On Ruled Surfaces Defined by Smarandache Curve

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Abstract

In the surfaces theory, it is well-known that a surface is called to be a ruled surface if it is generated by a continuously moving of a straight line in the space. Since a ruled surface is obtained by a line movement, its geometry has many nice properties and such surfaces have been studied by many authors, see: [4, 5, 6] and references therein. Ruled surfaces are also important subject in many applications. In particular, such surfaces have been used in computer aided engineering design (CAD) [7].

In differential geometry, one can obtain a new curve by using a regular curve [4, 5, 8]. In this direction, Smarandache curves have been defined and studied in [11]. More precisely, if the position vector of a curve β is composed by the Frenet frame's vectors of another curve α, then the curve β is called a Smarandache curve [11]. Special Smarandache curves in the Euclidean and Minkowski spaces are studied by many authors [1, 2, 9, 10, 11, 12].

In this talk, we first obtain binormal surface generated by TN-Smarandache curve and investigate singular point of this surface. Then we derive main properties of binormal surfaces such as striction curve, distribution parameter, Gauss curvature, mean curvature and geodesic curvature. By using these main properties, we obtain certain results on Gauss curvature and geodesic curvature of binormal surface. We also check the minimality of such surfaces by using its mean curvature. Finally, we give an example to illustrate our results.

Keywords: Frenet frame, Smarandache curve, Binormal surface, Euclidean space
Discipline: Mathematics

INTRODUCTION

Ruled surfaces are surfaces which are generated by moving a straight line continuously in the 3-dimensional Euclidean space. Ruled surfaces are one of the most important topics of differential geometry. Also, geometrical properties of these surface are richer than the other surface due to generated with line. In [3, 4, 5], many geometers have investigated the many properties of these surfaces.

There have been many studies on the differential geometry of curves in the 3-dimensional Euclidean space. In the differential geometry, a new curve produced from a known curve is one of the most studied areas of research. Thus, it is possible to compare generated curve by initial curve. In [11], the authors defined a regular curve in Minkowski space-time, whose position vector is composed by Frenet frame vectors on another regular curve, This new curve is called a Smarandache curve. After that, special Smarandache curves have been investigated in differential geometers [1, 9, 12]. Also, surfaces family with common Smarandache geodesic curve have been investigated in [2].

In this talk, we investigated singular points of the binormal surface defined TN-Smarandache curve according to Frenet frame in Euclidean space. And we obtained striction curves, distribution parameters, Gauss curvatures and mean curvatures of the this surfaces. We give some important results and illustrate an example.

PRELIMINARIES

In this section, we recall some basic topics from [5, 8].Euclidean 3-space provided with the standard flat metric given by

$$<,>= dx_1^2 + dx_2^2 + dx_3^2,$$

where x_1, x_2, x_3 is a rectangular coordinate system of E^3. Then, the norm of an arbitrary vector $\varphi \in E^3$ is defined by

$$\|\varphi\| = \sqrt{\langle \varphi, \varphi \rangle}.$$

Let $\varphi = \varphi(s)$ be a regular curve in E^3. If $\left\|\varphi'(s)\right\| = 1$, the curve is called the unit speed curve. Also, if the tangent vector of this curve forms a constant angle with a constant axes, the this curve is called a general helix.

Denote by $\{\vec{T}, \vec{N}, \vec{B}\}$ the moving Frenet-Serret frame along the curve $\vec{\varphi}$ in the space E^3. For an arbitrary unit speed curve with the first and the second curvature, κ and τ, the Frenet-Serret formulae are given by

$$\begin{bmatrix} \vec{T}' \\ \vec{N}' \\ \vec{B}' \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} . \begin{bmatrix} \vec{T} \\ \vec{N} \\ \vec{B} \end{bmatrix}$$

where

$$\langle \vec{T}, \vec{T} \rangle = \langle \vec{N}, \vec{N} \rangle = \langle \vec{B}, \vec{B} \rangle = 1$$
$$\langle \vec{T}, \vec{N} \rangle = \langle \vec{T}, \vec{B} \rangle = \langle \vec{N}, \vec{B} \rangle = 0$$

Definition 1. A regular curve in E^4, whose position vector is obtained by Frenet frame vectors on another regular curve, is called Smarandache curve [11].

