On neutrosophic paraconsistent topology

Francisco Gallego Lupiáñez
Department of Mathematics, University of Complutense, Madrid, Spain

Abstract

Purpose – Recently, F. Smarandache generalized the Atanassov's intuitionistic fuzzy sets and other kinds of sets to neutrosophic sets (NSs). Also, this author introduced a general definition of neutrosophic topology. On the other hand, there exist various kinds of paraconsistent logics, where some contradiction is admissible. The purpose of this paper is to show that a Smarandache's definition of neutrosophic paraconsistent topology is not a generalization of Çoker's intuitionistic fuzzy topology (IFT) or Smarandache's general neutrosophic topology.

Design/methodology/approach – The possible relations between the IFT and the neutrosophic paraconsistent topology are studied.

Findings – Relations on IFT and neutrosophic paraconsistent topology are shown.

Research limitations/implications – Clearly, the paper is confined to IFSs and NSs.

Practical implications – The main applications are in the mathematical field.

Originality/value – The paper shows original results on fuzzy sets and topology.

Keywords Cybernetics, Logic, Set theory, Topology, Fuzzy logic, Philosophy

Paper type Research paper

1. Introduction

In various recent papers, Smarandache (1998, 2002, 2003, 2005a) generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). In some distinctions between NSs and IFSs are underlined, (Smarandache, 2005a).

The notion of IFS defined by Atanassov (1983, 1986) has been applied by Çoker (1997) for study intuitionistic fuzzy topological spaces (IFTS). This concept has been developed by many authors (Bayhan and Çoker,2003; Çoker, 1996, 1997; Çoker and Eş,1995; Eş and Çoker,1996; Gürçay *et al.*, 1997; Hanafy, 2003; Hur *et al.*, 2004; Lee and Lee, 2000; Lupiáñez, 2004a, b, 2006a, b, 2007, 2008; Turanh and Çoker, 2000).

Smarandache (2005b) also defined the general neutrosophic topology on a NS.

On the other hand, various authors (Priest *et al.*, 1989) worked on "paraconsistent logics", that is, logics where some contradiction is admissible. We remark the theories exposed in Da Costa (1958), Routley *et al.* (1982) and Peña (1987).

Smarandache (2005a) defined also the neutrosophic paraconsistent sets and he proposed a natural definition of neutrosophic paraconsistent topology.

A problem that we consider is the possible relation between this concept of neutrosophic paraconsistent topology and the previous notions of general neutrosophic topology and intuitionistic fuzzy topology (IFT). We show in this paper that neutrosophic paraconsistent topology is not an extension of IFT.

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2. Basic definitions

First, we present some basic definitions.

Definition 1. Let X be a non-empty set. An IFS A, is an object having the form $A = \{\langle x, \mu_A, \gamma_A \rangle / x \in X\}$ where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\gamma_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$ (Atanassov, 1983).

Definition 2. Let X be a non-empty set, and the IFSs $A = \{\langle x, \mu_A, \gamma_A \rangle | x \in X\}$, $B = \{\langle x, \mu_B, \gamma_B \rangle | x \in X\}$. Let (Atanassov, 1988):

$$\overline{A} = \{\langle x, \gamma_A, \mu_A \rangle | x \in X\}$$

$$A \cap B = \{\langle x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B \rangle | x \in X\}$$

$$A \cup B = \{\langle x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B \rangle | x \in X\}.$$

Definition 3. Let X be a non-empty set. Let $0_\sim = \{\langle x, 0, 1 \rangle | x \in X\}$ and $1_\sim = \{\langle x, 1, 0 \rangle | x \in X\}$ (Çoker, 1997).

Definition 4. An IFT on a non-empty set X is a family τ of IFSs in X satisfying:

- $0_\sim, 1_\sim \in \tau$;
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$; and
- $\cup G_j \in \tau$ for any family $\{G_j | j \in J\} \subset \tau$.

In this case the pair (X, τ) is called an IFTS and any IFS in τ is called an intuitionistic fuzzy open set in X (Çoker, 1997).

Definition 5. Let T, I, F be real standard or non-standard subsets of the non-standard unit interval $]^{-}0, 1^{+}[$, with:
$\sup T = t_{\sup}$, $\inf T = t_{\inf}$
$\sup I = i_{\sup}$, $\inf I = i_{\inf}$
$\sup F = f_{\sup}$, $\inf F = f_{\inf}$ and $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}$ $n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$,
T, I, F are called neutrosophic components. Let U be an universe of discourse, and M a set included in U. An element x from U is noted with respect to the set M as $x(T, I, F)$ and belongs to M in the following way: it is $t\%$ true in the set, $i\%$ indeterminate (unknown if it is) in the set, and $f\%$ false, where t varies in T, i varies in I, f varies in F. The set M is called a NS (Smarandache, 2005a).

Remark. All IFS is a NS.

Definition 6. Let M be a non-empty set. A general neutrosophic topology on M is a family Ψ of NSs in M satisfying the following axioms:

- $0_\sim = x(0, 0, 1)$, $1_\sim = x(1, 0, 0) \in \Psi$;
- if $A, B \in \Psi$, then $A \cap B \in \Psi$; and
- if a family $\{A_j | j \in J\} \subset \Psi$, then $\cup A_j \in \Psi$ (Smarandache, 2005b).

Definition 7. A NS $x(T, I, F)$ is called paraconsistent if $inf(T) + \inf(I) + \inf(F) > 1$ (Smarandache, 2005a).

Definition 8. For neutrosophic paraconsistent sets $0\_ = x(0, 1, 1)$ and $1\_ = x(1, 1, 0)$ (Smarandache).

Remark. If we use the unary neutrosophic negation operator for NSs (Smarandache, 2005b), $n_N(x(T,I,F)) = x(F,I,T)$ by interchanging the truth T and falsehood F components, we have that $n_N(0_{-}) = 1_{-}$.

Definition 9. Let X be a non-empty set. A family Φ of neutrosophic paraconsistent sets in X will called a neutrosophic paraconsistent topology if:

- 0_{-} and $1_{-} \in \Phi$;
- if $A, B \in \Phi$, then $A \cap B \in \Phi$; and
- any union of a subfamily of paraconsistent sets of Φ is also in Φ (Smarandache).

3. Results

Proposition 1. The neutrosophic paraconsistent topology is not an extension of IFT.

Proof. We have that $0_{\sim} = \langle x, 0, 1\rangle$ and $1_{\sim} = \langle x, 1, 0\rangle$ are members of all IFT, but $x(0,0,1) \in j(0_{\sim}) \neq 0_{-}$, and, $x(1,0,0) \in j(1_{\sim}) \neq 1_{-}$.

Proposition 2. A neutrosophic paraconsistent topology is not a general neutrosophic topology.

Proof. Let the family $\{1_{-}, 0_{-}\}$. Clearly it is a neutrosophic paraconsistent topology, but $0_{\sim}$, $1_{\sim}$ are not in this family.

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