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On the Neutrosophic Counterpart of Bellman-Ford Algorithm

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Abstract. With the development of computing technologies, the methods of network flows and linear programming have been significantly applied. The shortest path problem has been always one of the most practical problem in network analysis. By the development of various mathematical models, different algorithms have been proposed for optimal routing, given the parameters, characteristic and structure of network. During the years 1950 to 1960, various successful algorithm were proposed by Bellman, Dijkstra, Johnson and Floyd, suggested the shortest path problem as considering a central position in a network. One of the good sounded algorithm is bellman- Ford algorithm, which has been applied in fuzzy network, since the last some years. Here in this work, we have proposed the neutrosophic version of bellman's algorithm based on the trapezoidal neutrosophic numbers. Also, one numeric example is presented.

Keywords: Bellman's algorithm · Trapezoidal neutrosophic numbers · Ranking method · Shortest path problem · Network

1 Introduction

Fuzzy set (FS) [1] is a tool that is defined in terms of an 'affiliation function' and generally deals with various real life situations, where the information possesses some sort of uncertainty. Atanassov [2] generalizes the concept of FS to intuitionistic fuzzy set (IFS) which is defined in terms of two characteristic functions known as membership and non-membership functions. In 1995, Smarandache introduced the idea of neutrosophic sets (NS) [3], which is a generalization of FS and IFS. The NS is a set with each element having a degree of membership, indeterminate-membership and non-membership. There is a restriction that sum of membership, indeterminate-membership and non-membership grade of an object is less or equal to 3 [3].

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Neutrosophic numbers is a special case of the neutrosophic sets that extends the domain of numbers from those of real numbers to neutrosophic numbers.

Several researchers have focused on fuzzy shortest path and intuitionistic fuzzy shortest path algorithms [5–8]. Based on the idea of Bellman's algorithm applied in fuzzy network [9] for solving shortest path problem. And so for the first time, the neutrosophic version of Bellman's algorithm is introduced here for solving the shortest path problems on a network with single valued trapezoidal neutrosophic numbers (SVTrNs). Some applications of neutrosophic set theory are listed in [16, 17]

In this paper, we are motivated to present a new version of Bellman's algorithm for solving the shortest path problem on a network where the edge weight is characterized by trapezoidal neutrosophic numbers. The rest of this paper is organized as follows. In Sect. 2, some concepts and theories are reviewed. Section 3 presents the neutrosphic version of Bellman algorithm. In Sect. 4, a numerical example is provided as an application of our proposed algorithm. Section 5, shows the advantages of the proposed algorithm. The last but not least the section, in which the conclusion is drawn and some hints for further research is given.

2 Introduction to Neutrosophic and Trapezoidal Neutrosophic Set

In this part, we review some basic concepts regarding neutrosophic sets, single valued neutrosophic sets, trapezoidal neutrosophic sets and some existing ranking functions for trapezoidal neutrosophic numbers which are the background of this study and will help us to further research.

Definition 2.1 [3]. Let ξ be a of points (objects) set and its generic elements denoted by x; we define the neutrosophic set A (NS A) as the form $\dddot{A} = \{ <x : T_{\dddot{A}}(x), I_{\dddot{A}}(x), F_{\dddot{A}}(x)> , x \in \xi\}$, where the functions T, I, F: $\xi \rightarrow \,]^{-}0,1^{+}[$ are called the truth-membership function, an indeterminacy-membership function, and a falsity-membership function respectively and they satisfy the following condition:

$$^{-}0 \leq T_{\dddot{A}}(x) + I_{\dddot{A}}(x) + F_{\dddot{A}}(x) \leq 3^{+}. \tag{1}$$

The values of these three membership functions $T_{\dddot{A}}(x)$, $I_{\dddot{A}}(x)$ and $F_{\dddot{A}}(x)$ are real standard or nonstandard subsets of $]^{-}0,1^{+}[$. As we have difficulty in applying NSs to practical problems. Wang et al. [4] proposes the concept of a SVNS that represents the simplification of a NS and can be applied to real scientific and technical applications

