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Exploring the Algebraic Structures of Q-Complex Neutrosophic Soft Fields

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Abstract

A field is a fundamental algebraic structure that finds extensive applications in algebra and various mathematical domains. On the other hand, a Q-complex neutrosophic soft set (Q-CNSS) is a unique hybrid model that combines the characteristics of soft sets and neutrosophic sets within a complex number framework. It utilizes the effectiveness of Q-set as a powerful tool in the domain of this particular model. In this article, we leverage this model to define fields under uncertainty. We present the Q-complex neutrosophic soft field (Q-CNSF) and examine the unique algebraic properties associated with this model. Additionally, we explore the relationships between Q-CNSF and Q-neutrosophic soft field (Q-NSF). Furthermore, we define the Cartesian product of Q-CNSFs and delve into the relevant properties. Through this comprehensive exploration, our aim is to enhance the understanding of Q-CNSFs and their properties, ultimately contributing to the field of algebraic analysis and its practical applications in handling uncertainty and vagueness.

Kewords: Complex neutrosophic soft set; Q-complex neutrosophic soft set; Q-neutrosophic soft field; Q-neutrosophic soft set.

1 introduction

In real-life situations, human thinking faces many situations that are fully hidden, uncertain, and impartial. To translate these positions and to handle the outlined uncertainties, Smarandache[1] provided the definition of a neutrosophic set (NS), since the preserve is not able to handle the outlined issues. On the other hand, one of the Russian researchers set out to introduce a new mathematical tool called the soft set (SS).[2] This set is distinguished by giving a more accurate description of the data on daily life issues. The NSs and SSs attracted the attention of researchers around the world to present many research works that have wide applications.

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As a combination of SS and NS in several environments, scholars proposed different models with a more powerful ability to process real-life problems. For instance,[3,4] discusses the topic in the context of soft computing, while[6,7] covers its import to analysis and[8,9] to graph theory, complex analysis,[10,11] and algebraic structures.[12] Additionally, Palanikumar et al.[13] proposed many methods to solve design-making problems. A multi-criteria decision-making approach was introduced by Broumi et al.[14] when they extended NSs to plithogenic set. Al-Sharqi et al.[15] came up with NSS in matrix form and showed its application to real-life problems. Working on NSSs Al-Quran et al.[16] also presented some studies with some applications. Some researchers[17–19] also give similarity measures between NS-sets and some of their properties and applications.

On the opposite side,since its inception in 1971 with the pioneering work of Rosenfeld,[20] the study of fuzzy abstract algebra has continued to captivate the attention of numerous researchers. Over the years, the academic landscape has been enriched by the emergence of numerous scholarly papers exploring a wide array of fuzzy substructures within algebraic systems. Within this extensive body of literature, one particular focus of investigation has been the fundamental algebraic structure known as fields. Extensive research has been conducted by Malik and Mordeson[21] on the subject of fuzzy subfields, where they thoroughly examined their fundamental properties. Based on this foundation, Mordeson[22] delved into the exploration of fuzzy field extensions, verifying the connection between fuzzy sets and finite fields. Anandh and Giri[23] conducted an in-depth analysis of the concept of intuitionistic fuzzy subfields in relation to (T, S)-norms. Bera and Mahapatra,[24] investigated the structural characteristics of neutrosophic soft field. This work was further expanded upon by Abu Qamar et al.,[25] who extended the study to the realm of Q-neutrosophic soft fields. In the complex setting, Gulzar et al.[26] extended the discourse on complex fuzzy sets by introducing a novel concept of complex fuzzy subfields. This advancement involved the incorporation of a second dimension into the membership function of a fuzzy set, resulting in a significant expansion of the concept and its applications in complex space. Khamis and Ahmad[27] introduced a novel structure called the Q-complex intuitionistic fuzzy subfield, which originated from the existing framework of complex fuzzy subfields. This new model expands the scope of investigation beyond TM values to include both TM and FM function values.

On the other hand, Ali and Smarandache[28] coined the concept of complex neutrosophic set (CNS) as an extension of neutrosophic sets (NSs) . To further enhance its practicality in solving decision-making problems, Broumi et al.[19] extended the CNS framework by proposing the notion of complex neutrosophic soft sets (CNSS). Expanding on these advancements, Al-Quran et al.[29,30] took the CNSS framework even further by introducing the concept of Q-complex neutrosophic soft sets (Q-CNSSs). In this paper, our objective is twofold. Firstly, we seek to extend the range of Q-NSF beyond the unit interval [0,1] in the real space to the unit disc in the complex space. Alternatively, we endeavor to incorporate the complex-valued IM function to the structure of Q-CIFS field. This enhancement will pave the way for the introduction of Q-CNSF as a novel concept deserving of exploration.

2 Preliminaries

Within this section, we provide an overview of the fundamental principles underlying Q-neutrosophic soft set (Q-NSS),[25] Q-CNSS with their operations, Q-NSF and Q-CIFS field. These essential concepts and operations serve as the building blocks for our forthcoming analysis in this article.

Definition 2.1. [25] Let Y and Q be two non-empty sets, and let $\mathbb{A}$ denote a set of parameters. The Q-NSS $(\mathbb{F}, \mathbb{A})$ in Y is unequivocally defined by the following distinct characteristics:

$(\mathbb{F}, \mathbb{A}) = \{< a; \Gamma_{F(\mathfrak{a})}(y,q), \Lambda_{F(\mathfrak{a})}(y,q), \Omega_{F(\mathfrak{a})}(y,q) >: a \in \mathbb{A}, y \in Y, q \in Q\}$. The resolute functions $\Gamma_{F(\mathfrak{a})}(y,q), \Lambda_{F(\mathfrak{a})}(y,q)$, and $\Omega_{F(\mathfrak{a})}(y,q)$, unyieldingly, represent the TM, IM and FM functions, respectively.

Definition 2.2. [29] Consider W and Q as two non-empty sets, and let $\mathbb{A}$ represent a set of parameters. We define a Q-CNSS $(\mathbb{H}, \mathbb{A})$ in W as follows.

$$(\mathbb{H}, \mathbb{A}) = \{< \mathfrak{a}; \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q) >: \mathfrak{a} \in \mathbb{A}, s \in W, q \in Q\},$$

where $\forall \mathfrak{a} \in \mathbb{A}, s \in W, q \in Q$, $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q) = \Omega_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(s,q)}$, are, respectively, the complex-valued TM, IM and FM functions.

