Possibility neutrosophic soft sets with applications in decision making and similarity measure

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Abstract

In this paper, concept of possibility neutrosophic soft set and its operations are defined, and their properties are studied. An application of this theory in decision making is investigated. Also a similarity measure of two possibility neutrosophic soft sets is introduced and discussed. Finally an application of this similarity measure in personal selection for a firm.

Keywords: Soft set, neutrosophic soft set, possibility neutrosophic soft set, similarity measure, decision making

1. Introduction

In this physical world problems in engineering, medical sciences, economics and social sciences the information involved are uncertainty in nature. To cope with these problems, researchers proposed some theories such as the theory of fuzzy set [35], the theory of intuitionistic fuzzy set [4], the theory of rough set [26], the theory of vague set [18]. In 1999, Molodtsov [20] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties as different from these theories. A wide range of applications of soft sets have been developed in many different fields, including the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory and measurement theory. Maji et al. [21, 22] applied soft set theory to decision making problem and in 2003, they introduced some new operations of soft sets. After Maji's work,

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works on soft set theory and its applications have been progressing rapidly. see [1, 2, 9, 10, 14, 15, 16, 24, 25, 29, 33, 34, 36].

Neutrosophy has been introduced by Smarandache [30, 31] as a new branch of philosophy and generalization of fuzzy logic, intuitionistic fuzzy logic, paraconsistent logic. Fuzzy sets and intuitionistic fuzzy sets are characterized by membership functions, membership and non-membership functions, respectively. In some real life problems for proper description of an object in uncertain and ambiguous environment, we need to handle the indeterminate and incomplete information. But fuzzy sets and intuitionistic fuzzy sets don't handle the indeterminant and inconsistent information. Thus neutrosophic set is defined by Samarandache [31], as a new mathematical tool for dealing with problems involving incomplete, indeterminacy, inconsistent knowledge.

Maji [23] introduced concept of neutrosophic soft set and some operations of neutrosophic soft sets. Karaaslan [19] redefined concept and operations of neutrosophic soft sets as different from Maji's neutrosophic soft set definition and operations. Recently, the properties and applications on the neutrosophic soft sets have been studied increasingly [6, 7, 8, 12, 13, 28].

Alkhazaleh et al [3] were firstly introduced concept of possibility fuzzy soft set and its operations. They gave applications of this theory in solving a decision making problem and they also introduced a similarity measure of two possibility fuzzy soft sets and discussed their application in a medical diagnosis problem. In 2012, Bashir et al. [5] introduced concept of possibility intuitionistic fuzzy soft set and its operations and discussed similarity measure of two possibility intuitionistic fuzzy sets. They also gave an application of this similarity measure.

This paper is organized as follow: in Section 2, some basic definitions and operations are given regarding neutrosophic soft set required in this paper. In Section 3, possibility neutrosophic soft set is defined as a generalization of possibility fuzzy soft set and possibility intuitionistic fuzzy soft set introduced by Alkhazaleh [3] and Bashir [5], respectively. In Section 4, a decision making method is given using the possibility neutrosophic soft sets. In Section 5, similarity measure is introduced between two possibility neutrosophic soft sets and in Section 6, an application related personal selection for a firm is given as regarding this similarity measure method.

2. Preliminary

In this paper, we recall some definitions, operation and their properties related to the neutrosophic soft set [19, 23] required in this paper.

Definition 1. *A neutrosophic soft set (or namely* ns-*set) f over U is a neutrosophic set valued function from E to $\mathcal{N}(U)$. It can be written as*

$$f = \Big\{(e, \{\langle u, t_{f(e)}(u), i_{f(e)}(u), f_{f(e)}(u)\rangle : u \in U\}) : e \in E\Big\}$$

where, $\mathcal{N}(U)$ denotes set of all neutrosophic sets over U. Note that if $f(e) = \{\langle u, 0, 1, 1\rangle : u \in U\}$, the element $(e, f(e))$ is not appeared in the neutrosophic soft set f. Set of all ns-*sets over U is denoted by $\mathbb{NS}$.*

Definition 2. *[19] Let $f, g \in \mathbb{NS}$. f is said to be neutrosophic soft subset of g, if $t_{f(e)}(u) \leq t_{g(e)}(u)$, $i_{f(e)}(u) \geq i_{g(e)}(u)$ $f_{f(e)}(u) \geq f_{g(e)}(u)$, $\forall e \in E$, $\forall u \in U$. We denote it by $f \sqsubseteq g$. f is said to be neutrosophic soft super set of g if g is a neutrosophic soft subset of f. We denote it by $f \sqsupseteq g$.*

If f is neutrosophic soft subset of g and g is neutrosophic soft subset of f. We denote it $f = g$

Definition 3. *[19] Let $f \in \mathbb{NS}$. If $t_{f(e)}(u) = 0$ and $i_{f(e)}(u) = f_{f(e)}(u) = 1$ for all $e \in E$ and for all $u \in U$, then f is called null* ns-*set and denoted by $\tilde{\Phi}$.*

Definition 4. *[19] Let $f \in \mathbb{NS}$. If $t_{f(e)}(u) = 1$ and $i_{f(e)}(u) = f_{f(e)}(u) = 0$ for all $e \in E$ and for all $u \in U$, then f is called universal* ns-*set and denoted by $\tilde{U}$.*

Definition 5. *[19] Let $f, g \in \mathbb{NS}$. Then union and intersection of ns-sets f and g denoted by $f \sqcup g$ and $f \sqcap g$ respectively, are defined by as follow*

$$\begin{aligned} f \sqcup g &= \Big\{(e, \{\langle u, t_{f(e)}(u) \vee t_{g(e)}(u), i_{f(e)}(u) \wedge i_{g(e)}(u), \\ &\quad f_{f(e)}(u) \wedge f_{g(e)}(u)\rangle : u \in U\}) : e \in E\Big\}. \end{aligned}$$

and ns-*intersection of f and g is defined as*

$$\begin{aligned} f \sqcap g &= \Big\{(e, \{\langle u, t_{f(e)}(u) \wedge t_{g(e)}(u), i_{f(e)}(u) \vee i_{g(e)}(u), \\ &\quad f_{f(e)}(u) \vee f_{g(e)}(u)\rangle : u \in U\}) : e \in E\Big\}. \end{aligned}$$

Definition 6. *[19] Let $f, g \in \mathbb{NS}$. Then complement of ns-set f, denoted by $f^{\tilde{c}}$, is defined as follow*

$$f^{\tilde{c}} = \Big\{(e, \{\langle u, f_{f(e)}(u), 1 - i_{f(e)}(u), t_{f(e)}(u)\rangle : u \in U\}) : e \in E\Big\}.$$

Proposition 1. *[19] Let $f, g, h \in \mathbb{NS}$. Then,*

i. $\tilde{\Phi} \sqsubseteq f$

ii. $f \sqsubseteq \tilde{U}$

iii. $f \sqsubseteq f$

iv. $f \sqsubseteq g$ *and* $g \sqsubseteq h \Rightarrow f \sqsubseteq h$

Proposition 2. *[19] Let $f \in \mathbb{NS}$. Then*

i. $\tilde{\Phi}^{\tilde{c}} = \tilde{U}$

ii. $\tilde{U}^{\tilde{c}} = \tilde{\Phi}$

iii. $(f^{\tilde{c}})^{\tilde{c}} = f$.

Proposition 3. *[19] Let $f, g, h \in \mathbb{NS}$. Then,*

i. $f \sqcap f = f$ *and* $f \sqcup f = f$

ii. $f \sqcap g = g \sqcap f$ *and* $f \sqcup g = g \sqcup f$

iii. $f \sqcap \tilde{\Phi} = \tilde{\Phi}$ *and* $f \sqcap \tilde{U} = f$

iv. $f \sqcup \tilde{\Phi} = f$ *and* $f \sqcup \tilde{U} = \tilde{U}$

v. $f \sqcap (g \sqcap h) = (f \sqcap g) \sqcap h$ *and* $f \sqcup (g \sqcup h) = (f \sqcup g) \sqcup h$

vi. $f \sqcap (g \sqcup h) = (f \sqcap g) \sqcup (f \sqcap h)$ *and* $f \sqcup (g \sqcap h) = (f \sqcup g) \sqcap (f \sqcup h)$.

Proof. The proof is clear from definition and operations of neutrosophic soft sets.

