

On a Pseudo Smarandache Ideals of BH-algebra

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Abstract

In this paper the notion of a pseudo Smarandache BH-algebra, a pseudo Smarandache ideal, a pseudo Smarandache closed ideal and a pseudo Smarandache completely closed ideal of a pseudo Smarandache BH-algebra are defined. These notions are studied. The relationships among these types of ideals are discussed.

Keywords: BCK-algebra, BH-algebra, ideal of BH-algebra, a Smarandache of BH-algebra, a pseudo BH-algebra, a pseudo ideal of a pseudo BH-algebra, a pseudo closed ideal of a pseudo BH-algebra, a pseudo completely closed ideal of a pseudo BH-algebra, a pseudo Smarandache ideal of BH-algebra, a pseudo Smarandache closed ideal of BH-algebra, a pseudo Smarandache completely closed ideal of BH-algebra.

1. Introduction

In 1966 by Y. Imai and K. Iseki introduced the notion of BCK-algebra [8]. In 1998, Y. B. Jun, E. H. Ragh and H. S. Kim introduced the notion of a BH-algebra, and the notion of ideal of a BH-algebra [6]. In 2005, Y. B. Jun introduced the notion Q-Smarandache of BCH-algebra and Q-Smarandache ideal of BCH-algebra [5]. In 2012, H. H. Abbass and H. A. Dahham introduced the notion of completely closed ideal of a BH-algebra [2]. In 2013 H. H. Abbass and S. J. Mohammed introduced the notion of Q-Smarandache closed ideal and Q-Smarandache completely closed ideal of a Smarandache BH-algebra [4]. In 2015, Y. B. Jun and S. S. Ahn introduced the notion of a pseudo BH-algebra and a pseudo ideal of a pseudo BH-algebra [7]. In 2017, H. H. Abbass and A. H. Nouri introduced the notion of a pseudo completely closed ideal of a pseudo BH-algebra [1].

In this paper, we define the concepts of a pseudo Smarandache completely closed ideal and a pseudo Smarandache closed ideal of a pseudo Smarandache BH-algebra. We stated and proved some theorems which determine the relationships between these notions and some types of a pseudo Smarandache ideals of a Smarandache BH-algebra.

2. Materials and Methods

In this section, some basic concepts about a BCK-algebra, a BH-algebra, a pseudo BH-algebra, pseudo ideal and a pseudo closed ideal of a pseudo BH-algebra are given.

Definition (1.1) [8] A BCK-algebra is an algebra $(X, *, 0)$, where X is a nonempty set, $*$ is a binary operation and 0 is a constant, satisfying the following axioms:

- $((x * y) * (x * z)) * (z * y) = 0, \quad \forall x, y, z \in X$
- $(x * (x * y)) * y = 0, \quad \forall x, y, z \in X.$ iii. $x * x = 0, \quad \forall x \in X.$
- $x * y = 0 \text{ and } y * x = 0 \Rightarrow x = y, \quad \forall x, y \in X$
- $0 * x = 0, \quad \forall x \in X$

Definition (1.2) [6] A BH-

algebra is a nonempty set X with constant 0 and a binary operation satisfying the following conditions: i. $x * x = 0, \quad \forall x \in X.$ ii. $x * 0 = x, \quad \forall x \in X.$ iii. $x * y = 0 \text{ and } y * x = 0 \Rightarrow x = y, \quad \forall x, y \in X.$

Definition (1.3) [4] A Smarandache BH-algebra is defined to be a BH-algebra X in which there exists a proper subset Q of X such that i. $0 \in Q$ and $|Q| \geq 2$

ii. Q is a BCK-algebra under the operation of X .

Definition (1.4) [6] Let I be a nonempty subset of a BH-algebra X and $I \neq \emptyset \subseteq X$. Then I is called an ideal of X if it satisfies: i. $0 \in I.$ ii. $x * y \in I \text{ and } y \in I \text{ imply } x \in I, \quad \forall x, y \in X$

Now, we define the a Smarandache ideal of X to the Smarandache BH algebra X .

