*Research Article*

# Novel Distance Measures for Single-Valued Neutrosophic Fuzzy Sets and Their Applications to Multicriteria Group Decision-Making Problem

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Single-valued neutrosophic sets are a hybrid of fuzzy sets that are used to represent uncertain, imprecise, partial, and inconsistent information in the actual world. The focus of this research is to develop two novel distance measures for single-valued neutrosophic fuzzy sets (SVNFSs). We introduced two new distance measures named $d_\eta(G, Y)$ and $d_\zeta(G, Y)$ for SVNFSs and apply these measures to different examples and also compare them with existing measures to show the validity of our proposed measures. Our results are reliable and useful for decision-making problems. We also proposed an algorithm for multicriteria group decision-making. Based on this algorithm, we find the ranking matrices using proposed distance measures. We also give an example to demonstrate the notion and concept of our algorithm.

## 1. Introduction

Decision-making is the process of determining the best option from a set of options based on their attributes. Multi-attribute decision-making (MADM) is a type of decision-making that is based on more than one characteristic. However, since the environment and problem are often complicated, errors and cognitive limitations of the mind may make it difficult for decision-makers to use crisp statistics as a decision-making tool in practise [1, 2]; these characteristics have not been observed in the case of decision-making information traditionally thought to be determinable and clear. As a result, MADM is limited in real-world applications. Fuzzy set theories provide a useful mathematical tool for dealing with ambiguous data in MADM situations [3, 4].

Zadeh first introduced fuzzy sets in 1965 [5]. There are a wide variety of implementations that use fuzzy sets and fuzzy logic so as to deal with unpredictability. A single real value is used in the classic fuzzy set $\mu_A(\rho) \in [0, 1]$ to represent the fuzzy set's membership grade.

A fuzzy set with interval values was developed in 1986 [6] as a way to reflect the ambiguity of membership grade. To deal with information imprecision and uncertainty, decision-making theory has created a variety of effective methods [7, 8]. Most of these methods rely on fuzzy soft sets that have been changed [9]. Many fields (for example, computer sciences, physical sciences, social sciences, and medical sciences) work with ambiguous material that necessitates the use of fuzzy sets [10, 11].

Molodtsov [12] proposed a revolutionary technique known as "soft set theory," which is crucial in a number of domains. Several scholars have developed soft set theory methods and operations based on Molodtsov's technique. On the other hand, Maji et al. present several symbols and

TABLE 1: Comparison of different distance measure downward case ($i = 1, 2, 3, 4$).

| Method | Case I | Case II | Case III | Case IV |
|---|---|---|---|---|
| $d_{SK}^Z$ | 0.3 | 0.21 | 0.41 | 0.19 |
| $d_{SK}^U$ | 0.29 | 0.31 | 0.41 | 0.18 |
| $d_G$ | 0.23 | 0.11 | 0.27 | 0.19 |
| $d_{W1}$ | 0.18 | 0.05 | 0.23 | 0.15 |
| $d_Y$ | 0.18 | 0.06 | 0.18 | 0.11 |
| $d_{W_1^2}$ | 0.18 | 0.05 | 0.23 | 0.06 |

TABLE 2: Two SVNFSs $G_i$, $Y_i$ downward case ($i = 1, 2, 3, 4$).

| SVNFSs | Case I | Case II |
|---|---|---|
| $A_i$ | $\{\langle \rho_1, 0.20, 0.30, 0.70 \rangle, \langle \rho_2, 0.30, 0.50, 0.80 \rangle\}$ | $\{\langle \rho_1, 0.20, 0.30, 0.70 \rangle, \langle \rho_2, 0.30, 0.50, 0.80 \rangle\}$ |
| $B_i$ | $\{\langle \rho_1, 0.50, 0.25, 0.45 \rangle, \langle \rho_2, 0.25, 0.35, 0.40 \rangle\}$ | $\{\langle \rho_1, 0.27, 0.35, 0.75 \rangle, \langle \rho_2, 0.35, 0.50, 0.18 \rangle\}$ |
| SVNFSs | Case III | Case IV |
| $A_i$ | $\{\langle \rho_1, 0.20, 0.35, 0.75 \rangle, \langle \rho_2, 0.38, 0.58, 0.80 \rangle\}$ | $\{\langle \rho_1, 0.20, 0.35, 0.75 \rangle, \langle \rho_2, 0.38, 0.58, 0.80 \rangle\}$ |
| $B_i$ | $\{\langle \rho_1, 0.10, 0.30, 0.50 \rangle, \langle \rho_2, 0.20, 0.15, 0.18 \rangle\}$ | $\{\langle \rho_1, 0.25, 0.50, 0.90 \rangle, \langle \rho_2, 0.40, 0.35, 0.65 \rangle\}$ |

sets and interval valued fuzzy sets are not suitable for such structure. In 1986, Atanassov [15] proposed intuitionistic fuzzy sets, which are a type of fuzzy set that is inferable similar to interval valued fuzzy sets.

The intuitionistic fuzzy sets review both truth-membership $\alpha_A(\rho)$ and falsity-membership $\beta_A(\rho)$, with $\alpha_A(\rho), \beta_A(\rho) \in [0, 1]$ and $0 \leq \alpha_A(\rho) + \beta_A(\rho) \leq 1$. Only imperfect information, not the improbable and contradictory information found in belief systems, can be deal with by intuitionistic fuzzy sets. By default, indeterminacy in intuitionistic fuzzy sets is consider as $1 - \alpha_A(\rho) - \beta_A(\rho)$. If we ask an expert for his/her opinion on a certain statement, he/she
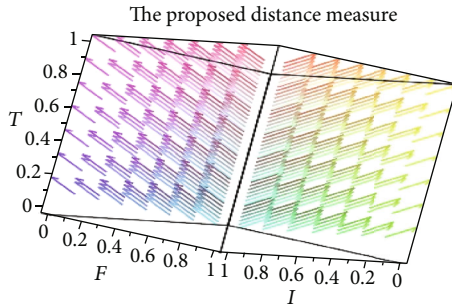


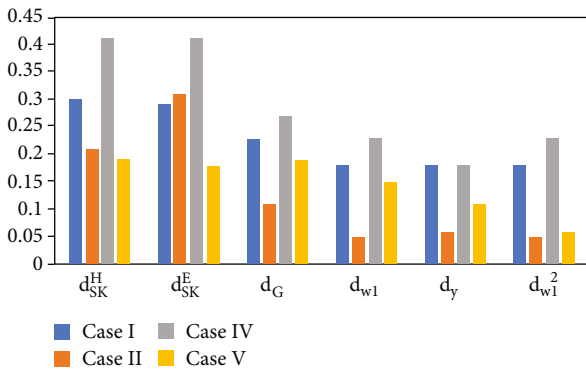FIGURE 1: Distance measures of Examples 1, 2, 3, and 4.