Theorem 1. *[19] Let $f, g \in \mathbb{NS}$. Then, De Morgan's law is valid.*

i. $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$

ii. $(f \sqcup g)^{\tilde{c}} = f^{\tilde{c}} \sqcap g^{\tilde{c}}$

Definition 7. *[19] Let $f, g \in \mathbb{NS}$. Then 'OR' product of ns-sets f and g denoted by $f \wedge g$, is defined as follow*

$$\begin{aligned} f \bigvee g &= \Big\{((e,e'), \{\langle u, t_{f(e)}(u) \vee t_{g(e)}(u), i_{f(e)}(u) \wedge i_{g(e)}(u), \\ &\quad f_{f(e)}(u) \wedge f_{g(e)}(u)\rangle : u \in U\}) : (e,e') \in E \times E\Big\}. \end{aligned}$$

Definition 8. *[19] Let $f, g \in \mathbb{NS}$. Then 'AND' product of ns-sets f and g denoted by $f \vee g$, is defined as follow*

$$\begin{aligned} f \bigwedge g &= \Big\{((e,e'), \{\langle u, t_{f(e)}(x) \wedge t_{g(e)}(u), i_{f(e)}(u) \vee i_{g(e)}(u), \\ &\quad f_{f(e)}(u) \vee f_{g(e)}(u)\rangle : u \in U\}) : (e,e') \in E \times E\Big\}. \end{aligned}$$

Proposition 4. *[19] Let $f, g \in \mathbb{NS}$. Then,*

1. $(f \bigvee g)^{\tilde{c}} = f^{\tilde{c}} \bigwedge g^{\tilde{c}}$
2. $(f \bigwedge g)^{\tilde{c}} = f^{\tilde{c}} \bigvee g^{\tilde{c}}$

PROOF. The proof is clear from Definition 7 and 8.

Definition 9. *[3] Let $U = \{u_1, u_2, ..., u_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to I^U$ and μ be a fuzzy subset of E, that is $\mu : E \to I^U$, where I^U is the collection of all fuzzy subsets of U. Let $F_\mu : E \to I^U \times I^U$ be a function defined as follows:*

$$F_\mu(e) = (F(e)(u), \mu(e)(u)), \forall u \in U.$$

Then F_μ is called a possibility fuzzy soft set (PFSS in short) over the soft universe (U, E). For each parameter e_i, $F_\mu(e_i) = (F(e_i)(u), \mu(e_i)(u))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $\mu(e_i)$.

Definition 10. *[5] Let $U = \{u_1, u_2, ..., u_n\}$ be the universal set of elements and $E = \{e_1, e_2, ..., e_m\}$ be the universal set of parameters. The pair (U, E) will be called a soft universe. Let $F : E \to (I \times I)^U \times I^U$ where $(I \times I)^U$ is* *the collection of all intuitionistic fuzzy subsets of U and I^U is the collection of all intuitionistic fuzzy subsets of U. Let p be a fuzzy subset of E, that is, $p : E \to I^U$ and let $F_p : E \to (I \times I)^U \times I^U$ be a function defined as follows:*

$$F_p(e) = (F(e)(u), p(e)(u)), F(e)(u) = (\mu(u), \nu(u)) \forall u \in U.$$

Then F_p is called a possibility intuitionistic fuzzy soft set (PIFSS in short) over the soft universe (U, E). For each parameter e_i, $F_p(e_i) = (F(e_i)(u), p(e_i)(u))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$, but also the degree of possibility of belongingness of the elements of U in $F(e_i)$, which is represented by $p(e_i)$.

3. Possibility neutrosophic soft sets

In this section, we introduced the concept of possibility neutrosophic soft set, possibility neutrosophic soft subset, possibility neutrosophic soft null set, possibility neutrosophic soft universal set and possibility neutrosophic soft set operations such as union, intersection and complement.

Throughout paper U is an initial universe, E is a set of parameters and Λ is an index set.

Definition 11. *Let U be an initial universe, E be a parameter set, $\mathcal{N}(\mathcal{U})$ be the collection of all neutrosophic sets of U and I^U is collection of all fuzzy subset of U. A possibility neutrosophic soft set (PNS-set) f_μ over U is defined by the set of ordered pairs*

$$f_\mu = \{(e_i, \{(\frac{u_j}{f(e_i)(u_j)}, \mu(e_i)(u_j)) : u_j \in U\}) : e_i \in E\}$$

where, $i, j \in \Lambda$, f is a mapping given by $f : E \to \mathcal{N}(\mathcal{U})$ and $\mu(e_i)$ is a fuzzy set such that $\mu : E \to I^U$. Here, $\tilde{f}_\mu$ is a mapping defined by $f_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$.

For each parameter $e_i \in E$, $f(e_i) = \{\langle u_j, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j)\rangle : u_j \in U\}$ indicates neutrosophic value set of parameter e_i and where $t, i, f : U \to [0, 1]$ are the membership functions of truth, indeterminacy and falsity respectively of the element $u_j \in U$. For each $u_j \in U$ and $e_i \in E$, $0 \leq t_{f(e_i)}(u_j) + i_{f(e_i)}(u_j) + f_{f(e_i)}(u_j) \leq 3$. Also $\mu(e_i)$, degrees of possibility of belongingness of elements of U in $f(e_i)$. So we can write

$$f_\mu(e_i) = \Big\{\Big(\frac{u_1}{f(e_i)(u_1)}, \mu(e_i)(u_1)\Big), \Big(\frac{u_2}{f(e_i)(u_2)}, \mu(e_i)(u_2)\Big), ..., \Big(\frac{u_n}{f(e_i)(u_n)}, \mu(e_i)(u_n)\Big)\Big\}$$

From now on, we will show set of all possibility neutrosophic soft sets over U with $\mathcal{PS}(U,E)$ such that E is parameter set.

Example 1. *Let $U = \{u_1, u_2, u_3\}$ be a set of three cars. Let $E = \{e_1, e_2, e_3\}$ be a set of qualities where e_1 =cheap, e_2 =equipment, e_3 =fuel consumption and let $\mu : E \to I^U$. We define a function $f_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ as follows:*

$$f_\mu = \left\{ \begin{array}{l} f_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.5,0.2,0.6)}, 0.8\right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.4\right), \left(\frac{u_3}{(0.4,0.5,0.8)}, 0.7\right) \right\} \\ f_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.8,0.4,0.5)}, 0.6\right), \left(\frac{u_2}{(0.5,0.7,0.2)}, 0.8\right), \left(\frac{u_3}{(0.7,0.3,0.9)}, 0.4\right) \right\} \\ f_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.6,0.7,0.5)}, 0.2\right), \left(\frac{u_2}{(0.5,0.3,0.7)}, 0.6\right), \left(\frac{u_3}{(0.6,0.5,0.4)}, 0.5\right) \right\} \end{array} \right\}$$

also we can define a function $g_\nu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ as follows:

$$g_\nu = \left\{ \begin{array}{l} g_\nu(e_1) = \left\{ \left(\frac{u_1}{(0.6,0.3,0.8)}, 0.4\right), \left(\frac{u_2}{(0.6,0.5,0.5)}, 0.7\right), \left(\frac{u_3}{(0.2,0.6,0.4)}, 0.8\right) \right\} \\ g_\nu(e_2) = \left\{ \left(\frac{u_1}{(0.5,0.4,0.3)}, 0.3\right), \left(\frac{u_2}{(0.4,0.6,0.5)}, 0.6\right), \left(\frac{u_3}{(0.7,0.2,0.5)}, 0.8\right) \right\} \\ g_\nu(e_3) = \left\{ \left(\frac{u_1}{(0.7,0.5,0.3)}, 0.8\right), \left(\frac{u_2}{(0.4,0.4,0.6)}, 0.5\right), \left(\frac{u_3}{(0.8,0.5,0.3)}, 0.6\right) \right\} \end{array} \right\}$$

For the purpose of storing a possibility neutrosophic soft set in a computer, we can use matrix notation of possibility neutrosophic soft set f_μ. For example, matrix notation of possibility neutrosophic soft set f_μ can be written as follows: for $m, n \in \Lambda$,

$$f_\mu = \begin{pmatrix} (\langle 0.5, 0.2, 0.6\rangle, 0.8) & (\langle 0.7, 0.3, 0.5\rangle, 0.4) & (\langle 0.4, 0.5, 0.8\rangle, 0.7) \\ (\langle 0.8, 0.4, 0.5\rangle, 0.6) & (\langle 0.5, 0.7, 0.2\rangle, 0.8) & (\langle 0.7, 0.3, 0.9\rangle, 0.4) \\ (\langle 0.6, 0.7, 0.5\rangle, 0.2) & (\langle 0.5, 0.3, 0.7\rangle, 0.6) & (\langle 0.6, 0.5, 0.4\rangle, 0.5) \end{pmatrix}$$

where the $m-$th row vector shows $f(e_m)$ and $n-$th column vector shows u_n.

