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## On Neutrosophic Z-algebras

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### Authors' contributions

*This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.*

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## Abstract

This study presents the notion of neutrosophic Z-algebra and neutrosophic pseudo Z-algebra explores some of its properties. Also studied are the neutrosophic Z-ideal, neutrosophic Z-sub algebra, and neutrosophic Z-filter. Several properties are discovered, and some findings from the study of homomorphism are discussed.

**Keywords:** Neutrosophic Z-algebra; neutrosophic pseudo Z-algebra; neutrosophic Z-sub algebra; neutrosophic Z-ideal; neutrosophic Z-filter.

## 1 Introduction

Smarandache established the area of philosophy known as neutrosophy, which has a many implementations in the real world and in mathematics, particularly in algebra [1].also gave more information about neutrosophy see [2,3]. making use of neutrosophic theory Kandasamy and Smarandache [4] in 2004 suggested a set-based algebraic structure of neutrosophic numbers of the type  $\mathcal{N} = \mathcal{Z} + \uparrow \mathcal{J}$  that they dubbed  $\mathcal{J}$ -Neutrosophic Algebraic Structure. ,where  $\mathcal{Z}, \uparrow \in \mathbb{R}$  or  $\mathbb{C}$ , and  $\mathcal{J}$  which means indetermined or uncertain thus that  $\mathcal{J}^2 = \mathcal{J}$ , is referred to as literal indeterminacy, here  $\mathcal{Z}$  is referred to as the  $\mathcal{N}$ 's determinate portion, and  $\uparrow \mathcal{J}$  is referred to as its indeterminate portion on  $\mathcal{N}$  ,with  $g\mathcal{J} + h\mathcal{J} = (g + h)\mathcal{J}$ ,  $0.\mathcal{J} = 0$ . Where  $\mathcal{J}$  is different from the

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imaginary  $i^2 = -1$ , in general,  $\mathcal{I}^j = \mathcal{I}$  if  $j > 0$ , and is unknown for  $j \leq 0$ . In 2006, the idea of neutrosophic algebraic structures was also proposed [5].

In [6,7,8,9], the idea of neutrosophic BCI/BCK –algebras, neutrosophic KU-algebras and neutrosophic B-algebras was presented.

Z-algebra is an unique algebraic structure based on logic that was first proposed in 2017 by Chandramouleeswaran et al. [10].

[11] and [12] They provided characteristics and further explanation of Z-algebra.

In this article, we explain the idea of neutrosophic Z-algebra, look at various relevant characteristics, examine a neutrosophic Z-homomorphism, and present some findings.

## 2 Preliminaries

**Definition 2.1:** [1] A neutrosophic set  $\mathcal{X}(\mathcal{I}) = \langle \mathcal{X}, \mathcal{I} \rangle = \{ \mathcal{Z} + \uparrow \mathcal{I} : \mathcal{Z}, \uparrow \in \mathcal{X} \}$ , where  $\mathcal{X} \neq \emptyset$  and  $\mathcal{I}$  an indeterminate.

**Definition 2.2:** [10] let  $\mathcal{Z} \neq \emptyset$  and  $*$  is a binary operation with constant 0 then the algebra  $(\mathcal{Z}, *, 0)$  named Z-Algebra if satisfying the following axiom:

- $\mathcal{Z}_1$ :  $\mathcal{Z} * 0 = 0$   
 $\mathcal{Z}_2$ :  $0 * \mathcal{Z} = \mathcal{Z}$   
 $\mathcal{Z}_3$ :  $\mathcal{Z} * \mathcal{Z} = \mathcal{Z}$   
 $\mathcal{Z}_4$ :  $\mathcal{Z} * \uparrow = \uparrow * \mathcal{Z}$  When  $\mathcal{Z} \neq 0$  and  $\uparrow \neq 0, \forall \mathcal{Z}, \uparrow \in \mathcal{Z}$ .

**Definition 2.3:** [10] Let  $\delta \neq \emptyset$  and  $\delta \subseteq \mathcal{Z}$  where  $(\mathcal{Z}, *, 0)$  is a Z-Algebra,  $\delta$  is named Z-subalgebra if  $\mathcal{Z} * \uparrow \in \delta, \forall \mathcal{Z}, \uparrow \in \delta$ .

**Definition 2.4:** [10] Let  $\mathcal{I} \neq \emptyset$  and  $\mathcal{I} \subseteq \mathcal{Z}$ , where  $(\mathcal{Z}, *, 0)$  is a Z-Algebra,  $\mathcal{I}$  is named

Z-ideal of  $\mathcal{Z}$  if satisfy (1)  $0 \in \mathcal{I}$  (2)  $\mathcal{Z} * \uparrow \in \mathcal{I}$ , and  $\uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}$ .

**Definition 2.5:** [11] Let  $\mathcal{I} \neq \emptyset$  and  $\mathcal{I} \subseteq \mathcal{Z}$ , where  $(\mathcal{Z}, *, 0)$  is a Z-Algebra,  $\mathcal{I}$  is named  $\mathcal{Z}_1$  – ideal of  $\mathcal{Z}$  if satisfy

- (1)  $0 \in \mathcal{I}$  (2)  $((\mathcal{Z} * \mathfrak{A}) * \mathcal{Z}) * \uparrow \in \mathcal{I}$ , and  $\uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}, \forall \mathcal{Z}, \uparrow, \mathfrak{A} \in \mathcal{Z}$ .