FIGURE 2: Graphical comparison of distance measures given in Table 2.

operators on soft sets in [13]. They also showed how soft sets used to handle fuzzy decision-making challenges [14]. It is vital to remember that the evident promotes truth membership and opposes falsehood membership in few applications, for example, belief systems and information fusion. Fuzzy

might say anything like that the likelihood of the statement being true is 0.5, the likelihood of the statement being false is 0.6, and the degree of uncertainty is 0.2. Indeterminacy is explicitly quantified in the neutrosophic set, and truth membership, indeterminacy membership, and falsity membership are all independent. If you are going to combine data from a variety of sensors, this assumption is crucial. Wang et al. first proposed the notion of Neutrosophy in 1995 [16]. In [17], Smarandache describes it as "a philosophical discipline that analysis the origin, nature and scope of neutralities, as well as their interactions with various ideational spectra." It is a powerful generic formal framework that expands on the idea of a classic set, fuzzy sets [5, 18], interval valued fuzzy sets [19], intuitionistic fuzzy sets [15], and so on. According to a philosophical perspective, the neutrosophic set is the expansion of the previous fuzzy sets. The neutrosophic set and set-theoretic operators must have a scientific or engineering definition. As a result, real-world application will be difficult.

Neutrosophic fuzzy sets (NFSs) are difficult to apply in real-life circumstances because they lack a particular explanation. As a result, single-valued neutrosophic sets (SVNFSs), a variant of NFSs, have been proposed. The single-valued neutrosophic set is an extended form of a neutrosophic set [20–22]. In addition, the information energy of SVNFSs and their correlation and correlation coefficient, as well as the decision-making approach that utilized them, were proposed by Ye [23]. Ye also developed an MCDM technique based on the distance measures and aggregation operators of SVNFSs, which can be characterised by three real integers in the real unit interval [0,1]. Uncertainty is a challenge in daily life. Distance measure is a useful tool that solves the uncertified issues in our life. However, the

Step 1. Make the decision matrices for each person who makes decisions and give the linguistic terms for each criteria.

Step 2. Make a scale for linguistic terms in the form of SVNF numbers and make decision matrices according to decision-makers.

Step 3. Combine the decision matrices using mathematics terms as

$x_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij})$, where $\alpha_{ij} = \min (\alpha_{ij}^k), \beta_{ij} = 1/k \sum_{k=1}^{k} \beta_{ij}^k, \gamma_{ij} = \max (\gamma_{ij}^k)$.

Step 4. Normalized the fuzzy decision matrix as

$\delta_{ij} = \alpha_{ij}/\gamma_j^*, \beta_{ij}/\gamma_j^*, \gamma_{ij}/\gamma_j^*$, here $\gamma_j^* = \max (\gamma_{ij})$ for benefit criteria,$\delta_{ij} = \alpha_j^*/\gamma_{ij}, \alpha_j^*/\beta_{ij}, \alpha_j^*/\alpha_{ij}$, here $\alpha_j^* = \min (\alpha_{ij})$ for non $-$ benefit criteria.

Step 5. Use the following formula to compute the weighted normalized fuzzy decision matrix: $v_{ij} = \delta_{ij} * w_j$.

Step 6. Calculate the positive and negative ideal solution for SVNFSs.

Step 7. By applying the derived distance formula, calculate $p_i^+ = \sum_{j=1}^{n} p(v_{ij}, v_j^+)$ and $p_i^- = \sum_{j=1}^{n} p(v_{ij}, v_j^-)$.

Step 8. Before ranking the options, calculate the similarity coefficient (sc) for each.

"The flow chart of the algorithm decision-making based on the proposed distance measures is give in Figure 3."

ALGORITHM

TABLE 3: Combined decision matrix.

| Criteria/ alternatives | Professional attitude | Work experience | Salary demand |
|---|---|---|---|
| Candidate-1 | $\xi_{11}$ | $\xi_{12}$ | $\xi_{13}$ |
| Candidate-2 | $\xi_{21}$ | $\xi_{22}$ | $\xi_{23}$ |
| Candidate-3 | $\xi_{31}$ | $\xi_{32}$ | $\xi_{33}$ |
| Candidate-4 | $\xi_{41}$ | $\xi_{42}$ | $\xi_{43}$ |

TABLE 4: Combined normalized decision matrix.

| Criteria/ alternatives | Professional attitude/ben | Work experience/ben | Salary demand/cost |
|---|---|---|---|
| Candidate-1 | 0.33, 0.62, 1 | 0.55, 0.92, 1 | 0.2, 0.14, 0.11 |
| Candidate-2 | 0.55, 0.77, 1 | 0.33, 0.77, 1 | 0.33, 0.2, 0.14 |
| Candidate-3 | 0.55, 0.92, 1 | 0.33, 0.55, 0.77 | 1, 0.33, 0.2 |
| Candidate-4 | 0.11, 0.25, 0.5 | 0.11, 0.47, 0.77 | 1, 0.43, 0.2 |

TABLE 5: Combined normalized weighted decision matrix.

| Criteria/ alternatives | Professional attitude/ben | Work experience/ben | Salary demand/cost |
|---|---|---|---|
| Candidate-1 | 0.16, 0.43, 0.9 | 0.38, 0.82, 0.9 | 0.06, 0.07, 0.07 |
| Candidate-2 | 0.27, 0.53, 0.9 | 0.23, 0.69, 0.9 | 0.09, 0.1, 0.9 |
| Candidate-3 | 0.27, 0.64, 0.9 | 0.23, 0.49, 0.69 | 0.3, 0.16, 0.14 |
| Candidate-4 | 0.05, 0.17, 0.5 | 0.07, 0.42, 0.69 | 0.3, 0.21, 0.14 |

SVNFSs' activities may be impractical in some circumstances. For example, the sum of any element and the maximum value should equal the maximum value, but when utilizing the SVNFSs' procedures, this does not happen. As a result, in this work, the operations and comparison approach for SVNFSs, as well as the distance measurements for SVNFSs, are redefined. Furthermore, based on the proposed distance measurements, a multicriteria group decision-making (MCGDM) method is developed.

The rest of this paper is structured as follows. In Section 2, we give the basic concepts of single-valued neutrosophic fuzzy sets and fuzzy decision-making. In Section 3, we discuss some distance measures proposed up to now. In Section 4, we discuss the new distance measures that we find in our work. Section 5 presents the examples of decision-making using proposed distance measures. Conclusions and some possible directions for future study are presented in Section 6.

## 2. Preliminaries

This section covers some of the most basic SVNFS ideas.

*Definition 1* [16, 24]. $M$ is a space, and $\rho$ is a generic element in $M$. To describe a neutrosophic set in $M$, truth membership functions can be used. The truth membership function, Indeterminacy-membership function $\alpha_A \beta_A$ and falsity-membership function$\gamma_A$, respectively.$\alpha_A(\rho)$, $\beta_A(\rho)$, and $\gamma_A(\rho)$ are there any genuine standard or nonstandard subsets of $]0^{-1}, 1^+[$. That is

$$\alpha_A : M \longrightarrow ]0^{-1}, 1^+[, \beta_A : M \longrightarrow ]0^{-1}, 1^+[, \gamma_A : M \longrightarrow ]0^{-1}, 1^+[. \tag{1}$$

The overall value is unrestricted, $\alpha_A(\rho)$, $\beta_A(\rho)$, and $\gamma_A(\rho)$, so $0^- \leq \sup \alpha_A(\rho) + \sup \beta_A(\rho) + \sup \gamma_A(\rho) \leq 3^+$.

