Reliability Engineering and System Safety 167 (2017) 302–309

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

New approach for failure mode and effect analysis using linguistic distribution assessments and TODIM method

Jia Huang[a], Zhaojun(Steven) Li[b], Hu-Chen Liu[a,*]

[a] School of Management, Shanghai University, Shanghai 200444, PR China

[b] Department of Industrial Engineering and Engineering Management, Western New England University, Springfield, MA 01119, USA

ARTICLE INFO

Keywords:
Failure mode and effect analysis
Risk analysis
Linguistic distribution assessments
TODIM method
Combination weight

ABSTRACT

As a proactive risk management instrument, failure mode and effect analysis (FMEA) has been broadly utilized to recognize, evaluate and eliminate failure modes of products, processes, systems and services. Nevertheless, the conventional FMEA method suffers from many important deficiencies when used in the real world. First, crisp numbers are adopted to describe the risk of failure modes; but, in many practical situations, it is difficult to obtain exact assessment values due to inherent vagueness in the human judgments. Second, the priority ranking of failure modes is determined based on the risk priority number (RPN), which is questionable and strongly sensitive to the variation of risk factor ratings. Therefore, this paper applies linguistic distribution assessments to represent FMEA team members' risk evaluation information and employs an improved TODIM (an acronym in Portuguese of interactive and multicriteria decision making) method to determine the risk priority of failure modes. Furthermore, both subjective weights and objective weights of risk factors are taken into account while conducting the risk analysis process. Finally, an empirical case concerning the risk evaluation of a grinding wheel system is presented to demonstrate the practicality and effectiveness of the proposed new FMEA model.

1. Introduction

Failure mode and effect analysis (FMEA) is a powerful tool applied for system safety and reliability analysis of products, processes, systems and services [1]. When it involves criticality analysis, it is also called failure mode, effect, and criticality analysis (FMECA). As a systematized, bottom-up engineering technique, the main purpose of FMEA is to recognize, evaluate, and eliminate known or potential failure modes in a given system and prevent them from happening [2,3]. The FMEA was originally performed by the NASA as a formal design approach to evaluate system reliability problems in the 1960s [4]. Since its introduction, FMEA is increasingly prevalent and has been used in a variety of industries, such as military, manufacturing, electronics, and healthcare industries [5–8].

The traditional FMEA employs risk priority number (RPN) to obtain the priority orders of the identified failure modes [9]. The concrete steps can be summarized as follows: First, a discrete numerical scale of 1–10 is utilized to evaluate the criticality grades of the risk factors: occurrence (O), severity (S), and detection (D). Second, the RPN value is determined by the multiplication of O, S and D, i.e., $RPN = O \times S \times D$. Lastly, the failure modes with larger RPN values will be given higher priorities for remedial actions [2,10]. Usually, the RPNs should be recalculated after the modified measures have been performed. Although FMEA is an efficient continuous quality improvement technique, the conventional RPN method suffers from many flaws when used in the real world. The major ones are restated as follows [8,9,11–13]: (1) Crisp numbers are utilized to evaluate the risk of failure modes, whereas in many practical applications, it is hard or even impossible to assess failures using exact values because of lack of data and people's limited expertise. (2) The priority orders of failure modes are determined based on the RPN; but its mathematical formulation is controversy and strongly sensitive to the variation of risk factor ratings. (3) The three risk factors, O, S and D, are assumed to be of equal importance in the risk calculation process. Thus, the traditional FMEA is not able to manage the situations in which the risk factors have different weights. (4) The prioritization of failure modes undertaken by the standard FMEA solves solely a ranking problem. However, under tight resources constraints, FMEA should be able to address a sorting problem as well, i.e. clustering of the failure modes into ordered classes [14,15].

Due to various subjective and objective conditions, it is often difficult for FMEA team members to provide precise values for the assessment of failure modes. Instead, they prefer to utilize linguistic labels to state their opinions regarding the risk of failure modes. In the literature, different techniques for computing with words were employed to deal with the linguistic assessment information in FMEA. Generally, the linguis-

* Corresponding author.
E-mail addresses: huchenliu@foxmail.com, huchenliu@t.shu.edu.cn (H.-C. Liu).

http://dx.doi.org/10.1016/j.ress.2017.06.014
Received 11 January 2017; Received in revised form 4 June 2017; Accepted 11 June 2017
Available online 13 June 2017
0951-8320/

tic computational methods used in current studies can be classified into three types: the method based on membership functions [16,17], the linguistic symbolic method based on ordinal scales [18], and the method based on linguistic 2-tuples [19,20]. The results of the former two methods both produce a consequent loss of information and the lack of precision. The last method can only guarantee accuracy in dealing with uniformly and symmetrically distributed linguistic term sets. As a generalization of the 2-tuple linguistic method, Zhang et al. [21] proposed a new linguistic method based on distribution assessments, where symbolic proportions can be assigned to all the linguistic terms in a linguistic term set [22]. Compared with other methodologies, the linguistic distribution assessment method can represent evaluation information more accurately and reflect decision makers' real experience. In this way, the linguistic method based on distribution assessments is of great value in managing linguistic risk evaluations in FMEA.

On the other hand, the determination of the priority order of failure modes in FMEA is a multi-criteria decision making (MCDM) problem [9,15,23], which requires MCDM methods for an effective problem-solving. The TODIM (an acronym in Portuguese of interactive and multicriteria decision making) method proposed by Gomes and Lima [24] is an effective behavioral decision making method derived from the prospect theory. The main advantage of the TODIM is that the decision maker's psychological character is taken into account [25,26]. This is because it is able to capture the loss and gain under uncertainty from the view of reference point and the decision maker is more sensitive to the loss. Moreover, in complete rationality decision making, the decision maker pursuits utility maximization, while in the TODIM method, the decision maker aims to value function maximization. Therefore, the TODIM method can be regarded as a useful bounded rationality behavioral decision making method. Recently, the TODIM method has been broadly applied for the optimal ranking of alternatives in various decision making problems. For example, Ji et al. [27] established a projection-based TODIM model for personnel selection under the multi-valued neutrosophic environment. Wang et al. [28] proposed a likelihood-based TODIM approach based on multi-hesitant fuzzy linguistic information to evaluate contractors in logistics outsourcing. Qin et al. [29] developed an extended TODIM method based on multigranularity linguistic terms and entropy measure for the selection of energy performance contracting business models, and Qin et al. [30] modified the TODIM method to accommodate interval type-2 fuzzy setting for solving green supplier selection problem. Tseng et al. [26] employed a fuzzy TODIM model for the quantitative evaluation of green supply chain practices, and Zhang and Xu [31] used a hesitant fuzzy TODIM method for the efficiency evaluation of sustainable water management. Besides, other extensions of the TODIM technique, such as the pythagorean fuzzy TODIM [25] and the interval TODIM [32], have been presented to multi-criteria decision making. Therefore, due to its characteristics and wide usage, the TODIM approach is utilized and modified to tackle the risk prioritization of failure modes in FMEA.

Based on the above discussions, this paper applies the linguistic distribution assessments and the TODIM approach to assess and rank failure modes in FMEA. The main contributions of this study can be summarized as follows: (1) The linguistic distribution assessment method is employed to quantify the risk ratings of failure modes against each risk factor. (2) Through incorporating subjective weights with the objective ones derived by the entropy method, a combination weighting method is utilized to determine the relative weights of risk factors. (3) The normal TODIM approach is extended to acquire the risk prioritization of the recognized failure modes in FMEA. Further, the feasibility and usefulness of the proposed risk ranking model are validated with a practical case study concerning the risk evaluation of a grinding wheel system. The results show that the proposed FMEA can not only capture the uncertainty and diversity of assessment information provided by domain experts, but also include the psychological behavior of the experts in the risk analysis process to derive a more accurate ranking of failure modes.