From [1], we have the following information for such curves:

Let $\sigma = \sigma(s)$ be a unit speed regular curve and $\{T_\sigma, N_\sigma, B_\sigma, \kappa_\sigma, \tau_\sigma\}$ be Frenet apparatus of this curve. TN-Smarandache curve of $\sigma(s)$ curve is defined by

$$\vec{\alpha}(s_\alpha) = \frac{1}{\sqrt{2}}(T_\sigma + N_\sigma).$$

Denote by $\{T_\alpha, N_\alpha, B_\alpha, \kappa_\alpha, \tau_\alpha\}$ the Frenet apparatus of a curve α. The tangent vector and principal normal of the curve α can be written as follows:

$$T_\alpha = \alpha' = \frac{-\kappa_\sigma \vec{T}_\sigma + \kappa_\sigma \vec{N}_\sigma + \tau_\sigma \vec{B}_\sigma}{\sqrt{2\kappa_\sigma^2 + \tau_\sigma^2}}, \tag{1}$$

$$N_\alpha = \frac{\lambda_1 \vec{T}_\sigma + \lambda_2 \vec{N}_\sigma + \lambda_3 \vec{B}_\sigma}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}, \tag{2}$$

where

$$\lambda_1 = -[\kappa_\sigma^2(2\kappa_\sigma^2 + \tau_\sigma^2) + \tau_\sigma(\tau_\sigma \kappa_\sigma' - \kappa_\sigma \tau_\sigma')],$$
$$\lambda_2 = -[\kappa_\sigma^2(2\kappa_\sigma^2 + 3\tau_\sigma^2) + \tau_\sigma(\tau_\sigma^3 - \tau_\sigma \kappa_\sigma' + \kappa_\sigma \tau_\sigma')],$$
$$\lambda_3 = \kappa_\sigma[\tau_\sigma(2\kappa_\sigma^2 + \tau_\sigma^2) - 2(\tau_\sigma \kappa_\sigma' - \kappa_\sigma \tau_\sigma')].$$

The binormal vector of the curve α is given by the following:

$$B_\alpha = \frac{[\kappa_\sigma \lambda_3 - \tau_\sigma \lambda_2]\vec{T}_\sigma + [\kappa_\sigma \lambda_3 + \tau_\sigma \lambda_1]\vec{N}_\sigma - \kappa_\sigma[\lambda_1 + \lambda_2]\vec{B}_\sigma}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}\sqrt{2\kappa_\sigma^2 + \tau_\sigma^2}} \tag{3}$$

[1].

Also, tangent of the binormal vector of the curve α which will be used in later calculations as

$$B_\alpha' = \frac{\sqrt{2}(\omega \vec{T}_\sigma + \phi \vec{N}_\sigma + \psi \vec{B}_\sigma)}{(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)^{\frac{3}{2}}(2\kappa_\sigma^2 + \tau_\sigma^2)^2}, \tag{4}$$

where

$$\omega=(\kappa_\sigma'\lambda_3+\kappa_\sigma\lambda_3'-\tau_\sigma'\lambda_2-\tau_\sigma\lambda_2'-\kappa_\sigma^2\lambda_3-\kappa_\sigma\tau_\sigma\lambda_1)(\lambda_1^2+\lambda_2^2+\lambda_3^2)(2\kappa_\sigma^2+\tau_\sigma^2)$$
$$-(\kappa_\sigma\lambda_3-\tau_\sigma\lambda_2)[(\lambda_1\lambda_1'+\lambda_2\lambda_2'+\lambda_3\lambda_3')(2\kappa_\sigma^2+\tau_\sigma^2)+(2\kappa_\sigma\kappa_\sigma'+\tau_\sigma\tau_\sigma')(\lambda_1^2+\lambda_2^2+\lambda_3^2)],$$
$$\phi=(\kappa_\sigma'\lambda_3+\kappa_\sigma\lambda_3'+\tau_\sigma'\lambda_1+\tau_\sigma\lambda_1'+\kappa_\sigma^2\lambda_3+\kappa_\sigma\tau_\sigma\lambda_1)(\lambda_1^2+\lambda_2^2+\lambda_3^2)(2\kappa_\sigma^2+\tau_\sigma^2)$$
$$-(\kappa_\sigma\lambda_3+\tau_\sigma\lambda_1)[(\lambda_1\lambda_1'+\lambda_2\lambda_2'+\lambda_3\lambda_3')(2\kappa_\sigma^2+\tau_\sigma^2)+(2\kappa_\sigma\kappa_\sigma'+\tau_\sigma\tau_\sigma')(\lambda_1^2+\lambda_2^2+\lambda_3^2)],$$
$$\psi=(-\kappa_\sigma'\lambda_1-\kappa_\sigma\lambda_1'-\kappa_\sigma'\lambda_2-\kappa_\sigma\lambda_2'+\tau_\sigma^2\lambda_1+\kappa_\sigma\tau_\sigma\lambda_3)(\lambda_1^2+\lambda_2^2+\lambda_3^2)(2\kappa_\sigma^2+\tau_\sigma^2)$$
$$+\kappa_\sigma(\lambda_1+\lambda_2)[(\lambda_1\lambda_1'+\lambda_2\lambda_2'+\lambda_3\lambda_3')(2\kappa_\sigma^2+\tau_\sigma^2)+(2\kappa_\sigma\kappa_\sigma'+\tau_\sigma\tau_\sigma')(\lambda_1^2+\lambda_2^2+\lambda_3^2)]$$