*Definition 2.* Assume that $M$ represents finite universe of discourse (UOD). The $N$ of an SVNFSs in $M$ is specified in [16, 25].

$$N = \{\rho, \alpha_A(\rho), \beta_A(\rho), \gamma_A(\rho)|\rho \in M\},$$
$$\alpha_A : M \longrightarrow [0, 1], \beta_A : M \longrightarrow [0, 1], \gamma_A : M \longrightarrow [0, 1], \tag{2}$$

such that,

$$0 \leq \alpha_A(\rho) + \beta_A(\rho) + \gamma_A(\rho) \leq 3. \tag{3}$$

*Definition 3.* Let $W$ and $T$ be two SVNFSs in $M$. The single-valued neutrosophic fuzzy relation (SVNR) from $W \longrightarrow T$ is $W \times T$ and is denoted as $T_{SVNR}$ and $W_{SVNR}$. As a result of

TABLE 6: Analyzed in comparison to the purposed distance measurement.

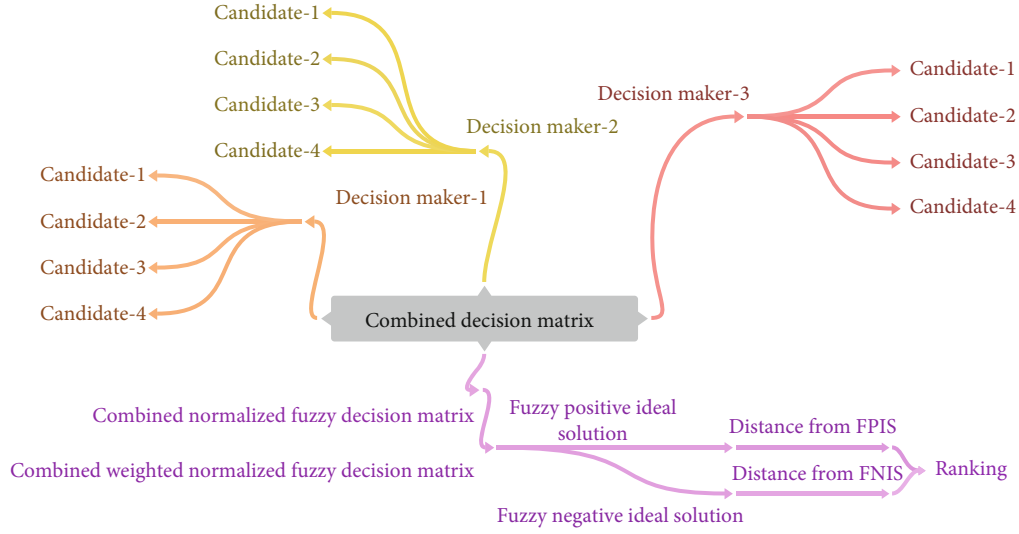| Sr. no. | $CC_i$ by Definition 6 | Rank | $CC_i$ by Definition 8 | Rank | $CC_i$ by Definition 9 | Rank | $CC_i$ by Definition 14 | Rank | $CC_i$ by Definition 15 | Rank |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.69106 | 3 | 0.64031 | 2 | 0.54167 | 3 | 0.64865 | 3 | 0.675 | 3 |
| 2 | 0.75 | 1 | 0.70851 | 1 | 0.63636 | 2 | 0.76000 | 1 | 0.7547 | 1 |
| 3 | 0.69880 | 2 | 0.63034 | 3 | 0.8 | 1 | 0.69231 | 2 | 0.4705 | 2 |
| 4 | 0.18219 | 4 | 0.11506 | 4 | 0.32 | 4 | 0.20000 | 4 | 0.1891 | 4 |



FIGURE 3: Flow chart of algorithm for decision-making.

the generation of fuzzy sets, the SVNFS is effective at dealing with uncertainty and has been widely applied in multiattribute decision-making tasks [26–28].

For probability distributions, the square root of the Jensen-Shannon divergence measure is used as a distance measure [29–31] and has been used in various disciplines.

*Definition 4* [29, 32]. Let $U$ and $V$ be the two probability distributions of a discrete random variable $Z$, where $U = \{e_1, e_2, \cdots, e_n\}$ and $V = \{f_1, f_2, \cdots, f_n\}$. $U$ and $V$ are separated by the Jehsen-Shannon divergence, which is calculated as:

$$\mathrm{JS}(U, V) = \frac{1}{2}\left[\mathrm{KL}\left(U, \frac{U+V}{2}\right) + \mathrm{KL}\left(V, \frac{U+V}{2}\right)\right], \quad (4)$$

where

$$\mathrm{KL}(U, V) = \sum_i e_i \log\left(\frac{e_i}{f_i}\right)(1 \leq i \leq n), \quad (5)$$

is the Kullback-Leibler divergence and $\sum_i e_i = \sum_i f_i = 1$.

$\mathrm{JS}(U, V)$ can also be expressed by

$$\mathrm{JS}(U, V) = Z\left(\frac{U+V}{2}\right) - \frac{1}{2}Z(U) - \frac{1}{2}Z(V) = \frac{1}{2}\left[\sum_i e_i \log\left(\frac{2e_i}{e_i+f_i}\right) + \sum_i f_i \log\left(\frac{2f_i}{e_i+f_i}\right)\right], \quad (6)$$

where

$$Z(U) = -\sum_i e_i \log e_i,$$
$$Z(V) = -\sum_i f_i \log f_i (1 \leq i \leq n). \quad (7)$$

The square root of JS is determined by $\mathrm{SR}_{\mathrm{JS}} = \sqrt{\mathrm{JS}(U, V)}$.

**Proposition 5** [33].

(1) $SR_{JS}(U, V) \geq 0$, where $SR_{JS}(U, V) = 0$ if and only if $U = V$, for $(U, V) \in Z$

(2) $SR_{JS}(U, V) = SR_{JS}(V, U)$, for $(U, V) \in Z$

(3) $SR_{JS}(U, G) + SR_{JS}(G, V) \geq SR_{JS}(U, V)$, for $(U, V, G) \in Z$

(4) $SR_{JS}(U, V) \leq 1$, for $U, V \in Z$

## 3. Existing Distance Measures

Here are some existing distance and similarity measurements between SVNFs.