The remaining part of this paper is arranged as follows: In Section 2, a brief review concerning various improvements of traditional FMEA is given. Some basic notions and operational laws related to the linguistic distribution assessments are presented in Section 3. In Section 4, the FMEA model using linguistic distribution assessments and an extended TODIM method is developed for ranking failure modes. A practical case is performed and some comparative methods are conducted in Section 5 to illustrate the proposed new risk priority method. To finish, some conclusions and future work are provided in Section 6.

2. Literature review

Over the last decade, lots of constructive risk priority methods have been proposed by researchers to enhance the performance of classical FMEA. In the following, the improvements of FMEA are reviewed briefly from the aspects of risk evaluation, risk factor weights and failure mode ranking.

First, many linguistic computational methods were used in previous studies to handle the risk evaluations of failure modes elicited from FMEA team members. For example, Sharma et al. [3] represented the risk factors as members of a fuzzy set fuzzified by membership functions and determined the criticality of failure modes based on rule base and fuzzy logic operations. Song et al. [16] proposed a failure evaluation structure using fuzzy technique for order preference by similarity to ideal solution (TOPSIS), which allowed experts to use linguistic grades for assessing failure modes. In [33], a FMEA technique based on intuitionistic fuzzy approach was suggested and the vague concepts and insufficient data were handled with the theory of intuitionistic fuzzy sets. Wang et al. [17] proposed a risk priority approach based on an extended COPRAS (COmplex PRoportional ASsessment) model and addressed the uncertainty and subjectivity of risk assessment information within the interval-valued intuitionistic fuzzy environment. Zhou et al. [18] treated the risk factors as linguistic variables and introduced a linguistic weighted geometric operator to perform algebraic calculations on the results of expert evaluations. In [34], an evidential FMEA method using linguistic terms was presented and the linguistic terms were utilized by experts to assign a more meaningful value for the considered failure modes. In addition, Liu et al. [19] applied the interval 2-tuple linguistic approach to deal with FMEA team members' risk evaluation results, and Liu et al. [20] proposed the concept of hesitant 2-tuple linguistic term sets to express various uncertainties in the assessment information of experts.

To circumvent the pitfall that the standard FMEA treats the risk factors equally, various weighting methods have been emerged in the literature to derive the weights of risk factors. The direct assessment [35], analytic hierarchy process (AHP) [36] and Delphi method [18] are common methods used to identify the subjective weights of risk factors. Besides, the ordered weight operator [37], data envelopment analysis [38] and minimum cut set [39] are usually utilized to deduce the objective risk factor weights. However, as reported in [23,37], either the subjective weighting method or the objective weighting method has its own limitations, and a hybrid weighting method is needed to indicate the relative importance of risk factors. In this regard, Liu et al. [23] utilized a combined weighting method to compute the weights of risk factors in FMEA, in which the objective weights were derived based on statistical distance. Liu et al. [40] applied fuzzy AHP and entropy method to determine the subjective and objective important degrees of risk factors, respectively. To fully reflect the importance of risk factors, Song et al. [16] considered integrating both subjective weights and objective weights and adopted the entropy-based weighting method to assign the objective weights of risk factors.

For improving the risk ranking of failure modes in FMEA, Chang et al. [41] proposed an exponential risk priority number (ERPN) method, in which the number of unique values for risk evaluation of failures has been increased. Using the universal generating function concept, Akbarzade et al. [42] further introduced a universal risk priority number

(URPN) approach to improve the assessment capability of the conventional FMEA in ranking. In addition, many MCDM methods have been applied in FMEA to determine the risk priority of failure modes. For instance, Emovon et al. [43] used two methods, VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) and compromise programming, for ordering the identified failure modes of a marine machinery system. Wang et al. [17] introduced a modified COPRAS method to enhance the credibility of ranking failure modes in FMEA. Selim et al. [44] employed fuzzy TOPSIS model to specify maintenance priorities of machines. Additionally, the MULTIMOORA (Multiplicative form of multi-objective optimization on the basis of ratio analysis) [45], the ELECTRE (ELimination Et Choix Traduisantla REalité) [23], the QUALIFLEX (Qualitative flexible multiple criteria method) [20], and the PROMETHEE (Preference ranking organization methods for enrichment evaluations) [15] were applied in FMEA for the risk prioritization of potential failure modes. Interested readers can refer to Liu [9] for more detailed procedures and applications of the MCDM methods in FMEA.

3. Preliminaries

3.1. Linguistic variables

The concept of linguistic variables was put forward by Zadeh [46] to handle the circumstances which are too complicated or too indeterminate to be expressed by quantitative means. Let $S = \{s_x | x = -t, \ldots, -1, 0, 1, \ldots, t\}$ be a linguistic term set, where s_x represents a possible value for a linguistic variable. Normally, the linguistic term set S should possess the following characteristics [47]:

(1) The set is ordered: $s_x > s_y$, if $x > y$;
(2) There is a negation operator: $Neg(s_x) = s_{-x}$;
(3) Maximization operator: $\max(s_x, s_y) = s_x$, if $s_x \geq s_y$;
(4) Minimization operator: $\min(s_x, s_y) = s_x$, if $s_x \leq s_y$.

For reserving the original linguistic information, Xu [48] generalized the discrete linguistic term set S to a consecutive one $\bar{S} = \{s_x | x \in [-r, r]\}$, where $r(r \geq t)$ is an adequately large positive integer. If $s_x \in S$, then s_x is called the original linguistic term used to evaluate alternatives; otherwise, it is called the virtual linguistic term applied for operation.

Definition 1. Let $s_x, s_y \in \bar{S}$ be any two linguistic terms and $\mu \in [0, 1]$, the basic operational laws of linguistic terms are described as follows [48,49]:

(1) $s_x \oplus s_y = s_{x+y}$;
(2) $s_x \otimes s_y = s_y \otimes s_x = s_{xy}$;
(3) $\mu s_x = s_{\mu x}$;
(4) $(s_x)^\mu = s_{x^\mu}$.

3.2. Linguistic distribution assessments

The linguistic distribution assessment method was initiated by Zhang et al. [21] for computing with words, in which all of the linguistic terms are assigned with proper symbolic proportions in a linguistic term set. In the following, some basic concepts associated to the linguistic distribution assessments are introduced.

Definition 2. Let $S = \{s_{-t}, \ldots, s_t\}$ be a linguistic term set, a linguistic distribution assessment of S is represented as [21]:

$$m = \left\{ \left(s_k, \beta_k\right) | k = -t, ..., t \right\}, \tag{1}$$

where $s_k \in S, \beta_k \geq 0, \sum_{k=-t}^{t} \beta_k = 1$ and β_k is the symbolic proportion of s_k.

Definition 3. Let $m = \{(s_k, \beta_k), k = -t, \ldots, t\}$ be a linguistic distribution assessment of S, where $s_k \in S$, $\beta_k \geq 0$ and $\sum_{k=-t}^{t} \beta_k = 1$. The expectation of m can be determined by [21]:

$$E(m) = \sum_{k=-t}^{t} \beta_k s_k. \tag{2}$$

Definition 4. Let m_1 and m_2 be any two linguistic distribution assessments of S, then [21]:

(1) if $E(m_1) > E(m_2)$, then m_1 is bigger than m_2.
(2) if $E(m_1) = E(m_2)$, then the expectation of m_1 equals to m_2.

To aggregate linguistic distribution assessments, the weighted averaging operator of linguistic distribution assessments (DAWA) and the ordered weighted averaging operator of linguistic distribution assessments (DAOWA) were defined in [21].

