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Certain Notions of Neutrosophic Topological K-Algebras

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Abstract: The concept of neutrosophic set from philosophical point of view was first considered by Smarandache. A single-valued neutrosophic set is a subclass of the neutrosophic set from a scientific and engineering point of view and an extension of intuitionistic fuzzy sets. In this research article, we apply the notion of single-valued neutrosophic sets to K-algebras. We introduce the notion of single-valued neutrosophic topological K-algebras and investigate some of their properties. Further, we study certain properties, including C_5-connected, super connected, compact and Hausdorff, of single-valued neutrosophic topological K-algebras. We also investigate the image and pre-image of single-valued neutrosophic topological K-algebras under homomorphism.

Keywords: K-algebras; single-valued neutrosophic sets; homomorphism; compactness; C_5-connectedness

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1. Introduction

A new kind of logical algebra, known as K-algebra, was introduced by Dar and Akram in [1]. A K-algebra is built on a group G by adjoining the induced binary operation on G. The group G is particularly of the type in which each non-identity element is not of order 2. This algebraic structure is, in general, non-commutative and non-associative with right identity element [1–3]. Akram et al. [4] introduced fuzzy K-algebras. They then developed fuzzy K-algebras with other researchers worldwide. The concepts and results of K-algebras have been broadened to the fuzzy setting frames by applying Zadeh's fuzzy set theory and its generalizations, namely, interval-valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, bipolar fuzzy sets and vague sets [5]. In handling information regarding various aspects of uncertainty, non-classical logic is considered to be a more powerful tool than the classical logic. It has become a strong mathematical tool in computer science, medical, engineering, information technology, etc. In 1998, Smarandache [6] introduced neutrosophic set as a generalization of intuitionistic fuzzy set [7]. A neutrosophic set is identified by three functions called truth-membership (T), indeterminacy-membership (I) and falsity-membership (F) functions. To apply neutrosophic set in real-life problems more conveniently, Smarandache [6] and Wang et al. [8] defined single-valued neutrosophic sets which takes the value from the subset of $[0, 1]$. Thus, a single-valued neutrosophic set is an instance of neutrosophic set.

Algebraic structures have a vital place with vast applications in various areas of life. Algebraic structures provide a mathematical modeling of related study. Neutrosophic set theory has also been applied to many algebraic structures. Agboola and Davazz introduced the concept of neutrosophic BCI/BCK-algebras and discuss elementary properties in [9]. Jun et al. introduced the notion of (ϕ, ψ) neutrosophic subalgebra of a BCK/BCI-algebra [10]. Jun et al. [11] defined interval neutrosophic sets on BCK/BCI-algebra [11]. Jun et al. [12] proposed neutrosophic positive implicative N-ideals and study their extension property [12] Several set theories and their topological structures have been introduced by many researchers to deal with uncertainties. Chang [13] was the first to introduce the notion of fuzzy topology. Later, Lowan [14], Pu and Liu [15], and Chattopadhyay and Samanta [16] introduced other concepts related to fuzzy topology. Coker [17] introduced the notion of intuitionistic fuzzy topology as a generalization of fuzzy topology. Salama and Alblowi [18] defined the topological structure of neutrosophic set theory. Akram and Dar [19] introduced the concept of fuzzy topological K-algebras. They extended their work on intuitionistic fuzzy topological K-algebras [20]. In this paper, we introduce the notion of single-valued neutrosophic topological K-algebras and investigate some of their properties. Further, we study certain properties, including C_5-connected, super connected, compact and Hausdorff, of single-valued neutrosophic topological K-algebras. We also investigate the image and pre-image of single-valued neutrosophic topological K-algebras under homomorphism.

2. Preliminaries

The notion of K-algebra was introduced by Dar and Akram in [1].

Definition 1. *[1] Let $(G, \cdot, e)$ be a group in which each non-identity element is not of order 2. A K-algebra is a structure $\mathcal{K} = (G, \cdot, \odot, e)$ over a particular group G, where $\odot$ is an induced binary operation $\odot : G \times G \to G$ is defined by $\odot(s,t) = s \odot t = s.t^{-1}$, and satisfy the following conditions:*

(i) $(s \odot t) \odot (s \odot u) = (s \odot ((e \odot u) \odot (e \odot t))) \odot s$;
(ii) $s \odot (s \odot t) = (s \odot (e \odot t) \odot s$;
(iii) $s \odot s = e$;
(iv) $s \odot e = s$; *and*
(v) $e \odot s = s^{-1}$

for all s, t, $u \in G$. The homomorphism between two K-algebras $\mathcal{K}_1$ and $\mathcal{K}_2$ is a mapping $f : \mathcal{K}_1 \to \mathcal{K}_2$ such that, for all $u, v \in \mathcal{K}_1$, $f(u \odot v) = f(u) \odot f(v)$.