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Definition 2.3. [29] The union of two Q-CNSSs $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ in W is a Q-CNSS $(\mathbb{M},\mathbb{E})$, where $\mathbb{E}=\mathbb{A}\cup\mathbb{B}$ and $\forall\, e\in\mathbb{E},\forall\,(s,q)\in W\times Q$,

$$\mathcal{T}_{\mathbb{M}(e)}(s,q)=\begin{cases}\Gamma_{\mathbb{H}(e)}(s,q).e^{i2\pi\mu_{\mathbb{H}(e)}(s,q)} & ,\text{if } e\in\mathbb{A}-\mathbb{B}\\ \Gamma_{\mathbb{G}(e)}(s,q).e^{i2\pi\mu_{\mathbb{G}(e)}(s,q)} & ,\text{if } e\in\mathbb{B}-\mathbb{A}\\ (\Gamma_{\mathbb{H}(e)}(s,q)\vee\Gamma_{\mathbb{G}(e)}(s,q)).e^{i2\pi(\mu_{\mathbb{H}(e)}(s,q)\vee\mu_{\mathbb{G}(e)}(s,q))} & ,\text{if } e\in\mathbb{A}\cap\mathbb{B},\end{cases}$$

$$\mathcal{I}_{\mathbb{M}(e)}(s,q)=\begin{cases}\Lambda_{\mathbb{H}(e)}(s,q).e^{i2\pi\nu_{\mathbb{H}(e)}(s,q)} & ,\text{if } e\in\mathbb{A}-\mathbb{B}\\ \Lambda_{\mathbb{G}(e)}(s,q).e^{i2\pi\nu_{\mathbb{G}(e)}(s,q)} & ,\text{if } e\in\mathbb{B}-\mathbb{A}\\ (\Lambda_{\mathbb{H}(e)}(s,q)\wedge\Lambda_{\mathbb{G}(e)}(s,q)).e^{i2\pi(\nu_{\mathbb{H}(e)}(s,q)\wedge\nu_{\mathbb{G}(e)}(s,q))} & ,\text{if } e\in\mathbb{A}\cap\mathbb{B},\end{cases}$$

$$\mathcal{F}_{\mathbb{M}(e)}(s,q)=\begin{cases}\Omega_{\mathbb{H}(e)}(s,q).e^{i2\pi\omega_{\mathbb{H}(e)}(s,q)} & ,\text{if } e\in\mathbb{A}-\mathbb{B}\\ \Omega_{\mathbb{G}(e)}(s,q).e^{i2\pi\omega_{\mathbb{G}(e)}(s,q)} & ,\text{if } e\in\mathbb{B}-\mathbb{A}\\ (\Omega_{\mathbb{H}(e)}(s,q)\wedge\Omega_{\mathbb{G}(e)}(s,q)).e^{i2\pi(\omega_{\mathbb{H}(e)}(s,q)\wedge\omega_{\mathbb{G}(e)}(s,q))} & ,\text{if } e\in\mathbb{A}\cap\mathbb{B},\end{cases}$$

Here, $\vee$ represents the maximum operator and $\wedge$ represents the minimum operator. The union of $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ is denoted as $(\mathbb{M},\mathbb{E})$, i.e., $(\mathbb{H},\mathbb{A})\cup(\mathbb{G},\mathbb{B})=(\mathbb{M},\mathbb{E})$.

Definition 2.4. [30] The intersection of two Q-CNSSs $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ in W is a Q-CNSS $(\mathbb{M},\mathbb{E})$, where $\mathbb{E}=\mathbb{A}\cap\mathbb{B}$ and $\forall\, e\in\mathbb{E},\forall\,(s,q)\in W\times Q$, the membership degrees of $(\mathbb{M},\mathbb{E})$ are:

$$\mathcal{T}_{\mathbb{M}(e)}(s,q)=(\Gamma_{\mathbb{H}(e)}(s,q)\wedge\Gamma_{\mathbb{G}(e)}(s,q)).e^{i2\pi(\mu_{\mathbb{H}(e)}(s,q)\wedge\mu_{\mathbb{G}(e)}(s,q))},$$

$$\mathcal{I}_{\mathbb{M}(e)}(s,q)=(\Lambda_{\mathbb{H}(e)}(s,q)\vee\Lambda_{\mathbb{G}(e)}(s,q)).e^{i2\pi(\nu_{\mathbb{H}(e)}(s,q)\vee\nu_{\mathbb{G}(e)}(s,q))},$$

$$\mathcal{F}_{\mathbb{M}(e)}(s,q)=(\Omega_{\mathbb{H}(e)}(s,q)\vee\Omega_{\mathbb{G}(e)}(s,q)).e^{i2\pi(\omega_{\mathbb{H}(e)}(s,q)\vee\omega_{\mathbb{G}(e)}(s,q))}.$$

Here, $\vee$ represents the maximum operator and $\wedge$ represents the minimum operator. The intersection of $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$ is denoted as $(\mathbb{M},\mathbb{E})$, i.e., $(\mathbb{H},\mathbb{A})\cap(\mathbb{G},\mathbb{B})=(\mathbb{M},\mathbb{E})$.

This part unequivocally elucidates the profound connection between Q-CNSS and Q-NSS.

Definition 2.5. [29] Let's consider the sets W and Q, and suppose $(\mathbb{H},\mathbb{A})$ represents a Q-CNSS in W. The Q-CNSS $(\mathbb{H},\mathbb{A})$ can be represented by a collection of elements in the form $\{<\mathfrak{a};\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q),\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)>:\mathfrak{a}\in\mathbb{A},s\in W,q\in Q\}$, where $\mathfrak{a}\in\mathbb{A}$, $s\in W$, and $q\in Q$. In this context, the TM, IM, and FM functions can be defined as follows: $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)=\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)=\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)=\Omega_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(s,q)}$. Based on this, $(\mathbb{H},\mathbb{A})$ generates two real Q-NSSs in W using the following formulations:

(1) The Q-NSS $(\mathfrak{h},\mathbb{A})$ is defined in the form of $\{<a;\Gamma_{\mathfrak{h}(\mathfrak{a})}(s,q),\Lambda_{\mathfrak{h}(\mathfrak{a})}(s,q),\Omega_{\mathfrak{h}(\mathfrak{a})}(s,q)>:a\in\mathbb{A},s\in W,q\in Q\}$, where $a\in\mathbb{A}$, $s\in W$, and $q\in Q$. Within this context, $\Gamma_{\mathfrak{h}(\mathfrak{a})}(s,q),\Lambda_{\mathfrak{h}(\mathfrak{a})}(s,q)$, and $\Omega_{\mathfrak{h}(\mathfrak{a})}(s,q)$ represent the amplitude terms associated with the complex valued membership functions $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)$, respectively.

(2) The Q-NSS $(\mathfrak{K},\mathbb{A})$ is defined in the form of $\{<a;\mu_{\mathfrak{K}(\mathfrak{a})}(s,q),\nu_{\mathfrak{K}(\mathfrak{a})}(s,q),\omega_{\mathfrak{K}(\mathfrak{a})}(s,q)>:a\in\mathbb{A},s\in W,q\in Q\}$, where $a\in\mathbb{A}$, $s\in W$, and $q\in Q$. Within this context, $\mu_{\mathfrak{K}(\mathfrak{a})}(s,q),\nu_{\mathfrak{K}(\mathfrak{a})}(s,q)$, and $\omega_{\mathfrak{K}(\mathfrak{a})}(s,q)$ represent the phase terms associated with the complex valued membership functions $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q),\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)$, respectively.