Let $G$ and $Y$ be two SVNFSs in a finite universe of discourse (UOD) $M = \rho_1, \rho_2, \cdots, \rho_n$, where

$$G = \{\rho_i, \alpha_G(\rho_i), \beta_G(\rho_i), \gamma_G(\rho_i)|\rho_i \in M\},$$
$$Y = \{\rho_i, \alpha_Y(\rho_i), \beta_Y(\rho_i), \gamma_Y(\rho_i)|\rho_i \in M\}. \tag{8}$$

*Definition 6* [34]. Szmidt and Kacprzyk defined the distance formula between two fuzzy sets called Szmidt and Kacprzyk's distance measures. The distance between two single-valued neutrosophic fuzzy sets is as follows:

$$d_{SK}^Z(G, Y) = \frac{1}{2n}\sum_{i=1}^{n}(|\alpha_G(\rho_i) - \alpha_Y(\rho_i)| + |\beta_G(\rho_i) - \beta_Y(\rho_i)|$$
$$+ |\gamma_G(\rho_i) - \gamma_Y(\rho_i)|),$$
$$d_{SK}^U(G, Y) = \left(\frac{1}{2n}\sum_{i=1}^{n}(\alpha_G(\rho_i) - \alpha_Y(\rho_i))^2 + (\beta_G(\rho_i) - \beta_Y(\rho_i))^2\right.$$
$$\left. + (\gamma_G(\rho_i) - \gamma_Y(\rho_i))^2\right)^{1/2}. \tag{9}$$

*Definition 7* [35]. The distance formula called Grzegorzewski's distance measure for two fuzzy sets, and it can be written for SVNFSs as:

$$d_G(G, Y) = \frac{1}{n}\sum_{i=1}^{n} \max\left[|\alpha_G(\rho_i) - \alpha_Y(\rho_i)|, |\beta_G(\rho_i) - \beta_Y(\rho_i)|\right]. \tag{10}$$

*Definition 8* [36]. For two single-valued neutrosophic fuzzy sets, Wang and Xin's distance measure is defined as

$$d_{W1}(G, Y) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{|\alpha_G(\rho_i) - \alpha_Y(\rho_i)| + |\beta_G(\rho_i) - \beta_Y(\rho_i)|}{4}\right.$$
$$\left. + \frac{\max(|\alpha_G(\rho_i) - \alpha_Y(\rho_i)|, |\beta_G(\rho_i) - \beta_Y(\rho_i)|)}{4}\right),$$
$$d_{W_1^2}(G, Y) = \frac{1}{n}\sum_{i=1}^{n}\frac{|\alpha_G(\rho_i) - \alpha_Y(\rho_i)| + |\beta_G(\rho_i) - \beta_Y(\rho_i)|}{2}. \tag{11}$$

*Definition 9* [36]. Yang, Yingjie, and Francisco Chiclana describe Yang and Francisco's distance measure as a distance formula among two fuzzy sets. It is as follows for a pair of single-valued neutrosophic fuzzy sets:

$$d_Y(G, Y) = \frac{1}{n}\sum_{i=1}^{n} \max\left[|\alpha_G(\rho_i) - \alpha_Y(\rho_i)|, |\beta_G(\rho_i) - \beta_Y(\rho_i)| \times |\gamma_G(\rho_i)\right.$$
$$\left. - \gamma_Y(\rho_i)|\right]. \tag{12}$$

## 4. Proposed Distance Measure of SVNFSs

Using the Jensen-Shannon divergence, we propose a novel distance measure for SVNFSs in this section. We also infer and establish the qualities and characteristics of proposed distance measures.

*Definition 10.* $G$ and $Y$ be two SVNFSs are given with a finite UOD.

$$G = \{(\rho, \alpha_G(\rho), \beta_G(\rho), \gamma_G(\rho))|\rho \in M\},$$
$$Y = \{(\rho, \alpha_Y(\rho), \beta_Y(\rho), \gamma_Y(\rho))|\rho \in M\}. \tag{13}$$

Here, $\gamma_G(\rho)$ and $\gamma_Y(\rho)$ are the hesitancy grades of $\rho$ to $G$ and $Y$, respectively; the divergence measure between the SVNFSs $G$ and $Y$ denotes as $JS_{SVNFSs}(G, Y)$ and defined as:

$$JS_{SVNFSs}(G, Y) = \frac{1}{2}\left[KL\left(G, \frac{G+Y}{2}\right) + KL\left(Y, \frac{G+Y}{2}\right)\right], \tag{14}$$

with

$$KL(G, Y) = \alpha_G(\rho) \log \frac{\alpha_G(\rho)}{\alpha_Y(\rho)} + \beta_G(\rho) \log \frac{\beta_G(\rho)}{\beta_Y(\rho)} + \gamma_G(\rho) \log \frac{\gamma_G(\rho)}{\gamma_Y(\rho)}, \tag{15}$$

where $KL(G, Y)$ is the Kullback-Leibler divergence. $JS_{SVNFSs}(G, Y)$ can alternatively be represented using the formula below:

$$JS_{SVNFSs}(G, Y) = Z\left(\frac{G+Y}{2}\right) - \frac{1}{2}Z(G) - \frac{1}{2}Z(Y) = \frac{1}{2}$$
$$\cdot\left[\alpha_G(\rho) \log \frac{2\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)}\right.$$
$$+ \alpha_Y(\rho) \log \frac{2\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)}$$
$$+ \beta_G(\rho) \log \frac{2\beta_G(\rho)}{\beta_G(\rho) + \beta_Y(\rho)}$$
$$+ \beta_Y(\rho) \log \frac{2\beta_Y(\rho)}{\beta_G(\rho) + \beta_Y(\rho)}$$
$$+ \gamma_G(\rho) \log \frac{2\gamma_G(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)}$$
$$\left. + \gamma_Y(\rho) \log \frac{2\gamma_Y(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)}\right], \tag{16}$$

such that

$$Z(G) = -(\alpha_G(\rho) \log \alpha_G(\rho) + \beta_G(\rho) \log \beta_G(\rho) + \gamma_G(\rho) \log \gamma_G(\rho)),$$

$$Z(Y) = -(\alpha_Y(\rho) \log \alpha_Y(\rho) + \beta_Y(\rho) \log \beta_Y(\rho) + \gamma_Y(\rho) \log \gamma_Y(\rho)),$$

$$(17)$$

where $Z(G)$ and $Z(Y)$ are the Shannon Entropy. Then, we defined a new distance measure for the SVNFSs in accordance with neutrosophic fuzzy divergence.

*Definition 11.* Suppose $G$ and $Y$ be two SVNFSs in finite UOD $M$. A new distance measure for SVNFSs, shown as $d_\rho(G, Y)$, between the SVNFSs $G$ and $Y$, is defined as

$$d_\rho(G, Y) = \sqrt{\text{JS}_{\text{SVNFSs}}(G, Y)}. \tag{18}$$

*Remark 12.* The higher the difference between the two SVNFSs $G$ and $Y$, the larger the distance measure $d_\rho(G, Y)$. The smaller $d_\rho(G, Y)$, on the other hand, the narrower the gap between the two SVNFSs $G$ and $Y$.

