On Imperfect Duplets In Some Refined Neutrosophic Rings

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Abstract: This paper solves the imperfect duplets problem in refined neutrosophic rings, where it presents the necessary and sufficient conditions for a pair (x, y) to be an imperfect duplet in any refined neutrosophic ring. Also, this work introduces a full description of the structure of imperfect duplets in numerical refined neutrosophic rings such as refined neutrosophic ring of integers $Z(I_1, I_2)$, refined neutrosophic ring of rationales $Q(I_1, I_2)$, and refined neutrosophic ring or real numbers $R(I_1, I_2)$.

Keywords: Refined Neutrosophic Ring, imperfect Duplet, Imperfect triplet

1. Introduction

Neutrosophy is a generalization of intuitionistic fuzzy logic founded by F.Smarandache to deal with indeterminacy in science and real life problems.

Neutrosophic algebra began with the efforts of Kandasamy and Smarandache where the concept of neutrosophic ring was presented in [7] as a generalization of classical rings. These rings were handled by many authors uch as [1,2,9,11-19].

Recently, there is an increasing interest in the generalizations of neutrosophic rings, where refined neutrosophic rings were defined by Agboola et.al [5,6].

If (R,+,.) is a ring, then the corresponding refined neutrosophic ring $R(I_1, I_2)$ is defined as follows:

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$R(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in R\}$. The operations on $R(I_1, I_2)$ are defined as follows:

Addition: (a, bI_1, cI_2)+(x, yI_1, zI_2)=$(a + x, (b + y)I_1, (c + z)I_2)$.

Multiplication: (a, bI_1, cI_2).(x, yI_1, zI_2)=$(ax, (ay + bx + by + bz + cy)I_1, (az + cz + cx)I_2)$

The notion of neutrosophic duplets and neutrosophic triplets was defined and handled by Smarandache et.al in [3,8,10], where they opened an interesting research direction about finding these elements in rings.

Through this paper, we extend the previous efforts to solve the problem of duplets into the case of refined neutrosophic rings, where we present the condition of imperfect duplets in any refined neutrosophic ring even when it is not commutative. In particular, we determine all possible imperfect duplets in the refined neutrosophic ring of integers $Z(I_1, I_2)$, refined neutrosophic ring of rationales $Q(I_1, I_2)$, and refined neutrosophic ring of real numbers $R(I_1, I_2)$.

2. Preliminaries

Definition 2.1: [5]

(a) If X is a set then X$(I_1, I_2) = \{(a, bI_1, cI_2): a , b , c \in X\}$ is called the refined neutrosophic set generated by X , I_1, I_2.

(b) Let (R,+,×) be a ring, (R(I_1, I_2) , + ,×) is called the refined neutrosophic ring generated by R ,I_1, I_2.

Definition 2.2: [12]

Let R be any ring, x, y are two arbitrary elements in R. We call them a duplet with y acts as an identity if and only if

$xy = yx = x$.

Definition 2.3: [10]

Let R be any ring, x, y, z three arbitrary elements in R. We call them a triplet with y acts as an identity if and only if

$xy = yx = x, zy = yz = z, xz = zx = y.$

3. Main discussion :

Definition 3.1:

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Let x, y, z be three elements in a refined neutrosophic ring, we have:

x, y are called an imperfect refined neutrosophic duplet with y acts as an identity if and only if $xy = yx = x$.

Example 3.2:

In $Z_{10}(I_1, I_2)$ the refined neutrosophic ring of integers modulo 10, we have:

$x = (2,0,0), y = (6,0,0)$ is an imperfect refined neutrosophic duplet with y acts as an identity, that is because $xy = yx = x$.

Theorem 3.3:

Let $x = (x_0, x_1 I_1, x_2 I_2), y = (y_0, y_1 I_1, y_2 I_2)$ be any two elements in $R(I_1, I_2)$, then (x, y) is an imperfect refined neutrosophic duplet with y acts as an identity if and only if

$(x_0, y_0), (x_0 + x_2, y_0 + y_2), (x_0 + x_1 + x_2, y_0 + y_1 + y_2)$ are three imperfect duplets in the classical ring R with $y_0, y_0 + y_2, y_0 + y_1 + y_2$ acts as identities.