**Definition 2.6:** [11] Let  $\mathcal{I} \neq \emptyset$  and  $\mathcal{I} \subseteq \mathcal{Z}$ , where  $(\mathcal{Z}, *, 0)$  is a Z-Algebra,  $\mathcal{I}$  is named  $\mathcal{Z}_2$  –ideal of  $\mathcal{Z}$  if satisfy

- (1)  $0 \in \mathcal{I}$  (2)  $(\mathcal{Z} * \mathfrak{A}) * (\mathcal{Z} * \uparrow) \in \mathcal{I}$ , and  $\uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}, \forall \mathcal{Z}, \uparrow, \mathfrak{A} \in \mathcal{Z}$

**Definition 2.7:** [12] Let  $\mathcal{I} \neq \emptyset$  and  $\mathcal{I} \subseteq \mathcal{Z}$ , where  $(\mathcal{Z}, *, 0)$  is a Z-Algebra,  $\mathcal{I}$  is named  $\mathcal{Z}_p$  –ideal of  $\mathcal{Z}$  if satisfy

- (1)  $0 \in \mathcal{I}$  (2)  $(\mathcal{Z} * \mathfrak{A}) * (\uparrow * \mathfrak{A}) \in \mathcal{I}$ , and  $\uparrow \in \mathcal{I} \Rightarrow \mathcal{Z} \in \mathcal{I}, \forall \mathcal{Z}, \uparrow, \mathfrak{A} \in \mathcal{Z}$ .

**Definition 2.8:** [10] let  $\mathcal{F} \neq \emptyset$  and  $\mathcal{F} \subseteq \mathcal{Z}$ , where  $(\mathcal{Z}, *, 0)$  is a Z-Algebra,  $\mathcal{F}$  is named Z-filter of  $\mathcal{Z}$  if  $\mathcal{Z}\mathfrak{A}\uparrow = \mathcal{Z} * (\mathcal{Z} * \uparrow) \in \mathcal{F}, \forall \mathcal{Z}, \uparrow \in \mathcal{F}, (\mathcal{Z} \neq \uparrow)$ .

**Example 2.9:** let  $\mathcal{Z} = \{0, \mathcal{Z}, \uparrow, \mathfrak{A}\}$  be set and  $*$  is a binary operation defined on  $\mathcal{Z}$  by the table:

*	0	$\mathcal{Z}$	$\uparrow$	$\mathfrak{A}$
0	0	$\mathcal{Z}$	$\uparrow$	$\mathfrak{A}$
$\mathcal{Z}$	0	$\mathcal{Z}$	$\uparrow$	$\uparrow$
$\uparrow$	0	$\uparrow$	$\uparrow$	$\uparrow$
$\mathfrak{A}$	0	$\uparrow$	$\uparrow$	$\mathfrak{A}$

Then  $(Z, *, 0)$  is  $Z$ -Algebra.  $\delta = \{0, \uparrow, \downarrow\}$  is  $Z$ -subalgebra and  $\mathcal{I} = \{0, \uparrow, \downarrow\}$  is a  $Z_1$ -ideal,  $\mathcal{I}^* = \{0, \uparrow, \downarrow\}$  is a  $(Z_2$ -ideal,  $Z_p$ -ideal)  $Z$ -ideal and  $\mathcal{F} = \{0, \uparrow, \downarrow\}$  is  $Z$ -filter.

Note : every  $(Z_1$ -ideal,  $Z_2$ -ideal) is an ideal of  $Z$ .

**Definition 2.10:** [11] Let  $Z \neq \emptyset$  with two binary operations  $*, \odot$  and constant 0 then the algebra  $(Z, *, \odot, 0)$  named pseudo  $Z$ -Algebra (briefly,  $PZ$ ) if satisfying the following axiom:

$$PZ_1: \mathcal{Z} * 0 = \mathcal{Z} \odot 0 = 0$$

$$PZ_2: 0 * \mathcal{Z} = 0 \odot \mathcal{Z} = \mathcal{Z}$$

$$PZ_3: \mathcal{Z} * \mathcal{Z} = \mathcal{Z} \odot \mathcal{Z} = \mathcal{Z}$$

$$PZ_4: \mathcal{Z} * \uparrow = \uparrow \odot \mathcal{Z} \text{ When } \mathcal{Z} \neq 0 \text{ and } \uparrow \neq 0, \forall \mathcal{Z}, \uparrow \in Z.$$

**Definition 2.11:** [11] Let  $\delta \neq \emptyset$  and  $\delta \subseteq Z$ , where  $(Z, *, \odot, 0)$  is  $PZ$  then  $\delta$  is named a pseudo  $Z$ -subalgebra if  $\mathcal{Z} * \uparrow, \mathcal{Z} \odot \uparrow \in \delta, \forall \mathcal{Z}, \uparrow \in \delta$ .

**Example 2.12:** Let  $Z = \{0, \uparrow, \downarrow\}$  be set and  $*, \odot$  are a binary operations defined on  $Z$  by the table as follows:

*	0	$\uparrow$	$\downarrow$	$\odot$	0	$\uparrow$	$\downarrow$
0	0	$\uparrow$	$\downarrow$	0	0	$\uparrow$	$\downarrow$
$\uparrow$	0	$\uparrow$	$\downarrow$	$\uparrow$	0	$\uparrow$	$\downarrow$
$\downarrow$	0	$\uparrow$	$\downarrow$	$\downarrow$	0	$\uparrow$	$\downarrow$

Then  $(Z, *, \odot, 0)$  is pseudo  $Z$ -algebra,  $\delta = \{\uparrow, \downarrow\}$  is a pseudo  $Z$ -sub algebra.