Definition 12. *Let f_μ, $g_\nu \in \mathcal{PS}(U,E)$. Then, f_μ is said to be a possibility neutrosophic soft subset (PNS-subset) of g_ν, and denoted by $f_\mu \subseteq g_\nu$, if*

i. $\mu(e)$ is a fuzzy subset of $\nu(e)$, for all $e \in E$

ii. f is a neutrosophic subset of g,

Example 2. *Let $U = \{u_1, u_2, u_3\}$ be a set of tree houses, and let $E = \{e_1, e_2, e_3\}$ be a set of parameters where e_1 =modern, e_2 =big and e_3 =cheap. Let f_μ be a PNS-set defined as follows:*

$$f_\mu = \left\{ \begin{array}{l} f_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.5,0.2,0.6)}, 0.8\right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.4\right), \left(\frac{u_3}{(0.4,0.5,0.9)}, 0.7\right) \right\} \\ f_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.8,0.4,0.5)}, 0.6\right), \left(\frac{u_2}{(0.5,0.7,0.2)}, 0.8\right), \left(\frac{u_3}{(0.7,0.3,0.9)}, 0.4\right) \right\} \\ f_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.6,0.7,0.5)}, 0.2\right), \left(\frac{u_2}{(0.5,0.3,0.8)}, 0.6\right), \left(\frac{u_3}{(0.6,0.5,0.4)}, 0.5\right) \right\} \end{array} \right\}$$

$g_\nu : E \to \mathcal{N}(U) \times I^U$ *be another PNS-set defined as follows:*

$$g_\nu = \left\{ \begin{array}{l} g_\nu(e_1) = \left\{ \left(\frac{u_1}{(0.6,0.1,0.5)}, 0.9\right), \left(\frac{u_2}{(0.8,0.2,0.3)}, 0.6\right), \left(\frac{u_3}{(0.7,0.5,0.8)}, 0.8\right) \right\} \\ g_\nu(e_2) = \left\{ \left(\frac{u_1}{(0.9,0.2,0.4)}, 0.7\right), \left(\frac{u_2}{(0.9,0.5,0.1)}, 0.9\right), \left(\frac{u_3}{(0.8,0.1,0.9)}, 0.5\right) \right\} \\ g_\nu(e_3) = \left\{ \left(\frac{u_1}{(0.6,0.5,0.4)}, 0.4\right), \left(\frac{u_2}{(0.7,0.1,0.7)}, 0.9\right), \left(\frac{u_3}{(0.8,0.2,0.4)}, 0.7\right) \right\} \end{array} \right\}$$

it is clear that f_μ is $PNS-subset$ of g_ν

Definition 13. *Let $f_\mu, g_\nu \in \mathcal{PN}(U, E)$. Then, f_μ and g_ν are called possibility neutrosophic soft equal set and denoted by $f_\mu = g_\nu$, if $f_\mu \subseteq g_\nu$ and $f_\mu \supseteq g_\nu$.*

Definition 14. *Let $f_\mu \in \mathcal{PN}(U, E)$. Then, f_μ is said to be possibility neutrosophic soft null set denoted by ϕ_μ, if $\forall e \in E$, $\phi_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ such that $\phi_\mu(e) = \{(\frac{u}{\phi(e)(u)}, \mu(e)(u)) : u \in U\}$, where $\phi(e) = \{\langle u, 0, 1, 1\rangle : u \in U\}$ and $\mu(e) = \{(u, 0) : u \in U\}$).*

Definition 15. *Let $f_\mu \in \mathcal{PN}(U, E)$. Then, f_μ is said to be possibility neutrosophic soft universal set denoted by U_μ, if $\forall e \in E$, $U_\mu : E \to \mathcal{N}(\mathcal{U}) \times I^U$ such that $U_\mu(e) = \{(\frac{u}{U(e)(u)}, \mu(e)(u)) : u \in U\}$, where $U(e) = \{\langle u, 1, 0, 0\rangle : u \in U\}$ and $\mu(e) = \{(u, 1) : u \in U\}$).*

Proposition 5. *Let f_μ, g_ν and $h_\delta \in \mathcal{PN}(U, E)$. Then,*

i. $\phi_\mu \subseteq f_\mu$

ii. $f_\mu \subseteq U_\mu$

iii. $f_\mu \subseteq f_\mu$

iv. $f_\mu \subseteq g_\nu$ *and* $g_\nu \subseteq h_\delta \Rightarrow f_\mu \subseteq h_\delta$

Proof. It is clear from Definition 13, 14 and 15.

Definition 16. *Let $f_\mu \in \mathcal{PN}(U)$, where $f_\mu(e_i) = \{(f(e_i)(u_j), \mu(e_i)(u_j)) : e_i \in E, u_j \in U\}$ and $f(e_i) = \{\langle u, t_{f(e_i)}(u_j), i_{f(e_i)}(u_j), f_{f(e_i)}(u_j)\rangle\}$ for all $e_i \in E$, $u \in U$. Then for $e_i \in E$ and $u_j \in U$,*

1. *f_μ^t is said to be truth-membership part of f_μ,*
   $f_\mu^t = \{(f_{ij}^t(e_i), \mu_{ij}(e_i))\}$ *and* $f_{ij}^t(e_i) = \{(u_j, t_{f(e_i)}(u_j))\}$, $\mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$

2. *f_μ^i is said to be indeterminacy-membership part of f_μ,*
   $f_\mu^i = \{(f_{ij}^i(e_i), \mu_{ij}(e_i))\}$ *and* $f_{ij}^i(e_i) = \{(u_j, i_{f(e_i)}(u_j))\}$, $\mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$

3. *f_μ^f is said to be truth-membership part of f_μ,*
   $f_\mu^f = \{(f_{ij}^f(e_i), \mu_{ij}(e_i))\}$ *and* $f_{ij}^f(e_i) = \{(u_j, f_{f(e_i)}(u_j))\}$, $\mu_{ij}(e_i) = \{(u_j, \mu(e_i)(u_j))\}$

We can write a possibility neutrosophic soft set in form $f_\mu = (f_\mu^t, f_\mu^i, f_\mu^f)$.

If considered the possibility neutrosophic soft set f_μ in Example 1, f_μ can be expressed in matrix form as follow:

$$f_\mu^t = \begin{pmatrix} (0.5, 0.8) & (0.7, 0.4) & (0.4, 0.7) \\ (0.8, 0.6) & (0.5, 0.8) & (0.7, 0.4) \\ (0.6, 0.2) & (0.5, 0.6) & (0.6, 0.5) \end{pmatrix}$$

$$f_\mu^i = \begin{pmatrix} (0.2, 0.8) & (0.3, 0.4) & (0.5, 0.7) \\ (0.4, 0.6) & (0.7, 0.8) & (0.3, 0.4) \\ (0.7, 0.2) & (0.3, 0.6) & (0.5, 0.5) \end{pmatrix}$$

$$f_\mu^f = \begin{pmatrix} (0.6, 0.8) & (0.5, 0.4) & (0.8, 0.7) \\ (0.5, 0.6) & (0.2, 0.8) & (0.9, 0.4) \\ (0.5, 0.2) & (0.7, 0.6) & (0.4, 0.5) \end{pmatrix}$$

Definition 17. *[27] A binary operation* $\otimes : [0,1]\times[0,1] \to [0,1]$ *is continuous* $t-$*norm if* $\otimes$ *satisfies the following conditions*

i. $\otimes$ *is commutative and associative,*

ii. $\otimes$ *is continuous,*

iii. $a \otimes 1 = a$, $\forall a \in [0,1]$,

iv. $a \otimes b \leq c \otimes d$ *whenever* $a \leq c, b \leq d$ *and* $a, b, c, d \in [0,1]$.

Definition 18. *[27] A binary operation* $\oplus : [0,1]\times[0,1] \to [0,1]$ *is continuous* $t-$*conorm (s-norm) if* $\oplus$ *satisfies the following conditions*

i. $\oplus$ *is commutative and associative,*

ii. $\oplus$ *is continuous,*

iii. $a \oplus 0 = a$, $\forall a \in [0,1]$,

iv. $a \oplus b \leq c \oplus d$ *whenever* $a \leq c, b \leq d$ *and* $a,b,c,d \in [0,1]$.

Definition 19. *Let* $I^3 = [0,1] \times [0,1] \times [0,1]$ *and* $N(I^3) = \{(a,b,c) : a,b,c \in [0,1]\}$. *Then* $(N(I^3), \oplus, \otimes)$ *be a lattices together with partial ordered relation* $\preceq$, *where order relation* $\preceq$ *on* $N(I^3)$ *can be defined by for* $(a,b,c),(d,e,f) \in N(I^3)$

$$(a,b,c) \preceq (e,f,g) \Leftrightarrow a \leq e, b \geq f, c \geq g$$

Definition 20. *A binary operation*

$$\tilde{\otimes} : \Big([0,1] \times [0,1] \times [0,1]\Big)^2 \to [0,1] \times [0,1] \times [0,1]$$

is continuous $n-$*norm if* $\tilde{\otimes}$ *satisfies the following conditions*

i. $\tilde{\otimes}$ *is commutative and associative,*

ii. $\tilde{\otimes}$ *is continuous,*

iii. $a\tilde{\otimes}0 = 0$, $a\tilde{\otimes}1 = a$, $\forall a \in [0,1] \times [0,1] \times [0,1]$, $(1 = (1,0,0))$ *and* $(0 = (0,1,1))$

iv. $a\tilde{\otimes}b \leq c\tilde{\otimes}d$ *whenever* $a \preceq c, b \preceq d$ *and* $a,b,c,d \in [0,1] \times [0,1] \times [0,1]$.

