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A new method for solving quadratic fractional programming problem in neutrosophic environment

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Abstract: In the current study, a neutrosophic quadratic fractional programming (NQFP) problem is investigated using a new method. The NQFP problem is converted into the corresponding quadratic fractional programming (QFP) problem. The QFP is formulated by using the score function and hence it is converted to the linear programming problem (LPP) using the Taylor series, which can be solved by LPP techniques or software (e.g., Lingo). Finally, an example is given for illustration.

Keywords: linear programming, quadratic fractional programming, neutrosophic set, trapezoidal neutrosophic numbers, Taylor series

1 Introduction

Fractional programming (FP) plays important roles in many applications such as economics, non-economic, and indirect applications. Many authors studied the LFP (Charnes and Cooper [1]).

Numerous authors have studied FP under uncertainty (for instance, Ammar and Khalifa [2], Effati and Pakdaman [3], Ammar and Khalifa [4], Tantawy [5], Odior [6], Pandey and Punnen [7], Dantzig [8], Mojtaba et al. [9], Gupta and Chakraborty [10], Dempe and Ruziyeua [11], Safaei [12], and Dutta and Kumar [13]). Saha et al. [14] developed a new method by converting the LFP into a single linear programming problem (LPP) for some cases of the nominator and denominator functions. Das et al. [15] introduced a note on a method presented by Safaei [12]. Liu et al. [16] proposed a novel methodology to determine the global optimization for a class of LFP on a large scale. Tas et al. [17] introduced all definitions and concepts related to metric spaces in neutrosophic environment. There are many researchers who introduced various topics in Neutrosophic environment (Topal et al. [18] and Topal et al., [19]). Alharbi and Khalifa [20] studied LFP in fuzzy environment.

Quadratic fractional programming (QFP) problems have enormous applications in operations research literature. It can be classified on the basis of the homogeneity of the constraints and the factorability of the objective function (Sharma and Singh [21]). Khurana and Arora [22] proposed a method for solving QFP problem including homogeneous constraints. Suleiman and Nawkhass [23] recommended that a modified simplex method is superior in addressing QFP problems. In addition, they solved the problem by applying Wolfe's method. Youness et al. [24] used a two-dimensional algorithm to introduce a parametric methodology for solving non-linear FP models.

Fuzzy sets were developed first by Zadeh [25] and then further studied by Dubois and Prade [26]. Kumar and Dutta [27] investigated an application of FP to determine the solution of an inventory management problem in fuzzy environment. Gupta et al. [28] introduced a model of multiple objective QFP model with a set of quadratic constraints and a methodology based on the iterative parametric functions to obtain a set of solutions of the problem. Khalifa et al. [29] used fuzzy set theory to solve the multi-objective fractional transportation problem.

Very recently, some applications of neutrosophic sets were discussed in various fields of operations research, for instance, assignment problem (Khalifa and Kumar [30]) and complex programming (Khalifa et al. [31]).

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The outline of the current study is organized as follows: in Section 2, some preliminaries needed are recalled. In neutrosophic environment, a quadratic fractional programming (NQFP) problem is developed in Section 3. In Section 4, a solution method to NQFP problem is investigated. In Section 5, a numerical experimentation is performed to show the efficiency of the suggested solution methodology. Section 6 introduces the discussion for the results obtained. In the end, some concluding remarks as well as future research directions are presented in Section 7.

2 Preliminaries

In this section, some fundamental terms associated with neutrosophic number and the arithmetic operation are recalled.

Definition 1. (Atanassov, [32]). Let X be a non-empty set. An intuitionistic fuzzy set $\bar{M}_I$ of X is defined as $\bar{M}_I = \{\langle x, \mu_{\bar{M}_I}(x), \eta_{\bar{M}_I}(x) : x \in X\rangle\}$, where $\mu_{\bar{M}_I}(x)$ and $\eta_{\bar{M}_I}(x)$ are membership and nonmembership functions, respectively, such that $\mu_{\bar{M}_I}(x), \eta_{\bar{M}_I}(x) : x \to [0, 1]$ and $0 \le \mu_{\bar{M}_I}(x) + \eta_{\bar{M}_I}(x) \le 1$; $\forall x \in X$.