Definition (1.5) [4] A nonempty subset I of a Smarandache BH-algebra X is called a Smarandache ideal of X if:

- $0 \in I$
- $x * y \in I \text{ and } y \in I \Rightarrow x \in I, \quad \forall x \in Q$

Proposition (1.6) [4] Every ideal of a Smarandache BH-algebra X is a Smarandache ideal of X .

Definition (1.7) [3] An ideal I of a BH-algebra X is called a closed ideal of X if and only if $0 * x \in I$ for all $x \in I$

Now, we define the Smarandache closed ideal of X to the Smarandache BH algebra X .

Definition (1.8) [4] A Smarandache ideal I of a Smarandache BH-algebra X is called a Smarandache closed ideal of X if: for all $x \in I, 0 * x \in I$

Proposition (1.9) [4] Every closed ideal of a Smarandache BH-algebra X is a Smarandache closed ideal of X .

Definition (1.10) [2] An ideal I of a BH-algebra X is called a completely closed ideal of X if it satisfies: $x * y \in I, \quad \forall x, y \in I$

Remark (1.11) [2] Every a completely closed ideal of BH-algebra X is closed ideal of X .

Now, we define the Smarandache a completely closed ideal of X to the Smarandache BH algebra X .

Definition (1.12) [4] A Smarandache ideal I of a Smarandache BH-algebra X is called a Smarandache completely closed ideal of X if: $x * y \in I, \quad \forall x, y \in I.$

Proposition (1.13) [4] Every completely closed ideal of a BH-algebra X is a Smarandache completely closed ideal of X .

Remarks (1.14) [4] Every a Smarandache completely closed ideal of Smarandache BH-algebra X is a Smarandache closed ideal of X .

Definition (1.15) [7]

A pseudoBH algebra is a nonempty set X with a constant 0 and two binary operations $*$ and $\#$ satisfying the following conditions: i. $x * x = x \# x = \forall x \in X$. ii. $x * 0 = x \# 0 = x, \forall x \in X$. iii. $x * y = y \# x = 0 \Rightarrow x = y, \forall x, y \in X$.

Definition (1.16) [7] Let $(X, *, \#, 0)$ be a pseudoBH-algebra, Then I is called pseudo ideal of X if it satisfies:

i. $0 \in I$. ii. $x * y, x \# y \in I, y \in I \Rightarrow x \in I, \forall x, y \in X$.

Definition (1.17) [7] A pseudoideal of a pseudoBH-algebra X is called a pseudo closed ideal of X , if for every $x \in I$, we have $0 * x, 0 \# x \in I$.

Definition (1.18) [1] A pseudoideal of a pseudoBH-algebra X is called a pseudo completely closed ideal of X , if satisfies: $x * y, x \# y \in I$, for all $x, y \in I$.

Remarks (1.19) [1] Every a pseudo completely closed ideal of a pseudo BH-algebra X is a pseudo closed ideal of X .

3. Main Results

In this section, the concepts a pseudo Smarandache BH-algebra, a pseudo Smarandache ideal, a pseudo Smarandache closed ideals and a pseudo Smarandache completely closed ideals of a pseudoSmarandache BH-algebra are given.

Definition (2.1) A pseudo Smarandache BH-algebra $(X, *, \#, 0)$ is defined to be a pseudo BH-algebra in which there exists a proper subset Q of X such that

i. $0 \in Q$ and $|Q| \geq 2$

ii. Q is BCK – algebra under the operations $*$ and $\#$ of X .

Example (2.2) the a pseudo BH- algebra $X = \{0, 1, 2, 3, 4\}$ with constant 0 and binary operations $*$ and $\#$ defined the following tables and $Q = \{0, 1, 2\}$ is a pseudo Smarandache BH-algebra.