Here,

$a\tilde{\otimes}b = \tilde{\otimes}(\langle t(a), i(a), f(a)\rangle, \langle t(b), i(b), f(b)\rangle) = \langle t(a)\otimes t(b), i(a)\oplus i(b), f(a)\oplus f(b)$

Definition 21. *A binary operation*

$$\tilde{\oplus} : \Big([0,1] \times [0,1] \times [0,1]\Big)^2 \to [0,1] \times [0,1] \times [0,1]$$

is continuous $n-$*conorm if* $\tilde{\oplus}$ *satisfies the following conditions*

i. $\tilde{\oplus}$ *is commutative and associative,*

ii. $\tilde{\oplus}$ *is continuous,*

iii. $a\tilde{\oplus}0 = a$, $a\tilde{\oplus}1 = 1$, $\forall a \in [0,1] \times [0,1] \times [0,1]$, $(1 = (1,0,0))$ *and* $(0 = (0,1,1))$

iv. $a\tilde{\oplus}b \leq c\tilde{\oplus}d$ *whenever* $a \leq c, b \leq d$ *and* $a, b, c, d \in [0,1] \times [0,1] \times [0,1]$.

Here,

$$a\tilde{\oplus}b = \tilde{\oplus}(\langle t(a), i(a), f(a)\rangle, \langle t(b), i(b), f(b)\rangle) = \langle t(a)\oplus t(b), i(a)\otimes i(b), f(a)\otimes f(b)$$

Definition 22. *Let* $f_\mu, g_\nu \in \mathcal{PN}(U,E)$*. The union of two possibility neutrosophic soft sets* f_μ *and* g_ν *over* U*, denoted by* $f_\mu \cup g_\nu$ *is defined by as follow:*

$$f_\mu \cup g_\nu = \left\{\left(e_i, \left\{\left(\frac{u_j}{(f^t_{ij}(e_i) \oplus g^t_{ij}(e_i), f^i_{ij}(e_i) \otimes g^i_{ij}(e_i), f^f_{ij}(e_i) \otimes g^f_{ij}(e_i))}, \mu_{ij}(e_i)\oplus\nu_{ij}(e_i)\right) : u_j \in U\right\}\right) : e_i \in E\right\}$$

Definition 23. *Let* $f_\mu, g_\nu \in \mathcal{PN}(U,E)$*. The intersection of two possibility neutrosophic soft sets* f_μ *and* g_ν *over* U*, denoted by* $f_\mu \cap g_\nu$ *is defined by as follow:*

$$f_\mu \cap g_\nu = \left\{\left(e_i, \left\{\left(\frac{u_j}{(f^t_{ij}(e_i) \otimes g^t_{ij}(e_i), f^i_{ij}(e_i) \oplus g^i_{ij}(e_i), f^f_{ij}(e_i) \oplus g^f_{ij}(e_i))}, \mu_{ij}(e_i)\otimes\nu_{ij}(e_i)\right) : u_j \in U\right\}\right) : e_i \in E\right\}$$

Example 3. *Let us consider the possibility neutrosophic soft sets* f_μ *and* g_ν *defined as in Example 1. Let us suppose that* $t-$*norm is defined by* $a \otimes b = min\{a, b\}$ *and the* $t-$*conorm is defined by* $a \oplus b = max\{a, b\}$ *for* $a, b \in [0,1]$*. Then,*

$$f_\mu \cup g_\nu = \left\{\begin{array}{l} (f_\mu \cup g_\nu)(e_1) = \left\{\left(\frac{u_1}{(0.6,0.2,0.6)}, 0.8\right), \left(\frac{u_2}{(0.7,0.3,0.5)}, 0.7\right), \left(\frac{u_3}{(0.4,0.5,0.4)}, 0.8\right)\right\} \\ (f_\mu \cup g_\nu)(e_2) = \left\{\left(\frac{u_1}{(0.8,0.4,0.3)}, 0.6\right), \left(\frac{u_2}{(0.5,0.6,0.2)}, 0.8\right), \left(\frac{u_3}{(0.7,0.2,0.5)}, 0.8\right)\right\} \\ (f_\mu \cup g_\nu)(e_3) = \left\{\left(\frac{u_1}{(0.7,0.3,0.3)}, 0.8\right), \left(\frac{u_2}{(0.5,0.3,0.6)}, 0.6\right), \left(\frac{u_3}{(0.8,0.5,0.3)}, 0.6\right)\right\} \end{array}\right\}$$

and

$$f_\mu \cap g_\nu = \left\{\begin{array}{l} (f_\mu \cap g_\nu)(e_1) = \left\{\left(\frac{u_1}{(0.5,0.3,0.8)}, 0.4\right), \left(\frac{u_2}{(0.6,0.5,0.5)}, 0.4\right), \left(\frac{u_3}{(0.2,0.6,0.8)}, 0.7\right)\right\} \\ (f_\mu \cap g_\nu)(e_2) = \left\{\left(\frac{u_1}{(0.5,0.4,0.5)}, 0.3\right), \left(\frac{u_2}{(0.4,0.7,0.5)}, 0.6\right), \left(\frac{u_3}{(0.7,0.3,0.9)}, 0.4\right)\right\} \\ (f_\mu \cap g_\nu)(e_3) = \left\{\left(\frac{u_1}{(0.6,0.7,0.5)}, 0.2\right), \left(\frac{u_2}{(0.4,0.5,0.7)}, 0.5\right), \left(\frac{u_3}{(0.6,0.5,0.4)}, 0.5\right)\right\} \end{array}\right\}$$

Proposition 6. *Let* $f_\mu, g_\nu, h_\delta \in \mathcal{PN}(U,E)$*. Then,*

i. $f_\mu \cap f_\mu = f_\mu$ *and* $f_\mu \cup f_\mu = f_\mu$

ii. $f_\mu \cap g_\nu = g_\nu \cap f_\mu$ *and* $f_\mu \cup g_\nu = g_\nu \cup f_\mu$

iii. $f_\mu \cap \phi_\mu = \phi_\mu$ *and* $f_\mu \cap U_\mu = f_\mu$

iv. $f_\mu \cup \phi = f_\mu$ *and* $f_\mu \cup U_\mu = U_\mu$

v. $f_\mu \cap (g_\nu \cap h_\delta) = (f_\mu \cap g_\nu) \cap h_\delta$ *and* $f_\mu \cup (g_\nu \cup h_\delta) = (f_\mu \cup g_\nu) \cup h_\delta$

vi. $f_\mu \cap (g_\nu \cup h_\delta) = (f_\mu \cap g_\nu) \cup (f_\mu \cap h_\delta)$ *and* $f_\mu \cup (g_\nu \cap h_\delta) = (f_\mu \cup g_\nu) \cap (f_\mu \cup h_\delta)$.

Proof. The proof can be obtained from Definitions 22. and 23.

Definition 24. *[17, 32] A function $N : [0,1] \to [0,1]$ is called a negation if $N(0) = 1$, $N(1) = 0$ and N is non-increasing ($x \leq y \Rightarrow N(x) \geq N(y)$). A negation is called a strict negation if it is strictly decreasing ($x < y \Rightarrow N(x) > N(y)$) and continuous. A strict negation is said to be a strong negation if it is also involutive, i.e. $N(N(x)) = x$*

Definition 25. *[30] A function $n_N : [0,1]\times[0,1]\times[0,1] \to [0,1]\times[0,1]\times[0,1]$ is called a negation if $n_N(\overline{0}) = \overline{1}$, $n_N(\overline{1}) = \overline{0}$ and n_N is non-increasing ($x \preceq y \Rightarrow n_N(x) \succeq n_N(y)$). A negation is called a strict negation if it is strictly decreasing ($x \prec y \Rightarrow N(x) \succ N(y)$) and continuous.*

Definition 26. *Let $f_\mu \in \mathcal{PN}(U,E)$. Complement of possibility neutrosophic soft set f_μ, denoted by f^c_μ is defined as follow:*

$$f^c_\mu = \Big\{ \Big(e, \{ (\frac{u_j}{n_N(f(e_i))}, N(\mu_{ij}(e_i)(u_j)) \Big) : u_j \in U \} \Big) : e \in E \Big\}$$

where $(n_N(f_{ij}(e_i)) = (N(f^t_{ij}(e_i)), N(f^i_{ij}(e_i)), N(f^f_{ij}(e_i))$ for all $i, j \in \Lambda$

Example 4. *Let us consider the possibility neutrosophic soft set f_μ define in Example 1. Suppose that the negation is defined by $N(f^t_{ij}(e_i)) = f^f_{ij}(e_i)$, $N(f^f_{ij}(e_i)) = f^t_{ij}(e_i)$, $N(f^i_{ij}(e_i)) = 1 - f^i_{ij}(e_i)$ and $N(\mu_{ij}(e_i)) = 1 - \mu_{ij}(e_i)$, respectively. Then, f^c_μ is defined as follow:*

$$f^c_\mu = \left\{ \begin{array}{l} f^c_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.6,0.8,0.5)}, 0.2 \right), \left(\frac{u_2}{(0.5,0.7,0.7)}, 0.6 \right), \left(\frac{u_3}{(0.8,0.5,0.4)}, 0.3 \right) \right\} \\ f^c_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.5,0.6,0.8)}, 0.4 \right), \left(\frac{u_2}{(0.2,0.3,0.5)}, 0.2 \right), \left(\frac{u_3}{(0.9,0.7,0.7)}, 0.6 \right) \right\} \\ f^c_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.5,0.3,0.6)}, 0.8 \right), \left(\frac{u_2}{(0.7,0.7,0.5)}, 0.4 \right), \left(\frac{u_3}{(0.4,0.5,0.6)}, 0.5 \right) \right\} \end{array} \right\}$$

Proposition 7. *Let $f_\mu \in \mathcal{PN}(U,E)$. Then,*

i. $\phi^c_\mu = U_\mu$

ii. $U^c_\mu = \phi_\mu$

iii. $(f^c_\mu)^c = f_\mu$.