Definition 5. Let $\{m_1, ..., m_n\}$ be a set of linguistic distribution assessments of S, where $m_i = \{(s_k, \beta_k^i) | k = -t, ..., t\}, i = 1, 2, ..., n$. Let $w = [w_1, w_2, ..., w_n]^T$ be a corresponding weighting vector with $w_i \geq 0$ and $\sum_{i=1}^{n} w_i = 1$. Then the DAWA operator is expressed as

$$\begin{aligned} \mathrm{DAWA}_w\left(m_1, ..., m_n\right) &= \left\{ \left(s_k, \beta_k\right), k = -t, ..., t \right\}, \\ \beta_k &= \sum\nolimits_{i=1}^{n} w_i \beta_k^i. \end{aligned} \tag{3}$$

Let $\omega = [\omega_1, \omega_2, \ldots, \omega_n]^T$ be the corresponding DAOWA weight vector that satisfies $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$, then the DAOWA operator is denoted as

$$\begin{aligned} \mathrm{DAOWA}_\omega\left(m_1, ..., m_n\right) &= \left\{ \left(s_k, \beta_k\right), k = -t, ..., t \right\}, \\ \beta_k &= \sum\nolimits_{i=1}^{n} \omega_i \beta_k^{\sigma(i)}, \end{aligned} \tag{4}$$

where $\{\sigma(1), \ldots, \sigma(n)\}$ is a permutation of $\{1, ..., n\}$ and $m_{\sigma(i-1)} \geq m_{\sigma(i)}$ for $i = 2, ..., n$.

Definition 6. Let $m = \{(s_k, \beta_k), k = -t, ..., t\}$ and $m' = \{(s_k, \beta'_k), k = -t, ..., t\}$ be two linguistic distribution assessments of S, then the distance between them is computed by [21]:

$$d\left(m, m'\right) = \frac{1}{2} \sum_{k=-t}^{t} \left|\beta_k - \beta'_k\right|. \tag{5}$$

Definition 7. Let $M = [m_{ij}]_{n\times n}$ and $M' = [m'_{ij}]_{n\times n}$ be two linguistic distribution assessments matrixes, where the elements $m_{ij} = \{(s_q, \beta_{q,ij}), q = -t, ..., t\}$ and $m'_{ij} = \{(s_q, \beta'_{q,ij}), q = -t, ..., t\}$ are linguistic distribution assessments of S. Then, the distance between M and M' can be determined via [21]:

$$d\left(M, M'\right) = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} d\left(m_{ij}, m'_{ij}\right) = \frac{1}{2n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{q=-t}^{t} \left|\beta_{q,ij} - \beta'_{q,ij}\right|. \tag{6}$$

For computational facility, the notion of numerical scale [50] was introduced to transform linguistic terms into real numbers.

Definition 8. Let $S = \{s_{-t}, \ldots, s_t\}$ be a linguistic term set and $N(s_k)$ is the numerical scale for s_k, then

$$N\left(s_k\right) = k. \tag{7}$$

Definition 9. Let s_k be a linguistic term of $S = \{s_{-t}, \ldots, s_t\}$ and $m = \{(s_k, \beta_k), k = -t, ..., t\}$ be a linguistic distribution assessment of S. Then the entropy measure for m is formed as [51]:

$$En(m) = -\frac{N(E(m)) + t}{2t} \ln\left(\frac{N(E(m)) + t}{2t}\right) - \frac{-N(E(m)) + t}{2t} \ln\left(\frac{-N(E(m)) + t}{2t}\right), \tag{8}$$

where $N(E(m))$ is the numerical scale of $E(m)$.

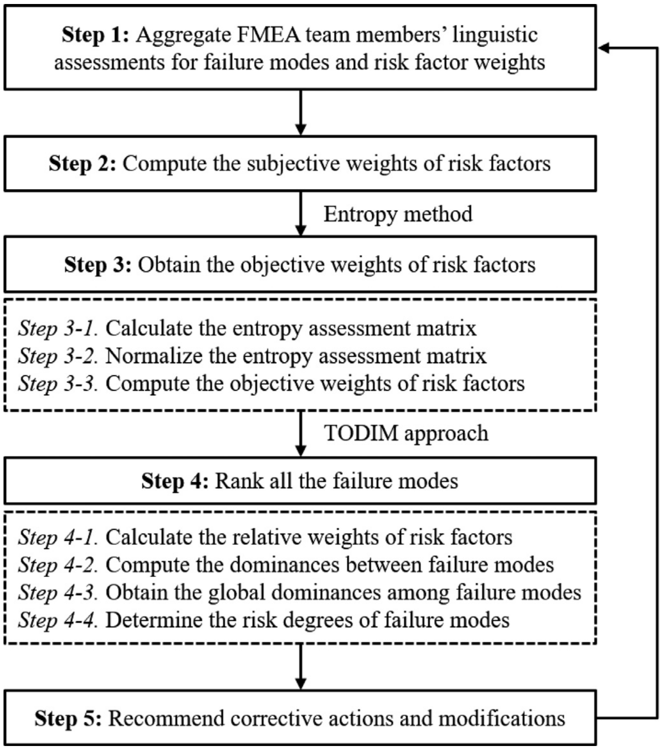

Fig. 1. Flow chart of the proposed FMEA model.

4. The proposed FMEA model

In this part, we devise a new risk priority model for FMEA using linguistic distribution assessment method and an extended TODIM approach. Also, both subjective weights and objective weights of risk factors are considered in order to conduct a more effective risk analysis. The flowchart of the proposed FMEA model is shown in Fig. 1. Note that the classical TODIM is modified in this study in the following ways: (1) In the standard TODIM method, the criteria values take the form of real numbers. This paper extends the TODIM method to accommodate the context of linguistic distribution assessments for ranking failure modes. (2) The classical TODIM method can only tackle the MCDM problems in which the criteria weights are directly given. While in this paper, both subjective and objective weights are considered for obtaining the relative weights of risk factors.

For a risk evaluation issue, assume there are n cross-functional members $\text{TM}_l (l = 1, 2, ..., n)$ in a FMEA team, who are responsible for the assessment of u failure modes $\text{FM}_i (i = 1, 2, ..., u)$ regarding v risk factors $\text{RF}_j (j = 1, 2, ..., v)$. Each team member TM_l is assigned a weight $\lambda_l > 0 (l = 1, 2, ..., n)$ satisfying $\sum_{l=1}^{n} \lambda_l = 1$ to denote his/her relative importance in the risk evaluation process. Let $M^l = [m_{ij}^l]_{u \times v}$ be the linguistic distribution assessment matrix of the lth team member, where $m_{ij}^l = \{(s_q^l, \beta_{q,ij}^l), q = -t, ..., t\}$ is the linguistic distribution expression given by TM_l for evaluating FM_i with respect to RF_j, and $s_q^l \in S = \{s_k | k = -t, ..., t\}$, $\beta_{q,ij}^l \geq 0$, $\sum_{q=-t}^{t} \beta_{q,ij}^l = 1$. Let $w^l = (w_1^l, w_2^l, ..., w_v^l)^T$ be the linguistic distribution weighting vector of the lth team member, where $w_j^l = \{(s_p^l, \beta_{pj}^l), p = -t, ..., t\}$ is the linguistic distribution weight of RF_j given by TM_l, and $s_p^l \in S = \{s_k | k = -t, ..., t\}$, $\beta_{pj}^l \geq 0$, $\sum_{p=-t}^{t} \beta_{pj}^l = 1$. According to above-mentioned descriptions, steps of the proposed FMEA model are presented as follows.