### 3 Neutrosophic $Z$ -algebra

**Definition 3.1:** A neutrosophic  $Z$ -algebra is the triple  $(Z(\mathcal{I}), *, (0, 0\mathcal{I}))$  (briefly,  $\mathcal{NZ}$ ) (where  $(Z, *, 0)$  be a  $Z$ -algebra,  $Z(\mathcal{I}) = \langle Z, \mathcal{I} \rangle$  a neutrosophic set)

if  $(\mathcal{Z}, \mathcal{h}\mathcal{I}), (\uparrow, \mathcal{q}\mathcal{I})$  are any two elements of  $Z(\mathcal{I})$  with  $\mathcal{Z}, \mathcal{h}, \uparrow, \mathcal{q} \in Z$  satisfies

$$(\mathcal{Z}, \mathcal{h}\mathcal{I}) * (\uparrow, \mathcal{q}\mathcal{I}) = (\mathcal{Z} * \uparrow, (\mathcal{Z} * \mathcal{q} \wedge \mathcal{h} * \uparrow \wedge \mathcal{h} * \mathcal{q})\mathcal{I})$$

An element  $\mathcal{Z} \in Z$  is represented by  $(\mathcal{Z}, 0\mathcal{I}) \in Z(\mathcal{I})$ ,

$$(\mathcal{Z}, 0\mathcal{I}) * (\mathcal{h}, 0\mathcal{I}) = (\mathcal{Z} * \mathcal{h}, 0\mathcal{I}) = (\mathcal{Z} \wedge \sim \mathcal{h}, 0) \text{ where } \sim \mathcal{h} \text{ is the negation of } \mathcal{h} \text{ in } Z$$

$$\text{And } (\mathcal{Z}, \mathcal{h}\mathcal{I}) = (\uparrow, \mathcal{q}\mathcal{I}) \Leftrightarrow (\mathcal{Z} = \uparrow \text{ and } \mathcal{h} = \mathcal{q})$$

**Definition 3.2:** A neutrosophic pseudo  $Z$ -algebra is  $(Z(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$  (briefly,  $\mathcal{NPZ}$ ) (where  $(Z, *, \odot, 0)$  be a pseudo  $Z$ -algebra

If  $(\mathcal{Z}, \mathcal{h}\mathcal{I}), (\uparrow, \mathcal{q}\mathcal{I})$  are any two elements of  $Z(\mathcal{I})$  with  $\mathcal{Z}, \mathcal{h}, \uparrow, \mathcal{q} \in Z$  satisfies

$$(\mathcal{Z}, \mathcal{h}\mathcal{I}) * (\uparrow, \mathcal{q}\mathcal{I}) = (\mathcal{Z} * \uparrow, (\mathcal{Z} * \mathcal{q} \wedge \mathcal{h} * \uparrow \wedge \mathcal{h} * \mathcal{q})\mathcal{I})$$

$$(\uparrow, \mathcal{q}\mathcal{I}) \odot (\mathcal{Z}, \mathcal{h}\mathcal{I}) = (\uparrow \odot \mathcal{Z}, (\mathcal{Z} \odot \mathcal{q} \wedge \mathcal{h} \odot \uparrow \wedge \mathcal{h} \odot \mathcal{q})\mathcal{I})$$

$$\text{Where } (\mathcal{Z}, \mathcal{h}\mathcal{I}) * (\uparrow, \mathcal{q}\mathcal{I}) = (\mathcal{Z}, \mathcal{h}\mathcal{I}) \odot (\uparrow, \mathcal{q}\mathcal{I})$$

$$\text{When } (\mathcal{Z}, \mathcal{h}\mathcal{I}) \neq (0, 0\mathcal{I}) \text{ and } (\uparrow, \mathcal{q}\mathcal{I}) \neq (0, 0\mathcal{I}), \forall (\mathcal{Z}, \mathcal{h}\mathcal{I}), (\uparrow, \mathcal{q}\mathcal{I}) \in Z(\mathcal{I})$$

**Theorem 3.3:** Every  $\mathcal{NZ} (Z(\mathcal{I}), *, (0, 0\mathcal{I}))$  with condition  $(0, 0\mathcal{I}) * (\mathcal{Z}, \mathcal{h}\mathcal{I}) = (\mathcal{Z}, \mathcal{h}\mathcal{I})$  is a  $Z$ -algebra and conversely, not.

**Proof:** let  $(\mathcal{X}(\mathcal{I}), *, (0, 0\mathcal{I}))$  is  $\mathcal{NZ}$

Let  $r = (\mathcal{Z}, \mathfrak{h}\mathcal{I})$  and  $0 = (0, 0\mathcal{I})$

$$\begin{aligned} Z_1: r * 0 &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (0, 0\mathcal{I}) = (\mathcal{Z} * 0, (\mathcal{Z} * 0 \wedge \mathfrak{h} * 0)\mathcal{I}) = (0, (0 \wedge 0)\mathcal{I}) = (0, 0\mathcal{I}) \\ Z_2: 0 * r &= (0, 0\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (0 * \mathcal{Z}, (0 * \mathfrak{h} \wedge 0 * \mathcal{Z})\mathcal{I}) = (\mathcal{Z}, (\mathfrak{h} \wedge \mathcal{Z})\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \\ Z_3: r * r &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (\mathcal{Z} * \mathcal{Z}, (\mathcal{Z} * \mathfrak{h} \wedge \mathfrak{h} * \mathcal{Z} \wedge \mathfrak{h} * \mathfrak{h})\mathcal{I}) \\ &= (\mathcal{Z}, (\mathcal{Z} \wedge \sim \mathfrak{h} \wedge \mathfrak{h} \wedge \sim \mathcal{Z} \wedge \mathfrak{h})\mathcal{I}) \\ &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \end{aligned}$$

$Z_4$ : if  $r * s = s * r$ , when  $r \neq 0$  &  $s \neq 0, \forall r, s \in \mathcal{Z}(\mathcal{I})$

let  $r = (\mathcal{Z}, \mathfrak{h}\mathcal{I}), s = (\mathfrak{t}, \mathfrak{q}\mathcal{I})$ ,

$$(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathfrak{t}, \mathfrak{q}\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I})$$

$$(\mathcal{Z} * \mathfrak{t}, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \mathfrak{t} \wedge \mathfrak{h} * \mathfrak{q})\mathcal{I}) = (\mathfrak{t} * \mathcal{Z}, (\mathfrak{t} * \mathfrak{h} \wedge \mathfrak{q} * \mathcal{Z} \wedge \mathfrak{q} * \mathfrak{h})\mathcal{I})$$

Suppose  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (0, 0\mathcal{I})$  &  $(\mathfrak{t}, \mathfrak{q}\mathcal{I}) \neq (0, 0\mathcal{I})$  we get

$$0 * \mathfrak{t} = \mathfrak{t} * 0 \Rightarrow \mathfrak{t} = 0$$

$$\text{and } 0 * \mathfrak{q} \wedge 0 * 0 = 0 * 0 \wedge \mathfrak{q} * 0 \Rightarrow \mathfrak{q} = 0$$

We get a contradiction.