Proof. *It is clear from Definition 26*

Proposition 8. *Let $f_\mu, g_\nu \in \mathcal{PN}(U,E)$. Then, De Morgan's law is valid.*

i. $(f_\mu \cup g_\nu)^c = f_\mu^c \cap g_\nu^c$

ii. $(f_\mu \cap g_\nu)^c = f_\mu^c \cup g_\nu^c$

Proof. *i.* Let $i, j \in \Lambda$

$$\begin{aligned}
(f_\mu \cup g_\nu)^c &= \Bigg\{\Big(e_i, \Big\{\big(\frac{u_j}{(f_{ij}^t(e_i) \oplus g_{ij}^t(e_i), f_{ij}^i(e_i) \otimes g_{ij}^i(e_i), f_{ij}^f(e_i) \otimes g_{ij}^f(e_i))},\\
&\quad \mu_{ij}(e_i) \oplus \nu_{ij}(e_i)\big) : u_j \in U\Big\}\Big) : e_i \in E\Bigg\}^c\\
&= \Bigg\{\Big(e_i, \Big\{\big(\frac{u_j}{(f_{ij}^f(e_i) \otimes g_{ij}^f(e_i), N(f_{ij}^i(e_i) \otimes g_{ij}^i(e_i)), f_{ij}^t(e_i) \oplus g_{ij}^t(e_i))},\\
&\quad N(\mu_{ij}(e_i) \oplus \nu_{ij}(e_i))\big) : u_j \in U\Big\}\Big) : e_i \in E\Bigg\}\\
&= \Bigg\{\Big(e_i, \Big\{\big(\frac{u_j}{(f_{ij}^f(e_i) \otimes g_{ij}^f(e_i), N(f_{ij}^i(e_i)) \oplus N(g_{ij}^i(e_i))), f_{ij}^t(e_i) \oplus g_{ij}^t(e_i))},\\
&\quad N(\mu_{ij}(e_i)) \otimes N(\nu_{ij}(e_i))\big) : u_j \in U\Big\}\Big) : e_i \in E\Bigg\}\\
&= \Bigg\{\Big(e_i, \Big\{\big(\frac{u_j}{(f_{ij}^f(e_i), N(f_{ij}^i(e_i)), f_{ij}^t(e_i)}, N(\mu_{ij}(e_i))\big) : u_j \in U\Big\}\Big) : e_i \in E\Bigg\}\\
&\cap \Bigg\{\Big(e_i, \Big\{\big(\frac{u_j}{g_{ij}^f(e_i), N(g_{ij}^i(e_i)), g_{ij}^t(e_i))}, N(\nu_{ij}(e_i))\big) : u_j \in U\Big\}\Big) : e_i \in E\Bigg\}\\
&= \Bigg\{\Big(e_i, \Big\{\big(\frac{u_j}{(f_{ij}^t(e_i), N(f_{ij}^i(e_i)), f_{ij}^f(e_i)}, \mu_{ij}(e_i)\big) : u_j \in U\Big\}\Big) : e_i \in E\Bigg\}^c\\
&\cap \Bigg\{\Big(e_i, \Big\{\big(\frac{u_j}{g_{ij}^t(e_i), g_{ij}^i(e_i), g_{ij}^f(e_i))}, \nu_{ij}(e_i)\big) : u_j \in U\Big\}\Big) : e_i \in E\Bigg\}^c\\
&= f_\mu^c \cap g_\nu^c
\end{aligned}$$

ii. The proof can be made with similar way.

Definition 27. *Let f_μ and $g_\nu \in \mathcal{PN}(U,E)$. Then 'AND' product of PNS-set f_μ and g_ν denoted by $f_\mu \wedge g_\nu$, is defined as follow:*

$$\begin{aligned}
f_\mu \wedge g_\nu &= \Big\{\Big((e_k, e_l), (f_{kj}^t(e_k) \wedge g_{lj}^t(e_l), f_{kj}^i(e_k) \vee g_{lj}^i(e_l), f_{kj}^f(e_k) \vee g_{lj}^f(e_l)),\\
&\quad \mu_{kj}(e_k) \wedge \nu_{lj}(e_l)\Big) : (e_k, e_l) \in E \times E, j, k, l \in \Lambda\Big\}
\end{aligned}$$

Definition 28. *Let f_μ and $g_\nu \in \mathcal{PN}(U,E)$. Then 'OR' product of PNS-set f_μ and g_ν denoted by $f_\mu \vee g_\nu$, is defined as follow:*

$$\begin{aligned} f_\mu \vee g_\nu &= \Big\{\Big((e_k,e_l),(f^t_{kj}(e_k)\vee g^t_{lj}(e_l), f^i_{kj}(e_k)\wedge g^i_{lj}(e_l), f^f_{kj}(e_k)\wedge g^f_{lj}(e_l)),\\ &\quad \mu_{kj}(e_k)\vee \nu_{lj}(e_l)\Big) : (e_k,e_l)\in E\times E, j,k,l\in\Lambda\Big\}\end{aligned}$$

4. Decision making method

In this section we will construct a decision making method over the possibility neutrosophic soft set. Firstly, we will define some notions that necessary to construct algorithm of decision making method. And then we will present an application of possibility neutrosophic soft set theory in a decision making problem.

Definition 29. *Let $f_\mu \in \mathcal{PN}(U,E)$, f^t_μ, f^i_μ and f^f_μ be the truth, indeterminacy and falsity matrices of $\wedge$-product matrix, respectively. Then, weighted matrices of f^t_μ, f^i_μ and f^f_μ are denoted by $\wedge^t, \wedge^i$ and $\wedge^f$, respectively and computed by as follows:*

$$\wedge^t(e_{ij},u_k) = t_{(f_\mu\wedge g_\nu)(e_{ij})}(u_k) + (\mu_{ik}(e_i)\wedge\nu_{jk}(e_j)) - t_{(f_\mu\wedge g_\nu)(e_{ij})}(u_k)\times(\mu_{ik}(e_i)\wedge\nu_{jk}(e_j))$$

$$\wedge^i(e_{ij},u_k) = i_{(f_\mu\wedge g_\nu)(e_{ij})}(u_k)\times(\mu_{ik}(e_i)\wedge\nu_{jk}(e_j))$$

$$\wedge^f(e_{ij},u_k) = f_{(f_\mu\wedge g_\nu)(e_{ij})}(u_k)\times(\mu_{ik}(e_i)\wedge\nu_{jk}(e_j))$$

for $i,j,k\in\Lambda$

Definition 30. *Let $f_\mu \in \mathcal{PN}(U,E)$, $\wedge^t, \wedge^i$ and $\wedge^f$ be the weighed matrices of f^t_μ, f^i_μ and f^f_μ, respectively. Then, for all $u_t \in U$ such that $t\in\Lambda$, scores of u_t is in the weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$ are denoted by $s^t(u_k)$, $s^i(u_k)$ and $s^f(u_k)$, and computed by as follow, respectively*

$$s^t(u_t) = \sum_{i,j\in\Lambda}\delta^t_{ij}(u_t)$$

$$s^i(u_t) = \sum_{i,j\in\Lambda}\delta^i_{ij}(u_t)$$

$$s^f(u_t) = \sum_{i,j\in\Lambda} \delta^f_{ij}(u_t)$$

here

$$\delta^t_{ij}(u_t) = \begin{cases} \wedge^t(e_{ij}, u_t), & \wedge^t(e_{ij}, u_t) = max\{\wedge^t(e_{ij}, u_k) : u_k \in U\} \\ 0, & otherwise \end{cases}$$

$$\delta^i_{ij}(u_t) = \begin{cases} \wedge^i(e_{ij}, u_t), & \wedge^i(e_{ij}, u_t) = max\{\wedge^i(e_{ij}, u_k) : u_k \in U\} \\ 0, & otherwise \end{cases}$$

$$\delta^f_{ij}(u_t) = \begin{cases} \wedge^f(e_{ij}, u_t), & \wedge^f(e_{ij}, u_t) = max\{\wedge^f(e_{ij}, u_k) : u_k \in U\} \\ 0, & otherwise \end{cases}$$

Definition 31. *Let $s^t(u_t), s^i(u_t)$ and $s^f(u_t)$ be scores of $u_t \in U$ in the weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$. Then, decision score of $u_t \in U$, denoted by $ds(u_t)$, is computed as follow:*

$$ds(u_t) = s^t(u_t) - s^i(u_t) - s^f(u_t)$$

Now, we construct a possibility neutrosophic soft set decision making method by the following algorithm;

Algorithm

Step 1: Input the possibility neutrosophic soft set f_μ,

Step 2: Construct the matrix $\wedge$-product

Step 3: Construct the matrices f^t_μ, f^i_μ and f^f_μ

Step 4: Construct the weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$,

Step 5: Compute score of $u_t \in U$, for each of the weighted matrices,

Step 6: Compute decision score, for all $u_t \in U$,

Step 7: The optimal decision is to select $u_t = maxds(u_i)$.