Definition 2. (Atanassov, [32]). An intuitionistic fuzzy subset $\bar{M}_I = \{\langle x, \mu_{\bar{M}_I}(x), \eta_{\bar{M}_I}(x) : x \in X\rangle\}$ of $\mathbb{R}$ is termed as an intuitionistic fuzzy number, when the following properties are satisfied:

(i) There exists $m \in \mathbb{R}$ such that $\mu_{\bar{M}_I}(m) = 1$ and $\eta_{\bar{M}_I}(m) = 0$,

(ii) $\mu_{\bar{M}_I}$ is continuous function from $\mathbb{R} \to [0, 1]$ such that $0 \le \mu_{\bar{M}_I} + \eta_{\bar{M}_I} \le 1$; $\forall x \in X$, and

(iii) The functions of membership and non-membership of $\bar{M}_I$ are defined by:

$$\mu_{\bar{M}_I} = \begin{cases} 0, & -\infty < x \le a_1, \\ f(x), & a_1 \le x \le a_2, \\ g(x), & a_2 \le x \le a_3, \\ 0, & a_3 \le x < \infty; \end{cases}$$

$$\eta_{\bar{M}_I} = \begin{cases} 0, & -\infty < x \le a_1^\circ, \\ f^\circ(x), & a_1^\circ \le x \le a_2^\circ, \\ g^\circ(x), & a_2^\circ \le x \le a_3^\circ, \\ 0, & a_3^\circ \le x < \infty; \end{cases}$$

where, f, f°, g, g° are functions from $\mathbb{R} \to [0, 1]$, f, and g° are strictly increasing functions, and g, f° are strictly decreasing functions with $0 \le f(x) + f^\circ(x) \le 1$ as well as $0 \le g(x) + g^\circ(x) \le 1$.

Definition 3. (Wang et al. [33]). A trapezoidal, intuitionistic fuzzy number is referred to as $\bar{M}_{IT} = (a_1, a_2, a_3, a_4)$, $(a_1^\circ, a_2, a_3, a_4^\circ)$, where $a_1^\circ \le a_1 \le a_2 \le a_3 \le a_4 \le a_4^\circ$, and the membership function and non-membership functions are

$$\mu_{\bar{M}_{IT}} = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_1, \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \le x \le a_4, \\ 0, & \text{otherwise}, \end{cases}$$

$$\eta_{\bar{M}_{IT}} = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1^\circ}, & a_1^\circ \le x \le a_1, \\ \dfrac{x - a_3}{a_4^\circ - a_3}, & a_3 \le x \le a_4^\circ, \\ 0, & \text{otherwise}. \end{cases}$$

Definition 4. (Smarandache, [34]). Let X be a non-empty set. Then, a neutrosophic set can be defined by $\bar{M}^N = \{\langle x, U_{\bar{M}^N}(x), V_{\bar{M}^N}(x), W_{\bar{M}^N}(x)\rangle : x \in X,\quad U_{\bar{M}^N}(x), V_{\bar{M}^N}(x), W_{\bar{M}^N}(x) \in]^-0, I^+[\}$, where $U_{\bar{M}^N}(x)$, $V_{\bar{M}^N}(x)$, and $W_{\bar{M}^N}(x)$ are truth, indeterminacy, and falsity membership functions, respectively. In addition, no restriction on the summation of them is imposed. Therefore, we obtain $^-0 \le U_{\bar{M}^N}(x) + V_{\bar{M}^N}(x) + W_{\bar{M}^N}(x) \le 3^+$ and $]-^0, 1^+[$ is non-standard unit interval.

