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On Neutrosophic Z-algebras

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This study presents the notion of neutrosophic Z-algebra and neutrosophic pseudo Z-algebra explores some of its properties. Also studied are the neutrosophic Z-ideal, neutrosophic Z-sub algebra, and neutrosophic Z-filter. Several properties are discovered, and some findings from the study of homomorphism are discussed.

Keywords: Neutrosophic Z-algebra; neutrosophic pseudo Z-algebra; neutrosophic Z-sub algebra; neutrosophic Z-ideal; neutrosophic Z-filter.

1 Introduction

Smarandache established the area of philosophy known as neutrosophy, which has a many implementations in the real world and in mathematics, particularly in algebra [1].also gave more information about neutrosophy see [2,3]. making use of neutrosophic theory Kandasamy and Smarandache [4] in 2004 suggested a set-based algebraic structure of neutrosophic numbers of the type $\mathcal{N} = \mathcal{Z} + \uparrow \mathcal{J}$ that they dubbed $\mathcal{J}$ -Neutrosophic Algebraic Structure. ,where $\mathcal{Z}, \uparrow \in \mathbb{R}$ or $\mathbb{C}$, and $\mathcal{J}$ which means indetermined or uncertain thus that $\mathcal{J}^2 = \mathcal{J}$, is referred to as literal indeterminacy, here $\mathcal{Z}$ is referred to as the $\mathcal{N}$'s determinate portion, and $\uparrow \mathcal{J}$ is referred to as its indeterminate portion on $\mathcal{N}$,with $g\mathcal{J} + h\mathcal{J} = (g + h)\mathcal{J}$, $0.\mathcal{J} = 0$. Where $\mathcal{J}$ is different from the

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imaginary $i^2 = -1$, in general, $\mathcal{I}^j = \mathcal{I}$ if $j > 0$, and is unknown for $j \leq 0$. In 2006, the idea of neutrosophic algebraic structures was also proposed [5].

In [6,7,8,9], the idea of neutrosophic BCI/BCK –algebras, neutrosophic KU-algebras and neutrosophic B-algebras was presented.

Z-algebra is an unique algebraic structure based on logic that was first proposed in 2017 by Chandramouleeswaran et al. [10].

[11] and [12] They provided characteristics and further explanation of Z-algebra.

In this article, we explain the idea of neutrosophic Z-algebra, look at various relevant characteristics, examine a neutrosophic Z-homomorphism, and present some findings.

2 Preliminaries

Definition 2.1: [1] A neutrosophic set $\mathcal{X}(\mathcal{I}) = \langle \mathcal{X}, \mathcal{I} \rangle = \{ \mathcal{Z} + \uparrow \mathcal{I} : \mathcal{Z}, \uparrow \in \mathcal{X} \}$, where $\mathcal{X} \neq \emptyset$ and $\mathcal{I}$ an indeterminate.

Definition 2.2: [10] let $\mathcal{Z} \neq \emptyset$ and $*$ is a binary operation with constant 0 then the algebra $(\mathcal{Z}, *, 0)$ named Z-Algebra if satisfying the following axiom:

$$\begin{aligned} \mathcal{Z}_1: & \mathcal{Z} * 0 = 0 \\ \mathcal{Z}_2: & 0 * \mathcal{Z} = \mathcal{Z} \\ \mathcal{Z}_3: & \mathcal{Z} * \mathcal{Z} = \mathcal{Z} \\ \mathcal{Z}_4: & \mathcal{Z} * \uparrow = \uparrow * \mathcal{Z} \text{ When } \mathcal{Z} \neq 0 \text{ and } \uparrow \neq 0, \forall \mathcal{Z}, \uparrow \in \mathcal{Z}. \end{aligned}$$

Definition 2.3: [10] Let $\delta \neq \emptyset$ and $\delta \subseteq \mathcal{Z}$ where $(\mathcal{Z}, *, 0)$ is a Z-Algebra, δ is named Z-subalgebra if $\mathcal{Z} * \uparrow \in \delta, \forall \mathcal{Z}, \uparrow \in \delta$.

Definition 2.4: [10] Let $\mathcal{I} \neq \emptyset$ and $\mathcal{I} \subseteq \mathcal{Z}$, where $(\mathcal{Z}, *, 0)$ is a Z-Algebra, $\mathcal{I}$ is named

Z-ideal of $\mathcal{Z}$ if satisfy (1) $0 \in \mathcal{I}$ (2) $\mathcal{Z} * \uparrow \in \mathcal{I}$, and $\uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}$.

Definition 2.5: [11] Let $\mathcal{I} \neq \emptyset$ and $\mathcal{I} \subseteq \mathcal{Z}$, where $(\mathcal{Z}, *, 0)$ is a Z-Algebra, $\mathcal{I}$ is named $\mathcal{Z}_1$ – ideal of $\mathcal{Z}$ if satisfy

$$(1) 0 \in \mathcal{I} \quad (2) ((\mathcal{Z} * \mathfrak{A}) * \mathcal{Z}) * \uparrow \in \mathcal{I}, \text{ and } \uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}, \forall \mathcal{Z}, \uparrow, \mathfrak{A} \in \mathcal{Z}.$$

