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Article *in* Journal of Intelligent and Fuzzy Systems · May 2019

DOI: 10.3233/JIFS-181742

Journal of Intelligent & Fuzzy Systems 37 (2019) 5743–5753
DOI:10.3233/JIFS-181742
IOS Press

On neutrosophic extended triplet groups (loops) and Abel-Grassmann's groupoids (AG-groupoids)

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Abstract. From the perspective of semigroup theory, the characterizations of a neutrosophic extended triplet group (NETG) and AG-NET-loop (which is both an Abel-Grassmann groupoid and a neutrosophic extended triplet loop) are systematically analyzed and some important results are obtained. In particular, the following conclusions are strictly proved: (1) an algebraic system is neutrosophic extended triplet group if and only if it is a completely regular semigroup; (2) an algebraic system is weak commutative neutrosophic extended triplet group if and only if it is a Clifford semigroup; (3) for any element in an AG-NET-loop, its neutral element is unique and idempotent; (4) every AG-NET-loop is a completely regular and fully regular Abel-Grassmann groupoid (AG-groupoid), but the inverse is not true. Moreover, the constructing methods of NETGs (completely regular semigroups) are investigated, and the lists of some finite NETGs and AG-NET-loops are given.

Keywords: Semigroup, neutrosophic extended triplet group (NETG), completely regular semigroup, Clifford semigroup, Abel-Grassmann's groupoid (AG-groupoid)

1. Introduction

Smarandache proposed the new concept of neutrosophic set, which is an extension of fuzzy set and intuitionistic fuzzy set [1]. Until now, neutrosophic sets have been applied to many fields [2–4], and some new theoretical studies are developed [5, 6].

As an application of the basic idea of neutrosophic sets (more general, neutrosophy), the new notion of neutrosophic triplet group (NTG) is introduced by Smarandache and Ali in [7, 8]. As a new algebraic structure, NTG is a generalization of classical group, but it has different properties from classical group. For NTG, the neutral element is relative and local, that is, for a neutrosophic triplet group $(N,^*)$, every element a in N has its own neutral element (denote by $neut\,(a)$) satisfying condition $a * neut\,(a) = neut\,(a)^* \, a = a$, and there exits at least one opposite element (denote by $anti\,(a)$) in N relative to $neut\,(a)$ such condition $a^* anti\,(a) = anti\,(a)^* \, a = neut\,(a)$. In the original definition of NTG in [8], $neut\,(a)$ is different from the traditional unit element. Later, the concept of neutrosophic extended triplet group (NETG) was introduced (see [7]), in which the neutral element may be traditional unit element, it is just a special case.

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For the structure of NETG, some exploratory research papers are published and a series of results are got [9–12]. Recently, we have analyzed these new results and studied them from the perspective of semigroup theory. Miraculously, we have obtained some unexpected results: every NETG is a completely regular semigroup, and the inverse is true. In fact, the research of completely regular semigroups originated from the study of Clifford [13], and have been greatly developed [14–16], and have been extended to a wide range of algebraic systems [17–20]. This paper will focus on the latest results of the authors, mainly discuss the relationships between neutrosophic extended triplet groups and completely regular semigroups.

Moreover, this paper also investigates the relationships between neutrosophic extended triplet loops and Abel-Grassmann's groupoids (AG-groupoids). The concept of an Abel-Grassmann's groupoid was first given by Kazim and Naseeruddin [21] in 1972 and they have called it a left almost semigroup (LA-semigroup). In [22], the same structure is called a left invertive groupoid. In [23–29], some properties and different classes of an AG-groupoid are investigated. In this paper, we combine the notions of neutrosophic extended triplet loop and AG-groupoid, introduce the new concept of Abel-Grassmann's neutrosophic extended triplet loop (AG-NET-loop), that is, AG-NET-loop is both AG-groupoid and neutrosophic extended triplet loop (NET-loop). We deeply analyze the internal connecting link between AG-NET-loop and completely regular AG-groupoid and obtain some important and interesting results.

2. Preliminaries

Definition 1. [7, 8] Let N be a non-empty set together with a binary operation*. Then, N is called a neutrosophic extended triplet set if for any$a \in N$, there exist a neutral of "a" (denote by $neut(a)$), and an opposite of "a" (denote by $anti(a)$), such that $neut(a) \in N$, $anti(a) \in N$ and:

$$a * neut(a) = neut(a)^* a = a;$$

$$a^* anti(a) = anti(a)^* a = neut(a).$$

The triplet $(a, neut(a), anti(a))$ is called a neutrosophic extended triplet.

Note that, for a neutrosophic triplet set $(N,^*)$, $a \in N$, $neut(a)$ and $anti(a)$ may not be unique. In order not to cause ambiguity, we use the following notations to distinguish:

$neut(a)$: denote any certain one of neutral of a;
$\{neut(a)\}$: denote the set of all neutral of a.
$anti(a)$: denote any certain one of opposite of a;
$\{anti(a)\}$: denote the set of all opposite of a.

Definition 2. [7, 8] Let $(N,^*)$ be a neutrosophic extended triplet set. Then, N is called a neutrosophic extended triplet group (NETG), if the following conditions are satisfied:

(1) $(N,^*)$ is well-defined, i.e., for any $a, b \in N$, one has $a * b \in N$.
(2) $(N,^*)$ is associative, i.e., $(a * b) * c = a * (b * c)$ for all $a, b, c \in N$.

N is called a commutative neutrosophic extended triplet group if for all $a, b \in N$, $a * b = b * a$.

Proposition 1. [11] *Let $(N,^*)$ be a NETG. Then*

(1) *$neut(a)$ is unique for any a in N.*
(2) *$neut(a) * neut(a) = neut(a)$ for any a in N.*
(3) *$neut(neut(a)) = neut(a)$ for any a in N.*

Definition 3. [11] Let $(N,^*)$ be a NETG. Then N is called a weak commutative neutrosophic extended triplet group (briefly, WCNETG) if $a * neut(b) = neut(b) * a$ *for all* $a,\ b \in N$.