[12].

Definition 2. A set of one-parameter of lines are called ruled surfaces. Such surfaces are represented by the vector equation

$$X(s,v)=\sigma(s)+v\beta(s), \tag{5}$$

where $\sigma(s)$ is its base curve and $\beta(s)$ is its direction of the ruled surface [4].

Definition 3. Let $M\subset E^3$ be a surface and $\sigma: I\subset R\to E^3$ be a regular curve. Also, let $\{T_\sigma,N_\sigma,B_\sigma,\kappa_\sigma,\tau_\sigma\}$ be Frenet apparatus of this curve. M is said to be the binormal surface of a curve σ if M can be parametrized as

$$X(s,v)=\sigma(s)+vB_\sigma(s),$$

[4].

The parametrization of the striction curve and distribution parameter on the ruled surface (5) are given by, respectively

$$\bar{\sigma}(s)=\sigma(s)-\frac{\langle T_\sigma,\beta'\rangle}{\langle \beta',\beta'\rangle}\beta(s), \tag{6}$$

$$P=\frac{\det(\beta,\beta',\sigma')}{\|\beta'\|^2}. \tag{7}$$

The standard unit normal vector field U on a surface $X(s,v)$ can be defined by

$$U=\frac{X_s\wedge X_v}{\|X_s\wedge X_v\|}=\frac{X_s\wedge X_v}{\sqrt{EG-F^2}},$$

where $X_s=\dfrac{\partial X(s,v)}{\partial s}$ and $X_v=\dfrac{\partial X(s,v)}{\partial v}$. Respectively, the first and second fundamental forms of the surface $X(s,v)$ are given by

$$I=Eds^2+2Fdsdv+Gdv^2,$$
$$II=eds^2+2fdsdv+gdv^2,$$

where

$$E=\langle X_s,X_s\rangle, F=\langle X_s,X_v\rangle, G=\langle X_v,X_v\rangle,$$
$$e=\langle U,X_{ss}\rangle, f=\langle U,X_{sv}\rangle, g=\langle U,X_{vv}\rangle.$$

The Gaussian curvature K and mean curvature H are defined by, respectively

$$K=\frac{eg-f^2}{EG-F^2}, \tag{8}$$

$$H=\frac{Eg+Ge-2Ff}{2(EG-F^2)} \quad (9)$$

[5].

BINORMAL SURFACES DEFINED BY TN-SMARANDACHE CURVE ACCORDING TO FRENET FRAME

Let $\sigma : I \rightarrow E^3$ be a curve. TN-Smarandache curve of $\sigma(s)$ curve is written as follows

$$\vec{\alpha}(s_\alpha)=\frac{1}{\sqrt{2}}(T_\sigma+N_\sigma).$$

We denote the Frenet apparatus of a curve α by $\{T_\alpha, N_\alpha, B_\alpha, \kappa_\alpha, \tau_\alpha\}$. We consider the following binormal surface

$$X(s,v)=\alpha+vB_\alpha \quad (10)$$

with TN-Smarandache curve of $\sigma(s)$.