Definition 5. (Wang et al. [33]). Let X be a non-empty set. The single-valued neutrosophic set $\bar{M}^{N_{SV}}$ of X can be defined as follows:

$$\bar{M}^{N_{SV}} = \left\{\langle x, U_{\bar{M}^N}(x), V_{\bar{M}^N}(x), W_{\bar{M}^N}(x)\rangle : x \in X\right\},$$

where $U_{\bar{M}^N}(x)$, $V_{\bar{M}^N}(x)$, as well as $U_{\bar{M}^N}(x) \in [0, 1]$ for each $x \in X$ and $0 \le U_{\bar{M}^N}(x) + V_{\bar{M}^N}(x) + W_{\bar{M}^N}(x) \le 3$.

Definition 6. (Thamaraiselvi and Santhi, [35]). Let $u_{\tilde{a}^N}$, $\zeta_{\tilde{a}^N}$, $\xi_{\tilde{a}^N} \in [0, 1]$, and a_1, a_2, a_3, $a_4 \in R$ such that $a_1 \le a_2 \le a_3 \le a_4$ The single-valued trapezoidal neutrosophic (SVTRN) number $\tilde{a}^N\langle(a_1, a_2, a_3, a_4); u_{\tilde{a}^N}, \zeta_{\tilde{a}^N}, \xi_{\tilde{a}^N}\rangle$ is a special neutrosophic set on real line R, where the truth, indeterminacy, and falsity membership functions are as follows:

$$\mu_{\tilde{a}^N}(x)=\begin{cases} u_{\tilde{a}^N}\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1\le x\le a_2,\\ u_{\tilde{a}^N}, & a_2\le x\le a_3,\\ u_{\tilde{a}^N}\left(\dfrac{a_4-x}{a_4-a_3}\right), & a_3\le x\le a_4,\\ 0, & \text{otherwise},\end{cases}$$

$$v_{\tilde{a}^N}(x)=\begin{cases} \dfrac{a_2-x+\zeta_{\tilde{a}^N}(x-a_1)}{a_2-a_1}, & a_1\le x\le a_2,\\ \zeta_{\tilde{a}^N}, & a_2\le x\le a_3,\\ \dfrac{x-a_3+\zeta_{\tilde{a}^N}(a_4-x)}{a_4-a_3}, & a_3\le x\le a_4,\\ 1, & \text{otherwise},\end{cases}$$

$$\pi_{\tilde{a}^N}(x)=\begin{cases} \dfrac{a_2-x+\zeta_{\tilde{a}^N}(x-a_1)}{a_2-a_1}, & a_1\le x\le a_2,\\ \zeta_{\tilde{a}^N}, & a_2\le x\le a_3,\\ \dfrac{x-a_3+\zeta_{\tilde{a}^N}(a_4-x)}{a_4-a_3}, & a_3\le x\le a_4,\\ 1, & \text{otherwise},\end{cases}$$

where $u_{\tilde{a}^N}$, $\zeta_{\tilde{a}^N}$, and $\xi_{\tilde{a}^N}$ represent the respective maximum truth, minimum indeterminacy, and minimum falsity membership degrees. An SVTRN number $\tilde{a}^N=\langle(a_1,a_2,a_3,a_4);\ u_{\tilde{a}^N},\zeta_{\tilde{a}^N},\xi_{\tilde{a}^N}\rangle$ can be expressed as an ill-defined quantity about a, which is approximately equal to $[a_2,a_3]$.

Definition 7. (Thamaraiselvi and Santhi, [35]). Suppose $\tilde{a}^N=\langle(a_1,a_2,a_3,a_4);\ u_{\tilde{a}^N},\zeta_{\tilde{a}^N},\xi_{\tilde{a}^N}\rangle$, as well as $\tilde{b}^N=\langle(b_1,b_2,b_3,b_4);\ u_{\tilde{b}^N},\zeta_{\tilde{b}^N},\xi_{\tilde{b}^N}\rangle$ are two SVTRN numbers with $c\ne 0$. Then,

(1) $\tilde{a}^N(+)\tilde{b}^N=\langle(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4);\ u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle$.

(2) $\tilde{a}^N(-)\tilde{b}^N=\langle(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1);\ u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle$.