Step 1: Aggregate the FMEA team members' evaluations to obtain the group linguistic distribution assessment matrix and the group linguistic distribution weighting vector of risk factors

By using the DAWA operator, the group linguistic distribution assessments m_{ij} of FM_i in relation to RF_j are calculated by

$$m_{ij} = \text{DAWA}_\lambda\left(m_{ij}^1, m_{ij}^2, ..., m_{ij}^n\right). \tag{9}$$

The group linguistic distribution weight w_j of RF_j is computed by

$$w_j^S = \text{DAWA}_\lambda\left(w_j^1, w_j^2, \dots, w_j^n\right). \tag{10}$$

As a result, the risk evaluation problem can be characterized by the following matrix format:

$$M = \begin{pmatrix} & \text{RF}_1 & \text{RF}_2 & \dots & \text{RF}_v \\ \text{FM}_1 & m_{11} & m_{12} & \dots & m_{1v} \\ \text{FM}_2 & m_{21} & m_{22} & \dots & m_{2v} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{FM}_u & m_{u1} & m_{u2} & \dots & m_{uv} \end{pmatrix}, w^S = \left(w_1^S, w_2^S, \dots, w_v^S\right)^T.$$

Step 2: Compute the subjective weights of risk factors

The group linguistic distribution weighting vector w^S is quantified and normalized by Eq. (11) to obtain the subjective weights of risk factors.

$$\bar{w}_j^S = \frac{E\left(w_j^S\right)}{\max\left\{E\left(w_j^S\right)\right\}} j = 1, 2, ..., v. \tag{11}$$

Step 3: Obtain the objective weights of risk factors

In this step, the entropy method [51] is applied to acquire the objective weights of risk factors based on the group linguistic distribution assessment matrix M.

Step 3.1. Compute the entropy assessment matrix M'

Using Eq. (8), the entropy value of each linguistic distribution assessment is calculated by

$$\begin{aligned} En_{ij} = En\left(m_{ij}\right) = &-\frac{N\left(E\left(m_{ij}\right)\right) + t}{2t} \ln\left(\frac{N\left(E\left(m_{ij}\right)\right) + t}{2t}\right) \\ &-\frac{-N\left(E\left(m_{ij}\right)\right) + t}{2t} \ln\left(\frac{-N\left(E\left(m_{ij}\right)\right) + t}{2t}\right). \end{aligned} \tag{12}$$

Then the entropy assessment matrix M' can be acquired and represented as

$$M' = \begin{pmatrix} & \text{RF}_1 & \text{RF}_2 & \dots & \text{RF}_v \\ \text{FM}_1 & En_{11} & En_{12} & \dots & En_{1v} \\ \text{FM}_2 & En_{21} & En_{22} & \dots & En_{2v} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{FM}_u & En_{u1} & En_{u2} & \dots & En_{uv} \end{pmatrix}. \tag{13}$$

Step 3.2. Obtain the normalized entropy assessment matrix $\bar{M}$

The entropy values of the entropy assessment matrix M' are normalized by the function:

$$\bar{E}n_{ij} = \frac{En_{ij}}{\max_{1 \leq i \leq u}\left\{En_{ij}\right\}}, i = 1, 2, \dots u, j = 1, 2, \dots v. \tag{14}$$

Then, the normalized entropy assessment matrix $\bar{M}$ can be described as

$$\bar{M} = \begin{pmatrix} & \text{RF}_1 & \text{RF}_2 & \dots & \text{RF}_v \\ \text{FM}_1 & \bar{E}n_{11} & \bar{E}n_{12} & \dots & \bar{E}n_{1v} \\ \text{FM}_1 & \bar{E}n_{21} & \bar{E}n_{22} & \dots & \bar{E}n_{2v} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{FM}_u & \bar{E}n_{u1} & \bar{E}n_{u2} & \dots & \bar{E}n_{uv} \end{pmatrix}. \tag{15}$$

Step 3.3. Calculate the objective weights of risk factors

The objective weight of the risk factor RF_j is derived by

$$w_j^O = \frac{1}{v - \tilde{E}n} \times \left(1 - \tilde{E}n_j\right), j = 1, 2, ..., v, \tag{16}$$

where $\tilde{E}n_j = \sum_{i=1}^{u} \bar{E}n_{ij}$ and $\tilde{E}n = \sum_{i=1}^{u} \sum_{j=1}^{v} \bar{E}n_{ij}$.

Table 1
FMEA for the considered grinding wheel system [18].

No.	Failure mode	Failure cause	Failure effect
FM_1	Electric spindle start-up difficulties	The air gap between the stator and the rotor is too small	Spindle cannot operate normally
FM_2	Spindle vibration	Lower accuracy or poor precision retention for bearing	Spindle function loss
FM_3	Spindle no action	Oil too dirty	Machine damage
FM_4	Timing belt loose	Mixed manufacturing precision for belt	Function loss of grinding wheel
FM_5	Grinding wheel collision	CNC system failure	Risk on task completion
FM_6	Grinding wheel dresser not automatically fall	Hydraulic cylinder failure	Grinding wheel cannot be repaired

Step 4: Rank the failure modes by applying the TODIM approach

Step 4.1. Calculate the relative weight w_{jr} of risk factor RF_j to the reference risk factor RF_r by

$$w_{jr} = \frac{w_j^C}{w_r}, \quad j = 1, 2, ..., v, \tag{17}$$

where $w_r = \max\{w_j^C | j = 1, 2, ..., v\}$ and w_j^C refers to the combination weight of the risk factor RF_j. Normally, the w_j^C can be computed by

$$w_j^C = \frac{\bar{w}_j^S \times w_j^O}{\sum_{j=1}^{v} \bar{w}_j^S \times w_j^O}. \tag{18}$$

Step 4.2. Obtain the dominance of failure mode FM_i over failure mode FM_p under the risk factor RF_j via Eq. (19).

$$\phi_j(\mathrm{FM}_i, \mathrm{FM}_p) = \begin{cases} \sqrt{\frac{w_{jr}}{\sum_{j=1}^{v} w_{jr}} d(m_{ij}, m_{pj})} & if\ m_{ij} > m_{pj} \\ 0 & if\ m_{ij} = m_{pj} \\ \frac{-1}{\theta}\sqrt{\frac{\sum_{j=1}^{v} w_{jr}}{w_{jr}} d(m_{ij}, m_{pj})} & if\ m_{ij} < m_{pj} \end{cases}, \tag{19}$$

where θ is a parameter denoting the attenuation factor of losses.

Step 4.3. Compute the global dominance of failure mode FM_i over failure mode FM_p by

$$\delta(\mathrm{FM}_i, \mathrm{FM}_p) = \sum_{j=1}^{v} \phi_j(\mathrm{FM}_i, \mathrm{FM}_p). \tag{20}$$

Step 4.4. Obtain the risk degree of failure mode FM_i using the following formula:

$$\xi_i = \frac{\sum_{p=1}^{u} \delta(\mathrm{FM}_i, \mathrm{FM}_p) - \min_i \left(\sum_{p=1}^{u} \delta(\mathrm{FM}_i, \mathrm{FM}_p)\right)}{\max_i \left(\sum_{p=1}^{u} \delta(\mathrm{FM}_i, \mathrm{FM}_p)\right) - \min_i \left(\sum_{p=1}^{u} \delta(\mathrm{FM}_i, \mathrm{FM}_p)\right)}. \tag{21}$$

For FMEA, the larger risk degree obtained from Eq. (21), the higher risk of the failure mode. In consequence, all the recognized failure modes can be ranked in terms of their risk degrees in descending order.

Step 5: Analyze the risk ranking results and take essential modified measures to enhance the reliability and safety of the considered system.

5. Case study

5.1. Application

In this section, a practical risk evaluation case regarding the grinding wheel system of a numerical-controlled (NC) machine [18] is provided to demonstrate the application and effectiveness of the proposed FMEA. Grinding wheel system is an important part in the NC machine MK2120 which affects the processing quality. In reality, there are many failure modes and causes related to the grinding wheel system. But only six failure modes with higher risk are selected for further analysis, which are named as FM_1, FM_2, ..., FM_6 in this study. These failure modes, the corresponding causes and possible effects are described in Table 1. A FMEA team consisted of four members $\mathrm{TM}_l (l = 1, 2, 3, 4)$ was established to conduct the risk evaluation of the failure modes. Since the four team members coming from different departments have distinct professional backgrounds, they are assigned distinct weights in the risk analysis process, i.e., $\lambda = (0.23, 0.26, 0.25, 0.26)$.

