OPTIMUM STATISTICAL TEST PROCEDURE

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Abstract. In this paper we search for the optimum tests that minimize the sum of two error probabilities.

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1. Introduction

Let X be a random variable having probability distribution $P(X/\theta)$, $\theta \in \Theta$. It is desired to test $H_0 : \theta \in \Theta$ against $H_1 : \theta \in \Theta_1 = \Theta - \Theta_0$. Let S denote the sample space of outcomes of an experiment and $\underline{x} = (x_1, x_2, ..., x_n)$ denote an arbitrary element of S. A test procedure consists in diving the sample space into two regions W and $S - W$ and deciding to reject H_0 if the observed $\underline{x} \in W$. The region W is called the critical region. The function $\gamma(\theta) = P_\theta(\underline{x} \in W) = P_\theta(W)$, say, is called the power function of the test.

We consider first the case where Θ_0 consists of a single element, θ_0 and its complement Θ_1 also has a single element θ_1. We want to test the simple hypothesis $H_0 : \theta = \theta_0$ against the simple alternative hypothesis $H_1 : \theta = \theta_1$.

Let $L_0 = L(X/H_0)$ and $L_1 = L(X/H_1)$ be the likelihood functions under H_0 and H_1 respectively. In the Neyman-Pearson set up the problem is to determine a critical region W such that

$$\gamma(\theta_0) = P_{\theta_0}(W) = \int_W L_0 dx = \alpha, \text{ an assigned value, and} \tag{1}$$

$$\gamma(\theta_1) = P_{\theta_1}(W) = \int_W L_1 dx \text{ is maximum} \tag{2}$$

compared to all other critical regions satisfying (1). If such a critical region exists it is called the most powerful critical region of level α.

By the Neyman-Pearson lemma, the most powerful critical region W_0 for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ is given by

$$W_0 = \{\underline{x} : L_1 \geq kL_0\}$$

provided there exists a k such that (1) is satisfied.

For this test $\gamma(\theta_0) = \alpha$ and $\gamma(\theta_1) \to 1$ as $n \to \alpha$.

But for any good test we must have $\gamma(\theta_0) \to 0$ and $\gamma(\theta_1) \to 1$ as $n \to \infty$ because complete discrimination between the hypotheses H_0 and H_1 should be possible as the sample size becomes indefinitely large.

Thus, for a good test it is required that the two error probabilities α and β should depend on the sample size n and both should tend to zero as $n \to \infty$.

We describe below test procedures which are optimum in the sense that they minimize the sum of the two error probabilities as compared to any other test. Note that minimizing $(\alpha + \beta)$ is equivalent to maximising

$$1 - (\alpha + \beta) = (1 - \beta) - \alpha = \text{ Power-Size.}$$

Thus, an optimum test maximises the difference of power and size as compared to any other test.

Definition 1. A critical region W_0 will be called *optimum* if

$$\int_{W_0} L_1 dx - \int_{W_0} L_0 dx \geq \int_W L_1 dx - \int_W L_0 dx, \tag{3}$$

for every other critical region W.

Lemma 1. *For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ the region*

$$W_0 = \{\underline{x} : L_1 \geq L_0\} \textit{ is optimum.}$$

Proof. W_0 is such that inside W_0, $L_1 \geq L_0$ and outside W_0, $L_1 < L_0$. Let W be any other critical region

$$\begin{aligned}\int_{W_0}(L_1 - L_0)dx - \int_W (L_1 - L_0)dx &= \int_{W_0 \cap W^c}(L_1 - L_0)dx - \int_{W \cap W_0^c}(L_1 - L_0)dx, \\ &\quad \text{by subtracting the integrals over the common region } W_0 \cap W \\ &\geq 0\end{aligned}$$

since the integrand of first integral is positive and the integrand under second integral is negative. Hence (3) is satisfied and W_0 is an optimum critical region.