Proof:

We assume that (x, y) is an imperfect refined neutrosophic duplet with y acts as an identity, then $xy = yx = x$. From this condition we get:

$xy = (x_0 y_0, I_1[x_0 y_1 + x_1 y_0 + x_1 y_1 + x_1 y_2 + x_2 y_1], I_2[x_0 y_2 + x_2 y_0 + x_2 y_2]) = (x_0 y_0, I_1[(x_0 + x_1 + x_2)(y_0 + y_1 + y_2) - (x_0 + x_2)(y_0 + y_2)], I_2[(x_0 + x_2)(y_0 + y_2) - x_0 y_0]) = x, and\ yx = (y_0 x_0, I_1[y_1 x_0 + y_0 x_1 + y_1 x_1 + y_1 x_2 + y_2 x_1], I_2[y_2 x_0 + y_0 x_2 + y_2 x_2]) = (y_0 x_0, I_1[(y_0 + y_1 + y_2)(x_0 + x_1 + x_2) - (y_0 + y_2)(x_0 + x_2)], I_2[(y_0 + y_2)(x_0 + x_2) - y_0 x_0]) = x$.

This implies that: $x_0 y_0 = x_0 = y_0 x_0$, $(x_0 + x_2)(y_0 + y_2) - x_0 y_0 = (y_0 + y_2)(x_0 + x_2) - y_0 x_0 = x_2$, . $(x_0 + x_1 + x_2)(y_0 + y_1 + y_2) - (x_0 + x_2)(y_0 + y_2) = (y_0 + y_1 + y_2)(x_0 + x_1 + x_2) - (y_0 + y_2)(x_0 + x_2) = x_1$

Thus $(x_0 + x_2)(y_0 + y_2) = (y_0 + y_2)(x_0 + x_2) = x_0 + x_2, and (x_0 + x_1 + x_2)(y_0 + y_1 + y_2) = (y_0 + y_1 + y_2)(x_0 + x_1 + x_2) = x_0 + x_1 + x_2$. Hence $(x_0, y_0), (x_0 + x_2, y_0 + y_2), (x_0 + x_1 + x_2, y_0 + y_1 + y_2)$ are three imperfect duplets in the classical ring R with $y_0, y_0 + y_2, y_0 + y_1 + y_2$ acts as identities.

The converse can be proved by the same.

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Now, we find the set of all imperfect duplets in some refined neutrosophic rings.

Theorem 3.4:

Let $Q(I_1, I_2)$ be the refined neutrosophic ring of rationales, the all non-trivial imperfect refined neutrosophic duplets have exactly the following 6 forms

1-)$\{(0, x_1I_1, 0), (y_0, y_1I_1, y_2I_2);\ y_0 + y_1 + y_2 = 1, x_1 \neq 0\}$,

2-)$\{(0, x_1I_1, -x_1I_1), (y_0, y_1I_1, y_2I_2); y_0 + y_2 = 1\ and\ x_1 \neq 0\}$,

3-)$\{(0, x_1I_1, x_2I_1), (y_0, 0, y_2I_2); y_0 + y_2 = 1\}$,

4-)$\{(x_0, 0, -x_0I_2),(1, y_1I_1, y_2I_2)\}$,

5-)$\{(x_0, x_1I_1, -x_0I_2),(1, y_1I_1, -y_1I_2); x_0 \neq 0\}$,

6-)$\{(x_0, x_1I_1, x_2I_2),(1, y_1I_1, 0); x_0 + x_1 + x_2 = 0\}$.

Proof:

According to Theorem 3.3, we have

$x_0y_0 = x_0\ (equation\ I), (x_0 + x_2)(y_0 + y_2) = x_0 + x_2\ (equation\ II), (x_0 + x_1 + x_2)(y_0 + y_1 + y_2) = x_0 + x_1 + x_2\ (equation\ III)$.

From equation I, we get $x_0 = 0$ or $y_0 = 1$. Firstly, we assume that $x_0 = 0$, from (II) we get

$x_2 = 0$ or $y_0 + y_2 = 1$, from equation (III), we get $x_1 + x_2 = 0$ or $y_0 + y_1 + y_2 = 1$. Now we discuss the following possible cases:

If($x_2 = 0, with\ x_1 + x_2 = 0$) we get a trivial duplet since $x = (0,0,0)$.

If ($x_2 = 0, with\ y_0 + y_1 + y_2 = 1$) we get the following duplet $x = (0, x_1I_1, 0), y = (y_0, y_1I_1, y_2I_2) with\ y_0 + y_1 + y_2 = 1\ and\ x_1 \neq 0$.

If ($x_2 \neq 0\ and\ y_0 + y_2 = 1, with\ x_1 + x_2 = 0$) we get the following duplet

$x = (0, x_1I_1, -x_1I_2), y = (y_0, y_1I_1, y_2I_2)$ with $y_0 + y_2 = 1$.

If($x_2 \neq 0\ and x_1 + x_2 \neq 0\ with\ y_0 + y_2 = 1, with y_0 + y_1 + y_2 = 1$) , then y_1=0, we get the following duplet

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$x = (0, x_1 I_1, x_2 I_2), y = (y_0, 0, y_2 I_2);\ y_0 + y_2 = 1$.

Now, we discuss the second case of equation (I), we suppose that $y_0 = 1\ and\ x_0 \neq 0$.

