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Plithogeny, Plithogenic Set, Logic, Probability, and Statistics (A Short Review)

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AQ1 AQ2 **Abstract** In this paper, one recalls our 2017 concepts of plithogeny and its derivative applications in set theory, logic, probability, and statistics. The plithogenic set, plithogenic logic, plithogenic probability, and plithogenic statistics are presented again.

1. Etymology of Plithogeny

Plithogeny etymologically comes from: (Gr.) πλήθος (plithos) = crowd, large number of, multitude, plenty of, and -geny < (Gr.) -γενιά (-geniá) = generation, the production of something, and γένεια (géneia) = generations, the production of something < -γένεση (-génesi) = genesis, origination, creation, development, according to Translate Google Dictionaries (https://translate.google.com/) and Webster's New World Dictionary of American English, Third College Edition, Simon & Schuster, Inc., New York, pp. 562–563, 1988.

Therefore, plithogeny is the genesis or origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory and/or neutrals and/or non-contradictory multiple old entities. Plithogenic means what is pertaining to plithogeny.

2. Plithogenic Set

A plithogenic set *P* is a set whose elements are characterized by one or more attributes, and each attribute may have many values. Each attribute's value *v* has a corresponding degree of appurtenance $d(x,v)$ of the element x to the set *P*, with respect to some given criteria.

In order to obtain a better accuracy for the plithogenic aggregation operators, a contradiction (dissimilarity) degree is defined between each attribute value and the dominant (most important) attribute value.

However, there are cases when such dominant attribute value may not be taken into consideration or may not exist (therefore it is considered zero by default), or there may be many dominant attribute values. In such cases, either the contradiction degree function is suppressed, or another relationship function between attribute values should be established.

The plithogenic aggregation operators (intersection, union, complement, inclusion, equality) are based on contradiction degrees between attributes' values, and the first two are linear combinations of the fuzzy operators' t_{norm} and t_{conorm}.

Plithogenic set was introduced by Smarandache in Uluta et al. (2022), Smarandache (2017, 2019, 2020), and Chavez et al. (2021), and it is a generalization of the crisp set, fuzzy set, intuitionistic fuzzy set, and neutrosophic set, since these types of sets are characterized by a single attribute value (appurtenance): which has one value (membership) – for the crisp set and fuzzy set, two values (membership and nonmembership) – for intuitionistic fuzzy set, or three values (membership, nonmembership, and indeterminacy) – for neutrosophic set.

2.1. Example

Let *P* be a plithogenic set, representing the students from a college. Let $x \in P$ be a generic student that is characterized by three attributes:

- altitude (a), whose values are {tall, short} = {*a*1, *a*2};
- weight (w), whose values are {obese, fat, medium, thin} = {*w*1, *w*2, *w*3, *w*4};
  and
- hair color, whose values are {blond, reddish, brown} = {h1, h2, h3}.

The multi-attribute of dimension 3 is

$$V_3 = \{(a_i, w_j, h_k) \text{ for all } 1 \le i \le 2,\ 1 \le j \le 4,\ 1 \le k \le 3\}.$$

Let us say *P* = {John(*a*1, *w*3, *h*2), Richard(*a*1, *w*3, *h*2)} = {John(tall, thin, reddish), Richard(tall, thin, reddish}.

From the view point of expert A, one has PA = {John(0.7, 0.2, 0.4), Richard(0.5, 0.8, 0.6)}, which means that, from the view point of expert A, John's fuzzy degrees of tallness, thinness, and reddishness are, respectively, 0.7, 0.2, and 0.4, while Richard's fuzzy degrees of tallness, thinness, and reddishness are, respectively, 0.5, 0.8, and 0.6.

While from the view point of expert B, one has PB = {John(0.8, 0.2, 0.5), Richard(0.3, 0.7, 0.4)}.

The *uni-dimensional attribute contradiction degrees* are

$$c(a_1, a_2) = 1$$

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$$c(w_1, w_2) = \frac{1}{3}$$

$$c(w_1, w_3) = \frac{2}{3}$$

$$c(w_1, w_4) = 1$$

$$c(h_1, h_2) = 0.5$$

$$c(h_1, h_3) = 1$$

Dominant attribute values are a_1, w_1, and h_1 for each corresponding uni-dimensional attribute *a, w,* and *h*, respectively. Let us use the fuzzy conjunction $a \wedge_F b = ab$ {where $\wedge_F$ means fuzzy conjunction}, and fuzzy disjunction $a \vee_F b = a + b - ab$ {where $\vee_F$ means fuzzy disjunction}.