Example 1. Consider a normal population $N(\theta, \sigma^2)$, where σ^2 is known. It is desired to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, $\theta_1 > \theta_0$.

$$L(x/\theta) = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\sum\limits_{i=1}^{n}\frac{(x_i-\theta)^2}{2\sigma^2}}$$

$$\frac{L_1}{L_0} = \frac{e^{-\frac{\sum(x_i-\theta_1)^2}{2\sigma^2}}}{e^{-\frac{\sum(x_i-\theta_0)^2}{2\sigma^2}}}$$

The optimum test rejects H_0 if $\dfrac{L_1}{L_0} \geq 1$,

i.e., if $\log\dfrac{L_1}{L_0} \geq 0$,

i.e., if $-\dfrac{\sum(x_i - \theta_1)^2}{2\sigma^2} + \dfrac{\sum(x_i - \theta_0)^2}{2\sigma^2} \geq 0$,

i.e., if $2\theta_1 \sum x_i - n\theta_1^2 - 2\mu_0 \sum x_i + n\theta_0^2 \geq 0$,

i.e., if $(2\theta_1 - 2\theta_0)\sum x_i \geq n(\theta_1^2 - \theta_0^2)$,

i.e., if $\dfrac{\sum x_i}{n} \geq \dfrac{\theta_1 + \theta_0}{2}$,

i.e., if $\overline{x} \geq \dfrac{\theta_1 + \theta_0}{2}\cdot$

Thus the optimum test rejects H_0 if $\overline{x} \geq \dfrac{\theta_1 + \theta_0}{2}\cdot$

We have $\alpha = P_{H_0}\left[\overline{x} \geq \dfrac{\theta_1 + \theta_0}{2}\right] = P_{H_0}\left[\dfrac{\overline{x} - \theta_0}{\sigma/\sqrt{n}} \geq \dfrac{\sqrt{n(\theta_1 - \theta_0)}}{2\sigma}\right]$.

Under H_0, $\dfrac{\overline{x} - \theta_0}{(\sigma/\sqrt{n})}$ follows $N(0, 1)$ distribution.

$$\therefore\ \alpha = 1 - \Phi\left(\frac{\sqrt{n}(\theta_1 - \theta_0)}{2\sigma}\right),$$

where Φ is the c.d.f. of an $N(0, 1)$ distribution

$$\beta = P_{H_1}\left[\overline{x} < \frac{\theta_1 + \theta_0}{2}\right] = P_{H_1}\left[\frac{\overline{x} - \theta_1}{\sigma/\sqrt{n}} < \frac{\sqrt{n}(\theta_1 - \theta_0)}{2\sigma}\right].$$

Under H_1, $\dfrac{\overline{x}-\theta_1}{(\sigma/\sqrt{n})}$ follows $N(0,1)$ distribution

$$\beta = 1-\Phi\left(\frac{\sqrt{n}(\theta_1-\theta_0)}{2\sigma}\right).$$

Here $\alpha=\beta$. It can be seen that $a=\beta\rightarrow 0$ as $n\rightarrow\infty$.

Example 2. For testing $H_0:\theta=\theta_0$ against $H_0:\theta=\theta_1, \theta_1<\theta_0$, the optimum test rejects H_0 when $\overline{x}\leq\dfrac{\theta_1+\theta_0}{2}\cdot$

Example 3. Consider a normal distribution $N(\theta,\sigma^2)$, θ known. It is desired to test $H_0:\sigma^2=\sigma_0^2$ against $H_1:\sigma^2=\sigma_1^2,\ \sigma_1^2>\sigma_0^2$. We have

$$\begin{aligned}
L(x/\sigma^2) &= \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{n}{2}} e^{-\sum\frac{(x_i-\theta)^2}{2\sigma^2}} \\
\frac{L_1}{L_0} &= \frac{\left(\dfrac{\sigma_0^2}{\sigma_1^2}\right)^{\frac{n}{2}} e^{-\frac{\sum(x_i-\theta)^2}{2\sigma_1^2}}}{e^{-\frac{\sum(x_i-\theta)^2}{2\sigma_0^2}}} \\
\log\frac{L_1}{L_0} &= -\frac{n}{2}\left(\log\sigma_1^2-\log\sigma_0^2\right)-\frac{\sum(x_i-\theta)^2}{2\sigma_1^2}+\frac{\sum(x_i-\theta)^2}{2\sigma_0^2} \\
&= -\frac{n}{2}\left(\log\sigma_1^2-\log\sigma_0^2\right)-\frac{\sum(x_i-\theta)^2}{2}\,\frac{\sigma_1^2-\sigma_0^2}{\sigma_1^2\sigma_0^2}
\end{aligned}$$