From equation (II) we get $x_0 + x_2 = 0$ or $y_2 = 0$. From equation (III) we get $x_0 + x_1 + x_2 = 0$ or $y_1 + y_2 = 0$. We discuss the possible cases

If $(x_0 + x_2 = 0\ and\ y_2 \neq 0\ with\ x_0 + x_1 + x_2 = 0)$ we get $x_1 = 0\ and\ x_2 = -x_0$, thus the corresponding duplet is

$x = (x_0, 0, -x_0 I_2), y = (1, y_1 I_1, y_2 I_2)$.

If $(x_0 + x_2 = 0\ and\ y_2 \neq 0\ with\ y_1 + y_2 = 0)$ we get $y_1 = -y_2, x_2 = -x_0$, thus the corresponding duplet is $x = (x_0, x_1 I_1, -x_0 I_2), y = (1, y_1 I_1 - y_1 I_2)$.

If $(x_0 + x_2 \neq 0\ and\ y_2 = 0\ with\ y_1 + y_2 = 0$) we get a trivial duplet since $y = (1,0,0)$.

If $(x_0 + x_2 \neq 0\ and\ y_2 = 0\ with\ x_0 + x_1 + x_2 = 0)$ we get the following duplet

$x = (x_0, x_1 I_1, x_2 I_2), y = (1, y_1 I_1 0)$. Thus the proof is complete.

Example 3.5:

In the first form of duplets we put $x_1 = 2$, $y_0 = y_1 = y_2 = \frac{1}{3}$. It is clear that $xy = yx = x$.

Theorem 3.6:

Let $R(I_1, I_2)$ be the refined neutrosophic ring of real numbers, the all non-trivial imperfect refined neutrosophic duplets have exactly the following 6 forms

1-)$\{(0, x_1 I_1, 0), (y_0, y_1 I_1, y_2 I_2);\ y_0 + y_1 + y_2 = 1, x_1 \neq 0\}$,

2-)$\{(0, x_1 I_1, -x_1 I_1), (y_0, y_1 I_1, y_2 I_2); y_0 + y_2 = 1\ and\ x_1 \neq 0\}$,

3-)$\{(0, x_1 I_1, x_2 I_1), (y_0, 0, y_2 I_2); y_0 + y_2 = 1\}$,

4-)$\{(x_0, 0, -x_0 I_2)$,$(1, y_1 I_1, y_2 I_2)\}$,

5-)$\{(x_0, x_1 I_1, -x_0 I_2)$,$(1, y_1 I_1, -y_1 I_2); x_0 \neq 0\}$,

6-)$\{(x_0, x_1 I_1, x_2 I_2)$,$(1, y_1 I_1, 0); x_0 + x_1 + x_2 = 0\}$.

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Proof:

The proof is exactly similar to Theorem 3.4 .

Theorem 3.7:

Let $Z(I_1, I_2)$ be the refined neutrosophic ring of integers, the all non-trivial imperfect refined neutrosophic duplets have exactly the following 6 forms

1-)$\{(0, x_1 I_1, 0), (y_0, y_1 I_1, y_2 I_2);\ y_0 + y_1 + y_2 = 1, x_1 \neq 0\}$,

2-)$\{(0, x_1 I_1, -x_1 I_1), (y_0, y_1 I_1, y_2 I_2); y_0 + y_2 = 1\ and\ x_1 \neq 0\}$,

3-)$\{(0, x_1 I_1, x_2 I_1), (y_0, 0, y_2 I_2); y_0 + y_2 = 1\}$,

4-)$\{(x_0, 0, -x_0 I_2), (1, y_1 I_1, y_2 I_2); x_0 \neq 0\}$,

5-)$\{(x_0, x_1 I_1, -x_0 I_2), (1, y_1 I_1, -y_1 I_2); x_0 \neq 0\}$,

6-)$\{(x_0, x_1 I_1, x_2 I_2), (1, y_1 I_1, 0); x_0 + x_1 + x_2 = 0\}$.

Proof:

The proof is exactly similar to Theorem 3.4.

Example 3.8:

We construct an example about an imperfect duplet in $Q(I_1, I_2)$.

Consider the corresponding imperfect duplet $x = \left(0, \frac{1}{3} I_1, -\frac{1}{3} I_2\right), y = (0, -I_1, I_2)$.

Example 3.9:

We construct an example about an imperfect duplet in $Z(I_1, I_2)$.

$x = (1, I_1, -2I_2), y = (1, I_1, 0)$.

Conclusion

In this paper, we have studied the problem of finding imperfect duplets in a refined neutrosophic ring. Where we have determined a general condition for imperfect duplets in such rings. In particular, we have

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presented the structure of all duplets in the refined neutrosophic rings of integers, reals, and rationales respectively.

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