Definition 2.6: [11] Let $\mathcal{I} \neq \emptyset$ and $\mathcal{I} \subseteq \mathcal{Z}$, where $(\mathcal{Z}, *, 0)$ is a Z-Algebra, $\mathcal{I}$ is named $\mathcal{Z}_2$ –ideal of $\mathcal{Z}$ if satisfy

$$(1) 0 \in \mathcal{I} \quad (2) (\mathcal{Z} * \mathfrak{A}) * (\mathcal{Z} * \uparrow) \in \mathcal{I}, \text{ and } \uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}, \forall \mathcal{Z}, \uparrow, \mathfrak{A} \in \mathcal{Z}$$

Definition 2.7: [12] Let $\mathcal{I} \neq \emptyset$ and $\mathcal{I} \subseteq \mathcal{Z}$, where $(\mathcal{Z}, *, 0)$ is a Z-Algebra, $\mathcal{I}$ is named $\mathcal{Z}_p$ –ideal of $\mathcal{Z}$ if satisfy

$$(1) 0 \in \mathcal{I} \quad (2) (\mathcal{Z} * \mathfrak{A}) * (\uparrow * \mathfrak{A}) \in \mathcal{I}, \text{ and } \uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}, \forall \mathcal{Z}, \uparrow, \mathfrak{A} \in \mathcal{Z}.$$

Definition 2.8: [10] let $\mathcal{F} \neq \emptyset$ and $\mathcal{F} \subseteq \mathcal{Z}$, where $(\mathcal{Z}, *, 0)$ is a Z-Algebra, $\mathcal{F}$ is named Z-filter of $\mathcal{Z}$ if $\mathcal{Z}\mathfrak{A}\uparrow = \mathcal{Z} * (\mathcal{Z} * \uparrow) \in \mathcal{F}, \forall \mathcal{Z}, \uparrow \in \mathcal{F}, (\mathcal{Z} \neq \uparrow)$.

Example 2.9: let $\mathcal{Z} = \{0, \mathcal{Z}, \uparrow, \mathfrak{A}\}$ be set and $*$ is a binary operation defined on $\mathcal{Z}$ by the table:

*	0	$\mathcal{Z}$	$\uparrow$	$\mathfrak{A}$
0	0	$\mathcal{Z}$	$\uparrow$	$\mathfrak{A}$
$\mathcal{Z}$	0	$\mathcal{Z}$	$\uparrow$	$\uparrow$
$\uparrow$	0	$\uparrow$	$\uparrow$	$\uparrow$
$\mathfrak{A}$	0	$\uparrow$	$\uparrow$	$\mathfrak{A}$

Then $(Z, *, 0)$ is Z -Algebra. $\delta = \{0, \uparrow, \downarrow\}$ is Z -subalgebra and $\mathcal{I} = \{0, \uparrow, \downarrow\}$ is a Z_1 -ideal, $\mathcal{I}^* = \{0, \uparrow, \downarrow\}$ is a $(Z_2$ -ideal, Z_p -ideal) Z -ideal and $\mathcal{F} = \{\uparrow, \downarrow\}$ is Z -filter.

Note : every $(Z_1$ -ideal, Z_2 -ideal) is an ideal of Z .

Definition 2.10: [11] Let $Z \neq \emptyset$ with two binary operations $*, \odot$ and constant 0 then the algebra $(Z, *, \odot, 0)$ named pseudo Z -Algebra (briefly, PZ) if satisfying the following axiom:

$$PZ_1: \mathcal{Z} * 0 = \mathcal{Z} \odot 0 = 0$$

$$PZ_2: 0 * \mathcal{Z} = 0 \odot \mathcal{Z} = \mathcal{Z}$$

$$PZ_3: \mathcal{Z} * \mathcal{Z} = \mathcal{Z} \odot \mathcal{Z} = \mathcal{Z}$$

$$PZ_4: \mathcal{Z} * \uparrow = \uparrow \odot \mathcal{Z} \text{ When } \mathcal{Z} \neq 0 \text{ and } \uparrow \neq 0, \forall \mathcal{Z}, \uparrow \in Z.$$

Definition 2.11: [11] Let $\delta \neq \emptyset$ and $\delta \subseteq Z$, where $(Z, *, \odot, 0)$ is PZ then δ is named a pseudo Z -subalgebra if $\mathcal{Z} * \uparrow, \mathcal{Z} \odot \uparrow \in \delta, \forall \mathcal{Z}, \uparrow \in \delta$.

Example 2.12: Let $Z = \{0, \uparrow, \downarrow\}$ be set and $*, \odot$ are a binary operations defined on Z by the table as follows:

*	0	$\uparrow$	$\downarrow$	$\odot$	0	$\uparrow$	$\downarrow$
0	0	$\uparrow$	$\downarrow$	0	0	$\uparrow$	$\downarrow$
$\uparrow$	0	$\uparrow$	$\downarrow$	$\uparrow$	0	$\uparrow$	$\downarrow$
$\downarrow$	0	$\uparrow$	$\downarrow$	$\downarrow$	0	$\uparrow$	$\downarrow$

Then $(Z, *, \odot, 0)$ is pseudo Z -algebra, $\delta = \{\uparrow, \downarrow\}$ is a pseudo Z -sub algebra.