In [6], Smarandache initiated the idea of neutrosophic set theory which is a generalization of intuitionistic fuzzy set theory. Later, Smarandache and Wang et al. introduced a single-valued neutrosophic set (SNS) as an instance of neutrosophic set in [8].

Definition 2. *[8] Let Z be a space of points with a general element $s \in Z$. A SNS $\mathcal{A}$ in Z is equipped with three membership functions: truth membership function ($\mathcal{T}_A$), indeterminacy membership function ($\mathcal{I}_A$) and falsity membership function($\mathcal{F}_A$), where $\forall\, s \in Z$, $\mathcal{T}_\mathcal{A}(s)$, $\mathcal{I}_\mathcal{A}(s)$, $\mathcal{F}_\mathcal{A}(s) \in [0,1]$. There is no restriction on the sum of these three components. Therefore, $0 \leq \mathcal{T}_\mathcal{A}(s) + \mathcal{I}_\mathcal{A}(s) + \mathcal{F}_\mathcal{A}(s) \leq 3$.*

Definition 3. *[8] A single-valued neutrosophic empty set ($\emptyset_{SN}$) and single-valued neutrosophic whole set (1_{SN}) on Z is defined as:*

- $\emptyset_{SN}(u) = \{u \in Z : (u, 0, 0, 1)\}$.
- $1_{SN}(u) = \{u \in Z : (u, 1, 1, 0)\}$.

Definition 4. *[8] If f is a mapping from a set Z_1 into a set Z_2, then the following statements hold:*

(i) Let $\mathcal{A}$ be a SNS in Z_1 and $\mathcal{B}$ be a SNS in Z_2, then the pre-image of $\mathcal{B}$ is a SNS in Z_1, denoted by $f^{-1}(\mathcal{B})$, defined as:
$f^{-1}(\mathcal{B}) = \{z_1 \in Z_1 : f^{-1}(\mathcal{T}_\mathcal{B})(z_1) = \mathcal{T}_\mathcal{B}(f(z_1)), f^{-1}(\mathcal{I}_\mathcal{B})(z_1) = \mathcal{I}_\mathcal{B}(f(z_1)), f^{-1}(\mathcal{F}_\mathcal{B})(z_1) = \mathcal{F}_\mathcal{B}(f(z_1))\}$.

(ii) *Let* $\mathcal{A} = \{z_1 \in Z_1 : \mathcal{T}_{\mathcal{A}}(z_1), \mathcal{I}_{\mathcal{A}}(z_1), \mathcal{F}_{\mathcal{A}}(z_1)\}$ *be a SNS in* Z_1 *and* $\mathcal{B} = \{z_2 \in Z_2 : \mathcal{T}_{\mathcal{B}}(z_2), \mathcal{I}_{\mathcal{B}}(z_2), \mathcal{F}_{\mathcal{B}}(z_2)\}$ *be a SNS in* Z_2. *Under the mapping* f, *the image of* $\mathcal{A}$ *is a SNS in* Z_2, *denoted by* $f(\mathcal{A})$, *defined as:* $f(\mathcal{A}) = \{z_2 \in Z_2 : f_{\sup}(\mathcal{T}_{\mathcal{A}})(z_2), f_{\sup}(\mathcal{I}_{\mathcal{A}})(z_2), f_{\inf}(\mathcal{F}_{\mathcal{A}})(z_2)\}$, *where for all* $z_2 \in Z_2$.

$$f_{\sup}(\mathcal{T}_{\mathcal{A}})(z_2) = \begin{cases} \sup_{z_1 \in f^{-1}(z_2)} \mathcal{T}_{\mathcal{A}}(Z_1), & \text{if } f^{-1}_{(z_2)} \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$f_{\sup}(\mathcal{I}_{\mathcal{A}})(z_2) = \begin{cases} \sup_{z_1 \in f^{-1}(z_2)} \mathcal{I}_{\mathcal{A}}(Z_1), & \text{if } f^{-1}_{(z_2)} \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$f_{\inf}(\mathcal{F}_{\mathcal{A}})(z_2) = \begin{cases} \inf_{z_1 \in f^{-1}(z_2)} \mathcal{F}_{\mathcal{A}}(Z_1), & \text{if } f^{-1}_{(z_2)} \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

We formulate the following proposition.