Then  $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$  is a  $\mathcal{Z}$ -algebra.

**Theorem 3.4:** Every  $\mathcal{NPZ}$ ,  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$  with condition  $(0, 0\mathcal{I}) * (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (0, 0\mathcal{I}) \odot (\mathcal{Z}, \mathfrak{h}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I})$  is a pseudo  $\mathcal{Z}$ -algebra and conversely, not.

**Proof:** it is easy as above.

**Definition 3.5:** Let  $\mathfrak{S}(\mathcal{I}) \neq \emptyset$  and  $\mathfrak{S}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ ,  $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$  is  $\mathcal{NZ}$ ,  $\mathfrak{S}(\mathcal{I})$  is named a neutrosophic  $\mathcal{Z}$ -subalgebra (briefly,  $\mathcal{NZ}^s$ ) of  $\mathcal{Z}(\mathcal{I})$  if

- 1)  $(0, 0\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 2)  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 3)  $\mathfrak{S}(\mathcal{I})$  Contains a proper sub set which a  $\mathcal{Z}$ -algebra.

**Definition 3.6:** Let  $\mathfrak{S}(\mathcal{I}) \neq \emptyset$  and  $\mathfrak{S}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ ,  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$  is  $\mathcal{NPZ}$ ,  $\mathfrak{S}(\mathcal{I})$  is called a neutrosophic pseudo  $\mathcal{Z}$ -subalgebra (briefly,  $\mathcal{NPZ}^s$ ) of  $\mathcal{Z}(\mathcal{I})$  if

- 1)  $(0, 0\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 2)  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$  &  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathfrak{S}(\mathcal{I})$
- 3)  $\mathfrak{S}(\mathcal{I})$  Contains a proper sub set which a pseudo  $\mathcal{Z}$ -algebra.

**Theorem 3.7:** If  $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) \neq \emptyset$  and  $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$  for  $\omega \neq 0$ ,  $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$  is  $\mathcal{NZ}$ , where  $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{Z}(\mathcal{I}): (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\omega, \omega\mathcal{I}) = (\omega, \omega\mathcal{I})\}$

Then 1)  $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I})$  is  $\mathcal{NZ}^s$ .

2)  $\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I}) \subseteq \mathcal{A}_{(0, 0\mathcal{I})}(\mathcal{I})$ .

**Proof:** 1) clearly  $(0, 0\mathcal{I}) \in \mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I})$

$\mathcal{A}_{(\omega, \omega\mathcal{I})}(\mathcal{I})$  contain a proper sub set which a  $\mathcal{Z}$ -algebra.

Let  $(\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \Rightarrow$

$$(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \quad , \quad (\mathfrak{f}, \mathfrak{q}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \Rightarrow$$

$$\mathcal{Z} * \omega = \omega \quad , \quad \mathcal{Z} * \omega \wedge \mathfrak{h} * \omega = \omega \quad \& \quad \mathfrak{f} * \omega = \omega \quad , \quad \mathfrak{f} * \omega \wedge \mathfrak{q} * \omega = \omega \quad \text{since } \omega \neq 0 \Rightarrow$$

$$\mathcal{Z} = \mathfrak{h} = \mathfrak{f} = \mathfrak{q} = \omega$$

$$\begin{aligned} [(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J})] * (\omega, \omega\mathcal{J}) &= [\mathcal{Z} * \mathfrak{f}, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \mathfrak{f})\mathcal{J}] * (\omega, \omega\mathcal{J}) \\ &= [(\mathcal{Z} * \mathfrak{f}) * \omega, ((\mathcal{Z} * \mathfrak{f}) * \omega \wedge (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \mathfrak{f}) * \omega)\mathcal{J}] \\ &= [\omega * \omega, (\omega * \omega \wedge \omega * \omega)\mathcal{J}] \\ &= (\omega, \omega\mathcal{J}) \end{aligned}$$

This shows that  $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$

Then  $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$  is  $\mathcal{NZ}^s$ .

(2) it's easy.

**Theorem 3.8:** If  $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \neq \emptyset$  and  $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$ , for  $\omega \neq 0$ ,

$(\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$  is  $\mathcal{NPZ}$ , where  $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}(\mathcal{J}) : (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \text{ \& } (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \odot \omega, \omega\mathcal{J} = \omega, \omega\mathcal{J}\}$

Then 1)  $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$  is  $\mathcal{NPZ}^s$ .

2)  $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{A}_{(0, 0\mathcal{J})}(\mathcal{J})$ .

**Proof:** it is easy as above.

**Theorem 3.9:** If  $\mathcal{Z}_\xi(\mathcal{J}) \neq \emptyset$  and  $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$ ,  $(\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$  is  $\mathcal{NZ}$ , where  $\mathcal{Z}_\xi(\mathcal{J}) = \{(\mathcal{Z}, \mathcal{Z}\mathcal{J}) : \mathcal{Z} \in \mathcal{Z}\}$  Then  $\mathcal{Z}_\xi(\mathcal{J})$  is a  $\mathcal{NZ}^s$  of  $\mathcal{Z}(\mathcal{J})$ .

**Proof:** clearly  $(0, 0\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$  and the third condition is satisfied for  $\mathcal{Z}_\xi(\mathcal{J})$

Let  $(\mathfrak{f}, \mathfrak{f}\mathcal{J}), (\mathfrak{h}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$ ,  $\mathfrak{f}, \mathfrak{h} \in \mathcal{Z} \Rightarrow$

$$(\mathfrak{f}, \mathfrak{f}\mathcal{J}) * (\mathfrak{h}, \mathfrak{h}\mathcal{J}) = (\mathfrak{f} * \mathfrak{h}, (\mathfrak{f} * \mathfrak{h})\mathcal{J})$$

This shows that  $(\mathfrak{f}, \mathfrak{f}\mathcal{J}) * (\mathfrak{h}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$

Then  $\mathcal{Z}_\xi(\mathcal{J})$  is a  $\mathcal{NZ}^s$  of  $\mathcal{Z}(\mathcal{J})$ .