Definition 2.6. [30] Let's consider two non-empty sets, W and Q. Suppose we have a Q-CNSS $(\mathbb{H},\mathbb{A})$ in W, characterized by complex-valued membership functions defined as:

$$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q)=\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(s,q)},\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q)=\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(s,q)},\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q)=\Omega_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(s,q)}.$$

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Now, for a set $(\mathbb{H}, \mathbb{A})$ to be considered a homogeneous Q-CNSS, the following conditions must hold for any s, t in W and $\mathfrak{a}$ in $\mathbb{A}$:

(1) $\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \Gamma_{\mathbb{H}(\mathfrak{a})}(t,q)$ if and only if $\mu_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \mu_{\mathbb{H}(\mathfrak{a})}(t,q)$,

(2) $\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \Lambda_{\mathbb{H}(\mathfrak{a})}(t,q)$ if and only if $\nu_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \nu_{\mathbb{H}(\mathfrak{a})}(t,q)$,

(3) $\Omega_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \Omega_{\mathbb{H}(\mathfrak{a})}(t,q)$ if and only if $\omega_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \omega_{\mathbb{H}(\mathfrak{a})}(t,q)$,

Definition 2.7. [30] Consider $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ as two Q-CNSSs in W, which are characterized by the following complex-valued membership functions:

For $(\mathbb{H}, \mathbb{A})$: $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(s,q)e^{i2\pi\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}$ and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q) = \Omega_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i2\pi\omega_{\mathbb{H}(\mathfrak{a})}(s,q)}$.
For $(\mathbb{G}, \mathbb{B})$: $\mathcal{T}_{\mathbb{G}(\mathfrak{a})}(s,q) = \Gamma_{\mathbb{G}(\mathfrak{a})}(s,q)e^{i2\pi\mu_{\mathbb{G}(\mathfrak{a})}(s,q)}$, $\mathcal{I}_{\mathbb{G}(\mathfrak{a})}(s,q) = \Lambda_{\mathbb{G}(\mathfrak{a})}(s,q)e^{i2\pi\nu_{\mathbb{G}(\mathfrak{a})}(s,q)}$ and $\mathcal{F}_{\mathbb{G}(\mathfrak{a})}(s,q) = \Omega_{\mathbb{G}(\mathfrak{a})}(s,q).e^{i2\pi\omega_{\mathbb{G}(\mathfrak{a})}(s,q)}$.

Q-CNSS $(\mathbb{H}, \mathbb{A})$ is said to be homogeneous with $(\mathbb{G}, \mathbb{B})$ if and only if for all $\mathfrak{a} \in \mathbb{A} \cap \mathbb{B}$, $s \in W$ and $q \in Q$, we have

(1) $\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \Gamma_{\mathbb{G}(\mathfrak{a})}(s,q)$ if and only if $\mu_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \mu_{\mathbb{G}(\mathfrak{a})}(s,q)$,

(2) $\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \Lambda_{\mathbb{G}(\mathfrak{a})}(s,q)$ if and only if $\nu_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \nu_{\mathbb{G}(\mathfrak{a})}(s,q)$,

(3) $\Omega_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \Omega_{\mathbb{G}(\mathfrak{a})}(s,q)$ if and only if $\omega_{\mathbb{H}(\mathfrak{a})}(s,q) \leq \omega_{\mathbb{G}(\mathfrak{a})}(s,q)$.

Definition 2.8. [29] Consider a Q-NSS $(\mathbb{D}, \mathbb{A})$ over a field $(\mathbb{F}, +, .)$. Then, $(\mathbb{D}, \mathbb{A})$ is said to be a Q-NSF if it is satisfying the following conditions for all $m, n \in \mathbb{F}$, $q \in Q$ and $\mathfrak{a} \in \mathbb{A}$:

1. $\Gamma_{\mathbb{D}(\mathfrak{a})}(m+n,q) \geq \min\{\Gamma_{\mathbb{D}(\mathfrak{a})}(m,q), \Gamma_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, $\Lambda_{\mathbb{D}(\mathfrak{a})}(m+n,q) \leq \max\{\Lambda_{\mathbb{D}(\mathfrak{a})}(m,q), \Lambda_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, and $\Omega_{\mathbb{D}(\mathfrak{a})}(m+n,q) \leq \max\{\Omega_{\mathbb{D}(\mathfrak{a})}(m,q), \Omega_{\mathbb{D}(\mathfrak{a})}(n,q)\}$.
2. $\Gamma_{\mathbb{D}(\mathfrak{a})}(-m,q) \geq \Gamma_{\mathbb{D}(\mathfrak{a})}(m,q)$, $\Lambda_{\mathbb{D}(\mathfrak{a})}(-m,q) \leq \Lambda_{\mathbb{D}(\mathfrak{a})}(m,q)$ and $\Omega_{\mathbb{D}(\mathfrak{a})}(-m,q) \leq \Omega_{\mathbb{D}(\mathfrak{a})}(m,q)$.
3. $\Gamma_{\mathbb{D}(\mathfrak{a})}(m.n,q) \geq \min\{\Gamma_{\mathbb{D}(\mathfrak{a})}(m,q), \Gamma_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, $\Lambda_{\mathbb{D}(\mathfrak{a})}(m.n,q) \leq \max\{\Lambda_{\mathbb{D}(\mathfrak{a})}(m,q), \Lambda_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, and $\Omega_{\mathbb{D}(\mathfrak{a})}(m.n,q) \leq \max\{\Omega_{\mathbb{D}(\mathfrak{a})}(m,q), \Omega_{\mathbb{D}(\mathfrak{a})}(n,q)\}$,
4. $\Gamma_{\mathbb{D}(\mathfrak{a})}(m^{-1},q) \geq \Gamma_{\mathbb{D}(\mathfrak{a})}(m,q)$, $\Lambda_{\mathbb{D}(\mathfrak{a})}(m^{-1},q) \leq \Lambda_{\mathbb{D}(\mathfrak{a})}(m,q)$ and $\Omega_{\mathbb{D}(\mathfrak{a})}(m^{-1},q) \leq \Omega_{\mathbb{D}(\mathfrak{a})}(m,q)$.

Theorem 2.9. [29] *Let's consider a Q-NSS denoted as $(\mathbb{D}, \mathbb{A})$ defined over a field $(\mathbb{F}, +, \cdot)$. In order for $(\mathbb{D}, \mathbb{A})$ to be classified as a Q-NSF, it must satisfy the following conditions for all $m, n \in \mathbb{F}$, $q \in Q$, and $\mathfrak{a} \in \mathbb{A}$:*

1. $\Gamma_{\mathbb{D}(\mathfrak{a})}(m-n,q) \geq \min\{\Gamma_{\mathbb{D}(\mathfrak{a})}(m,q), \Gamma_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, $\Lambda_{\mathbb{D}(\mathfrak{a})}(m-n,q) \leq \max\{\Lambda_{\mathbb{D}(\mathfrak{a})}(m,q), \Lambda_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, *and* $\Omega_{\mathbb{D}(\mathfrak{a})}(m-n,q) \leq \max\{\Omega_{\mathbb{D}(\mathfrak{a})}(m,q), \Omega_{\mathbb{D}(\mathfrak{a})}(n,q)\}$.
2. $\Gamma_{\mathbb{D}(\mathfrak{a})}(m.n^{-1},q) \geq \min\{\Gamma_{\mathbb{D}(\mathfrak{a})}(m,q), \Gamma_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, $\Lambda_{\mathbb{D}(\mathfrak{a})}(m.n^{-1},q) \leq \max\{\Lambda_{\mathbb{D}(\mathfrak{a})}(m,q), \Lambda_{\mathbb{D}(\mathfrak{a})}(n,q)\}$, *and* $\Omega_{\mathbb{D}(\mathfrak{a})}(m.n^{-1},q) \leq \max\{\Omega_{\mathbb{D}(\mathfrak{a})}(m,q), \Omega_{\mathbb{D}(\mathfrak{a})}(n,q)\}$,