(3) $$\tilde{a}^N\otimes\tilde{b}^N=\begin{cases}\langle(a_1b_1,a_2b_2,a_3b_3,a_4b_4);\ u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\\ \zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle, & a_4>0,\ b_4>0,\\ \langle(a_1b_4,a_2b_3,a_3b_2,a_4b_1);\ u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\\ \zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle, & a_4<0,\ b_4>0,\\ \langle(a_4b_4,a_3b_3,a_2b_2,a_1b_1);u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\\ \zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle, & a_4<0,\ b_4<0.\end{cases}$$

(4) $$\frac{\tilde{a}}{\tilde{b}}=\begin{cases}\langle(a_1/b_4,a_2/b_3,a_3/b_2,a_4/b_1);\ u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\\ \zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle, & a_4>0,\ b_4>0,\\ \langle(a_4/b_4,a_3/b_3,a_2/b_2,a_1/b_1);\ u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\\ \zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle, & a_4<0,\ b_4>0,\\ \langle(a_4/b_1,a_3/b_2,a_2/b_3,a_1/b_4);u_{\tilde{a}^N}\wedge u_{\tilde{b}^N},\\ \zeta_{\tilde{a}^N}\vee\zeta_{\tilde{b}^N},\xi_{\tilde{a}^N}\vee\xi_{\tilde{b}^N}\rangle, & a_4<0,\ b_4<0.\end{cases}$$

(5) $$c\tilde{a}^N=\begin{cases}\langle(ca_1,ca_2,ca_3,ca_4);\ u_{\tilde{a}^N},\zeta_{\tilde{a}^N},\xi_{\tilde{a}^N}\rangle, & c>0,\\ \langle(ca_4,ca_3,ca_2,ca_1);\ u_{\tilde{a}^N},\zeta_{\tilde{a}^N},\xi_{\tilde{a}^N}\rangle, & c<0.\end{cases}$$

(6) $$(\tilde{a}^N)^{-1}=\begin{cases}\left\langle\left(\frac{1}{a_4},\frac{1}{a_3},\frac{1}{a_2},\frac{1}{a_1}\right);\ u_{\tilde{a}^N},\zeta_{\tilde{a}^N},\xi_{\tilde{a}^N}\right\rangle,\\ \tilde{a}^N\ne 0.\end{cases}$$

Definition 8. (Thamaraiselvi and Santhi [35]). Suppose $\tilde{a}^N=\langle(a_1,a_2,a_3,a_4);\ u_{\tilde{a}^N},\zeta_{\tilde{a}^N},\xi_{\tilde{a}^N}\rangle$ is an SVTRN number. Then

(i) Score function

$$S(\tilde{a}^N)=(1/16)*(a+1+a_2+a_3+a_4)*\Big(\mu_{\tilde{a}^N}+\big(1-v_{\tilde{a}^N}\big)+\big(1-\pi_{\tilde{a}^N}\big)\Big).$$

(ii) Accuracy function

$$A(\tilde{a}^N)=(1/16)*(a+1+a_2+a_3+a_4)*\Big(\mu_{\tilde{a}^N}+\big(1-v_{\tilde{a}^N}\big)+\big(1+\pi_{\tilde{a}^N}\big)\Big).$$

Definition 9. (Thamaraiselvi and Santhi [35]). Suppose $\tilde{a}^N$, $\tilde{b}^N$ are two arbitrary SVTRN numbers. Then

(i) If $S(\tilde{a}^N)<S(\tilde{b}^N)$, then $\tilde{a}^N<\tilde{b}^N$.

(ii) If $S(\tilde{a}^N)=S(\tilde{b}^N)$ and if $\tilde{a}^N=\tilde{b}^N$,

(1) $A(\tilde{a}^N)<A(\tilde{b}^N)$, then $\tilde{a}^N<\tilde{b}$,

(2) $A(\tilde{a}^N)>A(\tilde{b}^N)$, then $\tilde{a}^N>\tilde{b}$, and

(3) $A(\tilde{a}^N)=A(\tilde{b}^N)$, then $\tilde{a}^N=\tilde{b}$.