**Theorem 3.10:** If  $\mathcal{Z}_\xi(\mathcal{J}) \neq \emptyset$  and  $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$ ,  $(\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$  is  $\mathcal{NPZ}$ , where

$\mathcal{Z}_\xi(\mathcal{J}) = \{(\mathcal{Z}, \mathcal{Z}\mathcal{J}) : \mathcal{Z} \in \mathcal{Z}\}$  Then  $\mathcal{Z}_\xi(\mathcal{J})$  is a  $\mathcal{NPZ}^s$  of  $\mathcal{Z}(\mathcal{J})$ .

**Proof:** it is easy as above.

**Example 3.11:** Let  $*$  is a binary operation defined on  $\mathcal{Z}_\xi(\mathcal{J}) = \{(0, 0\mathcal{J}), (\mathcal{Z}, \mathcal{Z}\mathcal{J}), (\mathfrak{f}, \mathfrak{f}\mathcal{J}), (\mathfrak{a}, \mathfrak{a}\mathcal{J})\}$  as follows:

*	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{Z}\mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{a}, \mathfrak{a}\mathcal{J})$
$(0, 0\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{Z}\mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{a}, \mathfrak{a}\mathcal{J})$
$(\mathcal{Z}, \mathcal{Z}\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{Z}\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{Z}\mathcal{J})$
$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(0, 0\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$
$(\mathfrak{a}, \mathfrak{a}\mathcal{J})$	$(0, 0\mathcal{J})$	$(\mathcal{Z}, \mathcal{Z}\mathcal{J})$	$(\mathfrak{f}, \mathfrak{f}\mathcal{J})$	$(\mathfrak{a}, \mathfrak{a}\mathcal{J})$

Then  $(\mathcal{Z}_\xi(\mathcal{J}), *, (0, 0\mathcal{J}))$  is a  $\mathcal{NZ}^s$  of  $\mathcal{Z}(\mathcal{J})$

**Theorem 3.12:** Let  $\{\mathcal{A}(\mathcal{J})_\gamma : \gamma \in \mathcal{S}\}$  and  $\mathcal{A}(\mathcal{J})_\gamma \neq \emptyset$  be a collection of  $\mathcal{NZ}^s$  of  $\mathcal{Z}(\mathcal{J})$  if

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \{(0,0\mathcal{I})\} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \text{ is a } \mathcal{NZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{I}).$$

**Proof:** since  $(0,0\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S} \Rightarrow$

$$(0,0\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset$$

And the third condition was achieved for  $\mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S} \Rightarrow$

The third condition was achieved for  $\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \{(0,0\mathcal{I})\} \Rightarrow \exists (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (0,0\mathcal{I}) \Rightarrow$$

$$\{(0,0\mathcal{I})\} \subseteq \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}, \text{ which is a } \mathcal{Z} - \text{algebra}$$

$$\text{Let } (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S}$$

Since  $\mathcal{A}(\mathcal{I})_{\gamma}$  is a  $\mathcal{NZ}^{\mathcal{S}}$ ,  $\forall \gamma \in \mathcal{S}$  of  $\mathcal{Z}(\mathcal{I})$  then

$$(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma}, \forall \gamma \in \mathcal{S}, \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$$

hence  $\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$  is a  $\mathcal{NZ}^{\mathcal{S}}$  of  $\mathcal{Z}(\mathcal{I})$ .

**Theorem 3.13:** Let  $\{\mathcal{A}(\mathcal{I})_{\gamma} : \gamma \in \mathcal{S}\}$  and  $\mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset$  be a collection of  $\mathcal{NPZ}^{\mathcal{S}}$  of  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$  is  $\mathcal{NPZ}$  if

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \{(0,0\mathcal{I})\} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \text{ is a } \mathcal{NPZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{I}).$$

**Proof:** it is easy as above.

**Theorem 3.14:** Let  $\{\mathcal{A}(\mathcal{I})_{\gamma} : \gamma \in \mathcal{S}\}$  and  $\mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset$  be a collection of  $\mathcal{NZ}^{\mathcal{S}}$  of  $\mathcal{Z}(\mathcal{I})$  if  $\mathcal{A}(\mathcal{I})_1 \subseteq \mathcal{A}(\mathcal{I})_2 \subseteq \dots$  then

$$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \text{ is a } \mathcal{NZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{I}).$$

$$\textbf{Proof:} \text{ obviously } (0,0\mathcal{I}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma} \neq \emptyset \Rightarrow \exists (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$$

$$\Rightarrow \text{For some } \gamma \in \mathcal{S} \text{ } (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma} \text{ and } (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{A}(\mathcal{I})_{\gamma \in \mathcal{S}}$$

$$\Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma}$$

Let  $\mathfrak{S}(\mathcal{I})_\gamma$  be a proper sub set of  $\mathcal{A}(\mathcal{I})_\gamma$ , for some  $\gamma \in \mathcal{S}$  which a  $\mathcal{Z}$ - algebra,  
then for any  $\gamma \in \mathcal{S}$ ,  $\mathfrak{S}(\mathcal{I})_\gamma \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$  then

$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_{\gamma \in \mathcal{S}}$  is  $\mathcal{NZ}^{\mathcal{S}}$  of  $\mathcal{Z}(\mathcal{I})$ .