Definition 2.10. [27] Let $(\mathbb{S}, \mathbb{A})$ be a homogeneous Q-complex intuitionistic fuzzy soft set(Q-CIFSS) over a field $(\mathbb{F}, +, .)$. Then $(\mathbb{S}, \mathbb{A})$ is said to be Q-CIFS field over $(\mathbb{F}, +, .)$, if for all $\mathfrak{a} \in \mathbb{A}$, $q \in Q$ and all $m, n \in \mathbb{F}$, the following conditions are fulfilled:

1. $\mathcal{T}_{\mathbb{S}(\mathfrak{a})}(-m,q) \geq \mathcal{T}_{\mathbb{S}(\mathfrak{a})}(m,q)$,
2. $\mathcal{F}_{\mathbb{S}(\mathfrak{a})}(-m,q) \leq \mathcal{F}_{\mathbb{S}(\mathfrak{a})}(m,q)$,
3. $\mathcal{T}_{\mathbb{S}(\mathfrak{a})}(m.n,q) \geq \min\{\mathcal{T}_{\mathbb{S}(\mathfrak{a})}(s,q), \mathcal{T}_{\mathbb{S}(\mathfrak{a})}(t,q)\}$,
4. $\mathcal{F}_{\mathbb{S}(\mathfrak{a})}(m.n,q) \leq \max\{\mathcal{F}_{\mathbb{S}(\mathfrak{a})}(s,q), \mathcal{F}_{\mathbb{S}(\mathfrak{a})}(t,q)\}$,
5. $\mathcal{T}_{\mathbb{S}(\mathfrak{a})}(m+n,q) \geq \min\{\mathcal{T}_{\mathbb{S}(\mathfrak{a})}(s,q), \mathcal{T}_{\mathbb{S}(\mathfrak{a})}(t,q)\}$,
6. $\mathcal{F}_{\mathbb{S}(\mathfrak{a})}(m+n,q) \leq \max\{\mathcal{F}_{\mathbb{S}(\mathfrak{a})}(s,q), \mathcal{F}_{\mathbb{S}(\mathfrak{a})}(t,q)\}$.
7. $\mathcal{T}_{\mathbb{S}(\mathfrak{a})}(m^{-1},q) \geq \mathcal{T}_{\mathbb{S}(\mathfrak{a})}(s,q)$,
8. $\mathcal{F}_{\mathbb{S}(\mathfrak{a})}(m^{-1},q) \leq \mathcal{F}_{\mathbb{S}(\mathfrak{a})}(s,q)$,

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3 Q-Complex Neutrosophic Soft Field

In this section, we introduce and rigorously define the powerful concept of Q-CNSF. We delve into an in-depth exploration of its various properties, thoroughly examining and analyzing their intricate interconnections.

Definition 3.1. Suppose that $(\mathbb{H}, \mathbb{A})$ is a homogeneous Q-CNSS over a field $(\mathbb{F}, +, .)$. It is stated that $(\mathbb{H}, \mathbb{A})$ is categorized as a Q-CNSF in $(\mathbb{F}, +, .)$ if, for every $\mathfrak{a} \in \mathbb{A}$, $q \in Q$ and all $s, t \in \mathbb{F}$, the following conditions are fulfilled:

1. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-s, q) \geq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-s, q) \leq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-s, q) \leq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
2. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.t, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,
3. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s + t, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s + t, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s + t, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$.
4. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) \geq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) \leq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) \leq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$,

Proposition 3.2. *Consider $(\mathbb{H}, \mathbb{A})$ is a Q-CNSF in $(\mathbb{F}, +, .)$. In this context, we establish the following properties for the additive identity $0_{\mathbb{F}}$ and the multiplicative identity $1_{\mathbb{F}}$. For all $\mathfrak{a} \in \mathbb{A}$, $q \in Q$ and all $s \in \mathbb{F}$, we have:*

1. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) \geq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) \leq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$ *and* $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) \leq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$.
2. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q) \geq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q) \leq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$ *and* $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q) \leq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$.
3. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) \geq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) \leq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)$ *and* $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) \leq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)$.

Proof. For all $\mathfrak{a} \in \mathbb{A}$, $q \in Q$ and all $s \in \mathbb{F}$, we have:

1. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s - s, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
   $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s - s, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
   $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s - s, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
2. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.s^{-1}, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q)\} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
   $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.s^{-1}, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
   $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.s^{-1}, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
3. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}} - 1_{\mathbb{F}}, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)\} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)$,
   $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}} - 1_{\mathbb{F}}, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)\} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)$,
   $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}} - 1_{\mathbb{F}}, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)\} = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q)$,

□

Definition 3.3. Consider two homogeneous Q-CNSSs denoted as $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$. It is asserted that $(\mathbb{H}, \mathbb{A})$ is designated as a Q-CNS subfield of $(\mathbb{G}, \mathbb{B})$ only when the following conditions are met with absolute certainty:

1 $(\mathbb{H}, \mathbb{A}) \subseteq (\mathbb{G}, \mathbb{B})$, where $\subseteq$ is a Q-CNS subset.

2 Both $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are Q-CNSFs.

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Definition 3.4. Suppose that $(\mathbb{H}, \mathbb{A})$ is a homogeneous Q-CNSS over a field $(\mathbb{F}, +, .)$. It is stated that $(\mathbb{H}, \mathbb{A})$ is categorized as an anti- Q-CNSF in $(\mathbb{F}, +, .)$ if, for every $\mathfrak{a} \in \mathbb{A}$, $q \in Q$ and all $s, t \in \mathbb{F}$, the following conditions are fulfilled:

1. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-s, q) \leq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-s, q) \geq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-s, q) \geq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$,
2. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t, q) \leq \max\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t, q) \geq \min\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.t, q) \geq \min\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,
3. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s + t, q) \leq \max\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s + t, q) \geq \min\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s + t, q) \geq \min\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$.
4. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) \leq \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$, $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) \geq \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$, and $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) \geq \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$,

Theorem 3.5. *Consider a homogeneous Q-CNSS denoted as $(\mathbb{H}, \mathbb{A})$ over a field $(\mathbb{F}, +, .)$. It is asserted that $(\mathbb{H}, \mathbb{A})$ can be classified as a Q-CNSF in $(\mathbb{F}, +, .)$, if and only if, for every $\mathfrak{a} \in \mathbb{A}$, $q \in Q$, and s , $t \in \mathbb{F}$, the following conditions are satisfied:*

1. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s - t, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,
2. $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s - t, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,
3. $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s - t, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,
4. $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,
5. $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,
6. $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$.

Proof. $\Rightarrow$ Consider that $(\mathbb{H}, \mathbb{A})$ is a Q-CNSF in $(\mathbb{F}, +, .)$. Then,
$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s - t, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s + (-t), q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-t, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s - t, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s + (-t), q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-t, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s - t, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s + (-t), q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-t, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$.

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t^{-1}, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t^{-1}, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t^{-1}, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$.

$\Leftarrow$ In the converse scenario, where conditions 1-6 are met, our objective is to establish that for every element $\mathfrak{a} \in \mathbb{A}$, the pair $(\mathbb{H}, \mathbb{A})$ fulfills the criteria to be classified as a Q- complex neutrosophic subfield.