Proposition 2. [11] *Let $(N,^*)$ be a NETG. Then $(N,^*)$ is weak commutative if and only if N satisfies the following conditions:*

(1) $neut(a) * neut(b) = neut(b) * neut(a)$ *for all* $a, b \in N$.
(2) $neut(a) * neut(b)^* a = a * neut(b)$ *for all* $a, b \in N$.

Proposition 3. [11] *Let $(N,^*)$ be a weak commutative NETG. Then (for all $a, b \in N$)*

(1) $neut(a) * neut(b) = neut(b^* a)$;
(2) $anti(a)^* anti(b) \in \{anti(b^* a)\}$.

Definition 4. [14] A semigroup $(S, *)$ will be called completely regular if there exists a unary operation $a_- \mapsto a^{-1}$ on S with the properties

$$(a^{-1})^{-1} = a, a^* a^{-1*} a = a, a^* a^{-1} = a^{-1*} a.$$

Proposition 4. [14] *Let $(S,^*)$ be a semigroup. Then the following statements are equivalent:*

(1) *S is completely regular;*
(2) *every element of S lies in a subgroup of S;*
(3) *every H-class in S is a group.*

Here, recall some basic concepts in semigroup theory. A non-empty subset A of a semigroup $(S,^*)$ is called a left ideal if $SA \subseteq A$, a right ideal if $AS \subseteq A$, and an ideal if it both a left and a right ideal. Evidently, every ideal (whether one- or two-sided) is a subsemigroup. If a is an element of a semigroup $(S,^*)$, the smallest left ideal containing a is $Sa \cup \{a\}$, which we may conveniently write as S^1a, and which we shall call the principle left ideal generated by a.

An equivalent relation L on S is defined by the rule that aLb if and only if $S^1a = S^1b$; an equivalent relation R on S is defined by the rule that aLb if and only if $aS^1 = bS^1$; denote $H = L \wedge R$, $D = L \vee R$, that is, aHb if and only if $S^1a = S^1b$ and $aS^1 = bS^1$; aDb if and only if $S^1a = S^1b$ or $aS^1 = bS^1$. An equivalent relation J on S is defined by the rule that aJb if and only if $S^1aS^1 = S^1bS^1$, where

$$S^1aS^1 = SaS \cup aS \cup Sa \cup \{a\}$$

That is, aJb if and only if there exists x, y, u, $v \in S^1$ for which $x^*a^*y = b$, $u^*b^*v = a$. The L-class (R-class, H-class, D-class, J-class) containing the element a will be written L_a (R_a, H_a, D_a, J_a).

Definition 5. [14] A semigroup $(S,^*)$ will be called Clifford semigroup, if it is completely regular and in which, for all x, y in S,

$$(x^*x^{-1})^*(y^*y^{-1}) = (y^*y^{-1})^*(x^*x^{-1}).$$

In an arbitrary semigroup S, we say that an element c is central if $c^*s = s^*c$ for every s in S. The set of central elements forms a subsemigroup of S, called the center of S.

Proposition 5. [14] *Let $(S,^*)$ be a semigroup. Then the following statements are equivalent:*

(1) *S is Clifford semigroup;*
(2) *S is a semilattice of groups;*
(3) *S is regular, and the idempotents of S are central.*

Abel-Grassmann's groupoid (AG-groupoid) [21, 22], is a groupoid $(S,^*)$ holding left invertive law, that is, for all $a, b, c \in S$, $(a^*b)^*\, c = (c^*b)^*\, a$. In an AG-groupoid the medial law holds, for all $a, b, c, d \in S$, $(a^*b)^*\, (c^*d) = (a^*c)^*\, (b^*d)$.

There can be a unique left identity in an AG-groupoid. In an AG-groupoid S with left identity the paramedial laws hold for all $a, b, c, d \in S$, $(a^*b)^*\, (c^*d) = (d^*c)^*\, (b^*a)$. Further if an AG-groupoid contain a left identity, hen he following law holds: for all $a, b, c \in S$, $a^*\, (b^*c) = b^*\, (a^*c)$.

An AG-groupoid is a non-associative algebraic structure midway between a groupoid and a commutative semigroup, because if an AG-groupoid contains right identity then it becomes a commutative semigroup.

Definition 6. [25] (1) An element a of an AG-groupoid $(S,^*)$ is called a regular if there exists $x \in S$ such that $a = (a^*x^*)^*a$ and S is called regular if all elements of S are regular.

(2) An element a of an AG-groupoid $(S,^*)$ is called a weakly regular if there exists $x, y \in S$ such that $a = (a * x) * (a * y)$ and S is called weakly regular if all elements of S are weakly regular.
(3) An element a of an AG-groupoid $(S,^*)$ is called an intra-regular if there exists $x, y \in S$ such that $a = \left(x^*a^2\right) * y$ and S is called an intra-regular if all elements of S are intra-regular.
(4) An element a of an AG-groupoid $(S,^*)$ *is* called a right regular if there exists $x \in S$ such that $a = a^2 * x = (a^*a) * x$ and S is called a right regular if all elements of S are right regular.
(5) An element a of an AG-groupoid $(S,^*)$ is called a left regular if there exists $x \in S$ such that $a = x^*a^2 = x * (a^*a)$ and S is called left regular if all elements of S are left regular.
(6) An element a of an AG-groupoid $(S,^*)$ is called a left quasi regular if there exists $x, y \in S$ such that $a = (x^*a) * (y^*a)$ and S is called left quasi regular if all elements of S are left quasi regular.
(7) An element a of an AG-groupoid $(S,^*)$ is called a completely regular if a is regular and left (right) regular. S is called completely regular if it is regular, left and right regular.