3 Neutrosophic Z-algebra

Definition 3.1: A neutrosophic Z -algebra is the triple $(Z(\mathcal{I}), *, (0, 0\mathcal{I}))$ (briefly, $\mathcal{NZ}$) (where $(Z, *, 0)$ be a Z -algebra, $Z(\mathcal{I}) = \langle Z, \mathcal{I} \rangle$ a neutrosophic set)

if $(\mathcal{Z}, \mathcal{H}\mathcal{I}), (\uparrow, \mathcal{V}\mathcal{I})$ are any two elements of $Z(\mathcal{I})$ with $\mathcal{Z}, \mathcal{H}, \uparrow, \mathcal{V} \in Z$ satisfies

$$(\mathcal{Z}, \mathcal{H}\mathcal{I}) * (\uparrow, \mathcal{V}\mathcal{I}) = (\mathcal{Z} * \uparrow, (\mathcal{Z} * \mathcal{V} \wedge \mathcal{H} * \uparrow \wedge \mathcal{H} * \mathcal{V})\mathcal{I})$$

An element $\mathcal{Z} \in Z$ is represented by $(\mathcal{Z}, 0\mathcal{I}) \in Z(\mathcal{I})$,

$$(\mathcal{Z}, 0\mathcal{I}) * (\mathcal{H}, 0\mathcal{I}) = (\mathcal{Z} * \mathcal{H}, 0\mathcal{I}) = (\mathcal{Z} \wedge \sim \mathcal{H}, 0) \text{ where } \sim \mathcal{H} \text{ is the negation of } \mathcal{H} \text{ in } Z$$

$$\text{And } (\mathcal{Z}, \mathcal{H}\mathcal{I}) = (\uparrow, \mathcal{V}\mathcal{I}) \Leftrightarrow (\mathcal{Z} = \uparrow \text{ and } \mathcal{H} = \mathcal{V})$$

Definition 3.2: A neutrosophic pseudo Z -algebra is $(Z(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$ (briefly, $\mathcal{NPZ}$) (where $(Z, *, \odot, 0)$ be a pseudo Z -algebra

If $(\mathcal{Z}, \mathcal{H}\mathcal{I}), (\uparrow, \mathcal{V}\mathcal{I})$ are any two elements of $Z(\mathcal{I})$ with $\mathcal{Z}, \mathcal{H}, \uparrow, \mathcal{V} \in Z$ satisfies

$$(\mathcal{Z}, \mathcal{H}\mathcal{I}) * (\uparrow, \mathcal{V}\mathcal{I}) = (\mathcal{Z} * \uparrow, (\mathcal{Z} * \mathcal{V} \wedge \mathcal{H} * \uparrow \wedge \mathcal{H} * \mathcal{V})\mathcal{I})$$

$$(\uparrow, \mathcal{V}\mathcal{I}) \odot (\mathcal{Z}, \mathcal{H}\mathcal{I}) = (\uparrow \odot \mathcal{Z}, (\mathcal{Z} \odot \mathcal{V} \wedge \mathcal{H} \odot \uparrow \wedge \mathcal{H} \odot \mathcal{V})\mathcal{I})$$

$$\text{Where } (\mathcal{Z}, \mathcal{H}\mathcal{I}) * (\uparrow, \mathcal{V}\mathcal{I}) = (\mathcal{Z}, \mathcal{H}\mathcal{I}) \odot (\uparrow, \mathcal{V}\mathcal{I})$$

$$\text{When } (\mathcal{Z}, \mathcal{H}\mathcal{I}) \neq (0, 0\mathcal{I}) \text{ and } (\uparrow, \mathcal{V}\mathcal{I}) \neq (0, 0\mathcal{I}), \forall (\mathcal{Z}, \mathcal{H}\mathcal{I}), (\uparrow, \mathcal{V}\mathcal{I}) \in Z(\mathcal{I})$$

Theorem 3.3: Every $\mathcal{NZ} (Z(\mathcal{I}), *, (0, 0\mathcal{I}))$ with condition $(0, 0\mathcal{I}) * (\mathcal{Z}, \mathcal{H}\mathcal{I}) = (\mathcal{Z}, \mathcal{H}\mathcal{I})$ is a Z -algebra and conversely, not.

Proof: let $(\mathcal{X}(\mathcal{I}), *, (0, 0\mathcal{I}))$ is $\mathcal{NZ}$

Let $r = (\mathcal{Z}, \mathfrak{h}\mathcal{I})$ and $0 = (0, 0\mathcal{I})$

$$\begin{aligned} Z_1: r * 0 &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (0, 0\mathcal{I}) = (\mathcal{Z} * 0, (\mathcal{Z} * 0 \wedge \mathfrak{h} * 0)\mathcal{I}) = (0, (0 \wedge 0)\mathcal{I}) = (0, 0\mathcal{I}) \\ Z_2: 0 * r &= (0, 0\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (0 * \mathcal{Z}, (0 * \mathfrak{h} \wedge 0 * \mathcal{Z})\mathcal{I}) = (\mathcal{Z}, (\mathfrak{h} \wedge \mathcal{Z})\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \\ Z_3: r * r &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (\mathcal{Z} * \mathcal{Z}, (\mathcal{Z} * \mathfrak{h} \wedge \mathfrak{h} * \mathcal{Z} \wedge \mathfrak{h} * \mathfrak{h})\mathcal{I}) \\ &= (\mathcal{Z}, (\mathcal{Z} \wedge \sim \mathfrak{h} \wedge \mathfrak{h} \wedge \sim \mathcal{Z} \wedge \mathfrak{h})\mathcal{I}) \\ &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \end{aligned}$$

