RANKING METHODS OF SINGLE VALUED NEUTROSOPHIC NUMBERS AND ITS APPLICATIONS TO MULTIPLE CRITERIA DECISION MAKING

Ieva Meidute-Kavaliauskiene
Vilnius Gediminas Technical University and General Jonas Žemaitis Military Academy of Lithuania

RANKING METHODS OF SINGLE VALUED NEUTROSOPHIC NUMBERS AND ITS APPLICATIONS TO MULTIPLE CRITERIA DECISION MAKING

Dragisa Stanujkic, Ieva Meidute-Kavaliauskiene, Darjan Karabasevic

[1]*Technical Faculty in Bor, University of Belgrade (SERBIA)*
[2]*Business Management Faculty, Vilnius Gediminas Technical University,Vilnius (LITHUANIA)*
[3]*Faculty of Applied Management, Economics and Finance, University Business Academy in Novi Sad (SERBIA)*

dstanujkic@tfbor.bg.ac.rs ieva.meidutekavaliauskiene@vgtu.lt darjan.karabasevic@mef.edu.rs

ABSTRACT

Decision making is a very important and actual process. It is exactly as such the most important to managers to whom it is a primary task. Taking into account the more possibilities of the action lines that can be implemented, the outcome of the decision-making process must be the solution of this situation, i.e., defining the directions for further action. The rapid development of the field of multiple-criteria decision making (MCDM), as one of the extremely important areas of operational research, has contributed to the development of many multiple-criteria decision-making methods. Therefore, the main objective of this article is to point out the usability of single-valued neutrosophic sets in solving multiple criteria decision-making problems. Three approaches for ranking of single-valued neutrosophic numbers are presented in the article, and its usability is demonstrated in numerical illustration.

Keywords: neutrosophy, neurosophic set, single-valued neutrosophic numbers, decision-making.

1 INTRODUCTION

Multiple Criteria Decision Making (MCDM) has become very important and fastest growing subfields of operations research and management science. As modern MCDM started to emerge about 50 years ago, and until now it is used for solving a number of different decision-making problems in different fields [1-3]. Multiple Criteria Decision Making can be defined as making choices in the presence of multiple conflicting criteria. More precise MCDM models usually leads to increasing number or evaluation criteria or use of more complex criteria that are later decomposed into sub-criteria. However, an increase in the number of criteria, as well as sub-criterion, can be less desirable in cases where data should be collected by the survey [4-7].

Hwang & Yoon [8] emphasize that MCDM can be divided into two basic categories: multi-attribute decision making which is mainly applied to selection problems and is always linked to a limited number of alternatives and ranking preferences, and multi-objective decision making which is usually applied to planning, i.e. to problems where the number of alternatives is infinite.

Having in mind the extremely dynamic development of the MCDM area, a number of methods of multiple-criteria decision-making have been developed over time, of which the most applied are: SAW, ELECTRE, MOORA, MULTIMOORA, TOPSIS, AHP, PROMETHEE, VIKOR, WASPAS and so on [9-10].

Significant progress in using the MCDM methods for solving complex decision-making problems was made after Zadeh [11-12], when he first proposed fuzzy sets, on which basis Bellman and Zadeh [13], somewhat later have proposed fuzzy MCDM. Since then, some extensions of fuzzy sets theory have been developed, such as: interval valued fuzzy sets [14], intuitionistic fuzzy sets [15] and interval-valued intuitionistic fuzzy sets [16].

In 1999, Smarandache [17] introduced the concept of neutrosophic sets, as generalization of the fuzzy sets theory and their extensions.

Fuzzy sets theory introduces partial membership to a set, expressed by membership function $\mu_{(x)}$, where membership function can have different forms, such as: bell-shaped, triangular, trapezoidal and singleton. Neutrosophic sets theory introduces three parameter that can be used to describe belonging to a set, that is; truth membership, indeterminacy membership, falsity membership. That is why neutrosophic sets could be more suitable for evaluating complex phenomena and events.