The optimum test rejects H_0 if $\dfrac{L_1}{L_0}\geq 1$

i.e., if $\dfrac{\sigma_1^2-\sigma_0^2}{2\sigma_1^2\sigma_0^2}\sum(x_i-\theta)^2\geq\dfrac{n}{2}\left(\log\sigma_1^2-\log\sigma_0^2\right)$

i.e., if $\dfrac{\sum(x_i-\theta)^2}{\sigma_0^2}\geq\dfrac{n\sigma_1^2}{\sigma_1^2-\sigma_0^2}\left(\log\sigma_1^2-\log\sigma_0^2\right)$

i.e., if $\sum\left(\dfrac{x_i-\theta}{\sigma_0}\right)^2\geq nc$,

where $c=\dfrac{\sigma_1^2}{\sigma_1^2-\sigma_0^2}\left(\log\sigma_1^2-\log\sigma_0^2\right)$.

Thus the optimum test rejects H_0 if $\sum\left(\dfrac{x_i-\theta}{\sigma_0}\right)^2\geq nc.$

Note that under $H_0 : \dfrac{x_i - \theta}{\sigma_0}$ follows $N(0,1)$ distribution. Hence $\sum \left(\dfrac{x_i - \theta}{\sigma_0}\right)^2$ follows under H_0, a chi-square distribution with n degrees of freedom (d.f.). Here

$$\alpha = P_{H_0}\left[\sum \left(\frac{x_i - \theta_0}{\sigma_0}\right)^2 \geq nc\right] = P\left[\chi^2_{(n)} \geq nc\right]$$

and

$$\begin{aligned} 1-\beta &= P_{H_1}\left[\sum \left(\frac{x_i - \theta}{\sigma_0}\right)^2 \geq nc\right] \\ &= P_{H_1}\left[\left(\frac{x_i - \theta}{\sigma_0}\right)^2 \geq \frac{nc\sigma_0^2}{\sigma_1^2}\right] \\ &= P_{H_1}\left[\chi^2_{(n)} \geq \frac{nc\sigma_0^2}{\sigma_1^2}\right]. \end{aligned}$$

Note that under H_1, $\sum \left(\dfrac{x_i - \sigma}{\sigma_1}\right)^2$ follows a chi-square distribution with n d.f. It can be seen that $\alpha \to 0$ and $\beta \to 0$ as $n \to \infty$.

Example 4. Let X follow the exponential family distributions

$$f(x/\theta) = c(\theta)e^{Q(\theta)T(x)h(x)}.$$

It is desired to test $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$

$$L(x/\theta) = [c(\theta)]^n e^{Q(\theta)T(x_i)} \prod_i h(x_i)$$

The optimum test rejects H_0 when $\log \dfrac{L_1}{L_0} \geq 0$

i.e., when $[Q(\theta_1) - q(\theta_0)] \sum t(x_i) \geq n \log \dfrac{c(\theta_0)}{c(\theta_1)}$

i.e., when $\sum T(x_i) \geq \dfrac{n \log \dfrac{c(\theta_0)}{c(\theta_1)}}{[Q(\theta_1) - Q(\theta_0)]}$ if $Q(\theta_1) - Q(\theta_0) > 0$

and rejects H_0, when $\sum T(x_i) \leq \dfrac{c(\theta_0)}{c(\theta_1)}[Q(\theta_1) - Q(\theta_0)]$ if $Q(\theta_1) - Q(\theta_0) < 0$.

Locally Optimum Tests

Let the random variable X have the probability distribution $P(x/\theta)$. We are interested in testing $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$. If W is any critical region, then the power of the test as a function of θ is

$$\gamma(\theta) = P_\theta(W) = \int_W L(x/\theta)dx.$$

We want to determine a region W for which

$$\gamma(\theta) - \gamma(\theta_0) = \int_W L(x/\theta)dx - \int_W L(x/\theta)$$

is a maximum.