Z_4 : if $r * s = s * r$, when $r \neq 0$ & $s \neq 0, \forall r, s \in \mathcal{Z}(\mathcal{I})$

let $r = (\mathcal{Z}, \mathfrak{h}\mathcal{I}), s = (\mathfrak{t}, \mathfrak{q}\mathcal{I})$,

$$(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathfrak{t}, \mathfrak{q}\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I})$$

$$(\mathcal{Z} * \mathfrak{t}, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \mathfrak{t} \wedge \mathfrak{h} * \mathfrak{q})\mathcal{I}) = (\mathfrak{t} * \mathcal{Z}, (\mathfrak{t} * \mathfrak{h} \wedge \mathfrak{q} * \mathcal{Z} \wedge \mathfrak{q} * \mathfrak{h})\mathcal{I})$$

Suppose $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (0, 0\mathcal{I})$ & $(\mathfrak{t}, \mathfrak{q}\mathcal{I}) \neq (0, 0\mathcal{I})$ we get

$$0 * \mathfrak{t} = \mathfrak{t} * 0 \Rightarrow \mathfrak{t} = 0$$

$$\text{and } 0 * \mathfrak{q} \wedge 0 * 0 = 0 * 0 \wedge \mathfrak{q} * 0 \Rightarrow \mathfrak{q} = 0$$

We get a contradiction.

Then $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$ is a $\mathcal{Z}$ -algebra.

Theorem 3.4: Every $\mathcal{NPZ}$, $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$ with condition $(0, 0\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (0, 0\mathcal{I}) \odot (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I})$ is a pseudo $\mathcal{Z}$ -algebra and conversely, not.

Proof: it is easy as above.

Definition 3.5: Let $\mathfrak{S}(\mathcal{I}) \neq \emptyset$ and $\mathfrak{S}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$ is $\mathcal{NZ}$, $\mathfrak{S}(\mathcal{I})$ is named a neutrosophic $\mathcal{Z}$ -subalgebra (briefly, $\mathcal{NZ}^s$) of $\mathcal{Z}(\mathcal{I})$ if

- 1) $(0, 0\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 2) $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 3) $\mathfrak{S}(\mathcal{I})$ Contains a proper sub set which a $\mathcal{Z}$ -algebra.

Definition 3.6: Let $\mathfrak{S}(\mathcal{I}) \neq \emptyset$ and $\mathfrak{S}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$ is $\mathcal{NPZ}$, $\mathfrak{S}(\mathcal{I})$ is called a neutrosophic pseudo $\mathcal{Z}$ -subalgebra (briefly, $\mathcal{NPZ}^s$) of $\mathcal{Z}(\mathcal{I})$ if

- 1) $(0, 0\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 2) $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$ & $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 3) $\mathfrak{S}(\mathcal{I})$ Contains a proper sub set which a pseudo $\mathcal{Z}$ -algebra.

Theorem 3.7: If $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) \neq \emptyset$ and $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ for $\omega \neq 0$, $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$ is $\mathcal{NZ}$, where $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{Z}(\mathcal{I}) : (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\omega, \omega\mathcal{I}) = (\omega, \omega\mathcal{I})\}$

Then 1) $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I})$ is $\mathcal{NZ}^s$.

2) $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) \subseteq \mathcal{A}_{(0, 0\mathcal{I})}(\mathcal{I})$.

Proof: 1) clearly $(0, 0\mathcal{I}) \in \mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I})$

$\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I})$ contain a proper sub set which a $\mathcal{Z}$ -algebra.

Let $(\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \Rightarrow$

$$(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \quad , \quad (\mathfrak{f}, \mathfrak{q}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \Rightarrow$$

$$\mathcal{Z} * \omega = \omega \quad , \quad \mathcal{Z} * \omega \wedge \mathfrak{h} * \omega = \omega \quad \& \quad \mathfrak{f} * \omega = \omega \quad , \quad \mathfrak{f} * \omega \wedge \mathfrak{q} * \omega = \omega \quad \text{since } \omega \neq 0 \Rightarrow$$

$$\mathcal{Z} = \mathfrak{h} = \mathfrak{f} = \mathfrak{q} = \omega$$

$$\begin{aligned} [(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J})] * (\omega, \omega\mathcal{J}) &= [\mathcal{Z} * \mathfrak{f}, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \mathfrak{f})\mathcal{J}] * (\omega, \omega\mathcal{J}) \\ &= [(\mathcal{Z} * \mathfrak{f}) * \omega, ((\mathcal{Z} * \mathfrak{f}) * \omega \wedge (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \mathfrak{f}) * \omega)\mathcal{J}] \\ &= [\omega * \omega, (\omega * \omega \wedge \omega * \omega)\mathcal{J}] \\ &= (\omega, \omega\mathcal{J}) \end{aligned}$$

This shows that $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$

Then $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$ is $\mathcal{NZ}^s$.

(2) it's easy.

Theorem 3.8: If $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \neq \emptyset$ and $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, for $\omega \neq 0$,

$(\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$ is $\mathcal{NPZ}$, where $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}(\mathcal{J}) : (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \text{ \& } (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \odot \omega, \omega\mathcal{J} = \omega, \omega\mathcal{J}\}$

Then 1) $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$ is $\mathcal{NPZ}^s$.