*	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	2	4
2	2	1	0	2	4
3	3	2	0	0	4
4	4	2	1	0	0

#	0	1	2	3	4
0	0	0	0	2	4
1	1	0	2	3	1
2	2	2	0	1	0
3	3	0	0	0	2
4	4	1	1	1	0

Definition (2.3) let X be a pseudo Smarandache BH- algebra. An empty subset I of X is called a pseudo Smarandache ideal of X related to Q (or briefly, a pseudo Smarandache ideal of X if.

i. $0 \in I$

ii. $\forall y \in I, x * y, x \# y \in I \Rightarrow x \in I, \forall x \in Q$

Example(2.4) Consider the pseudo Smarandache BH- algebra $X = \{0, 1, 2, 3, 4\}$ with the binary Operations $*$ and $\#$ defined by the tables.

*	0	1	2	3	4
0	0	0	0	3	4
1	1	0	0	2	3
2	2	1	0	2	4
3	3	3	2	0	4
4	4	2	1	0	0

#	0	1	2	3	4
0	0	0	0	3	4
1	1	0	0	2	1
2	2	2	0	1	0
3	3	3	2	0	2
4	4	1	2	1	0

And $Q = \{0, 1, 2\}$ the subset $I = \{0, 1, 3\}$ is a pseudo Smarandache ideal of X .

Proposition (2.5) Let X be a pseudo Smarandache – BH algebra. Then every a pseudo ideal of X is a pseudo Smarandache ideal of X .

Proof: It is clear

Remark (2.6) The following example shows that convers of proposition is not correct in general.

Example (2.7) Consider the a pseudo Smarandache BH- algebra $X = \{0, 1, 2, 3\}$ with binary operations $*$ and $\#$ defined by the following tables.

*	0	1	2	3
0	0	0	2	3
1	1	0	1	2
2	2	2	0	1
3	3	3	2	0

#	0	1	2	3
0	0	0	2	3
1	1	0	0	2
2	2	2	0	1
3	3	3	2	0

And $Q = \{0, 1\}$. The subset $I = \{0, 2\}$ is a pseudo Smarandache ideal of X but it is not a pseudo ideal of X

since $3 * 2 = 2 \in I$ and $3 \# 2 = 2 \in I$ but $3 \notin I$

Theorem (2.8) Let X be a pseudo Smarandache BH- algebra and I be a pseudo Smarandache ideal such that $x * y, x \# y \notin I$ for all $x \notin I$ and $y \in I$, then I is a pseudo ideal of X .

Proof:

Let I be a pseudo Smarandache ideal of X , $x \in X$, and $y \in I$. i. $0 \in I$. ii. let $x * y, x \# y \in I$ and $y \in I$

Then we have two cases. Case 1 if $x \in Q \Rightarrow x \in I$

Case 2 if $x \notin Q$, either $x \in I$, or $x \notin I$

If $x \in I \Rightarrow I$ is a pseudo ideal if $x \notin I, \Rightarrow x * y, x \# y \notin I$

And this contradiction since $x * y, x \# y \in I$

Therefore, I is a pseudo ideal

Definition (2.9) A pseudo Smarandache ideal of a pseudo Smarandache BH-algebra is called a pseudo Smarandache closed ideal of X if $0 * x, 0 \# x \in I, \forall x \in I$

Example(2.10) the a pseudo Smarandache ideal $I = \{0, 1, 3\}$ of X in example (2, 4) is a pseudo Smarandache closed ideal of X .

Definition (2.11) A pseudo Smarandache ideal of a pseudo Smarandache BH-algebra is called a pseudo Smarandache completely closed ideal of X if $x * y$ and $x \# y \in I, \forall x, y \in I$

Example (2.12) the a pseudo Smarandache ideal $I = \{0, 4\}$ of X in example (2.4) is a pseudo Smarandache completely closed ideal of X .

Proposition (2.13) Let X is a pseudo Smarandache BH – algebra. Then every a pseudo Smarandache completely closed ideal of X is a pseudo Smarandache closed ideal of X .

Proof: It is clear

Remark(2.14) The following example shows that convers of proposition is not correct in general.