When a uniformly optimum region does not exist, there is not a single region which is best for all alternatives. We may, however, find regions which are best for alternatives close to the null hypothesis and hope that such regions will also do well for distant alternatives. We shall call such regions locally optimum regions.

Let $\gamma(\theta)$ admit Taylor expansion about the point $\theta = \theta_0$. Then

$$\gamma(\theta) = \gamma(\theta_0) + (\theta - \theta_0)\gamma'(\theta_0) + \delta \quad \text{where } \delta \to 0 \text{ as } \theta \to \theta_0$$

$$\therefore \quad \gamma(\theta) - \gamma(\theta_0) = (\theta - \theta_0)\gamma'(\theta_0) + \delta.$$

If $|\theta - \theta_0|$ is small, $\gamma(\theta) - \gamma(\theta_0)$ is maximized when $\gamma'(\theta_0)$ is maximized.

Definition 2. A region W_0 will be called a *locally optimum critical region* if

$$\int_{W_0} L'(x/\theta_0)dx \geq \int_W L'(x/\theta_0)dx \tag{4}$$

for every other critical region W.

Lemma 2. *Let W_0 be the region $\{\underline{x} : L'(x/\theta_0) \geq L(x/\theta_0)\}$. Then W_0 is locally optimum.*

Proof. Let W_0 be the region such that inside it $L'(x/\theta_0) \geq L(x/\theta_0)$ and outside if $L'(x/\theta_0) < L(x/2_0)$. Let W be any other region.

$$\begin{aligned} &\int_{W_0} L'(x/\theta_0)dx - \int_W L'(x/\theta_0)dx \\ &\quad = \int_{W_0 \cap W^c} L'(x/\theta_0)dx - \int_{W_0^c \cap W} L'(x/\theta_0)dx \\ (*) \quad &\quad = \int_{W_0 \cap W^c} L'(x/\theta_0)dx + \int_{W_0^c \cup W} L'(x/\theta_0)dx \\ &\quad \geq \int_{W_0 \cap W^c} L(x/\theta_0)dx + \int_{W_0^c \cup W} L(x/\theta_0)dx \end{aligned}$$

since $L'(x/\theta_0)dx \geq L(x/\theta_0)$ inside both the regions of the integrals.

$\geq 0,$ since $L(x/\theta_0) \geq 0$ in all the regions.

Hence $\displaystyle\int_{W_0} L'(x/\theta_0)dx \geq \int_W L'(x/\theta_0)dx$ for every other region W.

To prove $(*)$ we have $\displaystyle\int_R L(x/\theta_0)dx + \int_{R^c} L(x/\theta_0)dx = 1$ for every region R. Differentiating, we have

$$\int_R L'(x/\theta_0)dx + \int_{R^c} L'(x/\theta_0)dx = 0$$

$$\int_R L'(x/\theta_0)dx = -\int_{R^c} L'(x/\theta_0)dx = 0.$$

In $(*)$, take $R^c = W_0^c \cap W$ and the relation is proved.

Similarly, if the alternatives are $H_1 : \theta < \theta_0$, the locally optimum critical region is

$$\{\underline{x} : L'(x/\theta_0) \leq L(x/\theta_0)\}.$$

Example 5. Consider $N(\theta, \sigma^2)$ distribution, σ^2 known. It is desired to test $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$

$$\begin{aligned}
L(x/\theta) &= \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{\frac{-\sum(x_i-\theta)^2}{2\sigma^2}} \\
\log L(x/\theta) &= n\log\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \frac{\sum(x_i-\theta)^2}{2\sigma^2} \\
\frac{L'(x/\theta)}{L(x/\theta)} &= \frac{\delta \log L(x/\theta)}{\delta\theta} = \frac{\sum(x_i-\theta)}{\sigma^2} = \frac{n(\overline{x}-\theta)}{\sigma^2} \\
\therefore\quad \frac{L'(x/\theta_0)}{L(x/\theta_0)} &= \frac{n(\overline{x}-\theta_0)}{\sigma^2}.
\end{aligned}$$