We first have following result

Theorem 1. If $\sigma(s)$ is helix, the binormal surface $X(s,v)$ defined by (10) have singular points. And, if $\sigma(s)$ isn't helix, the binormal surface $X(s,v)$ defined by (10) have non-singular surface.

Proof: We can calculate that

$$X_s \wedge X_v=\alpha' \wedge B_\alpha+vB_\alpha' \wedge B_\alpha=-N_\alpha+vB_\alpha' \wedge B_\alpha. \quad (11)$$

Then $P_0=X(s_0,v_0)$ is a singular point of binormal surface $X(s,v)$ defined by (10) if and only if

$$\left\|\frac{\partial X(s,v)}{\partial s}\wedge\frac{\partial X(s,v)}{\partial v}\right\|=0.$$

Thus, we write

$$vB_\alpha' \wedge B_\alpha=N_\alpha \Rightarrow v=\frac{1}{\det(B_\alpha', B_\alpha, N_\alpha)},$$

$$v=\frac{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{5}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}}{\begin{cases}\sqrt{2}(\kappa_\sigma\tau_\sigma'-\tau_\sigma\kappa_\sigma')[\lambda_3(\omega-\phi)-\psi(\lambda_1-\lambda_2)]\\+\sqrt{2}(2\kappa_\sigma^2+\tau_\sigma^2)(\tau_\sigma\lambda_2\psi-\tau_\sigma\lambda_3\phi+\kappa_\sigma\lambda_1\phi-\kappa_\sigma\lambda_2\omega)\end{cases}}, \quad (12)$$

$$B_\alpha' \wedge B_\alpha=\frac{\sqrt{2}\begin{cases}[\phi(-\kappa_\sigma\lambda_1-\kappa_\sigma\lambda_2)-\psi(\kappa_\sigma\lambda_3+\tau_\sigma\lambda_1)\\-\omega(-\kappa_\sigma\lambda_1-\kappa_\sigma\lambda_2)+\psi(\kappa_\sigma\lambda_3-\tau_\sigma\lambda_2)\\+\omega(\kappa_\sigma\lambda_3+\tau_\sigma\lambda_1)-\phi(\kappa_\sigma\lambda_3-\tau_\sigma\lambda_2)]\end{cases}}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{5}{2}}}. \quad (13)$$

Substituting (12) and (13) values in (11) equation, we obtain as

$$\frac{\lambda_1}{\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}=\begin{cases}\dfrac{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{5}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}}{\begin{cases}\sqrt{2}(\kappa_\sigma\tau_\sigma'-\tau_\sigma\kappa_\sigma')[\lambda_3(\omega-\phi)-\psi(\lambda_1-\lambda_2)]\\+(2\kappa_\sigma^2+\tau_\sigma^2)[\lambda_2(\tau_\sigma\psi-\kappa_\sigma\omega)+\phi(-\tau_\sigma\lambda_3+\kappa_\sigma\lambda_1)]]\end{cases}},\\ .\dfrac{\sqrt{2}[\phi\kappa_\sigma(2\kappa_\sigma^2+\tau_\sigma^2)-\psi(\kappa_\sigma\tau_\sigma'-\tau_\sigma\kappa_\sigma')]}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}}\end{cases}$$

$$\frac{\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}=\begin{cases}\dfrac{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{5}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}}{\begin{cases}\sqrt{2}(\kappa_\sigma\tau_\sigma^{'}-\tau_\sigma\kappa_\sigma^{'})[\lambda_3(\omega-\phi)-\psi(\lambda_1-\lambda_2)]\\+(2\kappa_\sigma^2+\tau_\sigma^2)[\lambda_2(\tau_\sigma\psi-\kappa_\sigma\omega)+\phi(\kappa_\sigma\lambda_1-\tau_\sigma\lambda_3)]]\end{cases}},\\ .\dfrac{\sqrt{2}[(2\kappa_\sigma^2+\tau_\sigma^2)(\psi\tau_\sigma-\omega\kappa_\sigma)+\psi(\kappa_\sigma\tau_\sigma^{'}-\tau_\sigma\kappa_\sigma^{'})]}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}}\end{cases}$$