Proposition 6. [25] *If $(S,^*)$ is regular (weakly regular, intra-regular, right regular, left regular, left quasi regular, completely regular) AG-groupoid, then $S = S^2$*

Proposition 7. [25] *In an AG-groupoid $(S,^*)$ with left identity, the following are equivalent:*

(i) *S is weakly regular.*
(ii) *S is an intra-regular.*
(iii) *S is right regular.*
(iv) *S is left regular.*
(v) *S is left quasi regular.*
(vi) *S is completely regular.*

Definition 7. [26] An element a of an AG-groupoid $(S,^*)$ is called a fully regular element of S if there exist some $p, q, r, s, t, u, v, w, x, y, z \in S$ ($p, q, ..., z$ may be repeated) such that

$$a = \left(p^* a^2\right) * q = \left(r^* a\right) * (a * s)$$
$$= (a * t) * (a * u) = \left(a^* a\right) * v$$
$$= w * \left(a^* a\right) = \left(x^* a\right) * \left(y^* a\right)$$
$$= \left(a^2 * z\right)^* a^2.$$

An AG-groupoid $(S,^*)$ is called fully regular if all elements of S are fully regular.

A non-empty subset A of an AG-groupoid $(S,^*)$ called left (right) ideal of S if and only if $SA \subseteq A(AS \subseteq A)$ and is called two-sided ideal or ideal of S if and only if it is both left and right ideal of S.

Definition 8. [26] A non-empty subset A of an AG-groupoid $(S,^*)$ called semiprime if and only if

$$a^2 \in A \Rightarrow a \in A.$$

Definition 9. [26] An AG-groupoid is called left (right) simple if and only if it has no proper left (right) ideal and is called simple if and only if it has no proper two-sided ideal.

Proposition 8. [26] *The following conditions are equivalent for an AG-groupoid $(S,^*)$ with left identity:*

(i) $aS = S$, *for some* $a \in S$.
(ii) $Sa = S$, *for some* $a \in S$.
(iii) S *is simple*.
(iv) $AS = S = SA$, *where A two-sided ideal of S*.
(v) S *is fully regular*.

3. NETG and completely regular semigroup

Theorem 1. *Let $(N,^*)$ be a NETG. Then for all $a \in N$,*

(1) $p * neut(a) \in \{anti(a)\}$, *for any* $p \in \{anti(a)\}$;
(2) $p * neut(a) = q * neut(a) = neut(a) * q$, *for any* $p, q \in \{anti(a)\}$;
(3) $neut(p * neut(a)) = neut(a)$, *for any* $p \in \{anti(a)\}$;
(4) $a \in \{anti(p * neut(a))\}$, *for any* $p \in \{anti(a)\}$;
(5) $anti(p * neut(a)) * neut(p * neut(a)) = a$, *for any* $p \in \{anti(a)\}$.

Proof. (1) Suppose $p \in \{anti(a)\}$, then $p^* a = a * p = neut(a)$.

From this, and applying Proposition 1, we get$(p * neut(a))^* a = p * \left(neut(a)^* a\right) = p^* a = neut(a)$, $a * (p * neut(a)) = (a * p) * neut(a) = neut(a) * neut(a) = neut(a)$.

It follows that $p * neut(a) \in \{anti(a)\}$.

(2) Suppose $p, q \in \{anti(a)\}$, then $p^* a = a * p = neut(a); q^* a = a * q = neut(a)$.
Thus,
$p * neut(a) = p * (a * q) = (p^* a) * q = neut(a) * q$
$= (q^* a) * q = q * (a * q) = q * neut(a)$.
That is, $p * neut(a) = neut(a) * q = q * neut(a)$.

(3) For any $p \in \{anti(a)\}$, by Proposition 1 and (2), we have
$(p * neut(a)) * neut(a) = p * (neut(a) * neut(a)) = p * neut(a)$,
$neut(a) * (p * neut(a)) = (neut(a) * p) * neut(a) = (p * neut(a)) * neut(a)$
$= p * (neut(a) * neut(a)) = neut(a)$.
Moreover, using Proposition 1,
$(p * neut(a))^* a = p * \left(neut(a)^* a\right)$
$= p^* a = neut(a), a * (p * neut(a)) = (a * p) * neut(a) = neut(a) * neut(a) = neut(a)$.
Applying Definition 1, $neut(a) = neut(p * neut(a))$.

(4) For any $p \in \{anti(a)\}$, by Proposition 1, we have
$a * (p * neut(a)) = (a * p) * neut(a)$
$= neut(a) * neut(a) = neut(a)$,
$(p * neut(a))^* a = p * (a * neut(a))$
$= p^* a = neut(a)$.
By Definition 1 we know that $a \in \{anti(p * neut(a))\}$.

(5) Assume $p \in \{anti(a)\}$. For all $anti(p * neut(a)) \in \{anti(p * neut(a))\}$, by (2) we know that $anti(p * neut(a)) * neut(p * neut(a))$ is unique. Applying (4), $a \in \{anti(p * neut(a))\}$, it follows that

$$anti(p * neut(a)) * neut(p * neut(a))$$
$$= a * neut(p * neut(a)).$$

Using (3), $neut(p * neut(a)) = neut(a)$. *Therefore,*

$$anti(p * neut(a)) * neut(p * neut(a))$$
$$= a * neut(p * neut(a))$$
$$= a * neut(a) = a.$$

□

Theorem 2. *Let $(N, *)$ be a groupoid. Then N is a NETG if and only if it is a completely regular semigroup.*

Proof. Assume that N is a NETG. By Theorem 1, we define a unary operation $a_{\mid} \to a^{-1}$ on N as follows:

$a^{-1} = anti(a) * neut(a)$, for any a in N.

By Theorem 1 (2), a^{-1} is unique. Applying Theorem 1 (5) we get

$$(a^{-1})^{-1} = anti(anti(a) * neut(a)) *neut(anti(a) * neut(a)) = a.$$

Moreover, by Proposition 1,

$$a^*a^{-1*}a = a^*anti(a) * neut(a)^* a = a,$$

$$a^*a^{-1} = a^*anti(a) * neut(a) = neut(a)^* anti(a) = neut(a) = anti(a)^* a = anti(a) * neut(a) ^*a = a^{-1*}a.$$

Thus, by Definition 4, N is a completely regular semigroup.

