0.8),(0.1,0.1,0.1),(0,0,0.1)\rangle\}.$$

For the RSVNS we use the equations (9)-(10) to get the following results of similarity measures between the alternatives P_s (s = 1, 2, 3, 4) and the ideal solution P^* in Table 4.

Table 4.
The similarity measure values between P_s and P^*.

Measure	Similarity measure value	Ranking order	The best choice
$W_1(P_s, P^*)$	$W_1(P_1, P^*) = 0.9106$ $W_1(P_2, P^*) = 0.9176$ $W_1(P_3, P^*) = 0.9012$ $W_1(P_4, P^*) = 0.9186$	$P_4>P_2>P_1>P_3$	P_4
$W_2(P_s, P^*)$	$W_2(P_1, P^*) = 0.8414$ $W_2(P_2, P^*) = 0.8536$ $W_2(P_3, P^*) = 0.8254$ $W_2(P_4, P^*) = 0.8557$	$P_4>P_2>P_1>P_3$	P_4

In Table 4, the attribute P_4 is considered as the best choice, which is the best alternative under RSVNS environment.

Case2. Under RINS environment, the evaluation values of the set of the alternatives $P = \{P_1, P_2, P_3, P_4\}$ over the attributes and sub-attributes are the sub-interval of the interval [0,1]. The weight vector of the given attribute set $Z = \{z_1, z_2, z_3\}$ is $\boldsymbol{w} = (0.3, 0.4, 0.3)$ and the weight vectors of the sub-attribute sets $\{z_{11}, z_{12}, z_{13}\}$, $\{z_{21}, z_{22}\}$ and $\{z_{31}, z_{32}, z_{33}\}$ are given as $\boldsymbol{w}_1 = (0.5, 0.3, 0.2)$, $\boldsymbol{w}_2 = (0.6, 0.4)$, and $\boldsymbol{w}_3 = (0.4, 0.2, 0.4)$, respectively. Then, the refined interval neutrosophic decision matrix $D=(p_{sji})_{4\times3}$ corresponding to the alternatives over the three attributes with three groups of the sub-attributes is given in Table 5.

Table 5.
The RINS decision matrix $D=(p_{sji})_{4\times3}$.

	z_1	z_2	z_3
	z_{11},z_{12},z_{13}	z_{21},z_{22}	z_{31},z_{32}, z_{33}
P_1	$\left\langle\begin{matrix}([0.5,0.6],[0.5,0.6],[0.6,0.7]),\\([0.3,0.4],[0.4,0.5],[0.2,0.3]),\\([0.2,0.3],[0.1,0.2],[0.2,0.3])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.7,0.8],[0.8,0.9]),\\([0.1,0.2],[0.2,0.3]),\\([0.1,0.2],[0.2,0.3])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.8,0.9],[0.8,0.9],[0.5,0.6]),\\([0.1,0.2],[0.1,0.2],[0.3,0.4]),\\([0,0.1],[0.1,0.2],[0.2,0.3])\end{matrix}\right\rangle$
P_2	$\left\langle\begin{matrix}([0.7,0.8],[0.6,0.7],[0.5,0.6]),\\([0.2,0.3],[0.2,0.3],[0.3,0.4]),\\([0.1,0.2],[0.2,0.3],[0.2,0.3])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.8,0.9],[0.5,0.6]),\\([0.1,0.2],[0.3,0.4]),\\([0.2,0.3],[0.2,0.3])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.7,0.8],[0.6,0.7],[0.8,0.9]),\\([0.1,0.2],[0.3,0.4],[0.1,0.2]),\\([0.2,0.3],[0.1,0.2],[0.1,0.2])\end{matrix}\right\rangle$
P_3	$\left\langle\begin{matrix}([0.8,0.9],[0.6,0.7],[0.8,0.9]),\\([0,0.1],[0.3,0.4],[0.1,0.2]),\\([0.2,0.3],[0.1,0.2],[0.1,0.2])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.7,0.8],[0.6,0.7]),\\([0.2,0.3],[0.1,0.2]),\\([0.3,0.4],[0.1,0.2])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.5,0.6],[0.6,0.7],[0.6,0.7]),\\([0.2,0.3],[0.2,0.3],[0.3,0.4]),\\([0.3,0.4],[0.2,0.3].[0.1,0.2])\end{matrix}\right\rangle$
P_4	$\left\langle\begin{matrix}([0.6,0.7],[0.7,0.8],[0.7,0.8]),\\([0.2,0.3],[0.2,0.3],[0.1,0.2]),\\([0.2,0.3],[0.1,0.2],[0.2,0.3])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.5,0.6],[0.8,0.9]),\\([0.3,0.4],[0.1,0.2]),\\([0.1,0.2],[0,0.1])\end{matrix}\right\rangle$	$\left\langle\begin{matrix}([0.8,0.9],[0.8,0.9],[0.6,0.7]),\\([0.1,0.2],[0.2,0.3],[0.2,0.3]),\\([0.1,0.2],[0,0.1],[0.2,0.3])\end{matrix}\right\rangle$