$$\frac{\lambda_3}{\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}=\begin{cases}\dfrac{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{5}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}}{\begin{cases}\sqrt{2}(\kappa_\sigma\tau_\sigma^{'}-\tau_\sigma\kappa_\sigma^{'})[\lambda_3(\omega-\phi)-\psi(\lambda_1-\lambda_2)]\\+(2\kappa_\sigma^2+\tau_\sigma^2)[\lambda_2(\tau_\sigma\psi-\kappa_\sigma\omega)+\phi(\kappa_\sigma\lambda_1-\tau_\sigma\lambda_3)]]\end{cases}}.\\ .\dfrac{\sqrt{2}[(\kappa_\sigma\tau_\sigma^{'}-\tau_\sigma\kappa_\sigma^{'})(\omega-\phi)-\psi\tau_\sigma(2\kappa_\sigma^2+\tau_\sigma^2)]}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}}\end{cases}$$

From this we have

$$\begin{cases}\omega-\phi=\psi\\ \lambda_1=-\lambda_2=-\lambda_3.\\ \kappa_\sigma=\tau_\sigma\end{cases}\tag{14}$$

respectively. Substituting ω,ϕ,ψ and $\lambda_1,\lambda_2,\lambda_3$ in the first and the second equations of (14), we obtain

$$\kappa_\sigma=\tau_\sigma\Rightarrow\frac{\tau_\sigma}{\kappa_\sigma}=1.$$

In that case, if $\sigma(s)$ is helix, the binormal surface $X(s,v)$ defined by (10) have singular points. And, if $\sigma(s)$ isn't helix, the binormal surface $X(s,v)$ defined by (10) have non-singular surface.

Theorem 2. The striction curve is the base curve of a surface $X(s,v)$ defined by (10).

Proof: Substituting (1) and (4) equations in (6), one can obtain the assertion.

Theorem 3. The distribution parameter of a surface $X(s,v)$ defined by (10) as follows

$$P=\frac{-\sqrt{2}(\lambda_1\omega+\lambda_2\phi+\lambda_3\psi)(\lambda_1^2+\lambda_2^2+\lambda_3^2)(2\kappa_\sigma^2+\tau_\sigma^2)}{2(\omega^2+\phi^2+\psi^2)}$$

Proof: It is obvious.

Theorem 4. The Gauss curvature and the mean curvature of a surface $X(s,v)$ defined by (10) are given by

$$K=\frac{-2(\lambda_1\omega+\lambda_2\phi+\lambda_3\psi)^2}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^4(2\kappa_\sigma^2+\tau_\sigma^2)^4E^2},$$

$$H=\frac{\begin{cases}\sqrt{2}v^2[(2\kappa_\sigma^2+\tau_\sigma^2)(\kappa_\sigma b_1\phi-\kappa_\sigma b_2\omega+\tau_\sigma b_2\psi-\tau_\sigma b_3\phi)\\ +(\kappa_\sigma\tau_\sigma^{'}-\tau_\sigma\kappa_\sigma^{'})(-b_1\psi+b_2\psi+b_3\omega-b_3\phi)]\\ -(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{9}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^4\sqrt{E}\\ +\sqrt{2}v[(2\kappa_\sigma^2+\tau_\sigma^2)(\kappa_\sigma\lambda_1\phi-\kappa_\sigma\lambda_2\omega+\tau_\sigma\lambda_2\psi-\tau_\sigma\lambda_3\phi)\\ .(\lambda_1^2+\lambda_2^2+\lambda_3^2)^3(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{5}{2}}\\ +\sqrt{2}v[(\kappa_\sigma\tau_\sigma^{'}-\tau_\sigma\kappa_\sigma^{'})(-\lambda_1\psi+\lambda_2\psi+\lambda_3\omega-\lambda_3\phi)]\\ .(\lambda_1^2+\lambda_2^2+\lambda_3^2)^3(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{5}{2}}\\ -v(\lambda_1b_1+\lambda_2b_2+\lambda_3b_3)(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{3}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{3}{2}}\sqrt{E}\end{cases}}{2(\lambda_1^2+\lambda_2^2+\lambda_3^2)^5(2\kappa_\sigma^2+\tau_\sigma^2)^6E^{\frac{3}{2}}},$$

respectively.