3 Problem formulation and solution concept

Consider the following single-valued neutrosophic QFP problem:

$$\max\tilde{Z}^N=\frac{f(x,\tilde{P}^N,\tilde{C}^N,\tilde{\alpha}^N)}{g(x,\tilde{Q}^N,\tilde{D}^N,\tilde{\beta}^N)} \tag{1}$$

subject to

$$x\in\tilde{X}^N=\{x\in R^n:\tilde{A}_j^N\otimes x_j(\le,=,\ge)\tilde{b}_i^N,\quad i,j=1,2,\ldots,n\}. \tag{2}$$

Here $x\in R^n$, $\tilde{b}_i^N\in R^{n\times 1}$, $\tilde{A}_i^N\in R^{n\times n}$, $\tilde{P}^N\in R^{n\times n}$, $\tilde{Q}^N\in R^{n\times n}$, $\tilde{C}^N\in R^{1\times n}$, and $\tilde{\alpha}^N$, $\tilde{\beta}^N$ are SVTRN numbers. It is observed that $f=x^T\tilde{P}^Nx+\tilde{\alpha}^N$, and $g=x^T\tilde{Q}^N+(\tilde{D}^N)^Tx+\tilde{\beta}^N$. Also, it is assumed that $g(x,\tilde{Q}^N,\tilde{D}^N,\tilde{\beta}^N)>0;\ \forall x\in\tilde{X}^N$.

Definition 10. A point x satisfies the constraints (2) is said to be a feasible point.

Definition 11. A feasible point x° is called an SVTRN optimal solution to problems (1)–(2), if $\tilde{Z}^N(x)(\geq)\tilde{Z}^N(x^\circ)$, for each feasible point x.

According to the score function in Definition 10, problem (1) is converted into the following deterministic form:

$$\max Z = \frac{f(x, P, C, \alpha)}{g(x, Q, D, \beta)} \tag{3}$$

subject to

$$\begin{aligned} x \in X = \{x \in R^n : A_j x_i (\leq, =, \geq) b_i,\, x_j \geq 0, \\ i, j = 1, 2, \ldots, n\}. \end{aligned} \tag{4}$$

Definition 12. (Sivri et al. [36]). Assume that the function G satisfies class $C^{(1)}$. The first two terms of the Taylor series generated by $G(x_1, x_2, \ldots, x_n)$ at $B(a_1, a_2, \ldots, a_n)$ are

$$\begin{aligned} F(B) + \frac{\partial}{\partial x_1} G(B)(x_1 - a_1) + \frac{\partial}{\partial x_2} G(B)(x_2 - a_2) \\ + c \cdots + \frac{\partial}{\partial x_n} G(B)(x_n - a_n) = 0. \end{aligned} \tag{5}$$

4 Proposed method

In this section, a solution method to NQFP problem is presented with the following steps:

Step 1: Convert the NQFP problems (1)–(2) into the corresponding crisp QFP problems (3)–(4).

Step 2: Choose an initial non-zero feasible point (arbitrary) (say, $x^\bullet$).

Step 3: Expand the objective function in (3) using Taylor series about the arbitrary non-zero feasible point $x^\bullet$ to linearize the objective function of problems (3)–(4).

Step 4: Solve the following LPP using any optimization techniques

$$\begin{aligned} &\max Z = E^T x, \quad \text{subject to} \\ &x \in X = \{x \in R^n : A_j x_j (\leq, =, \geq) b_i,\, x_j \geq 0, \\ &\qquad i, j = 1, 2, \ldots, n\}. \end{aligned} \tag{6}$$

Let the optimal solution be represented by $\hat{x}$.

Step 5: Expand the function of problem (3) using Taylor series about the optimal solution $\hat{x}$.

Step 6: Reconstruct the LPP based on step 5 following the same constrained.

Step 7: Solve the LPP obtained from step 6, let the optimal solution be $\hat{x}$.