Therefore, the applicability of neutrosophic sets in the MCDM model is considered in the rest of this article. The remainder of article is organized as follows: In Section 2 basic elements of neutrosophic sets and single valued neutrosophic numbers are considered. In Section 3 approaches for ranking single valued neutrosophic numbers are considered, and in Section 4 a multiple-criteria decision-making approach based on single valued neutrosophic numbers is presented. In Section 5 a numerical illustration is given in order to demonstrate proposed approach. Finally, conclusions are given.

2 BASIC ELEMENTS OF NEUTROSOPHIC SETS AND SINGLE VALUED NEUTROSOPHIC NUMBERS

Definition 1. Let *X* be a nonempty set, with a generic element in *X* denoted by *x*. Then, the Neutrosophic Set (NS) *A* in *X* is as follows [17-18]:

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \middle| \, x \in X \right\}, \tag{1}$$

with: $T_A : X \to]^-0,1^+[$; $I_A : X \to]^-0,1^+[$; $F_A : X \to]^-0,1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$

where: $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the indeterminacy-membership function and the falsity-membership function, respectively.

Definition 2. Let *X* be a nonempty set. The Single Valued Neutrosophic Set (SVNS) *A* in *X* is as follows [17-19]:

$$A = \left\{ \left\langle x, T_A(x), I_A(x), F_A(x) \right\rangle \middle| \, x \in X \right\}, \tag{2}$$

with: $T_A : X \to [0,1]$; $I_A : X \to [0,1]$; $F_A : X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 3. For an SVNS *A* in *X*, the triple $< t_A, i_A, f_A >$ is called the Single Valued Neutrosophic Number (SVNN) [17-18].

Definition 4. Let $x_1 =< t_1, i_1, f_1 >$ and $x_2 =< t_2, i_2, f_2 >$ be two SVNNs and $\lambda > 0$; then the basic operations are defined as follows:

$$x_1 + x_2 =< t_1 + t_2 - t_1t_2, i_1i_2, f_1f_2 >, \tag{3}$$

$$x_1 \cdot x_2 =< t_1t_2, i_1 + i_2 - i_1i_2, f_1 + f_2 - f_1f_2 >. \tag{4}$$

$$\lambda x_1 =< 1-(1-t_1)^\lambda, i_1^\lambda, f_1^\lambda >. \tag{5}$$

$$x_1^\lambda =< t_1^\lambda, i_1^\lambda, 1-(1-f_1)^\lambda >. \tag{6}$$

Definition 5. Let $x =< t, i, f >$ be an SVNN; then a score function $s_{(x)}$ of *x* can be as follows [20]:

$$s_{(x)} = \frac{1 + t_x - 2i_x - f_x}{2}. \tag{7}$$

Definition 6. Let $x =< t, i, f >$ be a SVNN; then a cosine similarity measure $c_{(x)}$ between SVNN *x* and the ideal alternative (point) <1,0,0> is as follows [20]:

$$c_{(x)} = \frac{t}{\sqrt{t^2 + i^2 + f^2}}. \tag{8}$$

Definition 7. Let $x =< t, i, f >$ a SVNN; then the Hamming distance $h_{(x)}$ between SVNN *x* and the ideal alternative (point) <1,0,0> is as follows:

$$h_{(x)} = \frac{1}{3}\left(|1-t| + |i-0| + |f-0| \right) = \frac{1}{3}(1-t+i+f). \tag{9}$$

Definition 8. Let $x=<t\ ,i\ ,f\ >$ a SVNN; then the Hamming distance $h_{(x)}$ between SVNN x and the ideal alternative (point) <1,0,0> is as follows:

$$h_{(x)}=\left(\frac{1}{3}(|1-t|^2+|i-0|^2+|\ f-0|^2)\right)^{\frac{1}{2}}=+\left(\frac{1}{3}\left((1-t)^2+i^2+f^2\right)\right)^{\frac{1}{2}}. \quad (10)$$