Example(2.15) Consider the a pseudo Smarandache BH- algebra $X = \{0, 1, 2, 3\}$ with binary operation $*$ and $\#$ defined by the following tables.

*	0	1	2	3
0	0	0	1	3
1	1	0	3	1
2	2	3	0	2
3	3	2	1	0

#	0	1	2	3
0	0	0	2	3
1	1	0	3	1
2	2	3	0	3
3	3	3	1	0

And $Q=\{0,1\}$, Then X a pseudo Smarandache BH- algebra where The pseudo Smarandache ideal $I=\{0,1,2\}$ is a pseudo Smarandache closed ideal of X .

But is not a pseudo Smarandache completely closed ideal of X . Since, $1*2=3 \notin I, 1\#2=3 \notin I$ and $1,2 \in I$

Remark (2.16) let X a pseudo Smarandache BH- algebra and I be a pseudo completely closed ideal of X then I is a pseudo Smarandache completely closed ideal of X .

Proposition (2.17) Let X be a pseudo Smarandache BH- algebra and I be a pseudo Smarandache closed ideal such that $x*y, x\#y \in I$ for all $x \notin I$ and $y \in I$, then I is a pseudo closed ideal of X .

Proof: Let I be a pseudo Smarandache closed ideal of X
 $\Rightarrow I$ is a pseudo Smarandache ideal of X

By theorem 1 I is a pseudo Smarandache closed ideal of X

Implies that $0*x, 0\#x \in I$

Therefore, I is a pseudo closed ideal of X

Proposition (2.18) Let X be a pseudo Smarandache BH- algebra and I be a pseudo Smarandache completely closed ideal such that $x*y, x\#y \notin I$ for all $x \notin I$ and $y \in I$, then I is pseudo completely closed ideal of X

Proof: Let I be a pseudo Smarandache completely closed ideal of X . This yield

I is a pseudo Smarandache ideal of X . by theorem (2.8) we have

I is a pseudo ideal of X [since I a pseudo Smarandache completely closed ideal of X]

It follows $x*y$ and $x\#y \in I$, for all $x, y \in I$

Hence, I is a pseudo completely closed ideal of X

Proposition (2.19) Let $\{I_i, i \in \lambda\}$ be a family of

A Pseudo Smarandache ideal of pseudo Smarandache BH – algebra. Then $\bigcap_{i \in \lambda} I_i$ is a pseudo Smarandache ideal of X .

Proof: i. since $0 \in I_i, \forall i \in \lambda \Rightarrow 0 \in \bigcap_{i \in \lambda} I_i$

ii. let $x*y, x\#y \in \bigcap_{i \in \lambda} I_i, y \in \bigcap_{i \in \lambda} I_i$

$\Rightarrow x*y, x\#y \in I_i$ and $y \in I_i, \forall i \in \lambda$

$\Rightarrow x \in I_i, \forall i \in \lambda$ [since I_i is a pseudo Smarandache ideal of $X, \forall i \in \lambda$]

$\Rightarrow x \in \bigcap_{i \in \lambda} I_i \Rightarrow \bigcap_{i \in \lambda} I_i$ is a pseudo Smarandache ideal of X .

Proposition (2.20) Let $\{I_i, i \in \lambda\}$ be a family of a pseudo Smarandache closed ideal of a pseudo Smarandache BH – algebra. Then $\bigcap_{i \in \lambda} I_i$ is a pseudo Smarandache closed ideal of X .

Proof: Since I_i is a pseudo Smarandache closed ideal of X ,

$\forall i \in \lambda$

$\Rightarrow I_i$ is a pseudo Smarandache ideal of $X, \forall i \in \lambda$. [From propos

$\bigcap_{i \in \lambda} I_i$ is a pseudo Smarandache ideal of X . Now, let

$x \in \bigcap_{i \in \lambda} I_i \Rightarrow x \in I_i, \forall i \in \lambda \Rightarrow 0*x, 0\#x \in I_i$

$\Rightarrow 0*x, 0\#x \in \bigcap_{i \in \lambda} I_i$

So, $\bigcap_{i \in \lambda} I_i$ is a pseudo Smarandache closed ideal of X .