Proposition 1. *Let* $f : Z_1 \to Z_2$ *and* $\mathcal{A}$, $(\mathcal{A}_j, j \in J)$ *be a SNS in* Z_1 *and* $\mathcal{B}$ *be a SNS in* Z_2. *Then,* f *possesses the following properties:*

(i) *If* f *is onto , then* $f(1_{SN}) = 1_{SN}$.
(ii) $f(\emptyset_{SN}) = \emptyset_{SN}$.
(iii) $f^{-1}(1_{SN}) = 1_{SN}$.
(iv) $f^{-1}(\emptyset_{SN}) = \emptyset_{SN}$.
(v) *If* f *is onto, then* $f(f^{-1}(\mathcal{B}) = \mathcal{B}$.
(vi) $f^{-1}(\bigcup_{i=1}^{n} \mathcal{A}_i) = \bigcup_{i=1}^{n} f^{-1}(\mathcal{A}_i)$.

3. Neutrosophic Topological *K*-algebras

Definition 5. *Let* Z *be a nonempty set. A collection* χ *of single-valued neutrosophic sets (SNSs) in* Z *is called a single-valued neutrosophic topology (SNT) on* Z *if the following conditions hold:*

(a) $\emptyset_{SN}, 1_{SN} \in \chi$
(b) *If* $\mathcal{A}, \mathcal{B} \in \chi$, *then* $\mathcal{A} \bigcap \mathcal{B} \in \chi$
(c) *If* $\mathcal{A}_i \in \chi$, $\forall i \in I$, *then* $\bigcup_{i \in I} \mathcal{A}_i \in \chi$

The pair (Z, χ) *is called a single-valued neutrosophic topological space (SNTS). Each member of* χ *is said to be* χ*-open or single-valued neutrosophic open set (SNOS) and compliment of each open single-valued neutrosophic set is a single-valued neutrosophic closed set (SNCS). A discrete topology is a topology which contains all single-valued neutrosophic subsets of* Z *and indiscrete if its elements are only* $\emptyset_{SN}$, 1_{SN}.

Definition 6. *Let* $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ *be a single-valued neutrosophic set in* $\mathcal{K}$. *Then,* $\mathcal{A}$ *is called a single-valued neutrosophic K-subalgebra of* $\mathcal{K}$ *if following conditions hold for* $\mathcal{A}$:

(i) $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$.
(ii) $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\}$,
$\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\}$,
$\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\} \ \forall\, s, t \in \mathcal{K}$.

Example 1. *Consider a K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given as:*

$\odot$	e	x	x^2	x^3	x^4	x^5	x^6	x^7	x^8
e	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x
x	x	e	x^8	x^7	x^6	x^5	x^4	x^3	x^2
x^2	x^2	x	e	x^8	x^7	x^6	x^5	x^4	x^3
x^3	x^3	x^2	x	e	x^8	x^7	x^6	x^5	x^4
x^4	x^4	x^3	x^2	x	e	x^8	x^7	x^6	x^5
x^5	x^5	x^4	x^3	x^2	x	e	x^8	x^7	x^6
x^6	x^6	x^5	x^4	x^3	x^2	x	e	x^8	x^7
x^7	x^7	x^6	x^5	x^4	x^3	x^2	x	e	x^8
x^8	x^8	x^7	x^6	x^5	x^4	x^3	x^2	x	e

If we define a single-valued neutrosophic set $\mathcal{A}, \mathcal{B}$ in $\mathcal{K}$ such that:

$$\mathcal{A} = \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.7)\},$$
$$\mathcal{B} = \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\}$$

$\forall\, s \neq e \in G$.

According to Definition 5, the family $\{\emptyset_{SN}, 1_{SN}, \mathcal{A}, \mathcal{B}\}$ of SNSs of K-algebra is a SNT on $\mathcal{K}$. We define a SNS $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$ in $\mathcal{K}$ such that $\mathcal{T}_{\mathcal{A}}(e) = 0.7, \mathcal{I}_{\mathcal{A}}(e) = 0.5, \mathcal{F}_{\mathcal{A}}(e) = 0.2$, $\mathcal{T}_{\mathcal{A}}(s) = 0.2, \mathcal{I}_{\mathcal{A}}(s) = 0.4, \mathcal{F}_{\mathcal{A}}(s) = 0.6$. Clearly, $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a SN K-subalgebra of $\mathcal{K}$.