Definition 9. Let $A_j=<t_j,i_j,f_j>$ be a collection of SVNSs and $W=(w_1,w_2,...,w_n)^T$ be an associated weighting vector. Then the Single Valued Neutrosophic Weighted Average (SVNWA) operator of A_j is as follows [20]:

$$SVNWA(A_1,A_2,...,A_n)=\sum_{j=1}^{n}w_jA_j=\left(1-\prod_{j=1}^{n}(1-t_j)^{w_j},\prod_{j=1}^{n}(i_j)^{w_j},\prod_{j=1}^{n}(f_j)^{w_j}\right). \quad (11)$$

where: w_j is the element j of the weighting vector, $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$.

3 RANKING OF SINGLE VALUED NEUTROSOPHIC NUMBERS

There are several approaches for ranking SVNNs. An approach based on the Score function is commonly used.

Definition 10. Let x_1 and x_2 be two SVNNs. Then, the ranking method based on the score function is as follows:

If $s_{(x_1)}>s_{(x_2)}$, then $x_1>x_2$. (12)

The next approach is based on the cosine similarity measure.

Definition 11. Let x_1 and x_2 be two SVNNs. Then, the ranking method based on the cosine similarity measure is as follows:

If $c_{(x_1)}>c_{(x_2)}$, then $x_1>x_2$. (13)

SVNNs can also be ranked on the basis of their distances from an ideal point.

Definition 12. Let x_1 and x_2 be two SVNNs. Then, the ranking method based on the Hamming distance is as follows:

If $h_{(x_1)}>h_{(x_2)}$, then $x_1<x_2$. (14)

4 A MULTIPLE CRITERIA DECISION MAKING APPROACH BASED ON SINGLE VALUED NEUTROSOPHIC NUMBERS

The procedure for solving multiple criteria decision-making problem that contain *m* alternatives that are evaluated based on *n* criteria by *K* experts can precisely be expressed by the following algorithm:

Step 1. Define a goal of evaluation and identify available alternatives.

Step 2. Define a set of evaluation criteria and determine their significance, i.e. criteria weights.

Step 3. Form a group of experts who will perform evaluation and perform evaluation.

Step 4. Construct a group decision-making matrix using Eq. (11).

Step 5. Calculate overall ratings using Eq. (11).

Step 6. Rank alternatives and select the best one using an approach presented in Section 3.

5 NUMERICAL ILLUSTRATION

In order to briefly demonstrate the usability of the SVNNs for solving MCDM problems, an example of supplier selection is presented in this section.

Assume that one company have to consider engaging of a new supplier. Therefore, a team of three experts if formed with the aim to select the most appropriate supplier from four alternatives on the basis on the following criteria:

C_1 – Delivery,
C_2 – Quality,
C_3 – Flexibility,
C_4 – Service, and
C_5 – Price.

The ratings obtained from three experts are shown in Tables 1, 2 and 3.

Table 1. The ratings obtained from the first of three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.8, 0.00, 0.10>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>	<0.5, 0.0, 0.5>
A_2	<0.7, 0.00, 0.20>	<0.8, 0.0, 0.2>	<0.8, 0.0, 0.2>	<0.8, 0.0, 0.2>	<0.8, 0.0, 0.2>
A_3	<0.5, 0.00, 0.20>	<0.5, 0.0, 0.5>	<0.6, 0.0, 0.4>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>
A_4	<0.7, 0.00, 0.30>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>	<0.5, 0.0, 0.5>	<0.5, 0.0, 0.5>

Table 2. The ratings obtained from the second of three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.6, 0.00, 0.40>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.5, 0.0, 0.0>	<0.6, 0.0, 0.0>
A_2	<0.8, 0.00, 0.20>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>	<0.8, 0.0, 0.2>	<0.6, 0.0, 0.0>
A_3	<0.7, 0.00, 0.30>	<0.8, 0.0, 0.2>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.7, 0.0, 0.3>
A_4	<0.6, 0.00, 0.40>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.6, 0.0, 0.4>	<0.5, 0.0, 0.5>