We use the notations: $\wedge_P$ and $\vee_P$ to denote the plithogenic intersection and the plithogenic union, respectively.

Then

$$\begin{aligned}(a, b, c) \wedge p\ (d, e, f) &= (ad, (1/2)\{(be) + (b + e - be)\}, c + f - cf) \\ &= (ad, (b + e)/2, c + f - cf).\end{aligned}$$

2.2. A plithogenic application to images

A pixel x may be characterized by colors $c_1, c_2, \ldots, c_n$. We write $x(c_1, c_2, \ldots, c_n)$, where $n \geq 1$. We may consider the degree of each color either fuzzy, intuitionistic fuzzy, or neutrosophic.

For example.

Fuzzy degree:

$$x(0.4,\ 0.6,\ 0.1,\ \ldots,\ 0.3).$$

Intuitionistic fuzzy degree:

$$x((0.1,\ 0.2),\ (0.3,\ 0.5),\ (0.0,\ 0.6),\ \ldots,\ (0.8,\ 0.9)).$$

Neutrosophic degree:

$$x((0.0,\ 0.3,\ 0.6),\ (0.2,\ 0.8,\ 0.9),\ (0.7,\ 0.4,\ 0.2),\ \ldots,\ (0.1,\ 0.1,\ 0.9)).$$

Then, we can use a plithogenic operator to combine them.

For example:

$$x(0.4,\ 0.6,\ 0.1, \ldots,\ 0.3) \wedge p\ \ x(0.1,\ 0.7,\ 0.5, \ldots,\ 0.2) = \ldots$$

We establish first the degrees of contradictions between all colors c_i and c_j in order to find the linear combinations of t-norm and t-conorm that one applies to each color (similar to the indeterminacy above).

3. Plithogenic Probability

Since in plithogenic probability each event E from a probability space U is characterized by many chances of the event to occur (not only one chance of the event E to occur: as in classical probability, imprecise probability, and neutrosophic probability), a plithogenic probability distribution function, *PP(x)*, of a random variable, *x*, is described by many plithogenic probability distribution sub-functions, where each sub-function represents the chance (with respect to a given attribute value) that value x occurs, and these chances of occurrence can be represented by classical, imprecise, neutrosophic probabilities, and in general any fuzzy-extension type (depending on the type of degree of a chance).

3.1. Example of plithogenic probabilistic

What is the plithogenic probability that Jenifer will graduate at the end of this semester in her program of electrical engineering, given that she is enrolled in and has to pass two courses of Mathematics (Non-Linear Differential Equations and Stochastic Analysis), and two courses of Mechanics (Fluid Mechanics, and Solid Mechanics)? We have four attribute values of plithogenic probability.

According to her adviser, Jenifer's plithogenic single-valued fuzzy probability of graduating at the end of this semester is: J(0.5, 0.6; 0.8, 0.4), which means 50% chance of passing the Non-Linear Differential Equations class, 60% chance of passing the Stochastic Analysis class (as part of Mathematics), and 80% chance of passing the Fluid Mechanics class and 40% chance of passing the Solid Mechanics class (as part of Physics).

Therefore, the plithogenic probability in this example is composed of four classical probabilities.

3.2. Subclasses of Plithogenic Probability (Smarandache and Smarandache, 2021) are

(i) If all probability distribution functions (PDFs) are classical, then we have a classical **MultiVariate Probability**.

(ii) If all PDFs are in the neutrosophic style, that is, of the form (T, I, F), where T is the chance that the event occurs, I is the indeterminate chance of the event to occur or not, and F is the chance that the event does not occur, with T, I, F $\in$ [0,1], $0 \leq T + I + F \leq 3$, then we have a **Plithogenic Neutrosophic Probability**.

(iii) If all PDFs are indeterminate functions (i.e. functions that have indeterminate data in the arguments, or in the values, or in both), then we have a **Plithogenic Indeterminate Probability**.

(iv) If all PDFs are Intuitionistic Fuzzy in the form of (T, F), where T is the chance that the event occurs, and F is the chance that the event does not occur, with $T, F \in [0, 1]$, $0 \leq T + F \leq 1$, then we have a **Plithogenic Intuitionistic Fuzzy Probability**.