Consequently, $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-s, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}} - s, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-s, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}} - s, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-s, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}} - s, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(0_{\mathbb{F}}, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)$.

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.(t^{-1})^{-1}, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t^{-1}, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.(t^{-1})^{-1}, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t^{-1}, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

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$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.t, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.(t^{-1})^{-1}, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t^{-1}, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$.

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s + t, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s - (-t), q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(-t, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s + t, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s - (-t), q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(-t, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s + t, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s - (-t), q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(-t, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t, q)\}$.

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}.s^{-1}, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\} \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}.s^{-1}, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\} \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$,

$\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s^{-1}, q) = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}.s^{-1}, q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(1_{\mathbb{F}}, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\} \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\} = \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s, q)\}$.

This completes the proof.

□

Theorem 3.6. *Let* $(\mathbb{F}, +, .)$ *be a field, and let* $(\mathbb{H}, \mathbb{A}) = \{< \mathfrak{a}; \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(r, q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(r, q) >: \mathfrak{a} \in \mathbb{A}, r \in W, q \in Q\}$ *be homogeneous Q-CNSS over* $(\mathbb{F}, +, .)$. *Suppose* $(\mathbb{H}, \mathbb{A})$ *generates the two Q-NSSs* $(\mathfrak{h}, \mathbb{A}) = \{< a; \Gamma_{\mathfrak{h}(\mathfrak{a})}(r, q), \Lambda_{\mathfrak{h}(\mathfrak{a})}(r, q), \Omega_{\mathfrak{h}(\mathfrak{a})}(r, q) >: a \in \mathbb{A}, r \in W, q \in Q\}$ *and* $(\mathfrak{K}, \mathbb{A}) = \{< a; \mu_{\mathfrak{K}(\mathfrak{a})}(r, q), \nu_{\mathfrak{K}(\mathfrak{a})}(r, q), \omega_{\mathfrak{K}(\mathfrak{a})}(r, q) >: a \in \mathbb{A}, r \in W, q \in Q\}$. *Then,* $(\mathbb{H}, \mathbb{A})$ *is a Q-CNS subfield of* $\mathbb{F}$ *if and only if both* $(\mathfrak{h}, \mathbb{A})$ *and* $(\mathfrak{K}, \mathbb{A})$ *are Q-NS subfields.*

Proof. $\Rightarrow$ In order to establish the validity of the first direction of this theorem, it is essential to demonstrate adherence to the conditions specified in Theorem (2.9).

Consider that $(\mathbb{H}, \mathbb{A})$ is a Q-CNS subfield of $\mathbb{F}$, then for all $a \in \mathbb{A}, s, t \in \mathbb{F}, q \in Q$, we have,

$\Gamma_{\mathbb{H}(\mathfrak{a})}(s-t, q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s-t,q)} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s-t, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s, q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(t, q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s, q), \Gamma_{\mathbb{H}(\mathfrak{a})}(t, q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(s,q), \mu_{\mathbb{H}(\mathfrak{a})}(t,q)\}}$. Thus,
$\Gamma_{\mathbb{H}(\mathfrak{a})}(s - t, q) \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s, q), \Gamma_{\mathbb{H}(\mathfrak{a})}(t, q)\}$ and $\mu_{\mathbb{H}(\mathfrak{a})}(s - t, q) \geq \min\{\mu_{\mathbb{H}(\mathfrak{a})}(s, q), \mu_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

Likewise, we can derive,

$\Lambda_{\mathbb{H}(\mathfrak{a})}(s-t, q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s-t,q)} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s-t, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s, q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(t, q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s, q), \Lambda_{\mathbb{H}(\mathfrak{a})}(t, q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(s,q), \nu_{\mathbb{H}(\mathfrak{a})}(t,q)\}}$. Thus,
$\Lambda_{\mathbb{H}(\mathfrak{a})}(s - t, q) \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s, q), \Lambda_{\mathbb{H}(\mathfrak{a})}(t, q)\}$ and $\nu_{\mathbb{H}(\mathfrak{a})}(s - t, q) \leq \max\{\nu_{\mathbb{H}(\mathfrak{a})}(s, q), \nu_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

By employing a similar approach, we can obtain $\Omega_{\mathbb{H}(\mathfrak{a})}(s-t, q) \leq \max\{\Omega_{\mathbb{H}(\mathfrak{a})}(s, q), \Omega_{\mathbb{H}(\mathfrak{a})}(t, q)\}$ and $\omega_{\mathbb{H}(\mathfrak{a})}(s-t, q) \leq \max\{\omega_{\mathbb{H}(\mathfrak{a})}(s, q), \omega_{\mathbb{H}(\mathfrak{a})}(t, q)\}$. Therefore, condition 1 is satisfied.

Next, $\Gamma_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q)} = \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t, q)\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s, q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(t, q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s, q), \Gamma_{\mathbb{H}(\mathfrak{a})}(t, q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(s,q), \mu_{\mathbb{H}(\mathfrak{a})}(t,q)\}}$. Thus,
$\Gamma_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s, q), \Gamma_{\mathbb{H}(\mathfrak{a})}(t, q)\}$ and $\mu_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \geq \min\{\mu_{\mathbb{H}(\mathfrak{a})}(s, q), \mu_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

Similarly, we can obtain

$\Lambda_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q)} = \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s, q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t, q)\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s, q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(t, q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s, q), \Lambda_{\mathbb{H}(\mathfrak{a})}(t, q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(s,q), \nu_{\mathbb{H}(\mathfrak{a})}(t,q)\}}$. Thus,
$\Lambda_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s, q), \Lambda_{\mathbb{H}(\mathfrak{a})}(t, q)\}$ and $\nu_{\mathbb{H}(\mathfrak{a})}(s.t^{-1}, q) \leq \max\{\nu_{\mathbb{H}(\mathfrak{a})}(s, q), \nu_{\mathbb{H}(\mathfrak{a})}(t, q)\}$, (Since $(\mathbb{H}, \mathbb{A})$ is homogeneous).

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In the same way we get,

$\Omega_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q) \leq \max\{\Omega_{\mathbb{H}(\mathfrak{a})}(s,q), \Omega_{\mathbb{H}(\mathfrak{a})}(t,q)\}$ and $\omega_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q) \leq \max\{\omega_{\mathbb{H}(\mathfrak{a})}(s,q), \omega_{\mathbb{H}(\mathfrak{a})}(t,q)\}$.

Therefore, condition 2 is satisfied. Which implies that $(\mathfrak{h}, \mathbb{A})$ and $(\mathfrak{K}, \mathbb{A})$ are Q-CNS fields.

In order to prove the second direction of this theorem, it should satisfy previously defined six conditions listed in Theorem (3.5).

$\Leftarrow$ Suppose that $(\mathfrak{h}, \mathbb{A})$ and $(\mathfrak{K}, \mathbb{A})$ are two Q-NS subfields. To prove that $(\mathbb{H}, \mathbb{A})$ is a Q-CNS subfield, we have to show that:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s-t,q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(s-t,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s-t,q)} \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(t,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(s,q),\mu_{\mathbb{H}(\mathfrak{a})}(t,q)\}} = \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(t,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s-t,q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$.

In a similar manner : $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s-t,q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(s-t,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s-t,q)} \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(t,q)\}.$
$e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(s,q),\nu_{\mathbb{H}(\mathfrak{a})}(t,q)\}} = \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(t,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$.
($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s-t,q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$.