For ease of evaluation, a seven-grade linguistic term set S is used by the experts to assess the failure modes, and a five-grade linguistic term set U is adopted to characterize the relative importance of the risk factors. These linguistic term sets are depicted as

$$S = \{s_{-3} = \text{Very low}, s_{-2} = \text{Low}, s_{-1} = \text{Moderately low}, s_0 = \text{Moderate}, s_1 = \text{Moderately high}, s_2 = \text{High}, s_3 = \text{Very high}\};$$

$$U = \{u_{-2} = \text{Very unimportant}, u_{-1} = \text{Unimportant}, u_0 = \text{Medium}, u_1 = \text{Important}, u_2 = \text{Very important}\}.$$

As a result, the linguistic distribution assessments on the six failure modes against every risk factor and the importance weights of the risk factors O, S and D are obtained as shown in Tables 2 and 3.

Based on the aforementioned evaluation results, the FMEA model being proposed is executed to rank the recognized failure modes as follows:

Step 1: According to Eqs. (9) and (10), the four team members' individual assessments are aggregated to get the group linguistic distribution assessment matrix M and the group linguistic distribution weighting vector w^S. The results are expressed as below:

$$M = \left(\begin{array}{l|lll} & \mathrm{RF}_1 & \mathrm{RF}_2 & \mathrm{RF}_3 \\ \hline \mathrm{FM}_1 & \{(s_{-1}, 0.396), (s_0, 0.514), (s_1, 0.091)\} & \{(s_0, 0.236), (s_1, 0.279), (s_2, 0.485)\} & \{(s_{-2}, 0.039), (s_{-1}, 0.722), (s_0, 239)\} \\ \mathrm{FM}_2 & \{(s_0, 0.052), (s_1, 0.501), (s_2, 0.448)\} & \{(s_1, 0.540), (s_2, 0.391), (s_3, 0.069)\} & \{(s_{-2}, 0.130), (s_{-1}, 0.448), (s_0, 0.423)\} \\ \mathrm{FM}_3 & \{(s_1, 0.389), (s_2, 0.535), (s_3, 0.076)\} & \{(s_2, 0.552), (s_3, 0.448)\} & \{(s_{-1}, 0.268), (s_0, 0.499), (s_1, 0.234)\} \\ \mathrm{FM}_4 & \{(s_0, 0.307), (s_1, 0.388), (s_2, 0.305)\} & \{(s_1, 0.204), (s_2, 0.302), (s_3, 0.494)\} & \{(s_{-3}, 0.230), (s_{-1}, 0.317), (s_0, 0.453)\} \\ \mathrm{FM}_5 & \{(s_{-2}, 0.499), (s_{-1}, 0.501)\} & \{(s_0, 0.234), (s_1, 0.704), (s_2, 0.062)\} & \{(s_0, 0.282), (s_1, 0.667), (s_2, 0.052)\} \\ \mathrm{FM}_6 & \{(s_{-2}, 0.298), (s_{-1}, 0.408), (s_0, 0.295)\} & \{(s_1, 0.052), (s_2, 0.443), (s_3, 0.551)\} & \{(s_{-3}, 0.420), (s_{-2}, 0.529), (s_{-1}, 0.052)\} \end{array}\right).$$

$$w^S = (\{(u_1, 0.753), (u_2, 0.247)\}, \{(u_1, 0.255), (u_2, 0.745)\}, \{(u_0, 0.948), (u_1, 0.052)\})^T.$$

Step 2: The group linguistic distribution weighting vector w^S is quantified and normalized by Eq. (11), and the subjective weights of the three risk factors are computed as w^S=$[0.410, 0.573, 0.017]^T$.

Step 3: In this step, the objective weights of the risk factors are derived by using the entropy method.

Firstly, the entropy value of each linguistic distribution assessment in the matrix M is calculated via Eq. (12). Thus, the matrix M is transformed

Table 2
Linguistic distribution assessments on the failure modes.

Team members	Failure modes	Risk factors O	S	D
TM_1	FM_1	$\{(s_{-1},0.85),(s_0,0.15)\}$	$\{(s_1,0.8),(s_2,0.2)\}$	$\{(s_{-1},0.3),(s_0,0.7)\}$
	FM_2	$\{(s_1,1)\}$	$\{(s_2,0.7),(s_3,0.3)\}$	$\{(s_{-1},0.25),(s_0,0.75)\}$
	FM_3	$\{(s_1,0.9),(s_2,0.1)\}$	$\{(s_2,0.76),(s_3,0.24)\}$	$\{(s_{-1},1)\}$
	FM_4	$\{(s_2,1)\}$	$\{(s_1,0.8),(s_2,0.2)\}$	$\{(s_{-3},1)\}$
	FM_5	$\{(s_{-2},0.8),(s_{-1},0.2)\}$	$\{(s_1,1)\}$	$\{(s_0,0.75),(s_1,0.25)\}$
	FM_6	$\{(s_0,1)\}$	$\{(s_2,0.15),(s_3,0.85)\}$	$\{(s_{-3},0.9),(s_{-2},0.1)\}$
TM_2	FM_1	$\{(s_0,0.8),(s_1,0.2)\}$	$\{(s_2,1)\}$	$\{(s_{-2},0.15),(s_{-1},0.85)\}$
	FM_2	$\{(s_0,0.2),(s_1,0.8)\}$	$\{(s_1,1)\}$	$\{(s_{-2},0.25),(s_{-1},0.75)\}$
	FM_3	$\{(s_1,0.7),(s_2,0.3)\}$	$\{(s_2,0.21),(s_3,0.79)\}$	$\{(s_0,0.1),(s_1,0.9)\}$
	FM_4	$\{(s_0,1)\}$	$\{(s_3,1)\}$	$\{(s_{-1},0.15),(s_0,0.85)\}$
	FM_5	$\{(s_{-2},0.25),(s_{-1},0.75)\}$	$\{(s_0,0.9),(s_1,0.1)\}$	$\{(s_1,0.8),(s_2,0.2)\}$
	FM_6	$\{(s_{-1},0.75),(s_0,0.25)\}$	$\{(s_1,0.2),(s_2,0.8)\}$	$\{(s_{-2},1)\}$
TM_3	FM_1	$\{(s_{-1},0.8),(s_0,0.2)\}$	$\{(s_1,0.1),(s_2,0.9)\}$	$\{(s_{-1},1)\}$
	FM_2	$\{(s_1,0.25),(s_2,0.75)\}$	$\{(s_1,0.86),(s_2,0.14)\}$	$\{(s_0,1)\}$
	FM_3	$\{(s_2,0.8),(s_3,0.2)\}$	$\{(s_2,1)\}$	$\{(s_{-1},0.15),(s_0,0.85)\}$
	FM_4	$\{(s_1,0.7),(s_2,0.3)\}$	$\{(s_1,0.08),(s_2,0.92)\}$	$\{(s_{-1},0.8),(s_0,0.2)\}$
	FM_5	$\{(s_{-2},1)\}$	$\{(s_1,1)\}$	$\{(s_0,0.28),(s_1,0.72)\}$
	FM_6	$\{(s_{-2},0.15),(s_{-1},0.85)\}$	$\{(s_2,0.8),(s_3,0.2)\}$	$\{(s_{-3},0.85),(s_{-2},0.15)\}$
TM_4	FM_1	$\{(s_0,0.85),(s_1,0.15)\}$	$\{(s_0,0.2),(s_1,0.8)\}$	$\{(s_{-1},0.7),(s_0,0.3)\}$
	FM_2	$\{(s_2,1)\}$	$\{(s_1,0.25),(s_2,0.75)\}$	$\{(s_{-2},0.25),(s_{-1},0.75)\}$
	FM_3	$\{(s_2,0.9),(s_3,0.1)\}$	$\{(s_2,0.28),(s_3,0.72)\}$	$\{(s_0,1)\}$
	FM_4	$\{(s_0,0.18),(s_1,0.82)\}$	$\{(s_2,0.1),(s_3,0.9)\}$	$\{(s_{-1},0.3),(s_0,0.7)\}$
	FM_5	$\{(s_{-1},1)\}$	$\{(s_1,0.76),(s_2,0.24)\}$	$\{(s_0,0.15),(s_1,0.85)\}$
	FM_6	$\{(s_{-2},1)\}$	$\{(s_3,1)\}$	$\{(s_{-2},0.8),(s_{-1},0.2)\}$

Table 3
Linguistic distribution assessments on risk factor weights.