From Table 5 we get the RINS P^* for the ideal solution by the formula (14) as follow:

$$P^* = \left\{ \left\langle\begin{matrix}([0.8,0.9],[0.7,0.8],[0.8,0.9]),\\([0,0.1],[0.2,0.3],[0.1,0.2]),\\([0.1,0.2],[0.1,0.2],[0.1,0.2])\end{matrix}\right\rangle \left\langle\begin{matrix}([0.8,0.9],[0.8,0.9]),\\([0.1,0.2],[0.1,0.2]),\\([0.1,0.2],[0,0.1])\end{matrix}\right\rangle \left\langle\begin{matrix}([0.8,0.9],[0.8,0.9],[0.8,0.9]),\\([0.1,0.2],[0.1,0.2],[0.1,0.2]),\\([0,0.1],[0,0.1],[0.1,0.2])\end{matrix}\right\rangle \right\}$$

Thus, we use the equations (11)-(12) to get the values of similarity measures between the ideal solution P^* and the alternatives P_s (s = 1, 2, 3, 4) and decision results, which are shown in Table 6.

Table 6.
The similarity measure values between P_s and P^* and decision results.

Measure	Similarity measure value	Ranking order	The best choice
$W_3(P_s, P^*)$	$W_3(P_1, P^*) = 0.9169$ $W_3(P_2, P^*) = 0.9208$ $W_3(P_3, P^*) = 0.9109$ $W_3(P_4, P^*) = 0.9282$	$P_4>P_2>P_1>P_3$	P_4
$W_4(P_s, P^*)$	$W_4(P_1, P^*) = 0.8531$ $W_4(P_2, P^*) = 0.8589$ $W_4(P_3, P^*) = 0.8328$ $W_4(P_4, P^*) = 0.8713$	$P_4>P_2>P_1>P_3$	P_4

In Table 6, the alternative P_4 is considered as the best choice, which is the best one.

However, the same ranking orders are shown in the two cases under RSNS environments. But existing literature [12, 19-22] cannot deal with such two cases with both attributes and sub-attributes in RSNS setting.

6. Conclusion

This study presented the tangent and cotangent functions-based similarity measures of RSNSs, and then proposed their decision making method, which is more suitable for the problems that have multiple attributes with sub-attributes, along with both the attribute weights and the sub-attribute weights.

By the similarity measure values between alternatives and the ideal solution, we can rank alternatives and choose the best one. Then, an illustrative example on the decision making problem of a construction project was provided in order to indicate the feasibility and effectiveness of the proposed method in RSNS (RSVNS and RINS) setting. Obviously, this study extends existing methods and provides a new way for the refined simplified neutrosophic MADM problems containing both the attribute weight and the sub-attribute weights. For the future study, the presented method will be extended to the similarity measures based on logarithm function for group decision-making.

Acknowledgments

This paper was supported by the National Natural Science Foundation of China (Nos. 71471172, 61703280).

Conflict of interest

The authors declare no conflict of interest.