Example 5. *Assume that $U = \{u_1, u_2, u_3\}$ is a set of houses and $E = \{e_1, e_2, e_3\} = \{cheap, large, moderate\}$ is a set of parameters which is attractiveness of houses. Suppose that Mr.X want to buy one the most suitable house according to to himself depending on three of the parameters only.*

Step 1: *Based on the choice parameters of $Mr.X$, let there be two observations f_μ and g_ν by two experts defined as follows:*

$$f_\mu = \left\{ \begin{array}{c} f_\mu(e_1) = \left\{ \left(\frac{u_1}{(0.5,0.3,0.7)}, 0.6\right), \left(\frac{u_2}{(0.6,0.2,0.5)}, 0.2\right), \left(\frac{u_3}{(0.7,0.6,0.5)}, 0.4\right) \right\} \\ f_\mu(e_2) = \left\{ \left(\frac{u_1}{(0.35,0.2,0.6)}, 0.4\right), \left(\frac{u_2}{(0.7,0.8,0.3)}, 0.5\right), \left(\frac{u_3}{(0.2,0.4,0.4)}, 0.6\right) \right\} \\ f_\mu(e_3) = \left\{ \left(\frac{u_1}{(0.7,0.2,0.5)}, 0.5\right), \left(\frac{u_2}{(0.4,0.5,0.2)}, 0.3\right), \left(\frac{u_3}{(0.5,0.3,0.6)}, 0.2\right) \right\} \end{array} \right\}$$

$$g_\nu = \left\{ \begin{array}{c} g_\nu(e_1) = \left\{ \left(\frac{u_1}{(0.3,0.4,0.5)}, 0.2\right), \left(\frac{u_2}{(0.7,0.3,0.4)}, 0.5\right), \left(\frac{u_3}{(0.4,0.5,0.2)}, 0.3\right) \right\} \\ g_\nu(e_2) = \left\{ \left(\frac{u_1}{(0.4,0.6,0.2)}, 0.3\right), \left(\frac{u_2}{(0.2,0.5,0.3)}, 0.7\right), \left(\frac{u_3}{(0.4,0.6,0.2)}, 0.8\right) \right\} \\ g_\nu(e_3) = \left\{ \left(\frac{u_1}{(0.2,0.1,0.6)}, 0.7\right), \left(\frac{u_2}{(0.8,0.4,0.5)}, 0.4\right), \left(\frac{u_3}{(0.6,0.4,0.3)}, 0.4\right) \right\} \end{array} \right\}$$

Step 2: *Let us consider possibility neutrosophic soft set $\wedge$-product which is the mapping $\wedge : E \times E \to \mathcal{N}(\mathcal{U}) \times I^U$ given as follows:*

$$\left(\begin{array}{c|ccc} \wedge & u_1, \mu & u_2, \mu & u_3, \mu \\ \hline e_{11} & (\langle 0.3, 0.4, 0.7\rangle, 0.2) & (\langle 0.6, 0.3, 0.5\rangle, 0.2) & (\langle 0.4, 0.6, 0.5\rangle, 0.3) \\ e_{12} & (\langle 0.4, 0.6, 0.7\rangle, 0.3) & (\langle 0.2, 0.5, 0.5\rangle, 0.2) & (\langle 0.4, 0.6, 0.5\rangle, 0.4) \\ e_{13} & (\langle 0.2, 0.3, 0.7\rangle, 0.6) & (\langle 0.6, 0.4, 0.5\rangle, 0.2) & (\langle 0.6, 0.6, 0.5\rangle, 0.4) \\ e_{21} & (\langle 0.3, 0.4, 0.6\rangle, 0.2) & (\langle 0.7, 0.8, 0.4\rangle, 0.5) & (\langle 0.2, 0.5, 0.5\rangle, 0.3) \\ e_{22} & (\langle 0.35, 0.6, 0.6\rangle, 0.3) & (\langle 0.2, 0.8, 0.3\rangle, 0.5) & (\langle 0.2, 0.6, 0.5\rangle, 0.6) \\ e_{23} & (\langle 0.2, 0.2, 0.6\rangle, 0.4) & (\langle 0.7, 0.8, 0.5\rangle, 0.4) & (\langle 0.2, 0.4, 0.5\rangle, 0.4) \\ e_{31} & (\langle 0.3, 0.4, 0.5\rangle, 0.2) & (\langle 0.4, 0.5, 0.4\rangle, 0.3) & (\langle 0.4, 0.5, 0.6\rangle, 0.2) \\ e_{32} & (\langle 0.4, 0.6, 0.5\rangle, 0.3) & (\langle 0.2, 0.5, 0.3\rangle, 0.3) & (\langle 0.4, 0.6, 0.6\rangle, 0.2) \\ e_{33} & (\langle 0.2, 0.2, 0.6\rangle, 0.5) & (\langle 0.4, 0.5, 0.5\rangle, 0.3) & (\langle 0.5, 0.4, 0.6\rangle, 0.2) \end{array} \right)$$

Matrix representation of $\wedge$-product

Step 3: *We construct matrices f^t_μ, f^i_μ and f^f_μ as follows:*

$\wedge$	u_1, μ	u_2, μ	u_3, μ
e_{11}	$(0.3, 0.2)$	$(0.6, 0.2)$	$(0.4, 0.3)$
e_{12}	$(0.4, 0.3)$	$(0.2, 0.2)$	$(0.4, 0.4)$
e_{13}	$(0.2, 0.6)$	$(0.6, 0.2)$	$(0.6, 0.4)$
e_{21}	$(0.3, 0.2)$	$(0.7, 0.5)$	$(0.2, 0.3)$
e_{22}	$(0.35, 0.3)$	$(0.2, 0.5)$	$(0.2, 0.6)$
e_{23}	$(0.2, 0.4)$	$(0.7, 0.4)$	$(0.2, 0.4)$
e_{31}	$(0.3, 0.2)$	$(0.4, 0.3)$	$(0.4, 0.2)$
e_{32}	$(0.4, 0.3)$	$(0.2, 0.3)$	$(0.4, 0.2)$
e_{33}	$(0.2, 0.5)$	$(0.4, 0.3)$	$(0.5, 0.2)$

Matrix f^t_μ of $\wedge$-product

$\wedge$	u_1, μ	u_2, μ	u_3, μ
e_{11}	$(0.4, 0.2)$	$(0.3, 0.2)$	$(0.6, 0.3)$
e_{12}	$(0.6, 0.3)$	$(0.5, 0.2)$	$(0.6, 0.4)$
e_{13}	$(0.3, 0.6)$	$(0.4, 0.2)$	$(0.6, 0.4)$
e_{21}	$(0.4, 0.2)$	$(0.8, 0.5)$	$(0.5, 0.3)$
e_{22}	$(0.6, 0.3)$	$(0.8, 0.5)$	$(0.6, 0.6)$
e_{23}	$(0.2, 0.4)$	$(0.8, 0.4)$	$(0.4, 0.4)$
e_{31}	$(0.4, 0.2)$	$(0.5, 0.3)$	$(0.5, 0.2)$
e_{32}	$(0.6, 0.3)$	$(0.5, 0.3)$	$(0.6, 0.2)$
e_{33}	$(0.2, 0.5)$	$(0.5, 0.3)$	$(0.4, 0.2)$

Matrix f^i_μ of $\wedge$-product

$\wedge$	u_1, μ	u_2, μ	u_3, μ
e_{11}	$(0.7, 0.2)$	$(0.5, 0.2)$	$(0.5, 0.3)$
e_{12}	$(0.7, 0.3)$	$(0.5, 0.2)$	$(0.5, 0.4)$
e_{13}	$(0.7, 0.6)$	$(0.5, 0.2)$	$(0.5, 0.4)$
e_{21}	$(0.6, 0.2)$	$(0.4, 0.5)$	$(0.5, 0.3)$
e_{22}	$(0.6, 0.3)$	$(0.3, 0.5)$	$(0.5, 0.6)$
e_{23}	$(0.6, 0.4)$	$(0.5, 0.4)$	$(0.5, 0.4)$
e_{31}	$(0.5, 0.2)$	$(0.4, 0.3)$	$(0.6, 0.2)$
e_{32}	$(0.5, 0.3)$	$(0.3, 0.3)$	$(0.6, 0.2)$
e_{33}	$(0.6, 0.5)$	$(0.5, 0.3)$	$(0.6, 0.2)$