**Theorem 3.15:** Let  $\{\mathcal{A}(\mathcal{I})_\gamma : \gamma \in \mathcal{S}\}$  and  $\mathcal{A}(\mathcal{I})_\gamma \neq \emptyset$  be a collection of  $\mathcal{NPZ}^{\mathcal{S}}$  of  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$  is  $\mathcal{NPZ}$  if  $\mathcal{A}(\mathcal{I})_1 \subseteq \mathcal{A}(\mathcal{I})_2 \subseteq \dots$  then

$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{I})_\gamma$  is  $\mathcal{NPZ}^{\mathcal{S}}$  of  $\mathcal{Z}(\mathcal{I})$ .

**Proof:** it is easy as above.

**Definition 3.16:** Let  $\mathcal{D}(\mathcal{I}) \neq \emptyset$  and  $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ ,  $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$  is  $\mathcal{NZ}$ ,  $\mathcal{D}(\mathcal{I})$  is named a neutrosophic  $\mathcal{Z}$ -ideal (briefly,  $\mathcal{NZ}^i$ ) of  $\mathcal{Z}(\mathcal{I})$  if :

- 1)  $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
- 2) If  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ , and  $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$

**Remark 3.17:** Let  $\mathcal{D}(\mathcal{I})$  is a  $\mathcal{NZ}^i$  of  $\mathcal{Z}(\mathcal{I})$  if

$(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$  and  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) = (0, 0\mathcal{I})$  then  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ .

**Proof:** let  $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$  and  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) = (0, 0\mathcal{I}) \Rightarrow$

$(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$  and  $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ ,  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$

Since  $\mathcal{D}(\mathcal{I})$  is a  $\mathcal{NZ}^i \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ .

**Definition 3.18:** Let  $\mathcal{D}(\mathcal{I}) \neq \emptyset$  and  $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ ,  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$  is  $\mathcal{NPZ}$ ,  $\mathcal{D}(\mathcal{I})$  is named a neutrosophic pseudo  $\mathcal{Z}$ -ideal (briefly,  $\mathcal{NPZ}^i$ ) of  $\mathcal{Z}(\mathcal{I})$  if :

- 1)  $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ .
- 2) If  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ , and  $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$

And  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ , and  $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ .

**Definition 3.19:** Let  $\mathcal{D}(\mathcal{I}) \neq \emptyset$  and  $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ ,  $(\mathcal{Z}(\mathcal{I}), *, (0, 0\mathcal{I}))$  is  $\mathcal{NZ}$ ,  $\mathcal{D}(\mathcal{I})$  is named a neutrosophic  $\mathcal{Z}_1$ -ideal (briefly,  $\mathcal{NZ}^{i1}$ ) of  $\mathcal{Z}(\mathcal{I})$  if :

- 1)  $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
- 2) If  $[((\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{r}, \mathfrak{u}\mathcal{I})) * (\mathcal{Z}, \mathfrak{h}\mathcal{I})] * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ , and  $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ ,  $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{r}, \mathfrak{u}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$

**Definition 3.20:** Let  $\mathcal{D}(\mathcal{I}) \neq \emptyset$  and  $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ ,  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0, 0\mathcal{I}))$  is  $\mathcal{NPZ}$ ,  $\mathcal{D}(\mathcal{I})$  is named a neutrosophic pseudo  $\mathcal{Z}_1$ -ideal (briefly,  $\mathcal{NPZ}^{i1}$ ) of  $\mathcal{Z}(\mathcal{I})$  if :

- 1)  $(0, 0\mathcal{I}) \in \mathcal{D}(\mathcal{I})$
- 2)  $[((\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{r}, \mathfrak{u}\mathcal{I})) * (\mathcal{Z}, \mathfrak{h}\mathcal{I})] * (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ , and  $(\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I}) \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$ ,  $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{r}, \mathfrak{u}\mathcal{I}), (\mathfrak{f}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$





- 1)  $(0,0\mathcal{I}) \notin \mathcal{D}(\mathcal{I})$
- 2)  $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$  and  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \Rightarrow$   
 $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * [(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$

**Definition 3.28:** Let  $\mathcal{D}(\mathcal{I}) \neq \emptyset$  and  $\mathcal{D}(\mathcal{I}) \subseteq \mathcal{Z}(\mathcal{I})$ ,  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$  is  $\mathcal{NPZ}$ ,  $\mathcal{D}(\mathcal{I})$  is named a neutrosophic pseudo Z-filter (briefly,  $\mathcal{NPZ}^f$ ) of  $\mathcal{Z}(\mathcal{I})$  if :

- 1)  $(0,0\mathcal{I}) \notin \mathcal{D}(\mathcal{I})$
- 2)  $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$  and  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \Rightarrow$   
 $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * [(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$

And  $\forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$  and  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \neq (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \Rightarrow$   
 $(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot [(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{t}, \mathfrak{q}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$

**Definition 3.29:** If  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$  &  $(\mathcal{Z}(\mathcal{J}), *, \odot, (\hat{0}, \hat{0}\mathcal{J}))$  be two  $\mathcal{NZ}$ , a mapping  $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\mathcal{J})$  is named a neutrosophic Z- homomorphism (briefly,  $\mathcal{NZ}^h$ ) if satisfied

- 1)  $f[(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] = f(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * f(\mathfrak{t}, \mathfrak{q}\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
- 2)  $f(0,0\mathcal{I}) = (\hat{0}, \hat{0}\mathcal{J})$
- 3) If  $f$  is 1-1  $\Rightarrow f$  is named a neutrosophic Z- monomorphism.
- 4) If  $f$  is onto  $\Rightarrow f$  is named a neutrosophic Z- epimorphism.
- 5) If  $f$  is 1-1 and onto  $\Rightarrow f$  is named a neutrosophic Z-isomorphism.