In the same manner we show that $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s-t,q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$. Therefore, conditions 1, 2, and 3 are satisfied.

To verify the validity of the conditions 4, 5, and 6, we have to show that:

$\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q) = \Gamma_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q)} \geq \min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q), \Gamma_{\mathbb{H}(\mathfrak{a})}(t,q)\}.e^{i\min\{\mu_{\mathbb{H}(\mathfrak{a})}(s,q),\mu_{\mathbb{H}(\mathfrak{a})}(t,q)\}} =$

$\min\{\Gamma_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Gamma_{\mathbb{H}(\mathfrak{a})}(t,q).e^{i\mu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q) \geq \min\{\mathcal{T}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{T}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$.

Similarly, we can obtain

$\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q) = \Lambda_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q)} \leq \max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q), \Lambda_{\mathbb{H}(\mathfrak{a})}(t,q)\}.e^{i\max\{\nu_{\mathbb{H}(\mathfrak{a})}(s,q),\nu_{\mathbb{H}(\mathfrak{a})}(t,q)\}} =$

$\max\{\Lambda_{\mathbb{H}(\mathfrak{a})}(s,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(s,q)}, \Lambda_{\mathbb{H}(\mathfrak{a})}(t,q).e^{i\nu_{\mathbb{H}(\mathfrak{a})}(t,q)}\} = \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$. ($(H, \mathbb{A})$ is homogeneous).

Thus, we obtain $\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q) \leq \max\{\mathcal{I}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{I}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$.

Using the same steps, we can show that $\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s.t^{-1},q) \leq \max\{\mathcal{F}_{\mathbb{H}(\mathfrak{a})}(s,q), \mathcal{F}_{\mathbb{H}(\mathfrak{a})}(t,q)\}$.

Thus, the six conditions listed in Theorem (3.5) have been verified. Which proves that $(\mathbb{H}, \mathbb{A})$ is Q-CNS subfield. □

Theorem 3.7. *Consider a field $(\mathbb{F}, +, .)$ and let $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ be two Q-CNSSs in $\mathbb{F}$, where $(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$. If both $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are Q-CNSFs in $\mathbb{F}$, then their intersection $(\mathbb{H}, \mathbb{A}) \cap (\mathbb{G}, \mathbb{B})$ is also a Q-CNSF in $\mathbb{F}$.*

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Proof. Suppose $(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are two Q-CNSFs. Let's begin by establishing the validity of the first three conditions of Theorem (3.5).

First, We examine the complex-valued truth membership function of the intersection.

For all $e \in \mathbb{A} \cap \mathbb{B}$, $q \in Q$, and $s, t \in \mathbb{F}$,

$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q) = \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q)}$
$= \min\{\Gamma_{\mathbb{H}(e)}(s-t,q), \Gamma_{\mathbb{G}(e)}(s-t,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(s-t,q),\mu_{\mathbb{G}(e)}(s-t,q)\}}$
$\geq \min\{\min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{H}(e)}(t,q)\}, \min\{\Gamma_{\mathbb{G}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{H}(e)}(t,q)\},\min\{\mu_{\mathbb{G}(e)}(s,q),\mu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}, \min\{\Gamma_{\mathbb{H}(e)}(t,q), \Gamma_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\},\min\{\mu_{\mathbb{H}(e)}(t,q),\mu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}.$
$e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}}, \min\{\Gamma_{\mathbb{H}(e)}(t,q), \Gamma_{\mathbb{G}(e)}(t,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(t,q),\mu_{\mathbb{G}(e)}(t,q)\}}\}$ ($(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$)
$= \min\{\Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}, \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(t,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)}\}$
$= \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$. Thus,
$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q) \geq \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$.

Second: We will verify whether the condition for the indeterminacy membership function of the intersection is met.

$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q) = \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q)}$
$= \max\{\Lambda_{\mathbb{H}(e)}(s-t,q), \Lambda_{\mathbb{G}(e)}(s-t,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(s-t,q),\nu_{\mathbb{G}(e)}(s-t,q)\}}$
$\leq \max\{\max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{H}(e)}(t,q)\}, \max\{\Lambda_{\mathbb{G}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{H}(e)}(t,q)\},\max\{\nu_{\mathbb{G}(e)}(s,q),\nu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}, \max\{\Lambda_{\mathbb{H}(e)}(t,q), \Lambda_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\},\max\{\nu_{\mathbb{H}(e)}(t,q),\nu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}.$
$e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}}, \max\{\Lambda_{\mathbb{H}(e)}(t,q), \Lambda_{\mathbb{G}(e)}(t,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(t,q),\nu_{\mathbb{G}(e)}(t,q)\}}\}$ ($(\mathbb{G}, \mathbb{A})$ is homogeneous with $(\mathbb{H}, \mathbb{B})$)
$= \max\{\Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}, \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(t,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)}\}$
$= \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(r,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$. Thus,
$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q) \leq \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$.

Third: Using the same steps as in the case of indeterminacy membership function, we obtain:

$\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s-t,q) \leq \max\{\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$.

Conditions 4-6 of Theorem (3.5) can be examined as follows.

For the complex-valued truth membership function of the intersection, we obtain

$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q) = \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q)}$
$= \min\{\Gamma_{\mathbb{H}(e)}(s.t^{-1},q), \Gamma_{\mathbb{G}(e)}(s.t^{-1},q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(s.t^{-1},q),\mu_{\mathbb{G}(e)}(s.t^{-1},q)\}}$
$\geq \min\{\min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{H}(e)}(t,q)\}, \min\{\Gamma_{\mathbb{G}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{H}(e)}(t,q)\},\min\{\mu_{\mathbb{G}(e)}(s,q),\mu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}, \min\{\Gamma_{\mathbb{H}(e)}(t,q), \Gamma_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\min\{\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\},\min\{\mu_{\mathbb{H}(e)}(t,q),\mu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \min\{\min\{\Gamma_{\mathbb{H}(e)}(s,q), \Gamma_{\mathbb{G}(e)}(s,q)\}.$
$e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(s,q),\mu_{\mathbb{G}(e)}(s,q)\}}, \min\{\Gamma_{\mathbb{H}(e)}(t,q), \Gamma_{\mathbb{G}(e)}(t,q)\}.e^{i2\pi\min\{\mu_{\mathbb{H}(e)}(t,q),\mu_{\mathbb{G}(e)}(t,q)\}}\}$ ($(\mathbb{G}, \mathbb{A})$ is homogeneous with $(\mathbb{H}, \mathbb{B})$)
$= \min\{\Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}, \Gamma_{\mathbb{H}\cap\mathbb{G}(e)}(t,q).e^{i2\pi\mu_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)}\}$
$= \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$. Thus,
$\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q) \geq \min\{\mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{T}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$.