Conversely, suppose that N is a completely regular semigroup. For any a in N, denote $neut(a) = a^*a^{-1}$, then

$$neut(a)^* a = a^*a^{-1*}a = a,$$

$$a * neut(a) = a^*a^*a^{-1} = a^*a^{-1*}a = a.$$

Moreover,

$$a^{-1*}a = a^*a^{-1} = neut(a).$$

By Definition 1, we know that N is a NETG, and$a^{-1} \in \{anti(a)\}$. □

Note that, in semigroup theory, a^{-1} is called inverse element, it is unique; in NETG, $anti(a)$ is called opposite element, it may be not unique, please see the following example.

Example 1. Let $N = \{a, b, c, d, e\}$, define operations $*$ on N as following Table 1. Then, $(N,^*)$ is a NETG and a completely regular semigroup. We can get that

$$a^{-1} = a; a^{-1*}a = a^*a^{-1} = a.$$

$$neut(a) = a, \{anti(a)\} = \{a, c, d, e\}.$$

Table 1
The operation * on N

*	a	b	c	d	e
a	a	b	a	a	a
b	b	a	b	b	b
c	a	b	d	c	a
d	a	b	c	d	a
e	a	b	a	a	e

4. Weak commutative NETG and Clifford semigroup

Applying Theorem 2 and Definition 5, we can get the following result (the proof is omitted).

Proposition 9. Let $(N,^*)$ *be a completely regular semigroup. Then N is a Clifford semigroup, if and only if it satisfies:*

$neut(a) * neut(b) = neut(b) * neut(a)$,
for all $a, b \in N$.

Theorem 3. *Let* $(N,^*)$ *be a groupoid. Then N is a weak commutative neutrosophic extended triplet group (NETG) if and only if it is a Clifford semigroup.*

Proof. Suppose that N is a weak commutative NETG. By Theorem 2, we know that N is a completely regular semigroup. Using Proposition 2, for any $a, b \in N$, $neut(a) * neut(b) = neut(b) * neut(a)$. Then, by Proposition 9 we know that N is a Clifford semigroup.

Conversely, assume that N is a Clifford semigroup. Applying Theorem 2 and Proposition 1, $neut(a) * neut(a) =$, for any a in . That is, $neut(a)$ is idempotent. Thus, by Proposition 3, $neut(a)$ is central. Therefore, for any b in N,

$$neut(a) * b = b * neut(a).$$

This means that N is a weak commutative NETG, by Definition 3. □

Applying Theorem 3 and Proposition 2, we can get the following result (the proof is omitted).

Proposition 10. *Let* $(N, *)$ *be a NETG. Then N is weak commutative, if and only if it satisfies:*

$$neut(a) * neut(b) = neut(b) * neut(a), \quad \textit{for all } a, b \in N.$$

In other words, in NETG, the following conditions are equivalent:

(1) $a * neut(b) = neut(b)^* a$, *for all* $a, b \in N$;
(2) $neut(a) * neut(b) = neut(b) * neut(a)$, *for all* $a, b \in N$

Now, we discuss the method of establishing Clifford semigroup (that is, weak commutative NETG) by two given groups.

Theorem 4. *Let* $(G_1, *_1)$ *and* $(G_2, *_2)$ *be two groups,* e_1 *and* e_2 *identity elements of* $(G_1, *_1)$ *and* $(G_2, *_2)$, $G_1 \cap G_2 = \emptyset$. *Denote* $N = G_1 \cup G_2$, *and define the operation* $*$ *in* N *as follows:*

(1) *if* $a, b \in G_1$, *then* $a * b = a *_1 b$;
(2) *if* $a, b \in G_2$, *then* $a * b = a *_2 b$;
(3) *if* $a \in G_1, b \in G_2$, *then* $a * b = a$;
(4) *if* $a \in G_2, b \in G_1$, *then* $a * b = b$.

Then $(N,^*)$ *is a Clifford semigroup (weak commutative NETG).*

Proof. *It is only necessary to prove that the associative law hold in* $(N,^*)$, *that is,* $(a * b) * c = a * (b * c)$ *for all* $a, b, c \in N$. *We will discuss the following situations separately.*
Case 1: $a, b, c \in G_1$, *or* $a, b, c \in G_2$. Since G_1 and G_2 are groups, so $(a * b) * c = *(b * c)$.
Case 2: $a \in G_1, b \in G_2$, *and* $c \in G_1$. Then, by the definition of $*$, we have $(a * b) * c = a * c = a * (b * c)$.
Case 3: $a \in G_1, b \in G_2$, *and* $c \in G_2$. Then, by the definition of $*$, we have $(a * b) * c = a * c = a = a * (b * c)$.
Case 4: $a \in G_2, b \in G_1$, *and* $c \in G_1$. Then, $(a * b) * c = b * c = a * (b * c)$.
Case 5: $a \in G_2, b \in G_1$, *and* $c \in G_2$. Then, $(a * b) * c = b * c = b = a * b = a * (b * c)$.
Case 6: $a \in G_1, b \in G_1$, *and* $c \in G_2$. From the definition of operation $*$ we have $(a * b) * c = a * b = a * (b * c)$.
Case 7: $a \in G_2, b \in G_2$, *and* $c \in G_1$. From the definition of operation $*$ we have $(a * b) * c = c = a * c = a * (b * c)$.

Therefore, $(N,^*)$ is a semigroup. Moreover, for any $a \in N$,

if $a \in G_1$, then $a * e_1 = e_1^* a = a$, and $a * (a^{-1}) = (a^{-1})^* a = e_1$, where a^{-1} is the inverse of a in group $(G_1, *_1)$;

if $a \in G_2$, then $a * e_2 = e_2^* a = a$, and $a * (a^{-1}) = (a^{-1})^* a = e_2$, where a^{-1} is the inverse of a in group $(G_2, *_2)$.

This means that $(N,^*)$ is a NETG by Definition 1. Moreover, by the definition of operation *, we have $x * e_1 = e_1 * x, x * e_2 = e_2 * x$, for any x in N. Hence, $(N,^*)$ is a weak commutative NETG by Definition 3. Using Theorem 3 we know that $(N,^*)$ is a Clifford semigroup. □

Similarly, we can get the following result.