Step 8: Check the two solutions $\hat{x}$ from step 5, and $\hat{x}$ from step 7. If there is an overlap between them, then it is an optimal solution of the problems (3)–(4). Otherwise, go to step 10.

Step 9: Estimate the neutrosophic optimal solution of problems (3)–(4).

Step 10: Stop.

A flowchart of the solution method is demonstrated in Figure 1.

5 Numerical experimentation

Consider the following NQFP problem:

$$\max \tilde{Z}^N = \frac{(x_1 \;\; x_2)\tilde{P}^N \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus (\tilde{C}^N)^T x \oplus \tilde{\alpha}^N}{(x_1 \;\; x_2)\tilde{Q}^N \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus (\tilde{D}^N)^T x \oplus \tilde{\beta}^N} \tag{7}$$

subject to

$$\begin{pmatrix} \tilde{a}_{11}^N & \tilde{a}_{12}^N \\ \tilde{a}_{21}^N & \tilde{a}_{22}^N \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} (\leq) \begin{pmatrix} \tilde{b}_1^N \\ \tilde{b}_2^N \end{pmatrix}; \quad x_1,\; x_2 \geq 0. \tag{8}$$

Here

$$\tilde{P}^N = \begin{pmatrix} \langle (0,1,2,3);\, 0.3, 0.5, 0.7 \rangle & \langle (0,1,3,6);\, 0.7, 0.5, 0.3 \rangle \\ \langle (1,2,3,4);\, 0.2, 0.9, 0.6 \rangle & \langle (1,3,4,5);\, 0.3, 0.6, 0.5 \rangle \end{pmatrix},$$

$$\tilde{Q}^N = \begin{pmatrix} \langle (1,2,4,5);\, 0.2, 0.8, 0.5 \rangle & \langle (1,2,3,4);\, 0.2, 0.7, 0.8 \rangle \\ \langle (2,3,4,6);\, 0.3, 0.7, 0.5 \rangle & \langle (0,1,2,3);\, 0.3, 0.7, 0.5 \rangle \end{pmatrix},$$

$$\tilde{C}^N = (\langle (1,2,4,5);\, 0.2, 0.8, 0.5 \rangle \;\; \langle (9,11,14,16);\, 0.5, 0.4, 0.7 \rangle),$$

$$\tilde{D}^N = \langle (15,17,19,22);\, 0.4, 0.8, 0.4 \rangle \; \langle (8,12,15,16);\, 0.5, 0.5, 0.8 \rangle,$$

$$\begin{pmatrix} \tilde{a}_{11}^N & \tilde{a}_{12}^N \\ \tilde{a}_{21}^N & \tilde{a}_{12}^N \end{pmatrix} = \begin{pmatrix} \langle (-6,-4,-3,-2);\, 0.3, 0.7, 0.5 \rangle & \langle (1,3,4,5);\, 0.3, 0.6, 0.5 \rangle \\ \langle (2,3,4,6);\, 0.3, 0.7;\, 0.5 \rangle & \langle (3,5,6,8);\, 0.6, 0.5, 0.4 \rangle \end{pmatrix},$$