**Proposition 13.** *Let $G$, $Y$, and $K$ be three SVNFSs in UOD $M$, then:*
  $(G1) d_\rho(G, Y) = 0$ *if* $G = Y$ *for* $G, Y \in M$
  $(G2) d_\rho(G, Y) = d_\rho(Y, G)$, *for* $G, Y \in M$
  $(G3) d_\rho(K, G) + d_\rho(G, Y) \geq d_\rho(K, Y)$
  $(G4) 0 \leq d_\rho(G, Y) \leq 1$ *for* $K, G, Y \in M$

*Proof* (G1). Given two SVNFSs $G = Y \in M$, we have $d_\rho(G, Y) = 0$

$$d_\rho(G, Y) = \left[ \frac{1}{2} \left( \alpha_G(\rho) \log \frac{2\alpha_G(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right) + \alpha_Y(\rho) \log \frac{2\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right.$$
$$\left. + \beta_G(\rho) \log \frac{2\beta_G(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} + \beta_Y(\rho) \log \frac{2\beta_Y(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} \right.$$
$$\left. + \gamma_G(\rho) \log \frac{2\gamma_G(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} + \gamma_Y(\rho) \log \frac{2\gamma_Y(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} \right]^{1/2} = 0.$$

$$(19)$$

Then, the following equations can be obtained:

$$\alpha_G(\rho) = \alpha_Y(\rho), \beta_G(\rho) = \beta_Y(\rho), \gamma_G(\rho) = \gamma_Y(\rho), \tag{20}$$

Therefore, we can conclude that

$$G = Y. \tag{21}$$

□

*Proof* (G2). Given $d_\rho(G, Y)$, we have

$$d_\rho(G, Y) = \left[ \frac{1}{2} \left( \alpha_G(\rho) \log \frac{2\alpha_G(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right) \right.$$
$$+ \alpha_Y(\rho) \log \frac{2\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)}$$
$$+ \beta_G(\rho) \log \frac{2\beta_G(\rho)}{\beta_G(\rho) + \beta_Y(\rho)}$$
$$+ \beta_Y(\rho) \log \frac{2\beta_Y(\rho)}{\beta_G(\rho) + \beta_Y(\rho)}$$
$$+ \gamma_G(\rho) \log \frac{2\gamma_G(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)}$$
$$\left. + \gamma_Y(\rho) \log \frac{2\gamma_Y(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} \right]^{1/2} = 0.$$

$$(22)$$

Next, given $d_\rho(Y, G)$, we have

$$d_\rho(Y, G) = \left[ \frac{1}{2} \left( \alpha_Y(\rho) \log \frac{2\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right) \right.$$
$$+ \alpha_G(\rho) \log \frac{2\alpha_G(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)}$$
$$+ \beta_Y(\rho) \log \frac{2\beta_Y(\rho)}{\beta_G(\rho) + \beta_Y(\rho)}$$
$$+ \beta_G(\rho) \log \frac{2\beta_G(\rho)}{\beta_G(\rho) + \beta_Y(\rho)}$$
$$+ \gamma_Y(\rho) \log \frac{2\gamma_Y(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)}$$
$$\left. + \gamma_G(\rho) \log \frac{2\gamma_G(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} \right]^{1/2} = 0.$$

$$(23)$$

Therefore,

$$d_\rho(G, Y) = d_\rho(Y, G), \tag{24}$$

which demonstrates that the Euclidean property for $d_\rho$ is satisfied. □

*Proof* (G3). Given four hypotheses to be tested, $A_1$, $A_2$, $A_3$, and $A_4$

$$A_1 : \alpha_K(\rho) \leq \alpha_G(\rho) \leq \alpha_Y(\rho)$$
$$A_2 : \alpha_Y \leq (\rho)\alpha_G(\rho) \leq \alpha_K(\rho)$$
$$A_3 : \alpha_G(\rho) \leq \min \{\alpha_K(\rho), \alpha_Y(\rho)\}$$
$$A_4 : \alpha_G(\rho) \geq \max \{\alpha_K(\rho), \alpha_Y(\rho)\}$$

$$(25)$$

Under assumption $A_1$ and $A_2$, it is self-evident that the triangle inequality is satisfied, where

$$|\alpha_K(\rho) - \alpha_Y(\rho)| \leq |\alpha_K(\rho) - \alpha_Y(\rho)| + |\alpha_G(\rho) - \alpha_Y(\rho)|. \tag{26}$$

under assumption $A_3$, we have

$$\alpha_K(\rho) - \alpha_G(\rho) \geq 0,$$
$$\alpha_Y(\rho) - \alpha_G(\rho) \geq 0. \tag{27}$$

Then, we can calculate

$$
\begin{aligned}
&|\alpha_K(\rho) - \alpha_G(\rho)| + |\alpha_G(\rho) - \alpha_Y(\rho)| - |\alpha_K(\rho) - \alpha_Y(\rho)|, \\
&\begin{cases}
\alpha_K(\rho) - \alpha_G(\rho) + \alpha_Y(\rho) - \alpha_G(\rho) - \alpha_K(\rho) + \alpha_Y(\rho) \, \text{if} \, \alpha_K(\rho) \geq \alpha_Y(\rho), \\
\alpha_K(\rho) - \alpha_G(\rho) + \alpha_Y(\rho) - \alpha_G(\rho) + \alpha_K(\rho) - \alpha_Y(\rho) \, \text{if} \, \alpha_K(\rho) \leq \alpha_Y(\rho), \\
\end{cases} \\
&= 2(\min\{\alpha_K(\rho), \alpha_Y(\rho)\} - \alpha_G(\rho)) \geq 0.
\end{aligned}
\tag{28}
$$

Analogously, under assumption $A_4$, we have

$$
\begin{aligned}
&|\alpha_K(\rho) - \alpha_G(\rho)| + |\alpha_G(\rho) - \alpha_Y(\rho)| - |\alpha_K(\rho) - \alpha_Y(\rho)| \\
&\begin{cases}
\alpha_G(\rho) - \alpha_K(\rho) + \alpha_G(\rho) - \alpha_Y(\rho) - \alpha_K(\rho) + \alpha_Y(\rho) \, \text{if} \, \alpha_K(\rho) \geq \alpha_Y(\rho), \\
\alpha_G(\rho) - \alpha_K(\rho) + \alpha_G(\rho) - \alpha_Y(\rho) + \alpha_K(\rho) - \alpha_Y(\rho) \, \text{if} \, \alpha_K(\rho) \leq \alpha_Y(\rho). \\
\end{cases} \\
&= 2(\alpha_G(\rho) - \max\{\alpha_K(\rho), \alpha_Y(\rho)\}) \geq 0.
\end{aligned}
\tag{29}
$$

As a result, the triangle inequality is satisfied under the implication $A_3$ and $A_4$, where

$$|\alpha_K(\rho) - \alpha_Y(\rho)| \leq |\alpha_K(\rho) - \alpha_G(\rho)| + |\alpha_G(\rho) - \alpha_Y(\rho)|. \tag{30}$$