Definition 7. *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra and let $\chi_{\mathcal{K}}$ be a topology on $\mathcal{K}$. Let $\mathcal{A}$ be a SNS in $\mathcal{K}$ and let $\chi_{\mathcal{K}}$ be a topology on $\mathcal{K}$. Then, an induced single-valued neutrosophic topology on $\mathcal{A}$ is a collection or family of single-valued neutrosophic subsets of $\mathcal{A}$ which are the intersection with $\mathcal{A}$ and single-valued neutrosophic open sets in $\mathcal{K}$ defined as $\chi_{\mathcal{A}} = \{\mathcal{A} \cap F : F \in \chi_{\mathcal{K}}\}$. Then, $\chi_{\mathcal{A}}$ is called single-valued neutrosophic induced topology on $\mathcal{A}$ or relative topology and the pair $(\mathcal{A}, \chi_{\mathcal{A}})$ is called an induced topological space or single-valued neutrosophic subspace of $(\mathcal{K}, \chi_{\mathcal{K}})$.*

Definition 8. *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two SNTSs and let $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$. Then, f is called single-valued neutrosophic continuous if following conditions hold:*

(i) *For each SNS $\mathcal{A} \in \chi_2$, $f^{-1}(\mathcal{A}) \in \chi_1$.*
(ii) *For each SN K-subalgebra $\mathcal{A} \in \chi_2$, $f^{-1}(\mathcal{A})$ is a SN K-subalgebra $\in \chi_1$.*

Definition 9. *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two SNTSs and let $(\mathcal{A}, \chi_{\mathcal{A}})$ and $(\mathcal{B}, \chi_{\mathcal{B}})$ be two single-valued neutrosophic subspaces over $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$. Let f be a mapping from $(\mathcal{K}_1, \chi_1)$ into $(\mathcal{K}_2, \chi_2)$, then f is a mapping from $(\mathcal{A}, \chi_{\mathcal{A}})$ to $(\mathcal{B}, \chi_{\mathcal{B}})$ if $f(\mathcal{A}) \subset \mathcal{B}$.*

Definition 10. *Let f be a mapping from $(\mathcal{A}, \chi_{\mathcal{A}})$ to $(\mathcal{B}, \chi_{\mathcal{B}})$. Then, f is relatively single-valued neutrosophic continuous if for every SNOS $Y_{\mathcal{B}}$ in $\chi_{\mathcal{B}}$, $f^{-1}(Y_{\mathcal{B}}) \cap \mathcal{A} \in \chi_{\mathcal{A}}$.*

Definition 11. *Let f be a mapping from $(\mathcal{A}, \chi_{\mathcal{A}})$ to $(\mathcal{B}, \chi_{\mathcal{B}})$. Then, f is relatively single-valued neutrosophic open if for every SNOS $X_{\mathcal{A}}$ in $\chi_{\mathcal{A}}$, the image $f(X_{\mathcal{A}}) \in \chi_{\mathcal{B}}$.*

Proposition 2. *Let $(\mathcal{A}, \chi_{\mathcal{A}})$ and $(\mathcal{B}, \chi_{\mathcal{B}})$ be single-valued neutrosophic subspaces of $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$, where $\mathcal{K}_1$ and $\mathcal{K}_2$ are K-algebras. If f is a single-valued neutrosophic continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$ and $f(\mathcal{A}) \subset \mathcal{B}$. Then, f is relatively single-valued neutrosophic continuous function from $\mathcal{A}$ into $\mathcal{B}$.*

Definition 12. *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two SNTSs. A mapping $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ is called a single-valued neutrosophic homomorphism if following conditions hold:*

(i) *f is a one-one and onto function.*
(ii) *f is a single-valued neutrosophic continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$.*
(iii) *f^{-1} is a single-valued neutrosophic continuous function from $\mathcal{K}_2$ to $\mathcal{K}_1$.*