Proposition (2.21) Let $\{I_i, i \in \lambda\}$ be a family of a pseudo Smarandache completely closed ideal of a pseudo Smarandache BH – algebra. Then $\bigcap_{i \in \lambda} I_i$ is a pseudo Smarandache completely closed ideal of X .

Proof Since I_i is a pseudo Smarandache completely closed ideal of $X, \forall i \in \lambda$ I_i is a pseudo Smarandache ideal of $X, \forall i \in \lambda$. [From proposition (2.9)] we get

$\bigcap_{i \in \lambda} I_i$ a pseudo Smarandache ideal of X , Now, let $x, y \in \bigcap_{i \in \lambda} I_i \Rightarrow$

$x, y \in I_i, \forall i \in \lambda$.

Then $x*y, x\#y \in I_i, \forall i \in \lambda$. [since I_i is a pseudo Smarandache completely closed ideal of X . Hence, $x*y, x\#y \in$

$\bigcap_{i \in \lambda} I_i, \forall i \in \lambda$

Therefore, $\bigcap_{i \in \lambda} I_i$ a pseudo Smarandache completely closed ideal of X .

Remark (2.22) Let $\{I_i, i \in \lambda\}$ be a family of a pseudo Smarandache ideal of a pseudo Smarandache BH-algebra X . $\bigcup_{i \in \lambda} I_i$

may not be a pseudo Smarandache ideal of X .

Example (2.23) Consider the a pseudo Smarandache BH- algebra $X=\{0,1,2,3,4\}$ with the binary operation "*" and "# defined by the following tables.

*	0	1	2	3	4
0	0	0	2	3	4
1	1	0	3	2	1
2	2	2	0	1	4
3	3	3	2	0	2
4	4	4	1	2	0

#	0	1	2	3	4
0	0	0	0	0	1
1	1	0	2	3	4
2	2	1	0	2	2
3	3	2	3	0	1
4	4	4	1	2	0

and $Q=\{0,1\}$ the subsets $I_1=\{0,2\}$ and $I_2=\{0,3\}$ are a pseudo Smarandache ideal of X .

But $I_1 \cup I_2 = \{0,2,3\}$ is not a pseudo Smarandache ideals of X .

Since $1*2=3 \in I_1 \cup I_2, 1\#2=2 \in I_1 \cup I_2$ but $1 \notin$

$I_1 \cup I_2$

Proposition (2.24) Let $\{I_i, i \in \lambda\}$ be a chain of a pseudo Smarandache ideal of a pseudo Smarandache BH –algebra X . Then $\bigcup_{i \in \lambda} I_i$ is a pseudo Smarandache ideal of X .

Proof

i. $0 \in I_i, \forall i \in \lambda$ [since each I_i is a pseudo Smarandache ideal

of $X, \forall i \in \lambda \Rightarrow 0 \in \bigcup_{i \in \lambda} I_i$

ii. let $x * y, x \# y \in \bigcup_{i \in \lambda} I_i$ and $y \in \bigcup_{i \in \lambda} I_i$. There exists $I_k \in \{I_i\} \in \lambda$ such that $x * y, x \# y \in I_k$ and $y \in I_i$ since $\{I_i\} \in \lambda$ a

chain. So, $x \in I_i$ since I_k is a pseudo Smarandache ideal of X .

Therefore $x \in \bigcup_{i \in \lambda} I_i$

$\bigcup_{i \in \lambda} I_i$ a pseudo Smarandache ideal of X

4. Conclusion

In this paper, the notions of a pseudo Smarandache BH-algebra, a pseudo Smarandache ideal of BH-algebra, a pseudo Smarandache closed ideal of BH-algebra, a pseudo Smarandache completely closed ideal of a BH-algebra are introduced. Furthermore, the results are examined in terms of the relationships between a pseudo Smarandache closed ideal of BH-algebra, a pseudo Smarandache completely closed ideal of BH-algebra

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