In the same manner, we can find that

$$
\begin{aligned}
&|\beta_K(\rho) - \beta_Y(\rho)| \leq |\beta_K(\rho) - \beta_G(\rho)| + |\beta_G(\rho) - \beta_Y(\rho)|, \\
&|\gamma_K(\rho) - \gamma_Y(\rho)| \leq |\gamma_K(\rho) - \gamma_G(\rho)| + |\gamma_G(\rho) - \gamma_Y(\rho)|.
\end{aligned}
\tag{31}
$$

As a result, it is demonstrated that the feature of triangle inequality for $d_\rho$ is truly in the sense that

$$d_\rho(K, G) + d_\rho(G, Y) \geq d_\rho(K, Y). \tag{32}$$

$\square$

*Proof* (G4). Given two SVNFSs $G$ and $Y$ in $M$, we have

$$
\begin{aligned}
d_{(G,Y)} &= \left[ \frac{1}{2}\left( \alpha_G(\rho) \log \frac{2\alpha_G(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right) + \alpha_Y(\rho) \log \frac{2\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right. \\
&\quad + \beta_G(\rho) \log \frac{2\beta_G(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} + \beta_Y(\rho) \log \frac{2\beta_Y(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} \\
&\quad \left. + \gamma_G(\rho) \log \frac{2\gamma_G(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} + \gamma_Y(\rho) \log \frac{2\gamma_Y(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} \right]^{1/2} = 0, \\
&= \left[ \frac{1}{2}(\alpha_G(\rho) + \alpha_Y(\rho))\left( \frac{\alpha_G(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \log \frac{2\alpha_G(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right. \right. \\
&\quad \left. + \frac{\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \log \frac{2\alpha_Y(\rho)}{\alpha_G(\rho) + \alpha_Y(\rho)} \right) + (\beta_G(\rho) + \beta_Y(\rho)) \\
&\quad \cdot \left( \frac{\beta_G(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} \log \frac{2\beta_G(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} + \frac{\beta_Y(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} \log \right. \\
&\quad \left. \cdot \frac{2\beta_Y(\rho)}{\beta_G(\rho) + \beta_Y(\rho)} \right) + (\gamma_G(\rho) + \gamma_Y(\rho)) \left( \frac{\gamma_G(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} \log \right. \\
&\quad \left. \left. \cdot \frac{2\gamma_G(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} + \frac{\gamma_Y(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} \log \frac{2\gamma_Y(\rho)}{\gamma_G(\rho) + \gamma_Y(\rho)} \right). \right. \\
&= \left[ \frac{1}{2}(G+Y)\left( 1 - Z\left( \frac{G}{G+Y}, \frac{Y}{G+Y} \right) \right) \right]^{1/2}.
\end{aligned}
\tag{33}
$$

It was proven as $0 \leq \varsigma \leq 1$

$$1 - Z(\varsigma, 1 - \varsigma) \leq |\varsigma - (1 - \varsigma)|. \tag{34}$$

Then, there is

$$d_\rho(G, Y) \leq \left[\frac{1}{2}(G + Y)\left|\frac{G}{G + Y} - \frac{Y}{G + Y}\right|\right]^{1/2} = \left[\frac{1}{2}V(G, Y)\right]^{1/2}. \tag{35}$$

Here, $V(G, Y)$ is the variational distance.

As proved, $0 \leq V(G, Y) \leq 2$.

Thus, it is obtain $0 \leq d_\rho(G, Y) \leq 1$. As a result, it is demonstrated that the property of boundedness for $d\rho$ is fulfilled. □

*Definition 14.* Let $G$ and $Y$ be two SVNFSs in a finite UOD $M = \{\rho_1, \rho_2, \rho_3, \cdots, \rho_n\}$, where

$$G = \{\langle \rho_i, \alpha_G(\rho), \beta_G(\rho), \gamma_G(\rho)\rangle | \rho_i \in M\} \text{ and } \\ Y = \{\langle \rho_i, \alpha_Y(\rho), \beta_Y(\rho), \gamma_Y(\rho)\rangle | \rho_i \in M\}. \tag{36}$$

Normalized distance $d_\eta$ measure between $G$ and $Y$ is defined by

$$d_\eta(G, Y) = \frac{1}{n}\Sigma_{i=1}^n d_\eta(G, Y) = \frac{1}{n} \\ \cdot \left[\frac{1}{2}\left(\alpha_G(\rho_i) \log \frac{2\alpha_G(\rho_i)}{\alpha_G(\rho_i) + \alpha_Y(\rho_i)} + \alpha_Y(\rho_i) \log \frac{2\alpha_Y(\rho_i)}{\alpha_G(\rho_i) + \alpha_Y(\rho_i)}\right.\right. \\ + \beta_G(\rho_i) \log \frac{2\beta_G(\rho_i)}{\beta_G(\rho_i) + \beta_Y(\rho_i)} + \beta_Y(\rho_i) \log \frac{2\beta_Y(\rho_i)}{\beta_G(\rho_i) + \beta_Y(\rho_i)} \\ \left.\left.+ \gamma_G(\rho_i) \log \frac{2\gamma_G(\rho_i)}{\gamma_G(\rho_i) + \gamma_Y(\rho_i)} + \gamma_Y(\rho_i) \log \frac{2\gamma_Y(\rho_i)}{\gamma_G(\rho_i) + \gamma_Y(\rho_i)}\right)\right]^{1/2}. \tag{37}$$

Following that, several numerical examples are presented to validate the new distance measure.

*Definition 15.* Let $G$ and $Y$ be two SVNFSs in a finite UOD $M = \{\rho_1, \rho_2, \rho_3, \cdots, \rho_n\}$, where

$$G = \{\langle \rho_i, \alpha_G(\rho), \beta_G(\rho), \gamma_G(\rho)\rangle | \rho_i \in M\}, \\ Y = \{\langle \rho_i, \alpha_Y(\rho), \beta_Y(\rho), \gamma_Y(\rho)\rangle | \rho_i \in M\}. \tag{38}$$

Normalized distance measure $d_\zeta$ from $G$ and $Y$ is defined as

$$d_\zeta(G, Y) = \frac{1}{4n}\Sigma_{i=1}^n d_\zeta(G, Y), = \frac{1}{4n}\Sigma_{i=1}^n (|\alpha_G(\rho_i) - \alpha_Y(\rho_i)| \\ + |\beta_G(\rho_i) - \beta_Y(\rho_i)| + |\gamma_G(\rho_i) - \gamma_Y(\rho_i)| \\ + 2 \max \{|\alpha_G(\rho_i) - \alpha_Y(\rho_i)|, |\beta_G(\rho_i) \\ - \beta_Y(\rho_i)|, |\gamma_G(\rho_i) - \gamma_Y(\rho_i)|\}). \tag{39}$$

After that, various numerical examples are presented below to verify the new distance measure.