Theorem 1. *Let $(\mathcal{K}_1, \chi_1)$ be a SNTS and $(\mathcal{K}_2, \chi_2)$ be an indiscrete SNTS on K-algebras $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively. Then, each function f defined as $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ is a single-valued neutrosophic continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$. If $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two discrete SNTSs $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively, then each homomorphism $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ is a single values neutrosophic continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$.*

Proof. Let f be a mapping defined as $f : \mathcal{K}_1 \to \mathcal{K}_2$. Let χ_1 be SNT on $\mathcal{K}_1$ and χ_2 be SNT on $\mathcal{K}_2$, where $\chi_2 = \{\emptyset_{SN}, 1_{SN}\}$. We show that $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_1$, i.e., for each $\mathcal{A} \in \chi_2$, $f^{-1}(\mathcal{A}) \in \chi_1$. Since $\chi_2 = \{\emptyset_{SN}, 1_{SN}\}$, then for any $u \in \chi_1$, consider $\emptyset_{SN} \in \chi_2$ such that $f^{-1}(\emptyset_{SN})(u) = \emptyset_{SN}(f(u)) = \emptyset_{SN}(u)$.

Therefore, $(f^{-1}(\emptyset_{SN})) = \emptyset_{SN} \in \chi_1$. Likewise, $(f^{-1}(1_{SN})) = 1_{SN} \in \chi_1$. Hence, f is a SN continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$.

Now, for the second part of the theorem, where both χ_1 and χ_2 are SNTSs on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively, and $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ is a homomorphism. Therefore, for all $\mathcal{A} \in \chi_2$ and $f^{-1}\mathcal{A} \in \chi_1$, where f is not a usual inverse homomorphism. To prove that $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic K-subalgebra in of $\mathcal{K}_1$. Let for $u, v \in \mathcal{K}_1$,

$$\begin{aligned}
f^{-1}(\mathcal{T}_{\mathcal{A}})(u \odot v) &= \mathcal{T}_{\mathcal{A}}(f(u \odot v)) \\
&= \mathcal{T}_{\mathcal{A}}(f(u) \odot f(v)) \\
&\geq \min\{\mathcal{T}_{\mathcal{A}}(f(u)) \odot \mathcal{T}_(f(v))\} \\
&= \min\{f^{-1}(\mathcal{T}_{\mathcal{A}})(u), f^{-1}(\mathcal{T}_{\mathcal{A}})(v)\}, \\
f^{-1}(\mathcal{I}_{\mathcal{A}})(u \odot v) &= \mathcal{I}_{\mathcal{A}}(f(u \odot v)) \\
&= \mathcal{I}_{\mathcal{A}}(f(u) \odot f(v)) \\
&\geq \min\{\mathcal{I}_{\mathcal{A}}(f(u)) \odot \mathcal{I}_(f(v))\} \\
&= \min\{f^{-1}(\mathcal{I}_{\mathcal{A}})(u), f^{-1}(\mathcal{I}_{\mathcal{A}})(v)\}, \\
f^{-1}(\mathcal{F}_{\mathcal{A}})(u \odot v) &= \mathcal{F}_{\mathcal{A}}(f(u \odot v)) \\
&= \mathcal{F}_{\mathcal{A}}(f(u) \odot f(v)) \\
&\leq \max\{\mathcal{F}_{\mathcal{A}}(f(u)) \odot \mathcal{F}_(f(v))\} \\
&= \max\{f^{-1}(\mathcal{F}_{\mathcal{A}})(u), f^{-1}(\mathcal{F}_{\mathcal{A}})(v)\}.
\end{aligned}$$

Hence, f is a single-valued neutrosophic continuous function from $\mathcal{K}_1$ to $\mathcal{K}_2$. □

Proposition 3. *Let χ_1 and χ_2 be two SNTSs on $\mathcal{K}$. Then, each homomorphism $f : (\mathcal{K}, \chi_1) \to (\mathcal{K}, \chi_2)$ is a single-valued neutrosophic continuous function.*