Proof: From (10), we have

$$X_s=\begin{cases}\left(\frac{-\kappa_\sigma}{\sqrt{2\kappa_\sigma^2+\tau_\sigma^2}}+\frac{\sqrt{2}v\omega}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{3}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^2}\right)\vec{T}_\sigma\\ +\left(\frac{\kappa_\sigma}{\sqrt{2\kappa_\sigma^2+\tau_\sigma^2}}+\frac{\sqrt{2}v\phi}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{3}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^2}\right)\vec{N}_\sigma,\\ +\left(\frac{\tau_\sigma}{\sqrt{2\kappa_\sigma^2+\tau_\sigma^2}}+\frac{\sqrt{2}v\psi}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^{\frac{3}{2}}(2\kappa_\sigma^2+\tau_\sigma^2)^2}\right)\vec{B}_\sigma\end{cases}$$

$$X_v=B_\alpha=\frac{[\kappa_\sigma\lambda_3-\tau_\sigma\lambda_2]\vec{T}_\sigma+[\kappa_\sigma\lambda_3+\tau_\sigma\lambda_1]\vec{N}_\sigma-\kappa_\sigma[\lambda_1+\lambda_2]\vec{B}_\sigma}{\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}\sqrt{2\kappa_\sigma^2+\tau_\sigma^2}}.$$

The components of the first and the second fundamental forms are obtain

$$\begin{cases}E=1+\frac{2v^2(\omega^2+\phi^2+\psi^2)}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^3(2\kappa_\sigma^2+\tau_\sigma^2)^4},\\ F=0\\ G=1,\end{cases}$$

and

$$
\begin{cases}
e = \dfrac{\begin{cases}[\lambda_1(\lambda_1^2+\lambda_2^2+\lambda_3^2)^3(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{5}{2}}+vb_1]u_1 \\ +[\lambda_2(\lambda_1^2+\lambda_2^2+\lambda_3^2)^3(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{5}{2}}+vb_2]u_2 \\ [\lambda_3(\lambda_1^2+\lambda_2^2+\lambda_3^2)^3(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{5}{2}}+vb_3]u_3\end{cases}}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^3(2\kappa_\sigma^2+\tau_\sigma^2)^{\frac{9}{2}}}, \\
f = \dfrac{-\sqrt{2}(\lambda_1\omega+\lambda_2\phi+\lambda_3\psi)}{(\lambda_1^2+\lambda_2^2+\lambda_3^2)^2(2\kappa_\sigma^2+\tau_\sigma^2)^2\sqrt{E}}, \\
g = 0
\end{cases}
$$

respectively, where u_1, u_2, u_3 are components of the $U(s,v)$ unit normal vector of $X(s,v)$ defined by (10). Making use of the data described above in (8) and (9) the Gauss curvature K and mean curvature H are obtain.

EXAMPLE

Let $\vec{\sigma}(s)$ be unit speed curve in E^3 defined by

$$\sigma = \vec{\sigma}(s) = (\cos s, \sin s, s). \tag{15}$$

See in Figure 1.

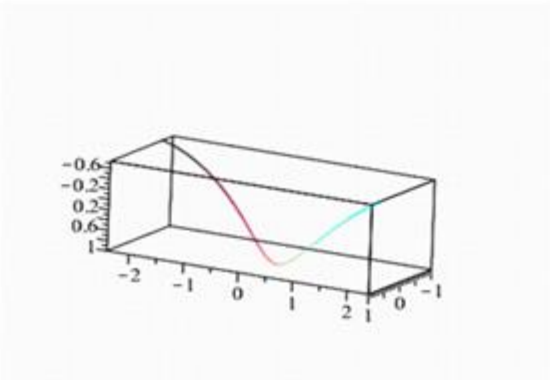

Figure 1. The curve $\sigma = \vec{\sigma}(s)$

One can calculate its Frenet-Serret apparatus as the following

$$\vec{T}(s) = (-\sin s, \cos s, 1),$$
$$\vec{N}(s) = (-\cos s, -\sin s, 0),$$
$$\vec{B}(s) = (\sin s, -\cos s, 0).$$

TN-Smarandache curve of $\vec{\sigma}(s)$ curve in (15) is defined by

$$\alpha(s_\alpha)=\frac{1}{\sqrt{2}}(T(s)+N(s))=(\frac{-\sin s-\cos s}{\sqrt{2}},\frac{\cos s-\sin s}{\sqrt{2}},\frac{1}{\sqrt{2}}).$$