*Example 1.* Suppose the SVNFSs $G$, $Y$, and $R$ in the UOD $M = \{\rho_1, \rho_2\}$

$$G = \{\langle \rho_1, 0.2, 0.3, 0.7\rangle, \langle \rho_2, 0.30, 0.50, 0.8\rangle\} \\ Y = \{\langle \rho_1, 0.2, 0.3, 0.7\rangle, \langle \rho_2, 0.30, 0.50, 0.8\rangle\},$$

$$R = \{\langle \rho_1, 0.5, 0.25, 0.45\rangle, \langle \rho_2, 0.25, 0.35, 0.4\rangle\}. \tag{40}$$

According to Definition 14, the distance between the SVNFSs $G$, $Y$, and $R$ is measured as follows:

$$d_\eta(G, Y) = \frac{1}{n}\Sigma_{i=1}^n d_\eta(G, Y) = \frac{1}{n}\left[\frac{1}{2}\left(\alpha_G(\rho_i) \log \frac{2\alpha_G(\rho_i)}{\alpha_G(\rho_i) + \alpha_Y(\rho_i)}\right.\right. \\ + \alpha_Y(\rho_i) \log \frac{2\alpha_Y(\rho_i)}{\alpha_G(\rho_i) + \alpha_Y(\rho_i)} + \beta_G(\rho_i) \log \frac{2\beta_G(\rho_i)}{\beta_G(\rho_i) + \beta_Y(\rho_i)} \\ + \beta_Y(\rho_i) \log \frac{2\beta_Y(\rho_i)}{\beta_G(\rho_i) + \beta_Y(\rho_i)} + \gamma_G(\rho_i) \log \frac{2\gamma_G(\rho_i)}{\gamma_G(\rho_i) + \gamma_Y(\rho_i)} \\ \left.\left.+ \gamma_Y(\rho_i) \log \frac{2\gamma_Y(\rho_i)}{\gamma_G(\rho_i) + \gamma_Y(\rho_i)}\right)\right]^{1/2},$$

$$d_\eta(G, Y) = 0.0000, d_\eta(Y, G) = 0.0000, d_\eta(G, R) = 0.09875,$$

$$d_\eta(R, G) = 0.09875, d_\eta(Y, R) = 0.09875, d_\eta(R, Y) = 0.09875. \tag{41}$$

We can see that $d_\eta(G, Y)$ is equal to zero and that $d_\eta(G, R) = d_\eta(Y, R) = 0.098$ because SVNFS $A$ is the same as SVNFS $B$. Moreover, we can also see that $d_\eta(G, Y) = d_\eta(Y, G) = 0.00$, $d_\eta(G, R) = d_\eta(R, G) = 0.098$, and $d_\eta(Y, R) = d_\eta(R, Y) = 0.098$. These results are convenient and validate the new distance measure's first two properties.

*Example 2.* Suppose the SVNFSs $G$, $Y$, and $R$ in the UOD $M = \{\rho_1, \rho_2\}$

$$G = \{\langle \rho_1, 0.3, 0.4, 0.6\rangle, \langle \rho_2, 0.30, 0.50, 0.8\rangle\} \\ Y = \{\langle \rho_1, 0.3, 0.4, 0.6\rangle, \langle \rho_2, 0.30, 0.50, 0.8\rangle\},$$

$$R = \{\langle \rho_1, 0.15, 0.35, 0.45\rangle, \langle \rho_2, 0.25, 0.35, 0.4\rangle\}. \tag{42}$$

According to Definition 14, the distance between the SVNFSs $G$, $Y$, and $R$ is measured as follows:

$$d_\eta(G, Y) = \frac{1}{n}\Sigma_{i=1}^n d_\eta(G, Y), = \frac{1}{n}\left[\frac{1}{2}\left(\alpha_G(\rho_i) \log \frac{2\alpha_G(\rho_i)}{\alpha_G(\rho_i) + \alpha_Y(\rho_i)}\right.\right. \\ + \alpha_Y(\rho_i) \log \frac{2\alpha_Y(\rho_i)}{\alpha_G(\rho_i) + \alpha_Y(\rho_i)} + \beta_G(\rho_i) \log \frac{2\beta_G(\rho_i)}{\beta_G(\rho_i) + \beta_Y(\rho_i)} \\ + \beta_Y(\rho_i) \log \frac{2\beta_Y(\rho_i)}{\beta_G(\rho_i) + \beta_Y(\rho_i)} + \gamma_G(\rho_i) \log \frac{2\gamma_G(\rho_i)}{\gamma_G(\rho_i) + \gamma_Y(\rho_i)} \\ \left.\left.+ \gamma_Y(\rho_i) \log \frac{2\gamma_Y(\rho_i)}{\gamma_G(\rho_i) + \gamma_Y(\rho_i)}\right)\right]^{1/2},$$

$$d_\eta(G, Y) = 0.0000, d_\eta(Y, G) = 0.0000, d_\eta(G, R) = 0.2892,$$

$$d_\eta(R, G) = 0.2892, d_\eta(Y, R) = 0.2892, d_\eta(R, Y) = 0.2892. \tag{43}$$

We can see that $d_\eta(G, Y)$ is equal to zero and that $d_\eta(G, R) = d_\eta(Y, R) = 0.2892$ because SVNFS $A$ is the same as SVNFS $B$. Moreover, we can also see that $d_\eta(G, Y) = d_\eta(Y, G) = 0.00$, $d_\eta(G, R) = d_\eta(R, G) = 0.2892$, and $d_\eta(Y, R) = d_\eta(R, Y) = 0.2892$. These results are obvious and confirm the new distance measure's first two characteristics.

*Example 3.* Suppose that SVNFSs $G$, $Y$, and $R$ in the UOD $M = \rho_1, \rho_2$

$$G = \{\langle \rho_1, 0.2, 0.3, 0.7 \rangle, \langle \rho_2, 0.30, 0.50, 0.8 \rangle\},$$
$$Y = \{\langle \rho_1, 0.2, 0.3, 0.7 \rangle, \langle \rho_2, 0.30, 0.50, 0.8 \rangle\},$$

$$R = \{\langle \rho_1, 0.5, 0.25, 0.45 \rangle, \langle \rho_2, 0.25, 0.35, 0.4 \rangle\}. \tag{44}$$

According to Definition 15, the distance between the SVNFSs $G$, $Y$, and $R$ is measured as follows:

$$d_\zeta(G, Y) = \frac{1}{4n} \Sigma_{i=1}^n (|\alpha_G(\rho_i) - \alpha_Y(\rho_i)| + |\beta_G(\rho_i) - \beta_Y(\rho_i)| + |\gamma_G(\rho_i) - \gamma_Y(\rho_i)| + 2 \max \{|\alpha_G(\rho_i) - \alpha_Y(\rho_i)|, |\beta_G(\rho_i) - \beta_Y(\rho_i)|, |\gamma_G(\rho_i) - \gamma_Y(\rho_i)|\}),$$

$$d_\zeta(G, Y) = 0.0000, d_\zeta(Y, G) = 0.0000, d_\zeta(G, R) = 0.3250,$$

$$d_\zeta(R, G) = 0.3250, d_\zeta(Y, R) = 0.3250, d_\zeta(R, Y) = 0.3250. \tag{45}$$

We can see that $d_\zeta(G, Y)$ is equal to zero and that $d_\zeta(G, R) = d_\zeta(Y, R) = 0.325$ because SVNFS $G$ is the same as SVNFS $Y$. Moreover, we can also see that $d_\zeta(G, Y) = d_\zeta(Y, G) = 0.00$, $d_\zeta(G, R) = d_\zeta(R, G) = 0.325$, and $d_\zeta(Y, R) = d_\zeta(R, Y) = 0.325$. These findings are all intuitive, and they back up the first two new distance measurements.