Matrix f^f_μ of $\wedge$-product

Step 4: *We obtain weighted matrices $\wedge^t, \wedge^i$ and $\wedge^f$ using Definition 29 as follows:*

$$\left(\begin{array}{c|ccc} \wedge^t & u_1 & u_2 & u_3 \\ \hline e_{11} & 0.44 & \underline{0.64} & 0.58 \\ e_{12} & 0.58 & 0.36 & \underline{0.64} \\ e_{13} & 0.68 & 0.68 & \underline{0.76} \\ e_{21} & 0.44 & \underline{0.85} & 0.44 \\ e_{22} & 0.55 & 0.60 & \underline{0.68} \\ e_{23} & 0.52 & \underline{0.82} & 0.48 \\ e_{31} & 0.44 & \underline{0.58} & 0.52 \\ e_{32} & \underline{0.58} & 0.44 & 0.52 \\ e_{33} & \underline{0.60} & 0.58 & \underline{0.60} \end{array}\right), \left(\begin{array}{c|ccc} \wedge^i & u_1 & u_2 & u_3 \\ \hline e_{11} & 0.08 & \underline{0.16} & 0.18 \\ e_{12} & 0.18 & 0.10 & \underline{0.24} \\ e_{13} & 0.18 & 0.08 & \underline{0.24} \\ e_{21} & 0.08 & \underline{0.40} & 0.15 \\ e_{22} & 0.18 & \underline{0.40} & 0.36 \\ e_{23} & 0.08 & \underline{0.32} & 0.16 \\ e_{31} & 0.08 & \underline{0.15} & 0.10 \\ e_{32} & \underline{0.18} & 0.15 & 0.12 \\ e_{33} & 0.10 & \underline{0.15} & 0.08 \end{array}\right), \left(\begin{array}{c|ccc} \wedge^f & u_1 & u_2 & u_3 \\ \hline e_{11} & 0.14 & 0.10 & \underline{0.15} \\ e_{12} & \underline{0.21} & 0.10 & 0.20 \\ e_{13} & \underline{0.42} & 0.10 & 0.20 \\ e_{21} & 0.12 & \underline{0.20} & 0.15 \\ e_{22} & 0.18 & 0.15 & \underline{0.30} \\ e_{23} & \underline{0.24} & 0.20 & \underline{0.20} \\ e_{31} & 0.10 & \underline{0.12} & \underline{0.12} \\ e_{32} & \underline{0.15} & 0.09 & 0.12 \\ e_{33} & \underline{0.30} & 0.15 & 0.12 \end{array}\right)$$

Weighed matrices of f^t_μ, f^i_μ and f^f_μ from left to right, respectively.

Step 5: *For all $u \in U$, we find scores using Definition 30 as follow:*

$$s^t(u_1) = 1,18, \quad s^t(u_2) = 2,89, \quad s^t(u_3) = 2,68$$
$$s^i(u_1) = 0,18 \quad s^i(u_2) = 1,42 \quad s^i(u_3) = 0,66$$
$$s^f(u_1) = 1,32 \quad s^f(u_2) = 0,32 \quad s^f(u_3) = 0,57$$

Step 5: *For all $u \in U$, we find scores using Definition 30 as follows:*

$$\begin{array}{rcl} ds(u_1) & = & 1,18 - 0,18 - 1,32 = -0,32 \\ ds(u_2) & = & 2,89 - 1,42 - 0,32 = 0,90 \\ ds(u_3) & = & 2,68 - 0,66 - 0,57 = 1,45 \end{array}$$

Step 5: *Then the optimal selection for $Mr.X$ is u_3.*

5. Similarity measure of possibility neutrosophic soft sets

In this section, we introduce a measure of similarity between two PNS-sets.

Definition 32. *Similarity between two PNS-sets* f_μ *and* g_ν*, denoted by* $S(f_\mu, g_\nu)$*, is defined as follows:*

$$S(f_\mu, g_\nu) = M(f(e), g(e)).M(\mu(e), \nu(e))$$

such that

$$M(f(e), g(e)) = \frac{1}{n} M_i(f(e), g(e)), M(\mu, \nu) = \frac{1}{n} \sum_{i=1}^{n} M(\mu(e_i), \nu(e_i)),$$

where

$$M_i(f(e), g(e)) = 1 - \frac{1}{\sqrt[p]{n}} \sqrt[p]{\sum_{i=1}^{n} (\phi_{f_\mu(e_i)}(u_j) - \phi_{g_\nu(e_i)}(u_j))^p}, 1 \leq p \leq \infty$$

such that and

$$\phi_{f_\mu(e_i)}(u_j) = \frac{f_{ij}^t(e_i) + f_{ij}^i(e_i) + f_{ij}^f(e_i)}{3}, \ \phi_{g_\mu(e_i)}(u_j) = \frac{g_{ij}^t(e_i) + g_{ij}^i(e_i) + g_{ij}^f(e_i)}{3},$$

$$M(\mu(e_i), \nu(e_i)) = 1 - \frac{\sum_{j=1}^{n} |\mu_{ij}(e_i) - \nu_{ij}(e_i)|}{\sum_{j=1}^{n} |\mu_{ij}(e_i) + \nu_{ij}(e_i)|}$$

Definition 33. *Let* f_μ *and* g_ν *be two PNS-sets over* U*. We say that* f_μ *and* g_ν *are significantly similar if* $S(f_\mu, g_\nu) \geq \frac{1}{2}$

Proposition 9. *Let* $f_\mu, g_\nu \in \mathcal{PN}(U, E)$*. Then,*

i. $S(f_\mu, g_\nu) = S(g_\mu, f_\nu)$

ii. $0 \leq S(f_\mu, g_\nu) \leq 1$

iii. $f_\mu = g_\nu \Rightarrow S(f_\mu, g_\nu) = 1$

Proof. The proof is straightforward and follows from Definition 32.

Example 6. *Let us consider PNS-sets* f_μ *and* g_ν *in Example 1 given as follows:*

$$\left\{\begin{array}{l} f_\mu(e_1)=\left\{\left(\frac{u_1}{(0.5,0.2,0.6)},0.8\right),\left(\frac{u_2}{(0.7,0.3,0.5)},0.4\right),\left(\frac{u_3}{(0.4,0.5,0.8)},0.7\right)\right\} \\ f_\mu(e_2)=\left\{\left(\frac{u_1}{(0.8,0.4,0.5)},0.6\right),\left(\frac{u_2}{(0.5,0.7,0.2)},0.8\right),\left(\frac{u_3}{(0.7,0.3,0.9)},0.4\right)\right\} \\ f_\mu(e_3)=\left\{\left(\frac{u_1}{(0.6,0.7,0.5)},0.2\right),\left(\frac{u_2}{(0.5,0.3,0.7)},0.6\right),\left(\frac{u_3}{(0.6,0.5,0.4)},0.5\right)\right\} \end{array}\right\}$$

and

$$\left\{\begin{array}{l} g_\nu(e_1)=\left\{\left(\frac{u_1}{(0.6,0.3,0.8)},0.4\right),\left(\frac{u_2}{(0.6,0.5,0.5)},0.7\right),\left(\frac{u_3}{(0.2,0.6,0.4)},0.8\right)\right\} \\ g_\nu(e_2)=\left\{\left(\frac{u_1}{(0.5,0.4,0.3)},0.3\right),\left(\frac{u_2}{(0.4,0.6,0.5)},0.6\right),\left(\frac{u_3}{(0.7,0.2,0.5)},0.8\right)\right\} \\ g_\nu(e_3)=\left\{\left(\frac{u_1}{(0.7,0.5,0.3)},0.8\right),\left(\frac{u_2}{(0.4,0.4,0.6)},0.5\right),\left(\frac{u_3}{(0.8,0.5,0.3)},0.6\right)\right\} \end{array}\right\}$$

then,

$$\begin{aligned} M(\mu(e_1),\nu(e_1)) &= 1-\frac{\sum_{j=1}^{3}|\mu_{1j}(e_1)-\nu_{1j}(e_1)|}{\sum_{j=1}^{3}|\mu_{1j}(e_1)+\nu_{1j}(e_1)|} \\ &= 1-\frac{|0.8-0.4|+|0.4-0.7|+|0.7-0.8|}{|0.8+0.4|+|0.4+0.7|+|0.7+0.8|}=0.79 \end{aligned}$$

Similarly we get $M(\mu(e_2),\nu(e_2))=0.74$ and $M(\mu(e_3),\nu(e_3))=0.75$, then

$$M(\mu,\nu)=\frac{1}{3}(0.79+0.75+0.74)=0.76$$

$$\begin{aligned} M_1(f(e),g(e)) &= 1-\frac{1}{\sqrt[p]{n}}\sqrt[p]{\sum_{i=1}^{n}(\phi_{f_\mu(e_i)}(u_j)-\phi_{g_\nu(e_i)}(u_j))^p} \\ &= 1-\frac{1}{\sqrt{3}}\sqrt{(0.43-0.57)^2+(0,50-0.53)^2+(0,57-0,40)^2}=0.73 \\ M_2(f(e),g(e)) &= 0.86 \\ M_3(f(e),g(e)) &= 0.94 \end{aligned}$$

$$M(f(e),g(e))=\frac{1}{3}(0.73+0.86+0.94)=0.84$$

and

$$S(f_\mu,g_\nu)=0.84\times 0.76=0.64$$

6. Decision-making method based on the similarity measure

In this section, we give a decision making problem involving possibility neutrosophic soft sets by means of the similarity measure between the possibility neutrosophic soft sets.