**Definition 3.30:** If  $(\mathcal{Z}(\mathcal{I}), *, \odot, (0,0\mathcal{I}))$  &  $(\mathcal{Z}(\mathcal{J}), *, \odot, (\hat{0}, \hat{0}\mathcal{J}))$  be two  $\mathcal{NPZ}$ , a mapping  $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\mathcal{J})$  is named a neutrosophic pseudo Z- homomorphism (briefly,  $\mathcal{NPZ}^h$ ) if satisfied

- 1)  $f[(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})] = f(\mathcal{Z}, \mathfrak{h}\mathcal{I}) * f(\mathfrak{t}, \mathfrak{q}\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
- 2)  $f[(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot (\mathfrak{t}, \mathfrak{q}\mathcal{I})] = f(\mathcal{Z}, \mathfrak{h}\mathcal{I}) \odot f(\mathfrak{t}, \mathfrak{q}\mathcal{I}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in \mathcal{Z}(\mathcal{I})$
- 3)  $f(0,0\mathcal{I}) = (\hat{0}, \hat{0}\mathcal{J})$
- 4) If  $f$  is 1-1  $\Rightarrow f$  is named "a neutrosophic pseudo Z- monomorphism".
- 5) If  $f$  is onto  $\Rightarrow f$  is named "a neutrosophic pseudo Z- epimorphism".
- 6) If  $f$  is 1-1 and onto  $\Rightarrow f$  is named a neutrosophic pseudo Z-isomorphism.

**Theorem 3.31:** Let  $\mathcal{Z}(\mathcal{I})$  &  $\mathcal{Z}(\mathcal{J})$  be two  $\mathcal{NZ}$ ,  $f: \mathcal{Z}(\mathcal{I}) \rightarrow \mathcal{Z}(\mathcal{J})$  be a neutrosophic Z- epimorphism .If  $\mathcal{D}(\mathcal{I})$  is a  $\mathcal{NZ}^f$  of  $\mathcal{Z}(\mathcal{I}) \Rightarrow f(\mathcal{D}(\mathcal{I}))$  is a  $\mathcal{NZ}^f$  of  $\mathcal{Z}(\mathcal{J})$ .

**Proof:** let  $(\mathcal{Z}, \mathfrak{h}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) \in f(\mathcal{D}(\mathcal{I})) \Rightarrow$

$$(\mathcal{Z}, \mathfrak{h}\mathcal{I}) = f(\mathfrak{a}, \mathfrak{u}\mathcal{I}), (\mathfrak{t}, \mathfrak{q}\mathcal{I}) = f(\mathfrak{q}, \mathfrak{e}\mathcal{I}) \text{ where } (\mathfrak{a}, \mathfrak{u}\mathcal{I}), (\mathfrak{q}, \mathfrak{e}\mathcal{I}) \in \mathcal{D}(\mathcal{I})$$

Since  $\mathcal{D}(\mathcal{I})$  is a  $\mathcal{NZ}^f$  of  $\mathcal{Z}(\mathcal{I})$ ,  $\Rightarrow$

$$(\mathfrak{a}, \mathfrak{u}\mathcal{I}) \Delta (\mathfrak{q}, \mathfrak{e}\mathcal{I}) = (\mathfrak{a}, \mathfrak{u}\mathcal{I}) * [(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * (\mathfrak{q}, \mathfrak{e}\mathcal{I})] \in \mathcal{D}(\mathcal{I})$$

$$\text{Also } f((\mathfrak{a}, \mathfrak{u}\mathcal{I}) \Delta (\mathfrak{q}, \mathfrak{e}\mathcal{I})) \in f(\mathcal{D}(\mathcal{I}))$$

$$\begin{aligned} (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) &= (\mathcal{Z}, \mathfrak{h}\mathcal{I}) * ((\mathcal{Z}, \mathfrak{h}\mathcal{I}) * (\mathfrak{t}, \mathfrak{q}\mathcal{I})) \\ &= f(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * (f(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * f(\mathfrak{q}, \mathfrak{e}\mathcal{I})) \\ &= f[(\mathfrak{a}, \mathfrak{u}\mathcal{I}) * ((\mathfrak{a}, \mathfrak{u}\mathcal{I}) * (\mathfrak{q}, \mathfrak{e}\mathcal{I}))] \\ &= f[(\mathfrak{a}, \mathfrak{u}\mathcal{I}) \Delta (\mathfrak{q}, \mathfrak{e}\mathcal{I})] \end{aligned}$$

$$\begin{aligned} (\mathcal{Z}, \mathfrak{h}\mathcal{I}) \Delta (\mathfrak{t}, \mathfrak{q}\mathcal{I}) &\in f(\mathcal{D}(\mathcal{I})) \Rightarrow \\ f(\mathcal{D}(\mathcal{I})) &\text{ is a } \mathcal{NZ}^f \text{ of } \mathcal{Z}(\mathcal{J}). \end{aligned}$$

**Theorem 3.32:** Let  $Z(\mathcal{J})$  &  $Z(\hat{\mathcal{J}})$  be two  $\mathcal{NPZ}$ ,  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  be a neutrosophic pseudo  $Z$ - epimorphism .If  $\mathcal{D}(\mathcal{J})$  is a  $\mathcal{NPZ}^f$  of  $Z(\mathcal{J}) \Rightarrow f(\mathcal{D}(\mathcal{J}))$  is a  $\mathcal{NPZ}^f$  of  $Z(\hat{\mathcal{J}})$ .

**Proof:** it is easy as above.

**Definition 3.33:** Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  be a  $\mathcal{NZ}^h$  then  $\ker(f) = \{(\mathcal{Z}, \mathcal{h}\mathcal{J}) \in Z(\mathcal{J}): f(\mathcal{Z}, \mathcal{h}\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J})\}$  is named the kernel of  $f$ .

**Definition 3.34:** Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  be a  $\mathcal{NPZ}^h$  then

$\ker(f) = \{(\mathcal{Z}, \mathcal{h}\mathcal{J}) \in Z(\mathcal{J}): f(\mathcal{Z}, \mathcal{h}\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J})\}$  is named the kernel of  $f$ .

**Remark 3.35:** (1) Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  is a  $\mathcal{NZ}^h$ , then  $\ker(f)$  is not a  $\mathcal{NZ}^f$  of  $Z(\mathcal{J})$ .  
 (2)  $\mathcal{NZ}^f$  is not  $\mathcal{NZ}^i$  and conversely .  
 (3)  $\mathcal{NZ}^f$  is not  $\mathcal{NZ}^s$  and conversely .