2) $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{A}_{(0, 0\mathcal{J})}(\mathcal{J})$.

Proof: it is easy as above.

Theorem 3.9: If $\mathcal{Z}_\xi(\mathcal{J}) \neq \emptyset$ and $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is $\mathcal{NZ}$, where $\mathcal{Z}_\xi(\mathcal{J}) = \{(\mathcal{Z}, \mathcal{J}) : \mathcal{Z} \in \mathcal{Z}\}$ Then $\mathcal{Z}_\xi(\mathcal{J})$ is a $\mathcal{NZ}^s$ of $\mathcal{Z}(\mathcal{J})$.

Proof: clearly $(0, 0\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$ and the third condition is satisfied for $\mathcal{Z}_\xi(\mathcal{J})$

Let $(\mathfrak{f}, \mathfrak{f}\mathcal{J}), (\mathfrak{h}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$, $\mathfrak{f}, \mathfrak{h} \in \mathcal{Z} \Rightarrow$

$$(\mathfrak{f}, \mathfrak{f}\mathcal{J}) * (\mathfrak{h}, \mathfrak{h}\mathcal{J}) = (\mathfrak{f} * \mathfrak{h}, (\mathfrak{f} * \mathfrak{h})\mathcal{J})$$

This shows that $(\mathfrak{f}, \mathfrak{f}\mathcal{J}) * (\mathfrak{h}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$

Then $\mathcal{Z}_\xi(\mathcal{J})$ is a $\mathcal{NZ}^s$ of $\mathcal{Z}(\mathcal{J})$.

Theorem 3.10: If $\mathcal{Z}_\xi(\mathcal{J}) \neq \emptyset$ and $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$ is $\mathcal{NPZ}$, where

$\mathcal{Z}_\xi(\mathcal{J}) = \{(\mathcal{Z}, \mathcal{J}) : \mathcal{Z} \in \mathcal{Z}\}$ Then $\mathcal{Z}_\xi(\mathcal{J})$ is a $\mathcal{NPZ}^s$ of $\mathcal{Z}(\mathcal{J})$.

Proof: it is easy as above.

Example 3.11: Let $*$ is a binary operation defined on $\mathcal{Z}_\xi(\mathcal{J}) = \{(0, 0\mathcal{J}), (\mathcal{Z}, \mathcal{J}), (\mathfrak{f}, \mathfrak{f}\mathcal{J}), (\mathfrak{h}, \mathfrak{h}\mathcal{J})\}$ as follows:

*	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{h}, \mathfrak{h}\mathcal{J})$
$(0, 0\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{h}, \mathfrak{h}\mathcal{J})$
$(\mathcal{Z}, \mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$
$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(0, 0\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$
$(\mathfrak{h}, \mathfrak{h}\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{h}, \mathfrak{h}\mathcal{J})$

Then $(\mathcal{Z}_\xi(\mathcal{J}), *, (0, 0\mathcal{J}))$ is a $\mathcal{NZ}^s$ of $\mathcal{Z}(\mathcal{J})$

Theorem 3.12: Let $\{\mathcal{A}(\mathcal{J})_\gamma : \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{J})_\gamma \neq \emptyset$ be a collection of $\mathcal{NZ}^s$ of $\mathcal{Z}(\mathcal{J})$ if

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \{(0,0\mathcal{I})\} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \text{ is a } \mathcal{NZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{I}).$$

Proof: since $(0,0\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S} \Rightarrow$

$$(0,0\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset$$

And the third condition was achieved for $\mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S} \Rightarrow$

The third condition was achieved for $\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \{(0,0\mathcal{I})\} \Rightarrow \exists (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (0,0\mathcal{I}) \Rightarrow$$

$$\{(0,0\mathcal{I})\} \subseteq \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}, \text{ which is a } \mathcal{Z} - \text{algebra}$$

$$\text{Let } (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S}$$

Since $\mathcal{A}(\mathcal{I})_{\gamma}$ is a $\mathcal{NZ}^{\mathcal{S}}$, $\forall \gamma \in \mathcal{S}$ of $\mathcal{Z}(\mathcal{I})$ then

$$(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S}, \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$$

hence $\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$ is a $\mathcal{NZ}^{\mathcal{S}}$ of $\mathcal{Z}(\mathcal{I})$.

Theorem 3.13: Let $\{\mathcal{A}(\mathcal{I})_{\gamma} : \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset$ be a collection of $\mathcal{NPZ}^{\mathcal{S}}$ of $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$ is $\mathcal{NPZ}$ if

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \{(0,0\mathcal{I})\} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \text{ is a } \mathcal{NPZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{I}).$$

Proof: it is easy as above.