Table 3. The ratings obtained from the third of three experts

	C_1	C_2	C_3	C_4	C_5
A_1	<0.8, 0.00, 0.20>	<0.6, 0.0, 0.4>	<0.5, 0.0, 0.5>	<0.6, 0.0, 0.4>	<0.8, 0.0, 0.2>
A_2	<0.6, 0.00, 0.10>	<0.6, 0.0, 0.4>	<0.8, 0.0, 0.2>	<0.5, 0.0, 0.5>	<0.7, 0.0, 0.3>
A_3	<0.6, 0.00, 0.40>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.2>	<0.6, 0.0, 0.4>	<0.5, 0.0, 0.5>
A_4	<0.7, 0.00, 0.30>	<0.8, 0.0, 0.2>	<0.7, 0.0, 0.3>	<0.6, 0.0, 0.4>	<0.6, 0.0, 0.1>

The group decision-making matrix, constructed using Eq. (11), are shown in Table 4. In this calculation all three experts had the same significance, that is w_j=0.333; j=1, 2 and 3.

Table 4. The group decision-making matrix

	C_1	C_2	C_3	C_4	C_5
A_1	<0.75, 0.0, 0.24>	<0.67, 0.0, 0.33>	<0.57, 0.0, 0.44>	<0.61, 0.0, 0.25>	<0.66, 0.0, 0.26>
A_2	<0.71, 0.0, 0.17>	<0.68, 0.0, 0.34>	<0.77, 0.0, 0.23>	<0.73, 0.0, 0.32>	<0.71, 0.0, 0.18>
A_3	<0.61, 0.0, 0.30>	<0.69, 0.0, 0.35>	<0.64, 0.0, 0.30>	<0.60, 0.0, 0.40>	<0.64, 0.0, 0.37>
A_4	<0.75, 0.0, 0.24>	<0.67, 0.0, 0.33>	<0.57, 0.0, 0.44>	<0.61, 0.0, 0.25>	<0.66, 0.0, 0.26>

In the next steep overall ratings was calculated using Eq. (11) and the following weights w_j = {0.18, 0.21, 0.20, 0.18, 0.23}. The overall ratings are shown in Table 5.

Table 5. The overall ratings obtained from the third of three experts

	Overall ratings
A_1	<0.65, 0.00, 0.30>
A_2	<0.72, 0.00, 0.24>
A_3	<0.64, 0.00, 0.34>
A_4	<0.65, 0.00, 0.30>

Finally, the ranking results obtained using three approaches, considered in Section 4, are encountered for in Table 6.

Table 6. The ranking results obtained using three approaches

	I		II		III	
	$s_{(i)}$	Rank	$c_{(i)}$	Rank	$h_{(i)}$	Rank
A_1	0.678	2	0.910	2	0.215	2
A_2	0.743	1	0.951	1	0.171	1
A_3	0.647	3	0.880	3	0.235	3
A_4	0.639	4	0.872	4	0.240	4

6 CONCLUSION

All current problems can be observed from a multiple-criteria decision-making perspective, because the problems are mainly related to the fulfillment of the objectives related to the larger number, usually conflicting criteria, which is a great approximation to the real tasks in the decision-making processes.

Taking into account previously stated, main objective of this manuscript is to emphasize the usability of single-valued neutrosophic sets in solving complex multiple-criteria decision-making problems. Therefore, in this manuscript three approaches for ranking of single-valued neutrosophic numbers are proposed. Usability and applicability of the approaches is demonstrated in conducted numerical example. Ranking results of the alternatives based on all of the three approaches are the same, alternative denoted as A_2 is the best in terms of evaluated criteria.

Neutrosophic sets theory introduces three parameters that can be used to describe belonging to a set, that is; truth membership, indeterminacy membership, falsity membership. That is why neutrosophic sets could be more suitable for evaluating complex phenomena and events. Thus, it is logical to expect greater application of neutrosophic sets in the area of MCDM, especially when it is necessary to solve complex problems.