Proof. Let $(\mathcal{K}, \chi_1)$ and $(\mathcal{K}, \chi_2)$ be two SNTSs, where $\mathcal{K}$ is a K-algebra. To prove the above result, it is enough to show that result is false for a particular topology. Let $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}},)$ and $\mathcal{B} = (\mathcal{T}_{\mathcal{B}}, \mathcal{I}_{\mathcal{B}}, \mathcal{F}_{\mathcal{B}})$ be two SNSs in $\mathcal{K}$. Take $\chi_1 = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$ and $\chi_2 = \{\emptyset_{SN}, 1_{SN}, \mathcal{B}\}$. If $f : (\mathcal{K}, \chi_1) \to (\mathcal{K}, \chi_2)$, defined by $f(u) = e \odot u$, for all $u \in \mathcal{K}$, then f is a homomorphism. Now, for $u \in \mathcal{A}$, $v \in \chi_2$, $(f^{-1}(\mathcal{B}))(u) = \mathcal{B}(f(u)) = \mathcal{B}(e \odot u) = \mathcal{B}(u)$,
$\forall\ u \in \mathcal{K}$, i.e., $f^{-1}(\mathcal{B}) = \mathcal{B}$. Therefore, $(f^{-1}(\mathcal{B})) \notin \chi_1$. Hence, f is not a single-valued neutrosophic continuous mapping. □

Definition 13. *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra and χ be a SNT on $\mathcal{K}$. Let $\mathcal{A}$ be a single-valued neutrosophic K-algebra (K-subalgebra) of $\mathcal{K}$ and $\chi_{\mathcal{A}}$ be a SNT on $\mathcal{A}$. Then, $\mathcal{A}$ is said to be a single-valued neutrosophic topological K-algebra (K-subalgebra) on $\mathcal{K}$ if the self mapping $\rho_a : (\mathcal{A}, \chi_{\mathcal{A}}) \to (\mathcal{A}, \chi_{\mathcal{A}})$ defined as $\rho_a(u) = u \odot a$, $\forall a \in \mathcal{K}$, is a relatively single-valued neutrosophic continuous mapping.*

Theorem 2. *Let χ_1 and χ_2 be two SNTSs on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively, and $f : \mathcal{K}_1 \to \mathcal{K}_2$ be a homomorphism such that $f^{-1}(\chi_2) = \chi_1$. If $\mathcal{A} = \{\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A}\}$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$, then $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$.*

Proof. Let $\mathcal{A} = \{\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A}\}$ be a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$. To prove that $f^{-1}(\mathcal{A})$ be a single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$. Let for any $u, v \in \mathcal{K}_1$,

$$\begin{aligned}
\mathcal{T}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{T}_\mathcal{A}(f(u \odot v)) \\
&\geq \min\{\mathcal{T}_\mathcal{A}(f(u)), \mathcal{T}_\mathcal{A}(f(v))\} \\
&= \min\{\mathcal{T}_{f^{-1}(\mathcal{A})}(u), \mathcal{T}_{f^{-1}(\mathcal{A})}(v)\}, \\
\mathcal{I}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{I}_\mathcal{A}(f(u \odot v)) \\
&\geq \min\{\mathcal{I}_\mathcal{A}(f(u)), \mathcal{I}_\mathcal{A}(f(v))\} \\
&= \min\{\mathcal{I}_{f^{-1}(\mathcal{A})}(u), \mathcal{I}_{f^{-1}(\mathcal{A})}(v)\}, \\
\mathcal{F}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{F}_\mathcal{A}(f(u \odot v)) \\
&\leq \max\{\mathcal{F}_\mathcal{A}(f(u)), \mathcal{F}_\mathcal{A}(f(v))\} \\
&= \max\{\mathcal{F}_{f^{-1}(\mathcal{A})}(u), \mathcal{F}_{f^{-1}(\mathcal{A})}(v)\}.
\end{aligned}$$

Hence, $f^{-1}(\mathcal{A})$ is a single-valued neutrosophic K-algebra of $\mathcal{K}_1$.

Now, we prove that $f^{-1}(\mathcal{A})$ is single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$. Since f is a single-valued neutrosophic continuous function, then by proposition 3.1, f is also a relatively single-valued neutrosophic continuous function which maps $(f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})})$ to $(\mathcal{A}, \chi_\mathcal{A})$.