Theorem 3.14: Let $\{\mathcal{A}(\mathcal{I})_{\gamma} : \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset$ be a collection of $\mathcal{NZ}^{\mathcal{S}}$ of $\mathcal{Z}(\mathcal{I})$ if $\mathcal{A}(\mathcal{I})_1 \subseteq \mathcal{A}(\mathcal{I})_2 \subseteq \dots$ then

$$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \text{ is a } \mathcal{NZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{I}).$$

$$\textbf{Proof:} \text{ obviously } (0,0\mathcal{I}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset \Rightarrow \exists (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$$

$$\Rightarrow \text{For some } \gamma \in \mathcal{S} \text{ } (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma} \text{ and } (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma \in \mathcal{S}}$$

$$\Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$$

Let $\mathfrak{S}(\mathcal{I})_\gamma$ be a proper sub set of $\mathcal{A}(\mathcal{I})_\gamma$, for some $\gamma \in \mathcal{S}$ which a Z- algebra,
then for any $\gamma \in \mathcal{S}$, $\mathfrak{S}(\mathcal{I})_\gamma \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$ then

$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma \in \mathcal{S}}$ is $\mathcal{NZ}^s$ of $\mathcal{Z}(\mathcal{I})$.

Theorem 3.15: Let $\{\mathcal{A}(\mathcal{I})_\gamma : \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{I})_\gamma \neq \emptyset$ be a collection of $\mathcal{NPZ}^s$ of $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$ is $\mathcal{NPZ}$ if $\mathcal{A}(\mathcal{I})_1 \subseteq \mathcal{A}(\mathcal{I})_2 \subseteq \dots$ then

$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$ is $\mathcal{NPZ}^s$ of $\mathcal{Z}(\mathcal{I})$.

Proof: it is easy as above.

Definition 3.16: Let $\mathcal{D}(\mathcal{I}) \neq \emptyset$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$ is $\mathcal{NZ}$, $\mathcal{D}(\mathcal{I})$ is named a neutrosophic Z-ideal (briefly, $\mathcal{NZ}^i$) of $\mathcal{Z}(\mathcal{I})$ if :

- 1) $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
- 2) If $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$

Remark 3.17: Let $\mathcal{D}(\mathcal{I})$ is a $\mathcal{NZ}^i$ of $\mathcal{Z}(\mathcal{I})$ if

$(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) = (0, 0\mathcal{I})$ then $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$.

Proof: let $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) = (0, 0\mathcal{I}) \Rightarrow$

$(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$

Since $\mathcal{D}(\mathcal{I})$ is a $\mathcal{NZ}^i \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$.

Definition 3.18: Let $\mathcal{D}(\mathcal{I}) \neq \emptyset$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$ is $\mathcal{NPZ}$, $\mathcal{D}(\mathcal{I})$ is named a neutrosophic pseudo Z-ideal (briefly, $\mathcal{NPZ}^i$) of $\mathcal{Z}(\mathcal{I})$ if :

- 1) $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$.
- 2) If $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$

And $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$.

Definition 3.19: Let $\mathcal{D}(\mathcal{I}) \neq \emptyset$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$ is $\mathcal{NZ}$, $\mathcal{D}(\mathcal{I})$ is named a neutrosophic $\mathcal{Z}_1$ -ideal (briefly, $\mathcal{NZ}^{i1}$) of $\mathcal{Z}(\mathcal{I})$ if :

- 1) $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
- 2) If $[((\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{r}, \mathfrak{w}\mathcal{I})) * (\mathcal{Z}, \mathfrak{h}\mathcal{I})] * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{r}, \mathfrak{w}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

Definition 3.20: Let $\mathcal{D}(\mathcal{I}) \neq \emptyset$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$ is $\mathcal{NPZ}$, $\mathcal{D}(\mathcal{I})$ is named a neutrosophic pseudo $\mathcal{Z}_1$ -ideal (briefly, $\mathcal{NPZ}^{i1}$) of $\mathcal{Z}(\mathcal{I})$ if :

- 1) $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
- 2) $[((\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{r}, \mathfrak{w}\mathcal{I})) * (\mathcal{Z}, \mathfrak{h}\mathcal{I})] * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$, $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{r}, \mathfrak{w}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

- 1) $(0,0\mathcal{I}) \notin \mathcal{D}(\mathcal{I})$
- 2) $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \Rightarrow$
 $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * [(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$

Definition 3.28: Let $\mathcal{D}(\mathcal{I}) \neq \emptyset$ and $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$, $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$ is $\mathcal{NPZ}$, $\mathcal{D}(\mathcal{I})$ is named a neutrosophic pseudo Z-filter (briefly, $\mathcal{NPZ}^f$) of $\mathcal{Z}(\mathcal{I})$ if :

- 1) $(0,0\mathcal{I}) \notin \mathcal{D}(\mathcal{I})$
- 2) $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \Rightarrow$
 $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * [(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$

And $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ and $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \Rightarrow$
 $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot [(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{t}, \mathfrak{q}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$

Definition 3.29: If $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$ & $(\mathcal{Z}(\mathcal{J}), *, \odot, (\hat{0}, \hat{0}\mathcal{J}))$ be two $\mathcal{NZ}$, a mapping $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\mathcal{J})$ is named a neutrosophic Z- homomorphism (briefly, $\mathcal{NZ}^h$) if satisfied

- 1) $f[(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] = f(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * f(\mathfrak{t}, \mathfrak{q}\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
- 2) $f(0,0\mathcal{I}) = (\hat{0}, \hat{0}\mathcal{J})$
- 3) If f is 1-1 $\Rightarrow f$ is named a neutrosophic Z- monomorphism.
- 4) If f is onto $\Rightarrow f$ is named a neutrosophic Z- epimorphism.
- 5) If f is 1-1 and onto $\Rightarrow f$ is named a neutrosophic Z-isomorphism.