Risk factors	Team members TM_1	TM_2	TM_3	TM_4
O	$\{(u_1,0.8), (u_2,0.2)\}$	$\{(u_1,0.9), (u_2,0.1)\}$	$\{(u_1,0.3), (u_2,0.7)\}$	$\{(u_1,1)\}$
S	$\{(u_1,0.1), (u_2,0.9)\}$	$\{(u_1,0.7), (u_2,0.3)\}$	$\{(u_1,0.2), (u_2,0.8)\}$	$\{(u_2,1)\}$
D	$\{(u_0,1)\}$	$\{(u_0,0.8), (u_1,0.2)\}$	$\{(u_0,1)\}$	$\{(u_0,1)\}$

into the entropy assessment matrix M' that is represented as

$$M' = \left(\begin{array}{c|ccc} & RF_1 & RF_2 & RF_3 \\ \hline FM_1 & 0.688 & 0.604 & 0.657 \\ FM_2 & 0.581 & 0.557 & 0.665 \\ FM_3 & 0.525 & 0.307 & 0.693 \\ FM_4 & 0.637 & 0.364 & 0.636 \\ FM_5 & 0.563 & 0.655 & 0.660 \\ FM_6 & 0.636 & 0.305 & 0.337 \end{array}\right).$$

Secondly, the element values in the entropy assessment matrix M' are normalized by Eq. (14) so that the normalized entropy assessment matrix $\bar{M}$ is obtained as below:

$$\bar{M} = \left(\begin{array}{c|ccc} & RF_1 & RF_2 & RF_3 \\ \hline FM_1 & 1.000 & 0.923 & 0.948 \\ FM_2 & 0.844 & 0.851 & 0.960 \\ FM_3 & 0.764 & 0.469 & 1.000 \\ FM_4 & 0.926 & 0.556 & 0.917 \\ FM_5 & 0.818 & 1.000 & 0.952 \\ FM_6 & 0.925 & 0.466 & 0.486 \end{array}\right).$$

Finally, using Eq. (16), the objective weighting vector of the three risk factors is calculated as $w_j^O=[0.362, 0.277, 0.361]^T$.

Step 4: This step is to identify the risk ranking of the identified failure modes with the extended TODIM approach.

First, the combination weights and the relative weights are calculated by Eqs. (17) and (18), respectively. The results here obtained are $w^C = [0.474, 0.506, 0.020]^T$ and $w_{jr} = [0.936, 1.000, 0.039]^T$.

Second, the dominance of failure mode FM_i over FM_p under each risk factor is acquired through Eq. (19). The sensitive coefficient θ is taken as 1 and the computed results are shown as follows:

$$\phi_1 = \left(\begin{array}{c|cccccc} & FM_1 & FM_2 & FM_3 & FM_4 & FM_5 & FM_6 \\ \hline FM_1 & 0 & -1.345 & -1.385 & -1.127 & 0.535 & 0.383 \\ FM_2 & 0.637 & 0 & -0.587 & 0.348 & 0.688 & 0.670 \\ FM_3 & 0.656 & 0.278 & 0 & 0.381 & 0.688 & 0.688 \\ FM_4 & 0.534 & -0.733 & -0.805 & 0 & 0.688 & 0.578 \\ FM_5 & -1.129 & -1.453 & -1.453 & -1.453 & 0 & -0.789 \\ FM_6 & -0.808 & -1.414 & -1.453 & -1.220 & 0.374 & 0 \end{array}\right),$$

$$\phi_2 = \left(\begin{array}{c|cccccc} & FM_1 & FM_2 & FM_3 & FM_4 & FM_5 & FM_6 \\ \hline FM_1 & 0 & -0.807 & -1.009 & -0.988 & 0.464 & -0.999 \\ FM_2 & 0.409 & 0 & -1.033 & -0.916 & 0.449 & -0.982 \\ FM_3 & 0.511 & 0.523 & 0 & 0.356 & 0.610 & -1.323 \\ FM_4 & 0.500 & 0.464 & -0.703 & 0 & 0.610 & -0.548 \\ FM_5 & -0.916 & -0.886 & -1.204 & -1.204 & 0 & -1.323 \\ FM_6 & 0.506 & 0.497 & 0.670 & 0.277 & 0.670 & 0 \end{array}\right),$$

$$\phi_3 = \left(\begin{array}{c|cccccc} & FM_1 & FM_2 & FM_3 & FM_4 & FM_5 & FM_6 \\ \hline FM_1 & 0 & -3.732 & -5.005 & 0.094 & -6.215 & 0.134 \\ FM_2 & 0.074 & 0 & -3.966 & 0.072 & -6.039 & 0.127 \\ FM_3 & 0.099 & 0.078 & 0 & 0.074 & -4.959 & 0.137 \\ FM_4 & -4.747 & -3.636 & -3.766 & 0 & -6.039 & 0.119 \\ FM_5 & 0.123 & 0.119 & 0.098 & 0.119 & 0 & 0.140 \\ FM_6 & -6.792 & -6.443 & -6.936 & -6.036 & -7.124 & 0 \end{array}\right).$$

Furthermore, the global dominances of failure mode FM_i over failure mode FM_p are calculated by Eq. (20) and the results are expressed as

Table 4
Ranking comparisons.

Failure modes	The RPN method		The linguistic FMEA method		The IWF-TOPSIS		The proposed method
	RPN	Ranking	L_i	Ranking	cc_i	Ranking	Ranking
FM_1	175	5	6.37	4	0.15	5	5
FM_2	320	2	10.09	2	0.23	2	2
FM_3	432	1	14.94	1	0.25	1	1
FM_4	252	3	9.66	3	0.16	4	4
FM_5	196	4	5.92	5	0.20	3	3
FM_6	72	6	3.24	6	0.10	6	6

below:

$$\delta = \begin{array}{c|cccccc} & FM_1 & FM_2 & FM_3 & FM_4 & FM_5 & FM_6 \\ \hline FM_1 & 0 & -5.884 & -7.398 & -2.021 & -5.216 & -0.482 \\ FM_2 & 1.120 & 0 & -5.586 & -0.497 & -4.901 & -0.185 \\ FM_3 & 1.266 & 0.879 & 0 & 0.812 & -3.661 & -0.497 \\ FM_4 & -3.713 & -3.905 & -5.274 & 0 & -4.741 & 0.149 \\ FM_5 & -1.923 & -2.220 & -2.559 & -2.537 & 0 & -1.971 \\ FM_6 & -7.094 & -7.360 & -7.719 & -6.979 & -6.080 & 0 \end{array}.$$

At last, using Eq. (21), the risk degrees of the six failure modes are calculated as $\xi_1 = 0.418, \xi_2 = 0.740, \xi_3 = 1.000, \xi_4 = 0.522, \xi_5 = 0.706, \xi_6 = 0$. In line with the descending order of the risk degrees, the ranking outcome of the failure modes is: $FM_3 > FM_2 > FM_5 > FM_4 > FM_1 > FM_6$. Consequently, FM_3 is the failure mode with the highest risk, which should be paid the most attention to, then followed by FM_2, FM_5, FM_4, FM_1, FM_6.