Theorem 5. *Let* $(G_1, *_1)$ *and* $(G_2, *_2)$ *be two groups,* e_1 *and* e_2 *identity elements of* $(G_1, *_1)$ *and* $(G_2, *_2)$, $G_1 \cap G_2 = \emptyset$. *Denote* $N = G_1 \cup G_2$, *and define the operation* * in N *as follows:*

(1) *if* $a, b \in G_1$, *then* $a * b = a *_1 b$;
(2) *if* $a, b \in G_2$, *then* $a * b = a *_2 b$;
(3) *if* $a \in G_1, b \in G_2$, *then* $a * b = b$;
(4) *if* $a \in G_2, b \in G_1$, *then* $a * b = a$.

Then $(N,^*)$ *is a Clifford semigroup (weak commutative NETG).*

Example 2. Let $G_1 = \{e, a, b, c\}$ and $G_2 = \{1, 2, 3, 4, 5, 6\}$. efine operations $*_1$ and $*_2$ on G_1, G_2 following Tables 2 and 3. Then, $N = G_1 \cup G_2 = \{e, a, b, c, 1, 2, 3, 4, 5, 6\}$ is $(N,^*)$ is a weak commutative NETG with the operation * in Table 4.

Moreover, according the method in Theorem 5, we can get another weak commutative NETG (Clifford semigroup) $(N,^{*'})$, in which the peration$*$' is defined as Table 5.

Table 2
Commutative group $(G_1, *_1)$

$*_1$	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Table 3
Non-commutative group $(G_2, *_2)$

$*_2$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	1	6	5	4	3
3	3	5	1	6	2	4
4	4	6	5	1	3	2
5	5	3	4	2	6	1
6	6	4	2	3	1	5

Table 4
First weak commutative NETG (Clifford semigroup) $(N,^*)$

*	e	a	b	c	1	2	3	4	5	6
e	e	a	b	c	e	e	e	e	e	e
a	a	e	c	b	a	a	a	a	a	a
b	b	c	e	a	b	b	b	b	b	b
c	c	b	a	e	c	c	c	c	c	c
1	e	a	b	c	1	2	3	4	5	6
2	e	a	b	c	2	1	6	5	4	3
3	e	a	b	c	3	5	1	6	2	4
4	e	a	b	c	4	6	5	1	3	2
5	e	a	b	c	5	3	4	2	6	1
6	e	a	b	c	6	4	2	3	1	5

Table 5
Second weak commutative NETG (Clifford semigroup) (N,* ')

*'	e	a	b	c	1	2	3	4	5	6
e	e	a	b	c	1	2	3	4	5	6
a	a	e	c	b	1	2	3	4	5	6
b	b	c	e	a	1	2	3	4	5	6
c	c	b	a	e	1	2	3	4	5	6
1	1	1	1	1	1	2	3	4	5	6
2	2	2	2	2	2	1	6	5	4	3
3	3	3	3	3	3	5	1	6	2	4
4	4	4	4	4	4	6	5	1	3	2
5	5	5	5	5	5	3	4	2	6	1
6	6	6	6	6	6	4	2	3	1	5

5. AG-NET-loops and completely regular AG-groupoids

Definition 10. Let $(N,^*)$ be a neutrosophic extended triplet set. Then, N is called a neutrosophic extended triplet loop (NET-loop), if $(N,^*)$ is ell-defined, i.e., for any$a, b \in N$, one has$a * b \in N$.

Remark 1. In [10, 12], the name of neutrosophic triplet loop is used. In order to be more rigorous and echoed with neutrosophic extended triplet group (NETG), the name of neutrosophic extended triplet loop (NET-loop) is used in this paper.

Definition 11. Let $(N,^*)$ be a neutrosophic extended triplet loop (NET-loop). Then, N is called an AG-NET-loop, if $(N,^*)$ is an AG-groupoid.

Theorem 6. *Assume that* $(N,^*)$ *is an AG-NET-loop. Then*

(1) *for all a in N, neut* (a) *is unique*
(2) *for all a in N,* $neut(a) * neut(a) = neut(a)$.

Proof. *Suppose that there exists* $x, y \in \{neut(a)\}$. *By Definition 1 and 10,* $a * x = x^*a = a$, $a * y = y^*a = a$, *and there exists* $u, v \in N$ *which satisfy* $a * u = u^*a = x$, $a * v = v^*a = y$. *Applying the invertive law, we have*

(i) $y * u = (v^*a) * u = (u^*a) * v = x * v$.
(ii) $x * y = (a * u) * y = (y * u)^*a = (x * v)^* a = (a * v) * x = y * x$. (by the invertive law and (i))
(iii) $x = a * u = (y^*a) * u = (u^*a) * y = x * y$.
(iv) $y = a * v = (x^*a) * v = (v^*a) * x = y * x$.
(v) $(x = x * y = y * x = y$. (by *iii*), (*ii*) and (*iv*))

Therefore, $neut(a)$ is unique. Moreover, by (v) and (iii) we get that $x = x * x$, that is, $neut(a) * neut(a) = neut(a)$. □

Theorem 7. *Let* $(N,^*)$ *be an AG-NET-loop. Then*

(1) *for any* $x, y \in \{anti(a)\}$, $neut(a) * x = neut(a) * y$, that is, $|neut(a) * \{anti(a)\}| = 1$;
(2) *for all a in N* $neut(neut(a)) * neut(a) = neut(a) = neut(a) * neut(neut(a))$;
(3) *for all a in N* $neut(neut(a)) = neut(a)$;
(4) *for any a in N and* $p \in anti(neut(a))$, $a * p = a$;
(5) *for any a in N* $q \in \{anti(a)\}$, $neut(a) * neut(q) = neut(a)$ and $neut(a) * q = q * neut(a)$;
(6) *for any a in N and any* $q \in \{anti(a)\}$, $neut(a)^*anti(q) = neut(q)^*a$;
(7) *for any a in N and for any* $q \in \{anti(a)\}$, $(q * neut(a))^*a = (neut(a) * q)^*a = neut(a)$;
(8) *for any a in N and for any* $q \in \{anti(a)\}$, $a * (q * neut(a)) = a * (neut(a) * q) = neut(a)$;
(9) *for any a in N and for any* $q \in \{anti(a)\}$, $q * neut(a) \in \{anti(a)\}$ and $neut(a) * q \in \{anti (a)\}$;
(10) *for any a in N* $q \in \{anti(a)\}$, $neut(q) * neut(a) = neut(a)$;
(11) *for any a in N* $q \in \{anti(a)\}$, $a * neut(q) = a$;
(12) *for any a in N* $q \in \{anti(a)\}$, $q * (a^*a) = a$;
(13) *for all a in N* $a * neut(a^*a) = a$.