*Example 4.* Suppose that SVNFSs $G$, $Y$, and $R$ in the UOD $M = \rho_1, \rho_2$

$$G = \{\langle \rho_1, 0.3, 0.4, 0.6 \rangle, \langle \rho_2, 0.30, 0.50, 0.8 \rangle\},$$
$$Y = \{\langle \rho_1, 0.2, 0.3, 0.7 \rangle, \langle \rho_2, 0.30, 0.50, 0.8 \rangle\},$$

$$R = \{\langle \rho_1, 0.15, 0.35, 0.45 \rangle, \langle \rho_2, 0.25, 0.35, 0.4 \rangle\}. \tag{46}$$

According to Definition 15, the distance between the SVNFSs $G$, $Y$, and $R$ is measured as follows:

$$d_\zeta(G, Y) = \frac{1}{4n} \Sigma_{i=1}^n (|\alpha_G(\rho_i) - \alpha_Y(\rho_i)| + |\beta_G(\rho_i) - \beta_Y(\rho_i)| + |\gamma_G(\rho_i) - \gamma_Y(\rho_i)| + 2 \max \{|\alpha_G(\rho_i) - \alpha_Y(\rho_i)|, |\beta_G(\rho_i) - \beta_Y(\rho_i)|, |\gamma_G(\rho_i) - \gamma_Y(\rho_i)|\}),$$

$$d_\zeta(G, Y) = 0.0000, d_\zeta(Y, G) = 0.0000, d_\zeta(G, R) = 0.2688,$$

$$d_\zeta(R, G) = 0.2688, d_\zeta(Y, R) = 0.2688, d_\zeta(R, Y) = 0.2688. \tag{47}$$

We can see that $d_\zeta(G, Y)$ is equal to zero and that $d_\zeta(G, R) = d_\zeta(Y, R) = 0.2688$ because SVNFS $G$ is the same as SVNFS $Y$. Moreover, we can also see that $d_\zeta(G, Y) = d_\zeta(Y, G) = 0.00$, $d_\zeta(G, R) = d_\zeta(R, G) = 0.2688$, and $d_\zeta(Y, R) = d_\zeta(R, Y) = 0.2688$. These results are straightforward and confirm the new distance measure's first two features.

Above all the examples that are solved and some existing distance measures given in literature, results that are in Table 1 using Table 2 and its graphical representation are shown in Figure 1. The graphical representation of Table 1 is shown in Figure 2.

## 5. Multicriteria Group Decision-Making in Single-Valued Neutrosophic Fuzzy Sets

The discovery of fuzzy set theory in the early seventeenth century paved the way for multicriteria decision-making. The idea has been well-suited to decision-making, resulting in the development of various unique fuzzy multicriteria decision-making approaches. The question of how to make decisions in an uncertain environment remains unanswered. Single-valued neutrosophic fuzzy sets are utilized to express the membership information of alternatives to characteristics, which is then merged with a decision-making matrix. The degree of the alternatives in regard to the ideal object is then assessed using the SVNFS distance measure. In this section, the decision-making problem is solved using a newly developed distance measures for SVNFSs.

*Example 5.* An interview panel consists of three members. They take the interview of four candidates and want to select the best candidate. The selection criteria are professional attitude, work experience, and salary demand. We here construct fuzzy decision-making to select the best candidate. These experts are provided the five linguistic variables, namely, very low, low, average, high, and very high. These experts provide information in terms of these variables and construct the decision matrices. Experts provide us some weighting information according to criteria that professional attitude has high weighted, work experience has very high weighting, and salary demand has average weighing. Linguistic variables are modeled as SVNFSs as v.low (0.1, 0.1, 0.3), low (0.1, 0.3, 0.5), average (0.3, 0.5, 0.7), high (0.5, 0.7, 0.9), and v.high (0.7, 0.9, 0.9). Then, the decision matrix in terms of SVNFSs is shown in Table 3.

We evaluate the above decision matrix using the algorithm and find the normalized combined decision matrix shown in Table 4.

The weighted normalized combined decision matrix shown in Table 5 is obtained by multiplying the weight values.

By using the algorithm, we compute fuzzy positive ideal solution $A^+ = \{\langle 0.27, 0.64, 0.9 \rangle, \langle 0.38, 0.82, 0.9 \rangle, \langle 0.3, 0.21, 0.14$

$\rangle\}$ and fuzzy negative ideal solution $A^- = \{\langle 0.05, 0.17, 0.5\rangle, \langle 0.07, 0.42, 0.69\rangle, \langle 0.06, 0.07, 0.07\rangle$.

After calculating the FPIS and FNIS by using the algorithm, then we apply the proposed distance measure to find the closeness coefficient using $CC_i = p_i^- / (p_i^+ + p_i^-)$ rank the candidates. According to the ranking, the sequence of the candidates is $2 > 3 > 1 > 4$.

## 6. Comparison Analysis

Szmidt and Kacprzyk's and Yang and Francisco's distance measures [36] are also used to address the challenges of selecting the best candidate in this part to emphasise the differences between the current and suggested distance measures. Assume four decision-maker opinions are presented in Table 5. Based on Szmidt and Kacprzyk's and Yang and Francisco's, the order in which four candidates are ranked is determined and shown in Table 6. The results based on our new distance measure are also shown in Table 6.

The ideal answer to the candidate selection quality evaluation challenges is always cadidate-1, as shown in Table 6. It shows that our new distance measurements are usable and practical. The ordering of alternatives based on the distance measure proposed in [36] is $2 > 3 > 1 > 4$, which corresponds to the ranking order of the new distance measure, despite the fact that the rating orders of the alternatives are identical.

## 7. Conclusion

To tackle multiattribute decision-making challenges, a decision-making approach is offered based on the recommended measure recommendations. We established novel method to find the distance between two single-valued neutrosophic fuzzy sets. We construct an algorithm and find some results using novel distance measures by the help of algorithm. We also compare our results with well-known existing measures. We see that our results are appropriate as compared to the other measures. A journalistic case study is used to demonstrate the process, and the results are compared to those obtained using alternative methods. According to studies, the proposed approach provides an innovative and uncomplicated method for dealing with ambiguous or doubtful data. It also offers a different approach to dealing with the SVNFSs' decision-making issues. In the future, our findings could be applied to more ambiguous and fuzzy circumstances.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare no conflicts of interest.

## Authors' Contributions

All the authors have contributed equally. All authors have read and agreed on the possible publication of the manuscript.

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