The tangent vector of the curve α can be written as follows

$$\alpha^{'}=\frac{d\alpha}{ds_\alpha}\frac{ds_\alpha}{ds}=(\frac{-\cos s+\sin s}{\sqrt{2}},\frac{-\sin s-\cos s}{\sqrt{2}},0) \quad (16)$$

and hence

$$T_\alpha=\frac{d\alpha}{ds_\alpha}\frac{ds_\alpha}{ds}=(\frac{-\cos s+\sin s}{\sqrt{2}},\frac{-\sin s-\cos s}{\sqrt{2}},0),$$

where

$$\left\langle T_\alpha\frac{ds_\alpha}{ds},T_\alpha\frac{ds_\alpha}{ds}\right\rangle=1\Rightarrow\frac{ds_\alpha}{ds}=1.$$

Thus, we have

$$N_\alpha=(\frac{\sin s+\cos s}{\sqrt{2}},\frac{-\cos s+\sin s}{\sqrt{2}},0), \quad (17)$$

$$B_\alpha=T_\alpha\wedge N_\alpha=(0,0,1).$$

Thus, we obtain the following binormal surface

$$X(s,v)=\alpha+vB_\alpha=(\frac{-\sin s-\cos s}{\sqrt{2}},\frac{\cos s-\sin s}{\sqrt{2}},\frac{1}{\sqrt{2}}+v). \quad (18)$$

with TN-Smarandache curve of $\vec{\sigma}(s)$.

See in Figure 2.

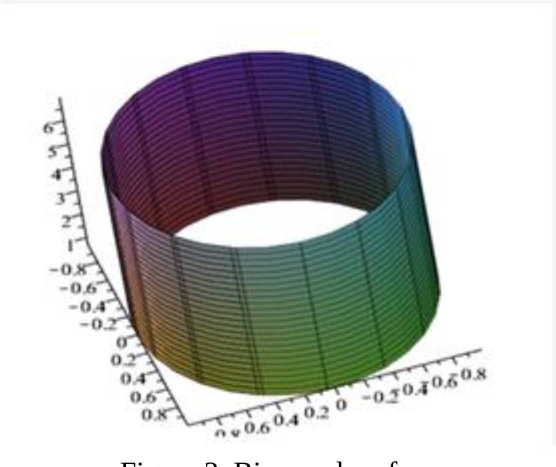

Figure 2. Binormal surface

From (18), we have

$$X_s=(\frac{-\cos s+\sin s}{\sqrt{2}},\frac{-\sin s-\cos s}{\sqrt{2}},0),$$

$$X_v=(0,0,1),$$

$$X_{ss}=(\frac{\sin s+\cos s}{\sqrt{2}},\frac{-\cos s+\sin s}{\sqrt{2}},0),$$

$$X_{vv}=0,$$

$$X_{sv} = 0.$$

The components of the first and second fundamental forms are obtain

$$\begin{cases} E = 1, \\ F = 0, \\ G = 1 \end{cases}$$

and

$$\begin{cases} e = -1, \\ f = 0, \\ g = 0 \end{cases}$$

respectively.

Thus, the striction curve $\gamma(s)$, distribution parameter P and mean curvature H of $X(s,v)$ defined by (18) as follows

$$\gamma(s) = \alpha - \frac{\langle \alpha', B_\alpha' \rangle}{\langle B_\alpha', B_\alpha' \rangle} B_\alpha$$

Substituting (16) and (17) values in above equation, we have

$$\langle \alpha', B_\alpha' \rangle = 0 \Rightarrow \gamma(s) = \alpha.$$

Therefore, the striction curve is the base curve of $X(s,v)$ defined by (18).

$$P = \frac{\det(B_\alpha, B_\alpha', \alpha')}{\left\| B_\alpha' \right\|^2}$$

Substituting (16) and (17) values in above equation, we have

$$P = 0.$$

Thus, $X(s,v)$ defined by (18) is developable.

$$H = \frac{Ge}{2(EG - F^2)} = \frac{-1}{2}.$$

$X(s,v)$ defined by (18) is not the minimal surface.

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