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For the complex-valued indeterminacy membership function of the intersection, we obtain:

$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q) = \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q)}$
$= \max\{\Lambda_{\mathbb{H}(e)}(s.t^{-1},q), \Lambda_{\mathbb{G}(e)}(s.t^{-1},q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(s.t^{-1},q),\nu_{\mathbb{G}(e)}(s.t^{-1},q)\}}$
$\leq \max\{\max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{H}(e)}(t,q)\}, \max\{\Lambda_{\mathbb{G}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{H}(e)}(t,q)\},\max\{\nu_{\mathbb{G}(e)}(s,q),\nu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}, \max\{\Lambda_{\mathbb{H}(e)}(t,q), \Lambda_{\mathbb{G}(e)}(t,q)\}\}.$
$e^{i2\pi\max\{\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\},\max\{\nu_{\mathbb{H}(e)}(t,q),\nu_{\mathbb{G}(e)}(t,q)\}\}}$
$= \max\{\max\{\Lambda_{\mathbb{H}(e)}(s,q), \Lambda_{\mathbb{G}(e)}(s,q)\}.$
$e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(s,q),\nu_{\mathbb{G}(e)}(s,q)\}}, \max\{\Lambda_{\mathbb{H}(e)}(t,q), \Lambda_{\mathbb{G}(e)}(t,q)\}.e^{i2\pi\max\{\nu_{\mathbb{H}(e)}(t,q),\nu_{\mathbb{G}(e)}(t,q)\}}\}$ ($(\mathbb{G},\mathbb{A})$ is homogeneous with $(\mathbb{H},\mathbb{B})$)
$= \max\{\Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(s,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(s,q)}, \Lambda_{\mathbb{H}\cap\mathbb{G}(e)}(t,q).e^{i2\pi\nu_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)}\}$
$= \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$. Thus,
$\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q) \leq \max\{\mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{I}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$.

In the similar way, we can show that $\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s.t^{-1},q) \leq \max\{\mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(s,q), \mathcal{F}_{\mathbb{H}\cap\mathbb{G}(e)}(t,q)\}$.

Thus, the six conditions listed in Theorem (3.5) have been verified. Which proves that the intersection $(\mathbb{H},\mathbb{A})$ $\cap$ $(\mathbb{G},\mathbb{B})$ is a Q-CNSF.

□

4 Cartesian product of Q- complex neutrosophic soft fields

Within this section, we establish the definition of the Cartesian product of Q-CNSFs and subsequently demonstrate its status as a Q-CNSF.

Definition 4.1. Consider two Q-CNSFs, $(\mathbb{H},\mathbb{A})$ and $(\mathbb{G},\mathbb{B})$, defined over $(\mathbb{F}_1,+,\cdot)$ and $(\mathbb{F}_2,+,\cdot)$, respectively. Let $(\mathbb{H},\mathbb{A})$ be homogeneous with $(\mathbb{G},\mathbb{B})$. We define their Cartesian product, denoted as $(\mathbb{M},\mathbb{A}\times\mathbb{B}) = (\mathbb{H},\mathbb{A})\times(\mathbb{G},\mathbb{B})$, where $\mathbb{M}(\alpha,\beta) = \mathbb{H}(\alpha)\times\mathbb{G}(\beta)$ for $(\alpha,\beta)\in\mathbb{A}\times\mathbb{B}$.

In an analytical representation, for $s\in\mathbb{F}_1$, $t\in\mathbb{F}_2$, and $q\in Q$, we have:

$\mathbb{M}(\alpha,\beta) = \{\langle((s,t),q), \mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s,t),q), \mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s,t),q), \mathcal{F}_{\mathbb{M}(\alpha,\beta)}((s,t),q)\rangle\}$, where:

$\mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s,t),q) = \min\{\mathcal{T}_{\mathbb{M}(\alpha)}(s,q), \mathcal{T}_{\mathbb{M}(\beta)}(t,q)\}$,

$\mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s,t),q) = \max\{\mathcal{I}_{\mathbb{M}(\alpha)}(s,q), \mathcal{I}_{\mathbb{M}(\beta)}(t,q)\}$,

$\mathcal{F}_{\mathbb{M}(\alpha,\beta)}((s,t),q) = \max\{\mathcal{F}_{\mathbb{M}(\alpha)}(s,q), \mathcal{F}_{\mathbb{M}(\beta)}(t,q)\}$.

Theorem 4.2. *Let's consider two Q-CNSFs,* $(\mathbb{H},\mathbb{A})$ *and* $(\mathbb{G},\mathbb{B})$*, which are defined over* $(\mathbb{F}_1,+,\cdot)$ *and* $(\mathbb{F}_2,+,\cdot)$*, respectively. Let* $(\mathbb{H},\mathbb{A})$ *be homogeneous with* $(\mathbb{G},\mathbb{B})$*. Then, the Cartesian product of these two Q-CNSFs, i.e.,* $(\mathbb{H},\mathbb{A})\times(\mathbb{G},\mathbb{B})$ *is also a Q-CNSF defined over* $\mathbb{F}_1\times\mathbb{F}_2$.

Proof. Let $(\mathbb{M},\mathbb{A}\times\mathbb{B}) = (\mathbb{H},\mathbb{A})\times(\mathbb{G},\mathbb{B})$, where $\mathbb{M}(\alpha,\beta) = \mathbb{H}(\alpha)\times\mathbb{G}(\beta)$ for $(\alpha,\beta)\in\mathbb{A}\times\mathbb{B}$. Then for $((s_1,t_1),q), ((s_2,t_2),q)\in(\mathbb{F}_1\times\mathbb{F}_2)\times Q$, we have for the complex-valued truth membership function:

$\mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s_1,t_1)-(s_2,t_2),q)$
$= \mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s_1-s_2,t_1-t_2),q)$

$= \Gamma_{\mathbb{H}(\alpha,\beta)}((s_1-s_2,t_1-t_2),q).e^{i\mu_{\mathbb{H}(\alpha,\beta)}((s_1-s_2,t_1-t_2),q)}$

$= \min\{\Gamma_{\mathbb{H}(\alpha)}((s_1-s_2),q), \Gamma_{\mathbb{G}(\beta)}((t_1-t_2),q)\}.e^{i\min\{\mu_{\mathbb{H}(\alpha)}((s_1-s_2),q),\mu_{\mathbb{G}(\beta)}((t_1-t_2),q)\}}$
$= \min\{\Gamma_{\mathbb{H}(\alpha)}((s_1-s_2),q).e^{i\mu_{\mathbb{H}(\alpha)}((s_1-s_2),q)}, \Gamma_{\mathbb{G}(\beta)}((t_1-t_2),q).e^{i\mu_{\mathbb{G}(\beta)}((t_1-t_2),q)}\}$ ($(\mathbb{H},\mathbb{A})$ is homogeneous with $(\mathbb{G},\mathbb{B})$)

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$= \min\{\mathcal{T}_{\mathbb{H}(\alpha)}((s_1 - s_2), q), \mathcal{T}_{\mathbb{G}(\beta)}((t_1 - t_2), q)\}$
$\geq \min\{\min\{\mathcal{T}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{T}_{\mathbb{H}(\alpha)}(s_2, q)\}, \min\{\mathcal{T}_{\mathbb{G}(\beta)}(t_1, q), \mathcal{T}_{\mathbb{G}(\beta)}(t_2, q)\}\}$. ($(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are Q-CNSFs)

$= \min\{\min\{\mathcal{T}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{T}_{\mathbb{G}(\beta)}(t_1, q)\}, \min\{\mathcal{T}_{\mathbb{H}(\alpha)}(s_2, q), \mathcal{T}_{\mathbb{G}(\beta)}(t_2, q)\}\}$.
$= \min\{\mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}$. Thus,

$$\mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1) - (s_2, t_2), q) \geq \min\{\mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}.$$

To validate the condition of complex-valued indeterminacy membership function, we employ a similar approach as that used for the complex-valued truth membership function.