5.2. Comparisons and discussions

To validate the efficiency of the proposed risk priority model, a comparative study is conducted with other methods based on the above case study. The new FMEA model based on linguistic distribution assessments and the modified TODIM method is proposed to alleviate the shortcomings of the classical RPN calculation. Hence, the conventional FMEA and the linguistic FMEA [18] methods are selected for comparision to investigate the advantages of the proposed risk priority approach. Furthermore, the weights of risk factors are determined with a combination weighting method in the improved risk analysis method. Therefore, a comparison with the integrated weight-based TOPSIS (IWF-TOPSIS) approach [16] was also performed to show the accuracy of the machine reliability risk evaluation using the proposed FMEA. Table 4 reveals the risk ranking results of the six failure modes derived from the listed FMEA methods.

From Table 4, some important results can be deduced distinctly. First, the ranking orders of FM_3, FM_2 and FM_6 produced by the proposed method are in agreement with those by the other three methods. In other words, the most two important failure modes and the least serious one remain the same according to the four FMEA methods. Moreover, the ranking results of the proposed FMEA are consistent with the ones obtained by the IWF-TOPSIS method. This proves the validity of our proposed risk priority approach. Furthermore, Fig. 2 portrays the deviations between each failure mode and the most serious one for the four methods. It can be found that the proposed FMEA fluctuates far more sharply than the three other methods. Therefore, the improved risk ranking model has higher discrimination degree among the failure modes than the traditional FMEA, the linguistic FMEA and the IWF-TOPSIS methods.

On the other side, there are still some differences between the applied FMEA methods. For example, FM_5 is given a prior consideration to FM_4 in the proposed method, which is contrary to the result by the RPN method. Meanwhile, apart from FM_3, FM_2 and FM_6, the ranking orders of other failure modes derived by the new risk priority method are different from the ones by the linguistic FMEA. These inconsistences can be explained by the following reasons:

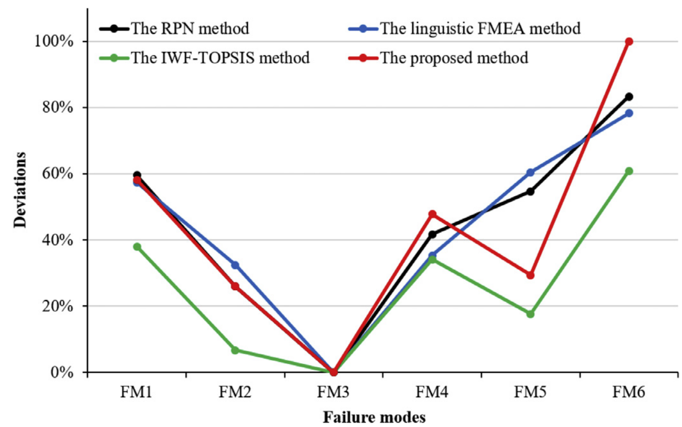

Fig. 2. Deviations of failure modes for the considered FMEA methods.

(1) Crisp numbers are adopted by the traditional FMEA, and single linguistic terms are utilized by the linguistic FMEA as well as the IWF-TOPSIS to assess the failure modes. In contrast, the linguistic distribution assessments are used in the proposed FMEA which enable the experts to provide their risk evaluations more accurate and closer to the real situation.
(2) The conventional FMEA considered equivalent importance for risk factors and the linguistic FMEA applied the AHP method to determine the subjective weights of risk factors. However, this paper employed the combined weighting method to identify the risk factor weights, which takes the advantages of both subjective and objective weighting methods.
(3) Both the traditional FMEA and the linguistic FMEA perform a similar procedure to reach the risk ranking, namely, according to the multiplication of risk factors. The IWF-TOPSIS uses the fuzzy TOPSIS to obtain the risk ranking of failure modes. By comparison, the proposed FMEA method based on the modified TODIM can reflect the team members' actual experience and exploit the relations of failure modes more accurately.

6. Conclusions

In this article, a new FMEA model was developed based on linguistic distribution assessments and a modified TODIM method to ameliorate the shortcomings of the traditional RPN method. The linguistic distribution assessment method was applied to quantify the risk evaluations of failure modes on each risk factor. Through integrating subjective and objective weights, a combination weighting method was used to indicate the relative importance of each risk factor. Lastly, the TODIM approach comprehensively taking experts' behaviors into account was extended to achieve the risk ranking of the recognized failure modes. A practical case concerning the risk evaluation of a grinding wheel system was performed and the results were compared with some existing methods to demonstrate the practicality and effectiveness of our proposed FMEA model.

In the further study, several research works deserve to be focused on. First, the weights of FMEA team members are given directly in the proposed model, which are, however, hard to be determined in some real situations. Thus, the approaches to derive the weights of experts should be explored in the future. Second, the sensitive coefficient θ of the TODIM method is set to 1 in the case study to simplify calculation. But risk analysts may have difficulty in setting the parameter value in a specific application. Hence, constructing an optimization model to obtain the sensitive coefficient objectively is another possible future study. Third, the computational process of the proposed FMEA is a little complex and cumbersome for the real community of FMEA practitioners. Therefore, a programming software is suggested to be developed to assist practitioners in applying the proposed FMEA model conveniently.

Acknowledgments

The authors are very grateful to the editor and reviewers for their insightful and constructive comments and suggestions, which are very helpful in improving the quality of the paper. This work was partially supported by the National Natural Science Foundation of China (Nos. 71671125 and 71402090), the NSFC Key Program (No. 71432007), and the Program for Professor of Special Appointment (Young Eastern Scholar) at Shanghai institutions of Higher Learning (No. QD2015019).