Definition 2.2 [4]. A single valued neutrosophic set $\dddot{A}$ (SVNS$\dddot{A}$) in the universe set ξ is defined by the set

$$\dddot{A} = \left\{ < x : T_{\dddot{A}}(x), I_{\dddot{A}}(x), F_{\dddot{A}}(x) > , x \in \xi \right\} \tag{2}$$

Where $T_{\dddot{A}}(x)$, $I_{\dddot{A}}(x)$, $F_{\dddot{A}}(x) \in [0,\ 1]$ satisfying the condition:

$$0 \leq T_{\dddot{A}}(x) + I_{\dddot{A}}(x) + F_{\dddot{A}}(x) \leq 3 \tag{3}$$

Definition 2.3 [10]. A single valued trapezoidal neutrosophic number SVTrNN $\tilde{a} = <(a_1, b_1, c_1, d_1); T_a, I_a, F_a >$ is a special NS on the real number set R, whose truth membership, indeterminacy-membership, and a falsity-membership are given as follows

$$T_{\tilde{a}}(x) = \begin{cases} (x - a_1)T_a/(b_1 - a_1) & (a_1 \leq x \leq b_1) \\ T_a & (b_1 \leq x \leq c_1) \\ (d_1 - x)T_a/(d_1 - c_1) & (c_1 \leq x \leq d_1) \\ 0 & otherwise \end{cases} \tag{4}$$

$$I_{\tilde{a}}(x) = \begin{cases} (b_1 - x + I_a(x - a_1))/(b_1 - a_1) & (a_1 \leq x \leq b_1) \\ I_a & (b_1 \leq x \leq c_1) \\ (x - c_1 + I_a(d_1 - x))/(d_1 - c_1) & (c_1 \leq x \leq d_1) \\ 1 & otherwise \end{cases} \tag{5}$$

$$F_{\tilde{a}}(x) = \begin{cases} (b_1 - x + F_a(x - a_1))/(b_1 - a_1) & (a_1 \leq x \leq b_1) \\ F_a & (b_1 \leq x \leq c_1) \\ (x - c_1 + F_a(d_1 - x))/(d_1 - c_1) & (c_1 \leq x \leq d_1) \\ 1 & otherwise \end{cases} \tag{6}$$

If $a_1 > 0$, $\tilde{a} = \langle [a_1, \mathrm{b}_1, c_1, d_1], (T_a, I_a, F_a) \rangle$ the SVTrNN is termed a positive number. Similarly, if $d_1 \leq 0$, $\tilde{a} = \langle [a_1, \mathrm{b}_1, c_1, d_1], (T_a, I_a, F_a) \rangle$ the SVTrNN is termed as a negative SVTrNN number. When $0 \leq a_1 \leq b_1 \leq c_1 \leq d_1 \leq 1$ and $T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}} \in [0, 1]$, SVTrNN $\tilde{a}$ is called a normalised number. When $I_{\tilde{a}} = 1 - T_{\tilde{a}} - F_{\tilde{a}}$, the SVTrNN number is reduced to triangular intuitionistic fuzzy numbers (TrIFN). When $a_I = c_1, \tilde{a} = \langle [a_1, \mathrm{b}_1, c_1, d_1], (T_{\tilde{a}}, I_{\tilde{a}}, F_{\tilde{a}}) \rangle$ transforming into a TNS number. When $I_{\tilde{a}} = 0, F_{\tilde{a}} = 0$, a TrN number is reduced to generalised TrFN, $\tilde{a} = \langle [a_1, \mathrm{b}_1, c_1, d_1], T_{\tilde{a}} \rangle$.

Definition 2.4 [10]. (Comparison of any two random SVTrNN): Let $\tilde{a} = <(a_1, b_1, c_1, d_1); T_a, I_a, F_a>$ be a SVTrNN, and then the score and accuracy function is defined, as follow:

$$s(\tilde{a}) = \frac{1}{12}[a_1, \mathrm{b}_1, c_1, d_1] \times [2 + T_a - I_a - F_a]$$

$$a(\tilde{a}) = \frac{1}{12}[a_1, \mathrm{b}_1, c_1, d_1] \times [2 + T_a - I_a + F_a]$$

Let $\widetilde{a}$ and $\widetilde{r}$ be two SVTrNNs, the ranking of $\widetilde{a}$ and $\widetilde{r}$ by score function and accuracy function is described as follows:

1. $if \;\; s(\hat{r}^N) \prec s(\hat{s}^N) \; then \; \hat{r}^N \prec \hat{s}^N$
2. $if \;\; s(\hat{r}^N) \approx s(\hat{s}^N) \;\; and \;\; if$
   a. $a(\hat{r}^N) \prec a(\hat{s}^N) \;\; then \;\; \hat{r}^N \prec \hat{s}^N$
   b. $a(\hat{r}^N) \succ a(\hat{s}^N) \; then \; \hat{r}^N \succ \hat{s}^N$
   c. $a(\hat{r}^N) \approx a(\hat{s}^N) \; then \; \hat{r}^N \approx \hat{s}^N$

3 Computation of Shortest Path Based on Trapezoidal Neutrosophic Number

This section presents an algorithmic approach to solving neutrosophic shortest path problem. It is assumed that we have n number nodes with node '1' as source node while node 'n' as destination node. We denote the neutrosophic distance between a node i and j by d_{ij}. In this case $M_{N(i)}$ denote the set of all nodes having a relation with i. Let's start with the following basic definition:

Bellman Dynamic Programming

Given G = (V, E) be an acyclic directed connected graph of n vertices numbered from 1 to n such that '1' is the source node and 'n' is the destination node. Here the nodes of the network are arranged with topological ordering (E_{ij}: i < j). Now the shortest path can be determined by Bellman dynamic programming formulation by forward pass computation method. The Bellman dynamic programming formulation is described as follows:

$$\mathrm{f}(1) = 0$$
$$\mathrm{f}(\mathrm{i}) = \min_{i<j}\{f(i) + d_{ij}\}$$

where d_{ij} = weight of the directed edge E_{ij}

$f(i)$ = length of the shortest path of i^{th} node from the sourcenode 1.

Applying the concept of Bellman's algorithm in neutrosophic environment, we have

Neutrosophic Bellman-Ford Algorithm:

1. *nrank[s] ← 0*
2. *ndist[s] ← Empty* neutrosophic number.
3. Add *s* into *Q*
4. **For** each node *i* (except the *s*) in the neutrosophic graph *G*
5. *rank[i] ← ∞*
6. *Add i* into Q
7. **End For**
8. *u←s*
9. **While**(*Q* is not empty)
10. remove the vertex *u* from *Q*
11. **For** each adjacent vertex *v* of vertex *u*
12. relaxed←*False*
13. *temp_ndist[v] ← ndist[u] ⊕ edge_weight(u,v) //⊕ represents the addition of* neutrosophic//
14. temp_nrank[*v*] ← rank_of_neutrosophic(*temp_ndist[v]*)
15. **If** temp_nrank[*v*]< nrank[v] **then**
16. *ndist[v] ←temp_ndist[v]*
17. *nrank[v] ← temp_nrank[v]*
18. *prev[v] ←u*
19. **End If**
20. **End For**
21. **If** relaxed equals False **then**
22. exit the loop
23. **End If**
24. *u* ←Node in *Q* with minimum rank value
25. **End While**
26. **For** each arc***(u,v)*** in neutrosophic graph *G* do
27. **If** *nrank[v]> rank_of_neutrosophic(ndist[u] ⊕ edge_weight(u,v))*
28. return false
29. **End If**
30. **End For**
31. The neutrosophic number *ndist[u]* is a neutrosophic number and its represents the shortest path between source node *s* and node *u*.

In the following, we will provide a simple example for a better understanding as follows-

4 Illustative Example

This part is based on a numerical problem adapted from [11] to show the potential application of the proposed algorithm.

Example 1: Consider a network Fig. 1 with six nodes and eight edges weights characterized by SVTrNNs, where node 1 is the source node and node 6 is the destination node. Trapezoidal neutrosophic distance is given in Table 1.

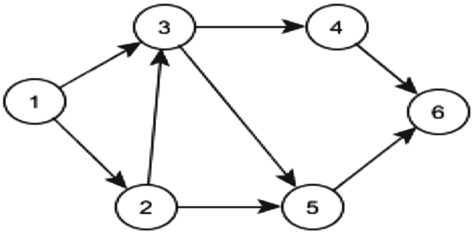

Fig. 1. A network with six vertices and eight edges Broumi et al. [11]

In this situation, we need to evaluate the shortest distance from source node i.e. node 1 to destination node i.e. node 6 (Table 2).