The locally optimum test rejects H_0,if

$$\frac{n(\overline{x}-\theta_0)}{\sigma^2} \geq 1 \quad \text{i.e.} \quad \overline{x} \geq \theta_0 + \frac{\sigma^2}{n}.$$

Now,

$$\begin{aligned}
\alpha &= P_{H_0}\left[\overline{x} \geq \theta_0 + \frac{\sigma^2}{n}\right] \\
&= P_{H_0}\left[\frac{\overline{x}-\theta_0}{\sigma/\sqrt{n}} \geq \sigma/\sqrt{n}\right] \\
&= 1 - \Phi\left(\sigma/\sqrt{n}\right), \text{ since under } H_0, \frac{\overline{x}-\theta_1}{\sigma/\sqrt{n}} \text{ follows } N(0,1) \text{ distribution.}
\end{aligned}$$

$$\begin{aligned}
1-\beta &= P_{H_1}\left[\overline{x} \geq \theta_0 + \sigma^2/n\right] \\
&= P_{H_1}\left[\frac{\overline{x}-\theta_1}{\sigma/\sqrt{n}} \geq -\frac{\theta_1-\theta_0}{\sigma/\sqrt{n}} + \frac{\sigma}{\sqrt{n}}\right] \\
&= 1 - \Phi\left[\frac{-(\theta_1-\theta_0\sqrt{n})}{\sigma} + \frac{\sigma}{\sqrt{n}}\right]
\end{aligned}$$

since under H_1, $\dfrac{\overline{x}-\theta_1}{\sigma/\sqrt{n}}$ follows $N(0,1)$ distribution.

Exercise. If $\theta_0 = 10,\ \theta_1 = 11,\ \sigma = 2,\ n = 16$, then $\alpha = 0.3085$, $1-\beta = 0.9337$, Power-Size $= 0.6252$.

If we reject H_0 when $\dfrac{\overline{x}-\theta_0}{\sigma/\sqrt{n}} > 1.64$, then $\alpha = 0.05,\ 1-\beta = 0.6406$, Power-Size $= 0.5906$. Hence Power-Size of locally optimum test is greater than Power-size of the usual test.

Locally Optimum Unbiased Test

Let the random variable X follows the probability distribution $P(x/\theta)$. Suppose it is desired to test $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. We impose the unbiasedness restriction

$$\gamma(\theta) \geq \gamma(\theta_0),\ \theta \neq \theta_0$$

and $\gamma(\theta) - \gamma(\theta_0)$ is a maximum as compared to all other regions. If such a region does not exist, we impose the unbiasedness restriction $\gamma'(\theta_0) = 0$.

Let $\gamma(\theta)$ admit Taylor expansion about the point $\theta = \theta_0$. Then

$$\gamma(\theta) = \gamma(\theta_0) + (\theta-\theta_0)\gamma'(\theta_0) + \frac{(\theta-\theta_0)^2}{2}\gamma''(\theta_0) + \eta$$

where $\eta \to 0$ as $\theta \to 0$.

$$\therefore\ \ \gamma(\theta) - \gamma(\theta_0) = (\theta-\theta_0)\gamma'(\theta_0) + \frac{(\theta-\theta_0)^2}{2}\gamma''(\theta_0) + \eta$$

Under the unbiasedness restriction $\gamma'(\theta_0) = 0$, if $|\theta - \theta_0|$ is small $\gamma(\theta) - \gamma(\theta_0)$ is maximized when $\gamma''(\theta_0)$ is maximized.

Definition 3. A region W_0 will be called a locally optimum unbiased region if

(5) $$\gamma'(\theta_0) = \int_{W_0} L'(x/\theta_0)dx = 0$$

and

(6) $$\gamma''(\theta_0) = \int_{W_0} L''(X/\theta_0)dx \geq \int_{W} L''(X/\theta_0)dx$$

for all other regions W satisfying (5).

Lemma 3. *Let W_0 be the region*

$$\{\underline{x} : L''(x/\theta_0) \geq L(x/\theta_0)\}.$$

Then W_0 is locally optimum unbiased.