References

1. L.A. Zadeh, Fuzzy sets. *Inf. Control* **8**, 1965, 338-353.
2. K.T. Atanassov, Intuitionistic fuzzy sets. *Fuzzy sets and Systems* **20**(1), 1986, 87-96.
3. F. Smarandache, Neutrosophy: Neutrosophic probability, set, and logic, American Research Press, Rehoboth, USA, 1998.
4. J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent & Fuzzy Systems* **26**(5), 2014, 2459-2466.
5. J. Ye, Vector Similarity Measures of Simplified Neutrosophic Sets and Their Application in Multicriteria Decision Making. *International Journal of Fuzzy Systems* **16**(2), 2014, 204-211.
6. J. Ye, Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. *Journal of Intelligent & Fuzzy Systems* **27**(6), 2014, 2927-2935.
7. P.D. Liu, Y. Wang, Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural Computing and Applications* **25**(7-8), 2014, 2001-2010.
8. J.J. Peng, J.Q. Wang, H.Y. Zhang, X.H. Chen, An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. *Appl. Soft Comput*. **25**, 2014, 336–346.
9. E.K. Zavadskas, R. Baušys, M. Lazauskas, Sustainable assessment of alternative sites for the construction of a waste incineration plant by applying WASPAS method with single-valued neutrosophic set. *Sustainability* **7**(12), 2015, 15923-15936.
10. J.J. Peng, J.Q. Wang, J. Wang, H.Y. Zhang, X.H. Chen, Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems, *Int. J. Systems Science* **47**(10), 2016, 2342-2358.
11. J.Q. Chen, J. Ye, Some single-valued neutrosophic dombi weighted aggregation operators for multiple attribute decision-making. *Symmetry* **9**(6), 2017, 82.
12. J. Ye, Single-valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. *Soft Computing* **21**(3), 2017, 817-825.
13. Z.K. Lu, J. Ye, Single-valued neutrosophic hybrid arithmetic and geometric aggregation operators and their decision-making method. *Information* **8**(3), 2017, 84.
14. J. Ye, The generalized Dice measures for multiple attribute decision making under simplified neutrosophic environments. *Journal of Intelligent & Fuzzy Systems* **31**, 2016, 663–671.
15. J. Ye, Simplified neutrosophic harmonic averaging projection-based method for multiple attribute decision making problems. *International Journal of Machine Learning and Cybernetics* **8**(3), 2017, 981-987.
16. D.S. Xu, C. Wei, G.W. Wei, TODIM method for single-valued neutrosophic multiple attribute decision making. *Information* **8**, 2017, 125.
17. S. Pramanik, P. Biswas, B.C. Giri, Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. *Neural Computing and Applications* **28**(5), 2017, 1163-1176.
18. F. Smarandache, n-Valued refined neutrosophic logic and its applications in physics. *Progress in Physics* **4**, 2013,143–146.
19. S. Broumi, I. Deli, Correlation measure for neutrosophic refined sets and its application in medical diagnosis. *Palestine Journal of Mathematics* **3**(1), 2014, 11-19.
20. S. Broumi, S. Smarandache, Extended Hausdorff Distance and Similarity measure of Refined Neutrosophic Sets and their Application in Medical Diagnosis. *Journal of New Theory* **1**(7), 2015, 64-78.

21. S. Broumi, I. Deli, F. Smarandache, N-Valued interval neutrosophic sets and their application in medical diagnosis, *Critical Review* **X**, 2015, 45-68.
22. K. Mondal, S. Pramanik, Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. *Journal of New theory* **8**, 2015, 41-50.
23. J. Ye, F. Smarandache, Similarity measure of refined single-valued neutrosophic sets and its multicriteria decision making method. *Neutrosophic Sets and Systems* **12**, 2016, 41-44.
24. J.Q. Chen, J. Ye, S.G. Du, Vector similarity measures between refined simplified neutrosophic sets and their multiple attribute decision-making method. *Symmetry* **9**(8), 2017, 153.
25. C.X. Fan, J. Ye, The cosine measure of refined-single valued neutrosophic sets and refined-interval neutrosophic sets for multiple attribute decision-making. *Journal of Intelligent & Fuzzy Systems* **33**(4), 2017, 2281-2289.