Definition 3.30: If $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$ & $(\mathcal{Z}(\mathcal{J}), *, \odot, (\hat{0}, \hat{0}\mathcal{J}))$ be two $\mathcal{NPZ}$, a mapping $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\mathcal{J})$ is named a neutrosophic pseudo Z- homomorphism (briefly, $\mathcal{NPZ}^h$) if satisfied

- 1) $f[(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] = f(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * f(\mathfrak{t}, \mathfrak{q}\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
- 2) $f[(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{t}, \mathfrak{q}\mathcal{I})] = f(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot f(\mathfrak{t}, \mathfrak{q}\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
- 3) $f(0,0\mathcal{I}) = (\hat{0}, \hat{0}\mathcal{J})$
- 4) If f is 1-1 $\Rightarrow f$ is named "a neutrosophic pseudo Z- monomorphism".
- 5) If f is onto $\Rightarrow f$ is named "a neutrosophic pseudo Z- epimorphism".
- 6) If f is 1-1 and onto $\Rightarrow f$ is named a neutrosophic pseudo Z-isomorphism.

Theorem 3.31: Let $\mathcal{Z}(\mathcal{I})$ & $\mathcal{Z}(\mathcal{J})$ be two $\mathcal{NZ}$, $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\mathcal{J})$ be a neutrosophic Z- epimorphism .If $\mathcal{D}(\mathcal{I})$ is a $\mathcal{NZ}^f$ of $\mathcal{Z}(\mathcal{I}) \Rightarrow f(\mathcal{D}(\mathcal{I}))$ is a $\mathcal{NZ}^f$ of $\mathcal{Z}(\mathcal{J})$.

Proof: let $(\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in f(\mathcal{D}(\mathcal{I})) \Rightarrow$

$$(\mathcal{Z}, \mathfrak{h}\mathcal{I}) = f(\mathfrak{a}, \mathfrak{u}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = f(\mathfrak{q}, \mathfrak{e}\mathcal{I}) \text{ where } (\mathfrak{a}, \mathfrak{u}\mathcal{I}), (\mathfrak{q}, \mathfrak{e}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$$

Since $\mathcal{D}(\mathcal{I})$ is a $\mathcal{NZ}^f$ of $\mathcal{Z}(\mathcal{I})$, $\Rightarrow$

$$(\mathfrak{a}, \mathfrak{u}\mathcal{I}) \Delta (\mathfrak{q}, \mathfrak{e}\mathcal{I}) = (\mathfrak{a}, \mathfrak{u}\mathcal{I}) * [(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * (\mathfrak{q}, \mathfrak{e}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$$

$$\text{Also } f((\mathfrak{a}, \mathfrak{u}\mathcal{I}) \Delta (\mathfrak{q}, \mathfrak{e}\mathcal{I})) \in f(\mathcal{D}(\mathcal{I}))$$

$$\begin{aligned} (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * ((\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})) \\ &= f(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * (f(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * f(\mathfrak{q}, \mathfrak{e}\mathcal{I})) \\ &= f[(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * ((\mathfrak{a}, \mathfrak{u}\mathcal{I}) * (\mathfrak{q}, \mathfrak{e}\mathcal{I}))] \\ &= f[(\mathfrak{a}, \mathfrak{u}\mathcal{I}) \Delta (\mathfrak{q}, \mathfrak{e}\mathcal{I})] \end{aligned}$$

$$\begin{aligned} (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) &\in f(\mathcal{D}(\mathcal{I})) \Rightarrow \\ f(\mathcal{D}(\mathcal{I})) &\text{ is a } \mathcal{NZ}^f \text{ of } \mathcal{Z}(\mathcal{J}). \end{aligned}$$

Theorem 3.32: Let $Z(\mathcal{J})$ & $Z(\dot{\mathcal{J}})$ be two $\mathcal{NPZ}$, $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ be a neutrosophic pseudo Z - epimorphism .If $\mathcal{D}(\mathcal{J})$ is a $\mathcal{NPZ}^f$ of $Z(\mathcal{J}) \Rightarrow f(\mathcal{D}(\mathcal{J}))$ is a $\mathcal{NPZ}^f$ of $Z(\dot{\mathcal{J}})$.

Proof: it is easy as above.

Definition 3.33: Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ be a $\mathcal{NZ}^h$ then $\ker(f) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \in Z(\mathcal{J}): f(\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\acute{0}, \acute{0}\mathcal{J})\}$ is named the kernel of f .

Definition 3.34: Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ be a $\mathcal{NPZ}^h$ then

$\ker(f) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \in Z(\mathcal{J}): f(\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\acute{0}, \acute{0}\mathcal{J})\}$ is named the kernel of f .

Remark 3.35: (1) Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ is a $\mathcal{NZ}^h$, then $\ker(f)$ is not a $\mathcal{NZ}^f$ of $Z(\mathcal{J})$.
 (2) $\mathcal{NZ}^f$ is not $\mathcal{NZ}^i$ and conversely .
 (3) $\mathcal{NZ}^f$ is not $\mathcal{NZ}^s$ and conversely .

Remark 3.36: (1) Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ is a $\mathcal{NPZ}^h$, then $\ker(f)$ is not a $\mathcal{NPZ}^f$ of $Z(\mathcal{J})$.
 (2) $\mathcal{NPZ}^f$ is not $\mathcal{NPZ}^i$ and conversely .
 (3) $\mathcal{NPZ}^f$ is not $\mathcal{NPZ}^s$ and conversely .