References

[1] Sankar NR, Prabhu BS. Modified approach for prioritization of failures in a system failure mode and effects analysis. Int J Qual Reliability Manage 2001;18:324–36.
[2] Stamatis DH. Failure mode and effect analysis: FMEA from theory to execution. 2nd ed. New York: ASQ Quality Press; 2003.
[3] Sharma RK, Kumar D, Kumar P. Systematic failure mode effect analysis (FMEA) using fuzzy linguistic modeling. Int J Qual Reliability Manage 2005;22:986–1004.
[4] Bowles JB, Peláez CE. Fuzzy logic prioritization of failures in a system failure mode, effects and criticality analysis. Reliability Eng Syst Saf 1995;50:203–13.
[5] Chanamool N, Naenna T. Fuzzy FMEA application to improve decision-making process in an emergency department. Appl Soft Comput 2016;43:441–53.
[6] Abrahamsen HB, Abrahamsen EB, Høyland S. On the need for revising healthcare failure mode and effect analysis for assessing potential for patient harm in healthcare processes. Reliability Eng Syst Saf 2016;155:160–8.
[7] Zhang Y, Andrews J, Reed S, Karlberg M. Maintenance processes modeling and optimisation. Reliability Eng Syst Saf 2017. doi:10.1016/j.ress.2017.02.011.
[8] Certa A, Hopps F, Inghilleri R, La Fata CM. A dempster-shafer theory-based approach to the failure mode, effects and criticality analysis (FMECA) under epistemic uncertainty: application to the propulsion system of a fishing vessel. Reliability Eng Syst Saf 2017;159:69–79.
[9] Liu HC. FMEA using uncertainty theories and MCDM methods. Singapore: Springer; 2016.
[10] Liu HC, Liu L, Liu N. Risk evaluation approaches in failure mode and effects analysis: a literature review. Expert Syst Appl 2013;40:828–38.
[11] Sayyadi Tooranloo H, Ayatollah A. A model for failure mode and effects analysis based on intuitionistic fuzzy approach. Appl Soft Comput 2016;49:238–47.
[12] Liu HC, You JX, Duan CY. An integrated approach for failure mode and effect analysis under interval-valued intuitionistic fuzzy environment. Int J Prod Econ 2017. doi:10.1016/j.ijpe.2017.03.008.
[13] Gargama H, Chaturvedi SK. Criticality assessment models for failure mode effects and criticality analysis using fuzzy logic. IEEE Trans Reliability 2011;60:102–10.
[14] Lolli F, Gamberini R, Rimini B, Pulga F. A revised FMEA with application to a blow moulding process. Int J Qual Reliability Manage 2016;33:900–19.
[15] Lolli F, Ishizaka A, Gamberini R, Rimini B, Messori M. FlowSort-GDSS – A novel group multi-criteria decision support system for sorting problems with application to FMEA. Expert Syst Appl 2015;42:6342–9.
[16] Song W, Ming X, Wu Z, Zhu B. Failure modes and effects analysis using integrated weight-based fuzzy TOPSIS. Int J Comput Integrated Manufacturing 2013;26:1172–86.
[17] Wang LN, Liu HC, Quan MY. Evaluating the risk of failure modes with a hybrid MCDM model under interval-valued intuitionistic fuzzy environments. Comput Ind Eng 2016;102:175–85.
[18] Zhou Y, Xia J, Zhong Y, Pang J. An improved FMEA method based on the linguistic weighted geometric operator and fuzzy priority. Qual Eng 2016;28:491–8.
[19] Liu HC, Li P, You JX, Chen YZ. A novel approach for FMEA: combination of interval 2-tuple linguistic variables and grey relational analysis. Qual Reliability Eng Int 2015;31:761–72.
[20] Liu HC, You JX, Li P, Su Q. Failure mode and effect analysis under uncertainty: an integrated multiple criteria decision making approach. IEEE Trans Reliability 2016;65:1380–92.
[21] Zhang G, Dong Y, Xu Y. Consistency and consensus measures for linguistic preference relations based on distribution assessments. Inf Fusion 2014;17:46–55.
[22] Dong Y, Wu Y, Zhang H, Zhang G. Multi-granular unbalanced linguistic distribution assessments with interval symbolic proportions. Knowl-Based Syst 2015;82:139–51.
[23] Liu HC, You JX, Chen S, Chen YZ. An integrated failure mode and effect analysis approach for accurate risk assessment under uncertainty. IIE Trans 2016;48:1027–42.
[24] Gomes LFAM, Lima MMPP. TODIM: basics and application to multicriteria ranking of projects with environmental impacts. Foundations Comput Decis Sci 1992;16:113–27.
[25] Ren P, Xu Z, Gou X. Pythagorean fuzzy TODIM approach to multi-criteria decision making. Appl Soft Comput 2016;42:246–59.
[26] Tseng ML, Lin YH, Tan K, Chen RH, Chen YH. Using TODIM to evaluate green supply chain practices under uncertainty. Appl Math Model 2014;38:2983–95.
[27] Ji P, Zhang HY, Wang JQ. A projection-based TODIM method under multi-valued neutrosophic environments and its application in personnel selection. Neural Comput Appl 2016. doi:10.1007/s00521-016-2436-z.
[28] Wang J, Wang JQ, Zhang HY. A likelihood-based TODIM approach based on multi-hesitant fuzzy linguistic information for evaluation in logistics outsourcing. Comput Ind Eng 2016;99:287–99.
[29] Qin Q, Liang F, Li L, Wei YM. Selection of energy performance contracting business models: a behavioral decision-making approach. Renewable Sustainable Energy Rev 2017;72:422–33.
[30] Qin J, Liu X, Pedrycz W. An extended TODIM multi-criteria group decision making method for green supplier selection in interval type-2 fuzzy environment. Eur J Operational Res 2017;258:626–38.
[31] Zhang Y, Xu Z. Efficiency evaluation of sustainable water management using the HF-TODIM method. Int Trans Operational Res 2016. doi:10.1111/itor.12318.
[32] Jiang Y, Liang X, Liang H. An I-TODIM method for multi-attribute decision making with interval numbers. Soft Comput 2016. doi:10.1007/s00500-016-2139-5.
[33] Tooranloo HS, Ayatollah A. A model for failure mode and effects analysis based on intuitionistic fuzzy approach. Appl Soft Comput 2016;49:238–47.
[34] Li Z, Xiao F, Fei L, Mahadevan S, Deng Y. An evidential failure mode and effects analysis using linguistic terms. Qual Reliability Eng Int 2016. doi:10.1002/qre.2075.
[35] Liu HC, Chen YZ, You JX, Li H. Risk evaluation in failure mode and effects analysis using fuzzy digraph and matrix approach. J Intell Manufacturing 2016;27:805–16.
[36] Yang Z, Wang J. Use of fuzzy risk assessment in FMEA of offshore engineering systems. Ocean Eng 2015;95:195–204.
[37] Liu HC, Liu L, Li P. Failure mode and effects analysis using intuitionistic fuzzy hybrid weighted euclidean distance operator. Int J Syst Sci 2014;45:2012–30.
[38] Chin KS, Wang YM, Poon GKK, Yang JB. Failure mode and effects analysis using a group-based evidential reasoning approach. Comput Oper Res 2009;36:1768–79.
[39] Xiao NC, Huang HZ, Li YF, He LP, Jin TD. Multiple failure modes analysis and weighted risk priority number evaluation in FMEA. Eng Fail Anal 2011;18:1162–70.
[40] Liu HC, You JX, You XY, Shan MM. A novel approach for failure mode and effects analysis using combination weighting and fuzzy VIKOR method. Appl Soft Comput 2015;28:579–88.
[41] Chang KH, Chang YC, Lai PT. Applying the concept of exponential approach to enhance the assessment capability of FMEA. J Intell Manufacturing 2014;25:1413–27.
[42] Akbarzade Khorshidi H, Gunawan I, Ibrahim MY. Applying UGF concept to enhance the assessment capability of FMEA. Qual Reliability Eng Int 2016;32:1085–93.
[43] Emovon I, Norman RA, J MA, Pazouki K. An integrated multicriteria decision making methodology using compromise solution methods for prioritising risk of marine machinery systems. Ocean Eng 2015;105:92–103.
[44] Selim H, Yunusoglu MG, Yılmaz Balaman Ş. A dynamic maintenance planning framework based on fuzzy TOPSIS and FMEA: application in an international food company. Qual Reliability Eng Int 2016;32:795–804.
[45] Zhao H, You JX, Liu HC. Failure mode and effect analysis using MULTIMOORA method with continuous weighted entropy under interval-valued intuitionistic fuzzy environment. Soft Comput 2016. doi:10.1007/s00500-016-2118-x.
[46] Zadeh LA. The concept of a linguistic variable and its application to approximate reasoning–I. Inf Sci 1975;8:199–249.
[47] Liu HC, Lin QL, Wu J. Dependent interval 2-tuple linguistic aggregation operators and their application to multiple attribute group decision making. Int J Uncertainty, Fuzziness Knowl-Based Syst 2014;22:717–35.
[48] Xu ZS. A method based on linguistic aggregation operators for group decision making with linguistic preference relations. Inf Sci 2004;166:19–30.
[49] Xu ZS. Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment. Inf Sci 2004;168:171–84.
[50] Dong Y, Xu Y, Yu S. Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model. IEEE Trans Fuzzy Syst 2009;17:1366–78.
[51] Farhadinia B. Determination of entropy measures for the ordinal scale-based linguistic models. Inf Sci 2016;369:63–79.