**Remark 3.36:** (1) Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  is a  $\mathcal{NPZ}^h$ , then  $\ker(f)$  is not a  $\mathcal{NPZ}^f$  of  $Z(\mathcal{J})$ .  
 (2)  $\mathcal{NPZ}^f$  is not  $\mathcal{NPZ}^i$  and conversely .  
 (3)  $\mathcal{NPZ}^f$  is not  $\mathcal{NPZ}^s$  and conversely .

**Theorem 3.37:** Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  be a  $\mathcal{NZ}^h$  then

- 1) If the identity of  $Z(\mathcal{J})$  is  $(0, 0\mathcal{J}) \Rightarrow$  the identity of  $Z(\hat{\mathcal{J}})$  is  $f(0, 0\mathcal{J})$ .
- 2) If  $\mathcal{U}$  is a  $\mathcal{NZ}^s$  of  $Z(\mathcal{J})$ , then  $f(\mathcal{U})$  is a  $\mathcal{NZ}^s$  of  $Z(\hat{\mathcal{J}})$ .
- 3) If  $\mathcal{U}$  is a  $\mathcal{NZ}^s$  of  $Z(\hat{\mathcal{J}})$ , then  $f^{-1}(\mathcal{U})$  is a  $\mathcal{NZ}^s$  of  $Z(\mathcal{J})$ .

**Proof:** it's clear.

**Theorem 3.38:** Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  be a  $\mathcal{NPZ}^h$  then

- 1) If the identity of  $Z(\mathcal{J})$  is  $(0, 0\mathcal{J}) \Rightarrow$  the identity of  $Z(\hat{\mathcal{J}})$  is  $f(0, 0\mathcal{J})$ .
- 2) If  $\mathcal{U}$  is a  $\mathcal{NPZ}^s$  of  $Z(\mathcal{J})$ , then  $f(\mathcal{U})$  is a  $\mathcal{NPZ}^s$  of  $Z(\hat{\mathcal{J}})$ .
- 3) If  $\mathcal{U}$  is a  $\mathcal{NPZ}^s$  of  $Z(\hat{\mathcal{J}})$ , then  $f^{-1}(\mathcal{U})$  is a  $\mathcal{NPZ}^s$  of  $Z(\mathcal{J})$ .

**Proof:** it's clear.

**Theorem 3.39:** Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  is a  $\mathcal{NZ}^h$  then  $f$  is a neutrosophic  $Z$ - monomorphism  $\Leftrightarrow \ker(f) = \{(0, 0\mathcal{J})\}$

**Proof:** it's clear.

**Theorem 3.40:** Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  is a  $\mathcal{NPZ}^h$  then  $f$  is a neutrosophic  $Z$ - monomorphism  $\Leftrightarrow \ker(f) = \{(0, 0\mathcal{J})\}$

**Proof:** it's clear.

**Theorem 3.41:** Let  $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$  is a  $\mathcal{NZ}^h$  then  $\ker(f)$  is a  $\mathcal{NZ}^i$  of  $Z(\mathcal{J})$ .

**Proof:**  $f(0, 0\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J}) \Rightarrow (0, 0\mathcal{J}) \in \ker(f)$

Let  $(\mathcal{Z}, \mathcal{h}\mathcal{J}) * [(\mathcal{T}, \mathcal{q}\mathcal{J}) * (\mathcal{A}, \mathcal{w}\mathcal{J})] \in \ker(f)$  and  $(\mathcal{T}, \mathcal{q}\mathcal{J}) \in \ker(f) \Rightarrow$

$$\begin{aligned} f((\mathcal{Z}, \mathcal{h}\mathcal{J}) * [(\mathcal{T}, \mathcal{q}\mathcal{J}) * (\mathcal{A}, \mathcal{w}\mathcal{J})]) &= (\hat{0}, \hat{0}\mathcal{J}) \text{ and } f(\mathcal{T}, \mathcal{q}\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J}) \\ (\hat{0}, \hat{0}\mathcal{J}) &= f((\mathcal{Z}, \mathcal{h}\mathcal{J}) * [(\mathcal{T}, \mathcal{q}\mathcal{J}) * (\mathcal{A}, \mathcal{w}\mathcal{J})]) \\ &= f(\mathcal{Z}, \mathcal{h}\mathcal{J}) \dot{*} [f(\mathcal{T}, \mathcal{q}\mathcal{J}) \dot{*} f(\mathcal{A}, \mathcal{w}\mathcal{J})] \\ &= f(\mathcal{Z}, \mathcal{h}\mathcal{J}) \dot{*} [(\hat{0}, \hat{0}\mathcal{J}) \dot{*} f(\mathcal{A}, \mathcal{w}\mathcal{J})] \end{aligned}$$

$$\begin{aligned} &= f(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * f(\mathfrak{a}, \mathfrak{u}\mathcal{J}) \\ &= f((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{u}\mathcal{J})) \end{aligned}$$

We get  $((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{u}\mathcal{J})) \in \ker(f)$ . then  $\ker(f)$  is a  $\mathcal{NZ}^i$  of  $\mathcal{Z}(\mathcal{J})$ .

**Theorem 3.42:** Let  $f: \mathcal{Z}(\mathcal{J}) \rightarrow \mathcal{Z}(\mathcal{J})$  is a  $\mathcal{NPZ}^h$  then  $\ker(f)$  is a  $\mathcal{NPZ}^i$  of  $\mathcal{Z}(\mathcal{J})$ .

**Proof:** it is easy as above.

## 4 Conclusion

We discussed the idea of a neutrosophic Z-algebra and neutrosophic pseudoZ – algebra looked into some of its properties, and the concept of neutrosophic Z-ideal, neutrosophic Z-sub algebra, neutrosophic Z-filter and neutrosophic Z- homomorphism are studied and a few properties are obtained.

## Competing Interests

Authors have declared that no competing interests exist.

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