Theorem 3.37: Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ be a $\mathcal{NZ}^h$ then

- 1) If the identity of $Z(\mathcal{J})$ is $(0, 0\mathcal{J}) \Rightarrow$ the identity of $Z(\dot{\mathcal{J}})$ is $f(0, 0\mathcal{J})$.
- 2) If $\mathcal{U}$ is a $\mathcal{NZ}^s$ of $Z(\mathcal{J})$, then $f(\mathcal{U})$ is a $\mathcal{NZ}^s$ of $Z(\dot{\mathcal{J}})$.
- 3) If $\mathcal{U}$ is a $\mathcal{NZ}^s$ of $Z(\dot{\mathcal{J}})$, then $f^{-1}(\mathcal{U})$ is a $\mathcal{NZ}^s$ of $Z(\mathcal{J})$.

Proof: it's clear.

Theorem 3.38: Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ be a $\mathcal{NPZ}^h$ then

- 1) If the identity of $Z(\mathcal{J})$ is $(0, 0\mathcal{J}) \Rightarrow$ the identity of $Z(\dot{\mathcal{J}})$ is $f(0, 0\mathcal{J})$.
- 2) If $\mathcal{U}$ is a $\mathcal{NPZ}^s$ of $Z(\mathcal{J})$, then $f(\mathcal{U})$ is a $\mathcal{NPZ}^s$ of $Z(\dot{\mathcal{J}})$.
- 3) If $\mathcal{U}$ is a $\mathcal{NPZ}^s$ of $Z(\dot{\mathcal{J}})$, then $f^{-1}(\mathcal{U})$ is a $\mathcal{NPZ}^s$ of $Z(\mathcal{J})$.

Proof: it's clear.

Theorem 3.39: Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ is a $\mathcal{NZ}^h$ then f is a neutrosophic Z - monomorphism $\Leftrightarrow \ker(f) = \{(0, 0\mathcal{J})\}$

Proof: it's clear.

Theorem 3.40: Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ is a $\mathcal{NPZ}^h$ then f is a neutrosophic Z - monomorphism $\Leftrightarrow \ker(f) = \{(0, 0\mathcal{J})\}$

Proof: it's clear.

Theorem 3.41: Let $f: Z(\mathcal{J}) \rightarrow Z(\dot{\mathcal{J}})$ is a $\mathcal{NZ}^h$ then $\ker(f)$ is a $\mathcal{NZ}^i$ of $Z(\mathcal{J})$.

Proof: $f(0, 0\mathcal{J}) = (\acute{0}, \acute{0}\mathcal{J}) \Rightarrow (0, 0\mathcal{J}) \in \ker(f)$

Let $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * [(\mathfrak{t}, \mathfrak{q}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})] \in \ker(f)$ and $(\mathfrak{t}, \mathfrak{q}\mathcal{J}) \in \ker(f) \Rightarrow$

$$\begin{aligned} f((\mathcal{Z}, \mathfrak{h}\mathcal{J}) * [(\mathfrak{t}, \mathfrak{q}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})]) &= (\acute{0}, \acute{0}\mathcal{J}) \text{ and } f(\mathfrak{t}, \mathfrak{q}\mathcal{J}) = (\acute{0}, \acute{0}\mathcal{J}) \\ (\acute{0}, \acute{0}\mathcal{J}) &= f((\mathcal{Z}, \mathfrak{h}\mathcal{J}) * [(\mathfrak{t}, \mathfrak{q}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})]) \\ &= f(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \dot{*} [f(\mathfrak{t}, \mathfrak{q}\mathcal{J}) \dot{*} f(\mathfrak{a}, \mathfrak{w}\mathcal{J})] \\ &= f(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \dot{*} [(\acute{0}, \acute{0}\mathcal{J}) \dot{*} f(\mathfrak{a}, \mathfrak{w}\mathcal{J})] \end{aligned}$$

$$\begin{aligned} &= f(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * f(\mathfrak{a}, \mathfrak{u}\mathcal{J}) \\ &= f((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{u}\mathcal{J})) \end{aligned}$$

We get $((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{u}\mathcal{J})) \in \ker(f)$. then $\ker(f)$ is a $\mathcal{NZ}^i$ of $\mathcal{Z}(\mathcal{J})$.

Theorem 3.42: Let $f: \mathcal{Z}(\mathcal{J}) \rightarrow \mathcal{Z}(\mathcal{J})$ is a $\mathcal{NPZ}^h$ then $\ker(f)$ is a $\mathcal{NPZ}^i$ of $\mathcal{Z}(\mathcal{J})$.

Proof: it is easy as above.

4 Conclusion

We discussed the idea of a neutrosophic $\mathcal{Z}$ -algebra and neutrosophic pseudo $\mathcal{Z}$ – algebra looked into some of its properties, and the concept of neutrosophic $\mathcal{Z}$ -ideal, neutrosophic $\mathcal{Z}$ -sub algebra, neutrosophic $\mathcal{Z}$ -filter and neutrosophic $\mathcal{Z}$ - homomorphism are studied and a few properties are obtained.

Competing Interests

Authors have declared that no competing interests exist.

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