Proof. (1) Assume $x, y \in \{anti(a)\}$, by Definition 1 and 10,

$$x^*a = a * x = neut(a), \ y^*a = a * y = neut(a).$$

Using the invertive law, we have

$$neut(a) * x = (y^*a) * x = (x^*a) * y = neut(a) * y.$$

(2) Since $neut(neut(a))$ is the neutral element of$neut(a)$, by Theorem 6 (1), Definition 1 and 10, we have$neut(neut(a)) * neut(a) = neut(a) = neut(a) * neut(neut(a))$.
(3) Let $p \in \{anti(neut(a))\}$, then
$neut(a) * p = neut(a)^* anti(neut(a)) = neut(neut(a))$.
$p * neut(a) = anti(neut(a)) * neut(a) = neut(neut(a))$.
By the invertive law,
$(p * x)^*a = (a * x) * p = neut(a) * p = neut(neut(a))$.
On the other hand, by the medial law and (2) we have
$(p * x)^*a = (p * x) * (neut(a)^* a) = (p * neut(a)) * (x^*a) = neut(neut(a)) * neut(a) = neut(a)$.

Therefore, $neut(neut(a)) = (p * x)^* a = neut(a)$.

(4) Let $p \in \{anti(neut(a))\}$, applying the invertive law and (3) we get
$a * p = (a * neut(a)) * p = (p * neut(a))^* a$
$= (anti(neut(a)) * neut(a))^* a = neut(neut(a))^* a$
$= neut(a)^* a = a$.

(5) Assume $q \in \{anti(a)\}$, *then* $a * q = q^* a = neut(a)$.
Applying the invertive law,
$neut(a) * neut(q) = (a * q) * neut(q)$
$= (neut(q) * q)^* a = q^* a = neut(a)$.
Moreover,
$neut(a) * q = (neut(a) * neut(q)) * q$
$= (q * neut(q)) * neut(a) = q * neut(a)$

(6) Assume $q \in \{anti(a)\}$, then $a * q = q^* a = neut(a)$, $q^* anti(q) = anti(q) * q = neut(q)$. Applying the invertive law and (5),
$neut(q)^* a = (anti(q) * q)^* a$
$= (a * q)^* anti(q) = neut(a)^* anti(q)$.

(7) Suppose $q \in \{anti(a)\}$, then
$(q * neut(a))^* a = (a * neut(a)) * q = a * q = neut(a)$.
And, applying (5), $(neut(a) * q)^* a = (q * neut(a))^* a = neut(a)$.

(8) Suppose $q \in \{anti(a)\}$, using the invertive law and (7) we have
$a * (q * neut(a)) = (a * neut(a)) * (q * neut(a))$
$= ((q * neut(a)) * neut(a))^* a$
$= ((neut(a) * neut(a)) * q)^* a$
$= (neut(a) * q)^* a$
$= neut(a)$.
Also, applying (5), $a * (neut(a) * q) = a * (q * neut(a)) = neut(a)$.

(9) If $q \in \{anti(a)\}$, by (7) and (8), we get that $q * neut(a) \in \{anti(a)\}$ *and* $neut(a) * q \in \{anti(a)\}$.

(10) If $q \in \{anti(a)\}$, then
$neut(q) * neut(a) = (q^* anti(q)) * neut(a)$
$= (neut(a)^* anti(q)) * q$..........
$= (neut(q)^* a) * q$...................... *(by* (6))
$= (q^* a) * neut(q)$
$= neut(a) * neut(q)$ ....*(by* $q \in \{anti(a)\}$)
$= neut(a)$ .... *(using* (5))

(11) Assume $q \in \{anti(a)\}$, then (applying (10))
$a * neut(q) = (a * neut(a)) * neut(q) = (neut(q) * neut(a))^* a = neut(a)^* a = a$.

Table 6
Non-Commutive AG-NET-loop

*	a	b	c	d	e
a	a	a	e	c	d
b	a	b	e	c	d
c	d	d	c	e	a
d	e	e	a	d	c
e	c	c	d	a	e

(12) Assume $q \in \{anti(a)\}$, then (applying (10))
$q * (a^* a) = (q * neut(q)) * (a^* a)$
$= (q^* a) * (neut(q)^* a)$ (applying the medial law)
$= (q^* a) * (a * neut(q))$......................... (by (5))
$= (q^* a) * (neut(a)^* anti(q))$.............. (by (6))
$= (q * neut(a)) * (a^* anti(q))$.... (by the medial law)
$= (neut(a) * q) * (a^* anti(q))$.............. (by (5))
$= (neut(a)^* a) * (q^* anti(q))$.... (by the medial law)
$= a * neut(q))$
$= a$................................ (by (11))

(13) For all a in N, there exists $q \in \{anti(a)\}$, then $a * neut(a^* a)$
$= (q * (a^* a)) * neut(a^* a)$........... (using (12))
$= (neut(a^* a) * (a^* a)) * q$... (by the invertive law)
$= (a^* a) * q$
$= (q^* a)^* a$......... (applying the invertive law)
$= neut(a)^* a$
$= a$.