(v) If all PDFs are in the Picture Fuzzy Set style, that is, of the form (T, N, F), where T is the chance that the event occurs, N is the neutral chance of the event to occur or not, and F is the chance that the event does not occur, with $T, N, F \in [0, 1]$, $0 \leq T + N + F \leq 1$, then we have a **Plithogenic Picture Fuzzy Probability**.

(vi) If all PDFs are in the Spherical Fuzzy Set style, that is, of the form (T, H, F), where T is the chance that the event occurs, H is the neutral chance of the event to occur or not, and F is the chance that the event does not occur, with $T, H, F \in [0, 1]$, $0 \leq T^2 + H^2 + F^2 \leq 1$, then we have a **Plithogenic Picture Fuzzy Probability**.

(vii) In general, if all PDFs are in any (fuzzy-extension set) style, then we have a **Plithogenic (*fuzzy-extension*) Probability**.

(viii) If some PDFs are in one of the above styles, while others are in different styles, then we have a **Plithogenic Hybrid Probability**.

3.3. Plithogenic refined probability

The most general form of probability is **Plithogenic Refined Probability** (Smarandache and Smarandache, 2021), when the components of T (Truth = Occurrence), I (Indeterminate-Occurrence), and F (Falsehood-NonOccurrence) are refined/split into sub-components: T_1, T_2, ..., T_p (sub-truths = sub-occurrences) and I_1, I_2, ..., I_r (sub-indeterminate-occurrences), and F_1, F_2, ..., F_s (sub-falsehoods = sub-nonoccurrences), where $p, r, s \geq 0$ are integers, and at least one of p, r, s is ≥ 2.

All the above sub-classes of plithogenic probability may be refined this way.

4. Plithogenic Statistics

As a generalization of classical statistics and neutrosophic statistics, the plithogenic statistics is the analysis of events described by the plithogenic probability.

In neutrosophic statistics, we have some degree of indeterminacy into the data or into the statistical inference methods. The neutrosophic probability (and similarly for classical probability and for the imprecise probability) of an event E to occur is calculated with respect to the chance of the event E to occur (i.e. it is calculated with respect to only ONE chance of occurrence), while the plithogenic probability of an event E to occur is calculated with respect to MANY chances of the event E to occur (it is calculated with respect to each event's attribute/parameter chance of occurrence). Therefore, the plithogenic probability is a multi-probability (i.e. multi-dimensional probability) – unlike the classical, and probabilities may be of any type, such as classical, imprecise, neutrosophic, and any other fuzzy-extension type that are uni-dimensional probabilities.

4.1. Example of plithogenic statistics

Let us consider the previous example of plithogenic probability that Jenifer will graduate at the end of this semester in her program of electrical engineering. Instead of defining only one probability distribution function (and drawing its curve), we do now draw four probability distribution functions (and draw four curves), when we consider the neutrosophic distribution as a uni-dimensional neutrosophic function. Therefore, plithogenic statistics is a multivariate statistics.

5. Conclusion

We have recalled the 2017 plithogenic set, logic, probability, and statistics of an event that is composed from the chances that the event occurs with respect to all random variables (parameters) that determine it. Each such a variable is described by a probability distribution (density) function, which may be a classical, (T, I, F)-neutrosophic, I-neutrosophic, (T, F)-intuitionistic fuzzy, (T, N, F)-picture fuzzy, (T, N, F)-spherical fuzzy, or (other fuzzy extension) distribution function.

Plithogenic statistics [2017] is the analysis of the events described by the plithogenic probability. Several examples were provided. AQ3

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AQ1: Please check if edit made to the sentence “In this paper, one recalls our . . . ” is fine.

AQ2: Please provide keywords.

AQ3: References “Smarandache, 2018a, b, c; Rana et al. 2019; Martin, 2020; Smarandache, 2020, Rana et al., 2020; Ahmad et al., 2020, Gayen et al., 2020; Martin and Smarandache, 2020a and 2020b, Quek et al., 2020; Bala, 2020; Alkhazaleh, 2020; Sujatha et al., 2020; Priyadharshini et al., 2020; Singh, 2020; Sankar et al., 2020; Martin and Priya, 2021; Abdel-Basset et al., 2021; Martin et al., 2021; Priyadharshini et al., 2021; Ahmad et al., 2022; Selcuk et al., 2020; Alwadani and Ndubisi, 2021; Priyadharshini and Nirmala Irudayam, 2021” have not been cited in the text. Please indicate where it should be cited.