REFERENCES

[1] Karabasevic, D., Stanujkic, D., Urosevic, S., Popovic, G., & Maksimovic, M. (2017). An approach to criteria weights determination by integrating the Delphi and the adapted SWARA methods. Management: Journal of Sustainable Business and Management Solutions in Emerging Economies, 22(3), pp. 15-25.

[2] Vesković, S., Stević, Ž., Stojić, G., Vasiljević, M., & Milinković, S. (2018). Evaluation of the railway management model by using a new integrated model DELPHI-SWARA-MABAC. Decision Making: Applications in Management and Engineering, 1(2), pp. 34-50.

[3] Stanujkić, D., & Karabašević, D. (2018). An extension of the WASPAS method for decision-making problems with intuitionistic fuzzy numbers: a case of website evaluation. Operational Research in Engineering Sciences: Theory and Applications, 1(1), pp. 29-39.

[4] Pamučar, D., Lukovac, V., Božanić, D., & Komazec, N. (2018). Multi-criteria FUCOM-MAIRCA model for the evaluation of level crossings: case study in the Republic of Serbia. Operational Research in Engineering Sciences: Theory and Applications, 1(1), pp. 108-129.

[5] Karabašević, D., Stanujkić, D., Đorđević, B., & Stanujkić, A. (2018). The weighted sum preferred levels of performances approach to solving problems in human resources management. Serbian Journal of Management, 13(1), pp. 145-156.

[6] Stanujkic, D., Zavadskas, E. K., Karabasevic, D., Urosevic, S., & Maksimovic, M. (2017). An approach for evaluating website quality in hotel industry based on triangular intuitionistic fuzzy numbers. Informatica, 28(4), pp. 725-748.

[7] Karabasevic, D., Popovic, G., Stanujkic, D., Maksimovic, M. & Sava, C. (2019). An approach for hotel type selection based on the single-valued intuitionistic fuzzy numbers. International Review, (1-2), pp. 9-16.

[8] Hwang, C. L. & Yoon, K. (1981). Multiple Attribute Decision Making, Methods and Applications: a State of the Art Survey. Lecture notes in economics and mathematical systems, Springer-Verlag, New York, NY.

[9] Popovic, G., Stanujkic, D., Brzakovic, M., & Karabasevic, D. (2019). A multiple-criteria decision-making model for the selection of a hotel location. Land Use Policy, 84, pp. 49-58.

[10] Popovic, G., Stanujkic, D., & Karabasevic, D. (2019). A framework for the evaluation of hotel property development projects. International Journal of Strategic Property Management, 23(2), pp. 96-107.

[11] Zadeh, L. A. (1965). Fuzzy sets. Information and control, 8(3), pp. 338-353.

[12] Karabasevic, D., Zavadskas, E. K., Turskis, Z., & Stanujkic, D. (2016). The framework for the selection of personnel based on the SWARA and ARAS methods under uncertainties. Informatica, 27(1), pp. 49-65.

[13] Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. Management science, 17(4), B-141.

[14] Turksen, I. B. (1986). Interval valued fuzzy sets based on normal forms. Fuzzy sets and systems, 20(2), pp. 191-210.

[15] Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, pp. 87-96.

[16] Atanassov, K., & Gargov, G. (1989). Interval-valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 31, pp. 343–349.

[17] Smarandache, F. (1999). A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. American Research Press: Rehoboth, DE, USA. pp. 141.

[18] Stanujkic, D., Smarandache, F., Zavadskas, E. K., & Karabasevic, D. (2016). Multiple Criteria Evaluation Model Based on the Single Valued Neutrosophic Set. Neutrosophic Sets and Systems, 14, pp. 3-6.

[19] Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. Multispace and Multistructure, 4, pp. 410-413.

[20] Şahin, R. (2014). Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment. arXiv preprint arXiv:1412.5202.