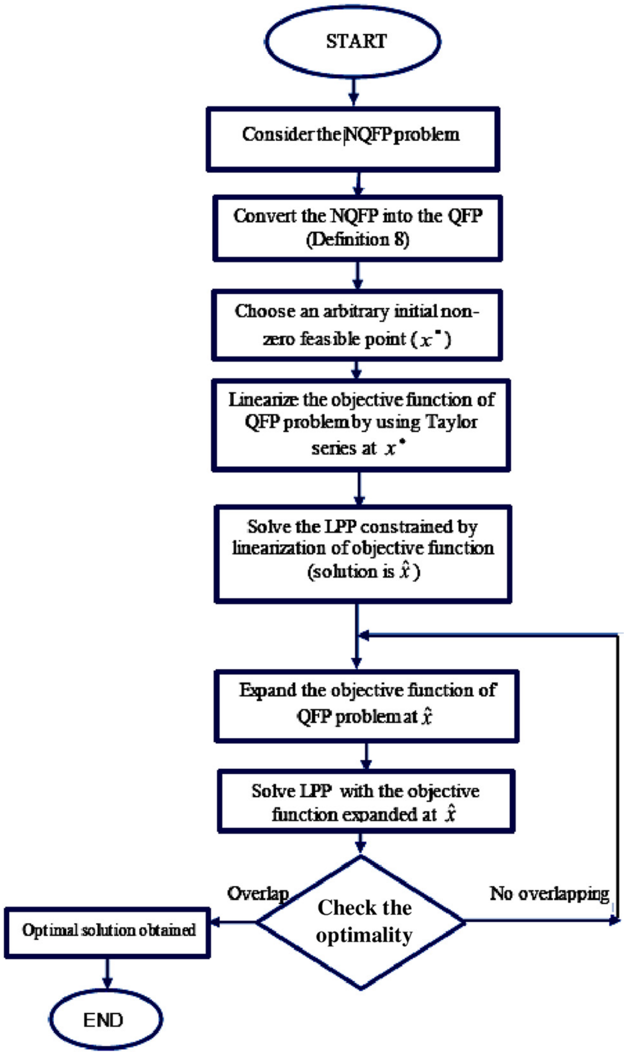

Figure 1: Flowchart of the solution method.

$$\begin{pmatrix} \tilde{b}_1^N \\ \tilde{b}_2^N \end{pmatrix} = \begin{pmatrix} \langle(1, 2, 4, 5); 0.2, 0.8, 0.5\rangle \\ \langle(12, 15, 19, 22); 0.6, 0.4, 0.5\rangle \end{pmatrix}$$

$\tilde{\alpha}^N = \langle(5, 8, 10, 14); 0.3, 0.6, 0.6\rangle$, $\tilde{\beta}^N = \langle(14, 16, 20, 27); 0.8, 0.4, 0.7\rangle$. The corresponding crisp QFP problem is presented as follows:

Step 1:

$$\max Z = \frac{(x_1 \ \ x_2)\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus (1 \ \ 4)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus 3}{(x_1 \ \ x_2)\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus (6 \ \ 4)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \oplus 8} \tag{9}$$

subject to

$$\begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 1 \\ 7 \end{pmatrix}; \quad x_1, x_2 \ge 0. \tag{10}$$

Equivalently,

$$\max Z = \frac{x_2^2 + x_1x_2 + x_1 + 4x_2 + 3}{x_1^2 + x_1x_2 + 6x_1 + 4x_2 + 8} \text{ subject to}$$
$$\begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 1 \\ 7 \end{pmatrix}; \tag{11}$$
$$x_1, x_2 \ge 0.$$

Step 2–3: Let $x^{\bullet} = (1, 1)$ be an initial feasible solution (an arbitrary) and depending on the Taylor series, expand the objective function of problem (5) about $x^{\bullet}$ as follows:

$$\begin{aligned} \frac{\partial Z}{\partial x_1} = {} & [(x_1^2 + x_1x_2 + 6x_1 + 4x_2 + 8)(x_2 + 1) \\ & - (x_2^2 + x_1x_2 + 4x_2 + 3)(2x_1 + x_2 \\ & + 6)]/[(x_1^2 + x_1x_2 + 6x_1 + 4x_2 + 8)^2], \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{\partial Z}{\partial x_2} = {} & [(x_1^2 + x_1x_2 + 6x_1 + 4x_2 + 8)(2x_2 + x_1 + 4) \\ & - (x_2^2 + x_1x_2 + x_1 + 4x_2 + 3)(x_1 \\ & + 4)]/[(x_1^2 + x_1x_2 + 6x_1 + 4x_2 + 8)^2]. \end{aligned} \tag{13}$$

Step 4: Construct and solve the following LPP:

$$\max Z = -0.125x_1 + 0.225x_2 \quad \text{subject to} \tag{14}$$

$$\begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 1 \\ 7 \end{pmatrix}; \quad x_1, x_2 \ge 0. \tag{15}$$

The optimal solution is $\hat{x} = (1.667, 2.667)$, and the corresponding optimum value is $\hat{Z} = 0.3917$.