The proof complete. □

Example 3. Let $N = \{a, b, c, d, e\}$. Define operation * on N as following Table 6. Then, $(N,^*)$ is a non-commutative AG-NET-loop. And,

$$neut(a) = a, \{anti(a)\} = \{a, b\};$$

$$neut(b) = b, \{anti(b)\} = \{b\};$$

$$neut(c) = c, \{anti(c)\} = \{c\}; neut(d) = d,$$

$$\{anti(d)\} = \{d\}; neut(e) = e, \{anti(e)\} = \{e\}.$$

Theorem 8. *Let $(N,^*)$ be an AG-NET-loop. Then N is a completely regular AG-groupoid.*

Table 7
Non-commutative completely regular AG-groupoid

$*_1$	1	2	3	4
1	1	1	1	1
2	1	2	3	4
3	1	4	2	3
4	1	3	4	2

Proof. For any a in N, by Definition 1 and 11 we have

$$(a^*anti\,(a))^*a = neut\,(a)^*\,a = a.$$

From this and Definition 6 (1), we know that N is a regular AG-groupoid.

Moreover, assume$a \in N$, we have

$$(a^*a)\ \ ^*anti\,(a) = (anti\,(a)^*\,a)$$
$$^*a = neut\,(a)^*\,a = a.$$

From this and Definition 6 (4), N is a right regular AG-groupoid.

For all $a \in N$, there exists $q \in \{anti\,(a)\}$, $a * q = q^*a = neut\,(a)$. Denote $x = q * neut\,(a)$, then (using the medial law)

$$\begin{aligned} x * (a^*a) &= (q * neut\,(a)) * (a^*a) \\ &= (q^*a) * (neut\,(a)^*\,a) \\ &= (q^*a)^*a = neut\,(a)^*\,a = a. \end{aligned}$$

From this and Definition 6 (5), N is a left regular AG-groupoid.

Therefore, by Definition 6 (7) we know that N is a completely regular AG-groupoid. □

The following example shows that a completely regular AG-groupoid may be not an AG- NET-loop.

Example 4. Let $N = \{1, 2, 3, 4\}$. Define operations $*$ on N as following Table 7. Then, $(N,^*)$ is a non-commutative completely regular AG-groupoid, but it is not an AG-NET-loop, since there is no $a \in N$ such that $a * 4 = 4^*a = 4$.

Theorem 9. *Let $(N,^*)$ be an AG-NET-loop. Then N is a fully regular AG-groupoid.*

Proof. Suppose $a \in N$. Then there exists $m \in \{anti\,(a)\}$, $a * m = m^*a = neut\,(a)$. Denote $p = m * neut\,(a)$, $q = neut\,(a)$; $r = m, s = neut\,(a)$; $t = m, u = neut\,(a)$; $v = m$; $w = m * neut\,(a)$; $x = m$, $y = neut\,(a)$. Then
$\left(p^*a^2\right) * q = ((m * neut\,(a))^*a^2)) * neut\,(a)$
$= ((a^{2*}neut\,(a)) * m)) * neut\,(a)$
$= (((a^*a) * neut\,(a)) * m)) * neut\,(a)$
$= (((neut\,(a)^*\,a)^*a) * m)) * neut\,(a)$
$= ((a^*a) * m)) * neut\,(a)$
$= ((w^*a)^*a)) * neut\,(a)$
$= (neut\,(a)^*\,a)) * neut\,(a)$
$= a * neut\,(a) = a.$
$(r^*a) * (a * s) = (m^*a) * (a * neut\,(a)) = neut\,(a)^*\,a = a$
$(a * t) * (a * u) = (a * m) * (a * neut\,(a)) = neut\,(a)^*\,a = a$
$(a^*a) * v = (a^*a) * m = (m^*a)^*\,a = neut\,(a)^*\,a = a$
$w * (a^*a) = (m * neut\,(a)) * (a^*a)$
$= ((a^*a) * neut\,(a)) * (m * neut\,(a))$
$= \left(\left(neut\,(a)^*\,a\right)^*\,a\right) * (m * neut\,(a))$
$= (a^*a) * (m * neut\,(a))$
$= \left((m * neut\,(a))^*\,a\right)^*\,a$
$= ((a * neut\,(a)) * m)^*\,a$
$= (a * m)^*\,a$
$= neut\,(a)^*\,a = a$
$(x^*a) * (y^*a) = (m^*a) * (neut\,(a)^*\,a) = neut\,(a)^*\,a = a.$

Moreover, for $a^2 \in N$, there exists $n \in \{anti(a^2)\}$. Denote$z = n * m$, then
$\left(a^2 * z\right)^*\,a^2 = ((a^*a) * z)^*\,a^2.$
$= \left((z^*a)^*\,a\right)^*\,a^2$..... (applying the invertive law)
$= \left(a^{2*}a\right) * (z^*a)$..... (applying the invertive law)
$= \left(a^{2*}a\right) * ((n * m)^*a)$
$= \left(a^{2*}a\right) * ((a * m)^*n)$.......*(by the invertive law)*
$= \left(a^{2*}a\right) * (neut\,(a)^*\,n)(by\ m \in \{anti\,(a)\})$
$= ((a^*a) * (neut\,(a)^*\,a)) * (neut\,(a)^*\,n)$
$= ((a * neut\,(a)) * (a^*a)) * (neut\,(a)^*\,n)$...(applying the medial law)
$= \left(a^*a^2\right) * (neut\,(a)^*\,n)$........(by the medial law)
$= (a * neut\,(a)) * \left(a^{2*}n\right)$..(applying the medial law)
$= a * neut(a^2)$ (by the definition of $n \in \{anti(a^2)\}$)
$= a$......................... (by Theorem 7 (13))

Therefore, combing above results, by Definition 7, we know that N is a fully regular AG- groupoid. □

The following example shows that a fully regular AG-groupoid may be not an AG-NET-loop.

Example 5. Let $N = \{1, 2, 3, 4, 5, 6, 7\}$. Define operations $*$ on N as following Table 8. Then, $(N,^*)$ is a non- commutative fully regular AG-groupoid (see [26]), but it is not an AG-NET-loop, since there is no $x \in N$ such that $x * 3 = 3 * x = 3$.