Table 1. The details of edges information in term SVTrNs

Edges	SVTrN weight	Edges	SVTrN weight
e_{12}	<(1, 2, 3, 4); 0.4, 0.6, 0.7>	e_{34}	<(2, 4, 8, 9); 0.5, 0.3, 0.1>
e_{13}	<(2, 5, 7, 8); 0.2, 0.3, 0.4>	e_{35}	<(3, 4, 5, 10); 0.3, 0.4, 0.7>
e_{23}	<(3, 7, 8, 9); 0.1, 0.4, 0.6>	e_{46}	<(7, 8, 9, 10); 0.3, 0.2, 0.6>
e_{25}	<(1, 5, 7, 9); 0.7, 0.6, 0.8>	e_{56}	<(2, 4, 5, 7); 0.6, 0.5, 0.3>

Table 2. The details of Deneutrosophication value of edge (i, j)

Edges	Score function	Edges	Score function
e_{12}	0.92	e_{34}	4.03
e_{13}	2.75	e_{35}	2.2
e_{23}	2.48	e_{46}	4.53
e_{25}	2.38	e_{56}	2.7

According to the algorithm method proposed in Sect. 3, the shortest path from node one to node six can be computed as follows:

f(1) = 0
$f(2) = \min_{i<2}\{f(1) + c_{12}\} = c^*_{12} = 0,92$

$$\begin{aligned}
\mathrm{f}(3) &= \min_{i<3}\{f(i)+c_{i3}\} = \min\{f(1)+c_{13}, f(2)+c_{23}\} \\
&= \{0+2,75, 0,92+2,48\} = \{2,75,3,4\} = 2,75 \\
\mathrm{f}(4) &= \min_{i<4}\{f(i)+c_{i4}\} = \min\{f(3)+c_{34}\} = \{2,75+4,03\} = 6,78 \\
\mathrm{f}(5) &= \min_{i<5}\{f(i)+c_{i5}\} = \min\{f(2)+c_{25}, f(3)+c_{35}\} \\
&= \{0.92+2,38,\ 2,75+2,2\} = \{3.3,\ 4,95\} = 3.3 \\
\mathrm{f}(6) &= \min_{i<6}\{f(i)+c_{i6}\} = \min\{f(4)+c_{46}, f(5)+c_{56}\} \\
&= \{6,78+4,53,\ 3,3+2,7\} = \{11.31,\ 6\} = 6
\end{aligned}$$

thus,

$$\begin{aligned}
\mathrm{f}(6) &= f(5)+c_{56} = f(2)+c_{25}+c_{56} = f(1)+c_{12}+c_{25} \\
&= c_{12}+c_{25}+c_{56}.
\end{aligned}$$

Therefore, the path P: $1 \to 2 \to 5 \to 6$ is identified as the neutrosophic shortest path, and the crisp shortest path is 6.

5 Advantages of the Proposed Algorithm

1. By comparing our proposed algorithm with Broumi et al. [11] for solving the same problem we conclude that proposed approach lead to the same path $1 \to 2 \to 5 \to 6$
2. This is the single algorithm which helps for solving the problem of finding the shortest path from a single source when the value of arc takes negative trapezoidal neutrosophic numbers.
3. No doubt that this present method might be slower than neutrosophic Dijkstra Algorithm [12], but is more versatile for the same problem, due to its exceptional handling power with the edge weights as negative TrNS.
4. Clearly this method is more generalized than the classical counterpart [13–15]. And hence is of greater visibility and importance.

6 Conclusion

In this paper, we have introduced the 'Neutrosophic version of Bellman-ford Algorithm for solving the shortest path problems', for the first time. And the present algorithm has the specialty of handling with the weights having negative TrNS as their weights. Also, we have provided a basic but stronger example for describing the same. And this new proposed method might be very helpful in risk evaluation, multi-criteria decision-making, portfolio selection, product adoption, and efficient network selection in heterogeneous wireless network. This paper is a moderate attempt in this direction.

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