Step 5–7: Expand the objective function of problem (11) again about the obtained optimal $\hat{x} = (1.667, 2.667)$, and establish the following LPP:

$$\max Z = -0.148x_1 + 0.188x_2 \quad \text{subject to}$$
$$\begin{pmatrix} -1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \le \begin{pmatrix} 1 \\ 7 \end{pmatrix}; \quad x_1, x_2 \ge 0.$$

The optimal solution is $\hat{x} = (1.667, 2.667)$, and the corresponding optimum value is $\hat{Z} = 0.25468$.

Step 8: The optimal solutions from step 4, and steps 5–7, are the same. Therefore, the optimal solution for the problem (2) is $x^{\circ} = (x_1^{\circ}, x_2^{\circ}) = (1.667, 2.667)$, and the corresponding optimum value is $Z^{\circ} = 0.74923$.

Thus, the solutions are presented in Table 1 as follows:

It is observed that the results obtained by the suggested method is the same as obtained by Sivri et al. [36].

Table 1: Solution of numerical example

In crisp environment	In neutrosophic environment
$x_1^\circ = 1.667$	$x_1 = \langle(3, 5, 6, 8);\ 0.6, 0.5, 0.4\rangle$
$x_2^\circ = 2.667$	$x_2 = \langle(5, 8, 10, 14);\ 0.3, 0.6, 0.6\rangle$
$Z^\circ = 0.749239$	$x_2 = \langle(0.227, 0.583, 1.031, 2.226);\ 0.2, 0.9, 0.9\rangle$

6 Discussion of the results

In the optimum value, as obtained in Section 5, $Z = \langle(0.227, 0.583, 1.031, 2.006);\ 0.2, 0.9, 0.9\rangle$, the total minimum value is greater than 0.227, less than 2.006, and the total minimum value lies in the range from 0.583 to 1.031, the overall level of acceptance or satisfaction or the truthfulness is 20%. In addition, for the values of total minimum value, the degrees of truthfulness, indeterminacy, and falsity are as follows:

$$\mu(x) = \begin{cases} 0.2\left(\dfrac{x - 0.227}{0.583 - 0.227}\right), & 0.227 \le x \le 0.583, \\ 0.2 & 0.583 \le x \le 1.031, \\ 0.2\left(\dfrac{2.006 - x}{2.006 - 1.031}\right) & 1.031 \le x \le 2.006, \\ 0, & \text{otherwise}, \end{cases}$$

$$\nu(x) = \begin{cases} \dfrac{0.583 - x + 0.9(x - 0.227)}{2.006 - 0.227}, & 0.227 \le x \le 0.583, \\ 0.9 & 0.583 \le x \le 1.031, \\ \dfrac{x - 1.031 + 0.9(2.006 - x)}{2.006 - 1.031} & 1.031 \le x \le 2.006, \\ 1, & \text{otherwise}, \end{cases}$$

$$\pi(x) = \begin{cases} \dfrac{0.583 - x + 0.9(x - 0.227)}{0.583 - 0.227}, & 0.227 \le x \le 0.583, \\ 0.9 & 0.583 \le x \le 1.031, \\ \dfrac{x - 1.031 + 0.9(2.006 - x)}{2.006 - 1.031} & 1.031 \le x \le 2.006, \\ 1, & \text{otherwise}. \end{cases}$$

Thus, the decision maker concludes that the total minimum value is between 0.227 and 2.006 with their truth degree, indeterminacy, and falsity degrees.

7 Conclusion

In this article, NQFP problem has been studied and a solution methodology has been proposed. The advantages of the suggested methodology can be viewed in terms of its applicability. In other words, the suggested method can handle QFP with homogeneous or non-homogenous constraints, and also, for factorized or non-factorized objective function, can be applied to solve enormous types of QFP with different types of parameter input data, easier than algebraic methods, and reduced the effort to obtain final solution. Finally, the article would be extended in different topics of Operations Research in future work.

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