6. On finite NETGs and finite AG-NET-loops

The instances with finite order and their constructions are of great significance for exploring structural

Table 8
Non-commutative fully regular AG-groupoid

*	1	2	3	4	5	6	7
1	2	4	6	1	3	5	7
2	5	7	2	4	6	1	3
3	1	3	5	7	2	4	6
4	4	6	1	3	5	7	2
5	7	2	4	6	1	3	5
6	3	5	7	2	4	6	1
7	6	1	3	5	7	2	4

features of abstract algebraic systems. By designing the MATLAB program, we have found all NTEGs of order 3, 4 and 5, which have 13, 67 and 353 respectively and they are not isomorphic to each other. Moreover, we obtained all AG-NET-loops of order 3, 4 and 5, which have 5, 17 and 54 respectively and they are not isomorphic to each other. In this section, we present our results in the form of theorems for the sake of further study. For NETGs with order 5, we only list all of commutative NETGs, a total of 51.

Theorem 10. *Let* $(N, *)$ *be a NETG with order 3 and denote* $N = \{1, 2, 3\}$. *Then N must be isomorphic to one of the NETGs represented by the following tables, and these NETGs are not mutually isomorphic:*

(1) $T3_1 = \{\{1, 1, 1\}, \{2, 2, 2\}, \{3, 3, 3\}\}$;
(2) $T3_2 = \{\{1, 2, 3\}, \{2, 2, 2\}, \{3, 3, 3\}\}$;
(3) $T3_3 = \{\{1, 3, 3\}, \{2, 2, 2\}, \{3, 3, 3\}\}$;
(4) $T3_4 = \{\{3, 2, 1\}, \{2, 2, 2\}, \{1, 2, 3\}\}$;
(5) $T3_5 = \{\{1, 2, 3\}, \{1, 2, 3\}, \{1, 2, 3\}\}$;
(6) $T3_6 = \{\{1, 2, 3\}, \{2, 2, 3\}, \{3, 2, 3\}\}$;
(7) $T3_7 = \{\{1, 3, 3\}, \{3, 2, 3\}, \{3, 3, 3\}\}$;
(8) $T3_8 = \{\{1, 2, 1\}, \{2, 2, 2\}, \{3, 2, 3\}\}$;
(9) $T3_9 = \{\{1, 2, 3\}, \{2, 2, 3\}, \{3, 3, 3\}\}$;
(10) $T3_{10} = \{\{3, 1, 1\}, \{1, 2, 3\}, \{1, 3, 3\}\}$;
(11) $T3_{11} = \{\{1, 2, 3\}, \{2, 2, 2\}, \{1, 2, 3\}\}$;
(12) $T3_{12} = \{\{1, 3, 3\}, \{1, 2, 3\}, \{1, 3, 3\}\}$;
(13) $T3_{13} = \{\{3, 1, 2\}, \{1, 2, 3\}, \{2, 3, 1\}\}$.

Theorem 11. *Let* $(N, *)$ *be a NETG with order 4 and denote* $N = \{1, 2, 3, 4\}$. *Then N must be isomorphic to one of the NETGs represented by the following 67 tables, and these NETGs are not mutually isomorphic: (the tables are omitted).*

Theorem 12. *Let* $(N, *)$ *be a commutative NETG with order 5 and denote* $N = \{1, 2, 3, 4, 5\}$. *Then N must be isomorphic to one of the NETGs represented by the following 51 tables, and these NETGs are not mutually isomorphic: (the tables are omitted).*

Theorem 13. *Let* $(N, *)$ *be an AG-NET-loop with order 3 and denote* $N = \{1, 2, 3\}$. *Then N must be isomorphic to one of the AG-NET-loops represented by the following tables, and these AG-NET-loops are not mutually isomorphic:*

(1) $L3_1 = \{\{1, 1, 1\}, \{1, 2, 1\}, \{1, 1, 3\}\}$;
(2) $L3_2 = \{\{1, 1, 1\}, \{1, 2, 2\}, \{1, 2, 3\}\}$;
(3) $L3_3 = \{\{1, 1, 1\}, \{1, 2, 3\}, \{1, 3, 2\}\}$;
(4) $L3_4 = \{\{1, 1, 3\}, \{1, 2, 3\}, \{3, 3, 1\}\}$;
(5) $L3_5 = \{\{1, 2, 3\}, \{2, 3, 1\}, \{3, 1, 2\}\}$.

Theorem 14. *Let* $(N, *)$ *be an AG-NET-loop order 4 and denote* $N = \{1, 2, 3, 4\}$. *Then N must be isomorphic to one of the AG-NET-loops represented by the following 17 tables, and these AG-NET-loops are not mutually isomorphic: (the tables are omitted).*

Theorem 15. *Let* $(N, *)$ *be an AG-NET-loop order 5 and denote* $N = \{1, 2, 3, 4, 5\}$. *Then N must be isomorphic to one of the AG-NET-loops represented by the following 54 tables, and these AG-NET-loops are not mutually isomorphic: (the tables are omitted).*

Table 9
Finite NETGs and AG-NET-loops

Order	NETGs	AG-NET-loops
3	13	5
4	67	17
5	353	54

7. Conclusions

In the paper, from the perspective of semigroup theory, we studied neutrosophic extended triplet group (NETG) and AG-NET-loop which is both an AG-groupoid and a neutrosophic extended triplet loop, and obtained some important results. We proved that the notion of NETG is equal to the notion of completely regular semi group, and the notion of weak commutative NETG is equal to the notion of Clifford semigroup. Moreover, we investigated the relationships among AG-NET-loops, and completely regular AG-groupoids and fully regular AG-groupoids, we proved that every AG-NET-loop is a completely regular and fully regular AG-groupoid, but the inverse is not true by constructing some counter examples. We also give some construction methods and low order instances of finite NETGs and AG-NET-loops (the order ≤ 5), see Table 9. These results are interesting for exploring the structure characterizations of NETGs and AG-NET-loops.

As a direction of future research, we will discuss the integration of the related topics, such as the combination of neutrosophic set, fuzzy set, soft set and algebra systems (see [30–34]).

Acknowledgments

This work was supported by National Natural Science Foundation of China (Grant No. 61573240).

Author contributions

Xiaohong Zhang, Xiaoying Wu and Xiaoyan Mao initiated the research and wrote the paper, Florentin Smarandache and Choonkil Park supervised the research work and provided helpful suggestions.

Conflicts of interest

The authors declare no conflict of interest.

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