Proof. Let W be any other region

$$\int_{W_0} L''(x/\theta_0)dx - \int_W L''(x/\theta_0)dx$$

$$= \int_{W_0\cap W^c} L''(x/\theta_0)dx + \int_{W_0\cap W} L''(x/\theta_0)dx$$

by subtracting the common area of W and W_0.

$$= \int_{W_0\cap W^c} L''(x/\theta_0)dx + \int_{W_0\cap W^c} L''(x/\theta_0)dx,$$

since $L''(x/\theta_0) \geq L(x/\theta)$ inside W_0 and outside W.

≥ 0 since $L(x/\theta) \geq 0$.

Example 6. Consider $N(\theta, \sigma^2)$ distribution, σ^2 known.

$$H_0 : \theta = \theta_0, \quad H_1 = \theta \neq \theta_0$$

$$L = \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n \ e^{-\frac{\sum(x_i-\theta)^2}{2\sigma^2}}$$

$$\frac{L''(x/\theta)}{L(x/\theta)} = \frac{n^2(\overline{x}-\theta)^2}{\sigma^4} - \frac{n}{\sigma^2}.$$

Locally optimum unbiased test rejects H_0

if $\dfrac{n^2(\overline{x}-\theta_0)^2}{\sigma^4} - \dfrac{n}{\sigma^2} \geq 1$ i.e., $\dfrac{n(\overline{x}-\theta_0)^2}{\sigma^2} \geq 1 + \dfrac{\sigma^2}{n}$

Under H_0, $\dfrac{n(\overline{x}-\theta_0)^2}{\sigma^2}$ follows $\chi^2_{(1)}$ distribution.

Testing Mean of a normal population when variance is unknown

Consider $N(\theta, \sigma^2)$ distribution, σ^2 known. For testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$, the critical function of the optimum test is given by

$$\phi_m(x) = \begin{cases} 1 & \text{if } L(x/\theta_1) \geq L(x/\theta_0) \\ 0 & \text{otherwise.} \end{cases}$$

On simplification, we get

$$\phi_m(x) = \begin{cases} 1 & \text{if } \overline{x} \geq \dfrac{\theta_0+\theta_1}{2} \text{ if } \mu_1 > \mu_0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\phi_m(x) = \begin{cases} 1 & \text{if } \overline{x} \geq \dfrac{\theta_0+\theta_1}{2} \text{ if } \mu_1 < \mu_0 \\ 0 & \text{otherwise.} \end{cases}$$

Consider the case when σ^2 is unknown.

For this case we propose a test which rejects H_0 when

$$\frac{\widehat{L}(x/\theta_1)}{\widehat{L}(x/\theta_0)} \geq 1$$

where $\widehat{L}(x/\theta_i)$, $(i = 0, 1)$, is the maximum of the likelihood under H_i obtained from $L(x/\theta_i)$ by replacing σ^2 by its maximum likelihood estimate

$$\widehat{\sigma}_i^2 = \frac{1}{n}\sum_{j=1}^{n}(x_j - \theta_i)^2; \ i = 0, 1.$$

Let $\phi_p(x)$ denote the critical function of the proposed test, then

$$\phi_p(x) = \begin{cases} 1 & \text{if } \widehat{L}(x/\theta_1) \geq \widehat{L}(x/\theta_0) \\ 0 & \text{otherwise} \end{cases}$$

On simplification, we get

$$\phi_p(x) = \begin{cases} 1 & \text{if } \overline{x} \geq \dfrac{\theta_0 + \theta_1}{2} \text{ if } \mu_1 > \mu_0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\phi_p(x) = \begin{cases} 1 & \text{if } \overline{x} \geq \dfrac{\theta_0 + \theta_1}{2} \text{ if } \mu_1 < \mu_0 \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the proposed test $\phi_p(x)$ is equivalent to $\phi_m(x)$, which is the optimum test that minimizes the sum of two error probabilities $(\alpha + \beta)$. Thus, we see that one gets the same test which minimizes the sum of the two error probabilities irrespective of whether σ^2 is known or unknown.

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