$\mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1) - (s_2, t_2), q)$
$= \mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s_1 - s_2, t_1 - t_2), q)$
$= \Lambda_{\mathbb{H}(\alpha,\beta)}((s_1 - s_2, t_1 - t_2), q).e^{i\nu_{\mathbb{H}(\alpha,\beta)}((s_1 - s_2, t_1 - t_2), q)}$
$= \max\{\Lambda_{\mathbb{H}(\alpha)}((s_1 - s_2), q), \Lambda_{\mathbb{G}(\beta)}((t_1 - t_2), q)\}.e^{i\max\{\nu_{\mathbb{H}(\alpha)}((s_1 - s_2), q), \nu_{\mathbb{G}(\beta)}((t_1 - t_2), q)\}}$
$= \max\{\Lambda_{\mathbb{H}(\alpha)}((s_1 - s_2), q).e^{i\nu_{\mathbb{H}(\alpha)}((s_1 - s_2), q)}, \Lambda_{\mathbb{G}(\beta)}((t_1 - t_2), q).e^{i\nu_{\mathbb{G}(\beta)}((t_1 - t_2), q)}\}$
($(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$)
$= \max\{\mathcal{I}_{\mathbb{H}(\alpha)}((s_1 - s_2), q), \mathcal{I}_{\mathbb{G}(\beta)}((t_1 - t_2), q)\}$
$\leq \max\{\max\{\mathcal{I}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{I}_{\mathbb{H}(\alpha)}(s_2, q)\}, \max\{\mathcal{I}_{\mathbb{G}(\beta)}(t_1, q), \mathcal{I}_{\mathbb{G}(\beta)}(t_2, q)\}\}$. ($(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are Q-CNSFs)
$= \max\{\max\{\mathcal{I}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{I}_{\mathbb{G}(\beta)}(t_1, q)\}, \max\{\mathcal{I}_{\mathbb{H}(\alpha)}(s_2, q), \mathcal{I}_{\mathbb{G}(\beta)}(t_2, q)\}\}$.
$= \max\{\mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}$. Thus,
$\mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1) - (s_2, t_2), q) \leq \max\{\mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}$.

Similarly, we can demonstrate that: $\mathcal{F}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1) - (s_2, t_2), q) \leq \max\{\mathcal{F}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{F}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}$.

Next, we will proceed to prove the remaining conditions as follows:

$\mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1).(s_2, t_2)^{-1}, q)$
$= \mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s_1.s_2^{-1}, t_1.t_2^{-1}), q)$
$= \Gamma_{\mathbb{H}(\alpha,\beta)}((s_1.s_2^{-1}, t_1.t_2^{-1}), q).e^{i\mu_{\mathbb{H}(\alpha,\beta)}((s_1.s_2^{-1}, t_1.t_2^{-1}), q)}$
$= \min\{\Gamma_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q), \Gamma_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)\}.e^{i\min\{\mu_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q), \mu_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)\}}$
$= \min\{\Gamma_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q).e^{i\mu_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q)}, \Gamma_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q).e^{i\mu_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)}\}$ ($(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$)
$= \min\{\mathcal{T}_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q), \mathcal{T}_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)\}$
$\geq \min\{\min\{\mathcal{T}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{T}_{\mathbb{H}(\alpha)}(s_2, q)\}, \min\{\mathcal{T}_{\mathbb{G}(\beta)}(t_1, q), \mathcal{T}_{\mathbb{G}(\beta)}(t_2, q)\}\}$. ($(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are Q-CNSFs)
$= \min\{\min\{\mathcal{T}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{T}_{\mathbb{G}(\beta)}(t_1, q)\}, \min\{\mathcal{T}_{\mathbb{H}(\alpha)}(s_2, q), \mathcal{T}_{\mathbb{G}(\beta)}(t_2, q)\}\}$.
$= \min\{\mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}$. Thus,

$$\mathcal{T}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1).(s_2, t_2)^{-1}, q) \geq \min\{\mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{T}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}.$$

To verify the condition of the complex-valued indeterminacy membership function, we follow a similar approach as we did for the complex-valued truth membership function.

$\mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1).(s_2, t_2)^{-1}, q)$
$= \mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s_1.s_2^{-1}, t_1.t_2^{-1}), q)$
$= \Lambda_{\mathbb{H}(\alpha,\beta)}((s_1.s_2^{-1}, t_1.t_2^{-1}), q).e^{i\nu_{\mathbb{H}(\alpha,\beta)}((s_1.s_2^{-1}, t_1.t_2^{-1}), q)}$
$= \max\{\Lambda_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q), \Lambda_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)\}.e^{i\max\{\nu_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q), \nu_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)\}}$
$= \max\{\Lambda_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q).e^{i\nu_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q)}, \Lambda_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q).e^{i\nu_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)}\}$ ($(\mathbb{H}, \mathbb{A})$ is homogeneous with $(\mathbb{G}, \mathbb{B})$)
$= \max\{\mathcal{I}_{\mathbb{H}(\alpha)}((s_1.s_2^{-1}), q), \mathcal{I}_{\mathbb{G}(\beta)}((t_1.t_2^{-1}), q)\}$
$\leq \max\{\max\{\mathcal{I}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{I}_{\mathbb{H}(\alpha)}(s_2, q)\}, \max\{\mathcal{I}_{\mathbb{G}(\beta)}(t_1, q), \mathcal{I}_{\mathbb{G}(\beta)}(t_2, q)\}\}$. ($(\mathbb{H}, \mathbb{A})$ and $(\mathbb{G}, \mathbb{B})$ are Q-CNSFs)
$= \max\{\max\{\mathcal{I}_{\mathbb{H}(\alpha)}(s_1, q), \mathcal{I}_{\mathbb{G}(\beta)}(t_1, q)\}, \min\{\mathcal{I}_{\mathbb{H}(\alpha)}(s_2, q), \mathcal{I}_{\mathbb{G}(\beta)}(t_2, q)\}\}$.
$= \max\{\mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}$. Thus,

$$\mathcal{I}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1).(s_2, t_2)^{-1}, q) \leq \max\{\mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{I}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}.$$

Likewise, we can establish that: $\mathcal{F}_{\mathbb{M}(\alpha,\beta)}((s_1, t_1).(s_2, t_2)^{-1}, q) \leq \max\{\mathcal{F}_{\mathbb{M}(\alpha,\beta)}(s_1, t_1), \mathcal{F}_{\mathbb{M}(\alpha,\beta)}(s_2, t_2)\}$.

Thus, the six conditions listed in Theorem (3.5) have been verified. Which proves that $(\mathbb{H}, \mathbb{A}) \times (\mathbb{G}, \mathbb{B})$ is a Q-CNSF.

□

5 Conclusion

This paper presented the notion of Q-CNSF. We defined and developed the algebraic structures pertaining to fields for the Q-complex neutrosophic soft model. We explored the relationship between neutrosophic fields in both real space and complex space, shedding light on their interplay. Additionally, we examined the cartesian product of Q-complex neutrosophic soft fields. The proposed notion of Q-CNSF presents a novel and promising idea within the realm of algebraic structure theory. It has the potential to be extensively utilized in the future for solving a wide range of algebraic problems, making it a significant contribution to the field.

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