Let our universal set contain only two elements "yes" and "no", that is $U = y, n$. Assume that $P = \{p_1, p_2, p_3, p_4, p_5\}$ are five candidates who fill in a form in order to apply formally for the position. There is a decision maker committee. They want to interview the candidates by model possibility neutrosophic soft set determined by committee. So they want to test similarity of each of candidate to model possibility neutrosophic soft set.

Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ be the set of parameters, where e_1=experience, e_2=computer knowledge, e_3=training, e_4=young age, e_5=higher education, e_6=marriage status and e_7=good health.

Our model possibility neutrosophic soft set determined by committee for suitable candidates properties f_μ is given in Table 1.

f_μ	e_1, μ	e_2, μ	e_3, μ	e_4, μ
y	$(\langle 1,0,0\rangle,1)$	$(\langle 1,0,0\rangle,1)$	$(\langle 0,1,1\rangle,1)$	$(\langle 0,1,1\rangle,1)$
n	$(\langle 0,1,1\rangle,1)$	$(\langle 1,0,0\rangle,1)$	$(\langle 0,1,1\rangle,1)$	$(\langle 1,0,0\rangle,1)$

f_μ	e_5, μ	e_6, μ	e_7, μ
y	$(\langle 1,0,0\rangle,1)$	$(\langle 0,1,1\rangle,1)$	$(\langle 1,0,0\rangle,1)$
n	$(\langle 0,1,1\rangle,1)$	$(\langle 1,0,0\rangle,1)$	$(\langle 0,1,1\rangle,1)$

Table 1: *The tabular representation of model possibility neutrosophic soft set*

g_ν	e_1, ν	e_2, ν	e_3, ν	e_4, ν
y	$(\langle 0.7,0.2,0.5\rangle,0.4)$	$(\langle 0.5,0.4,0.6\rangle,0.2)$	$(\langle 0.2,0.3,0.4\rangle,0.5)$	$(\langle 0.8,0.4,0.6\rangle,0.3)$
n	$(\langle 0.3,0.7,0.1\rangle,0.3)$	$(\langle 0.7,0.3,0.5\rangle,0.4)$	$(\langle 0.6,0.5,0.3\rangle,0.2)$	$(\langle 0.2,0.1,0.5\rangle,0.4)$

g_ν	e_5, ν	e_6, ν	e_7, ν
y	$(\langle 0.2,0.4,0.3\rangle,0.5)$	$(\langle 0,1,1\rangle,0.3)$	$(\langle 0.1,0.4,0.7\rangle,0.2)$
n	$(\langle 0.1,0.5,0.2\rangle,0.6)$	$(\langle 1,0,0\rangle,0.5)$	$(\langle 0.3,0.5,0.1\rangle,0.4)$

Table 2: *The tabular representation of possibility neutrosophic soft set for p_1*

h_δ	e_1, δ	e_2, δ	e_3, δ	e_4, δ
y	$(\langle 0.8,0.2,0.1\rangle,0.3)$	$(\langle 0.4,0.2,0.6\rangle,0.1)$	$(\langle 0.7,0.2,0.4\rangle,0.2)$	$(\langle 0.3,0.2,0.7\rangle,0.6)$
n	$(\langle 0.2,0.4,0.3\rangle,0.5)$	$(\langle 0.6,0.3,0.2\rangle,0.3)$	$(\langle 0.4,0.3,0.2\rangle,0.1)$	$(\langle 0.8,0.1,0.3\rangle,0.3)$

h_δ	e_5, δ	e_6, δ	e_7, δ
y	$(\langle 0.5,0.2,0.4\rangle,0.5)$	$(\langle 0,1,1\rangle,0.5)$	$(\langle 0.3,0.2,0.5\rangle,0.4)$
n	$(\langle 0.4,0.5,0.6\rangle,0.2)$	$(\langle 1,0,0\rangle,0.2)$	$(\langle 0.7,0.3,0.4\rangle,0.2)$

Table 3: *The tabular representation of possibility neutrosophic soft set for* p_2

r_θ	e_1,θ	e_2,θ	e_3,θ	e_4,θ
y	$(\langle 0.3,0.2,0.5\rangle,0.4)$	$(\langle 0.7,0.1,0.5\rangle,0.6)$	$(\langle 0.6,0.5,0.3\rangle,0.2)$	$(\langle 0.3,0.1,0.4\rangle,0.5)$
n	$(\langle 0.1,0.7,0.6\rangle,0.3)$	$(\langle 0.4,0.2,0.3\rangle,0.7)$	$(\langle 0.7,0.4,0.3\rangle,0.5)$	$(\langle 0.7,0.1,0.2\rangle,0.1)$

r_θ	e_5,θ	e_6,θ	e_7,θ
y	$(\langle 0.6,0.4,0.3\rangle,0.2)$	$(\langle 0,1,1\rangle,0.3)$	$(\langle 0.9,0.1,0.1\rangle,0.5)$
n	$(\langle 0.4,0.5,0.9\rangle,0.1)$	$(\langle 1,0,0\rangle,0.3)$	$(\langle 0.2,0.1,0.7\rangle,0.6)$

Table 4: *The tabular representation of possibility neutrosophic soft set for* p_3

s_α	e_1,α	e_2,α	e_3,α	e_4,α
y	$(\langle 0.2,0.1,0.4\rangle,0.5)$	$(\langle 0.7,0.5,0.4\rangle,0.8)$	$(\langle 0.8,0.1,0.2\rangle,0.4)$	$(\langle 0.5,0.4,0.5\rangle,0.4)$
n	$(\langle 0.6,0.5,0.1\rangle,0.1)$	$(\langle 0.3,0.7,0.2\rangle,0.2)$	$(\langle 0.7,0.5,0.1\rangle,0.7)$	$(\langle 0.1,0.3,0.7\rangle,0.5)$

s_α	e_5,α	e_6,α	e_7,α
y	$(\langle 0.3,0.2,0.5\rangle,0.8)$	$(\langle 1,0,0\rangle,0.7)$	$(\langle 0.1,0.8,0.9\rangle,0.7)$
n	$(\langle 0.2,0.1,0.5\rangle,0.3)$	$(\langle 0,1,1\rangle,0.2)$	$(\langle 0.5,0.1,0.4\rangle,0.1)$

Table 5: *The tabular representation of possibility neutrosophic soft set for* p_4

m_γ	e_1,γ	e_2,γ	e_3,γ	e_4,γ
y	$(\langle 0.1,0.2,0.1\rangle,0.3)$	$(\langle 0.2,0.3,0.5\rangle,0.8)$	$(\langle 0.4,0.1,0.3\rangle,0.9)$	$(\langle 0.7,0.3,0.2\rangle,0.3)$
n	$(\langle 0.4,0.5,0.3\rangle,0.2)$	$(\langle 0.7,0.6,0.1\rangle,0.3)$	$(\langle 0.2,0.3,0.4\rangle,0.5)$	$(\langle 0.5,0.2,0.3\rangle,0.6)$

m_γ	e_5,γ	e_6,γ	e_7,γ
y	$(\langle 0.4,0.2,0.8\rangle,0.1)$	$(\langle 1,0,0\rangle,0.5)$	$(\langle 0.3,0.2,0.1\rangle,0.7)$
n	$(\langle 0.5,0.4,0.7\rangle,0.2)$	$(\langle 0,1,1\rangle,0.5)$	$(\langle 0.3,0.2,0.1\rangle,0.9)$

Table 6: *The tabular representation of possibility neutrosophic soft set for* p_5

Now we find the similarity between the model possibility neutrosophic soft set and possibility neutrosophic soft set of each person as follow

$S(f_\mu,g_\nu)\cong 0,49<\frac{1}{2}$, $S(f_\mu,h_\delta)\cong 0,47<\frac{1}{2}$, $S(f_\mu,r_\theta)\cong 0,51>\frac{1}{2}$, $S(f_\mu,s_\alpha)\cong 0,54>\frac{1}{2}$, $S(f_\mu,m_\gamma)\cong 0,57>\frac{1}{2}$,

Consequently, p_5 is should be selected by the committee.

7. Conclusion

In this paper we have introduced the concept of possibility neutrosophic soft set and studied some of the related properties. Applications of this theory have been given to solve a decision making problem. We also presented a new method to find out the similarity measure of two possibility neutrosophic soft sets and we applied to a decision making problem. In future, these seem to have natural applications and algebraic structure.

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