Let $a \in \mathcal{K}_1$ and Y be a SNS in $\chi_\mathcal{A}$, and let X be a SNS in $\chi_{f^{-1}(\mathcal{A})}$ such that

$$f^{-1}(Y) = X. \tag{1}$$

We are to prove that $\rho_a : (f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})}) \to (f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})})$ is relatively single-valued neutrosophic continuous mapping, then for any $a \in \mathcal{K}_1$, we have

$$\begin{aligned}
\mathcal{T}_{\rho_a^{-1}(X)}(u) &= \mathcal{T}_{(X)}(\rho_a(u)) = \mathcal{T}_{(X)}(u \odot a) \\
&= \mathcal{T}_{f^{-1}(Y)}(u \odot a) = \mathcal{T}_{(Y)}(f(u \odot a)) \\
&= \mathcal{T}_{(Y)}(f(u) \odot f(a)) = \mathcal{T}_{(Y)}(\rho_{f(a)}(f(u))) \\
&= \mathcal{T}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{T}_{f^{-1}}(\rho_{f(a)}^{-1}(Y)(u)),
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_{\rho_a^{-1}(X)}(u) &= \mathcal{I}_{(X)}(\rho_a(u)) = \mathcal{I}_{(X)}(u \odot a) \\
&= \mathcal{I}_{f^{-1}(Y)}(u \odot a) = \mathcal{I}_{(Y)}(f(u \odot a)) \\
&= \mathcal{I}_{(Y)}(f(u) \odot f(a)) = \mathcal{I}_{(Y)}(\rho_{f(a)}(f(u))) \\
&= \mathcal{I}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{I}_{f^{-1}}(\rho_{f(a)}^{-1}(Y)(u)),
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\rho_a^{-1}(X)}(u) &= \mathcal{F}_{(X)}(\rho_a(u)) = \mathcal{F}_{(X)}(u \odot a) \\
&= \mathcal{F}_{f^{-1}(Y)}(u \odot a) = \mathcal{F}_{(Y)}(f(u \odot a)) \\
&= \mathcal{F}_{(Y)}(f(u) \odot f(a)) = \mathcal{F}_{(Y)}(\rho_{f(a)}(f(u))) \\
&= \mathcal{F}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{F}_{f^{-1}}(\rho_{f(a)}^{-1}(Y)(u)).
\end{aligned}$$

It concludes that $\rho_a^{-1}(X) = f^{-1}(\rho_{f(a)}^{-1}(Y))$. Thus, $\rho_a^{-1}(X) \cap f^{-1}(\mathcal{A}) = f^{-1}(\rho_{f(a)}^{-1}(Y)) \cap f^{-1}(\mathcal{A})$ is a SNS in $f^{-1}(\mathcal{A})$ and a SNS in $\chi_{f^{-1}(\mathcal{A})}$. Hence, $f^{-1}(\mathcal{A})$ and a single-valued neutrosophic topological K-algebra of $\mathcal{K}$. Hence, the proof. □

Theorem 3. *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two SNTSs on $\mathcal{K}_1$ and $\mathcal{K}_2$, respectively, and let f be a bijective homomorphism of $\mathcal{K}_1$ into $\mathcal{K}_2$ such that $f(\chi_1) = \chi_2$. If $\mathcal{A}$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_1$, then $f(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$.*

Proof. Suppose that $\mathcal{A} = \{\mathcal{T}_\mathcal{A}, \mathcal{I}_\mathcal{A}, \mathcal{F}_\mathcal{A}\}$ is a SN topological K-algebra of $\mathcal{K}_1$. To prove that $f(\mathcal{A})$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}_2$, let, for $u, v \in \mathcal{K}_2$,

$$f(\mathcal{A}) = (f_{\sup}(\mathcal{T}_{\mathcal{A}})(v), f_{\sup}(\mathcal{I}_{\mathcal{A}})(v), f_{\inf}(\mathcal{F}_{\mathcal{A}})(v)).$$

Let $a_o \in f^{-1}(u), b_o \in f^{-1}(v)$ such that

$$\begin{array}{l} \sup_{x \in f^{-1}(u)} \mathcal{T}_{\mathcal{A}}(x) = \mathcal{T}_{\mathcal{A}}(a_o), \sup_{x \in f^{-1}(v)} \mathcal{T}_{\mathcal{A}}(x) = \mathcal{T}_{\mathcal{A}}(b_o), \\ \sup_{x \in f^{-1}(u)} \mathcal{I}_{\mathcal{A}}(x) = \mathcal{I}_{\mathcal{A}}(a_o), \sup_{x \in f^{-1}(v)} \mathcal{I}_{\mathcal{A}}(x) = \mathcal{I}_{\mathcal{A}}(b_o), \\ \inf_{x \in f^{-1}(u)} \mathcal{F}_{\mathcal{A}}(x) = \mathcal{F}_{\mathcal{A}}(a_o), \inf_{x \in f^{-1}(v)} \mathcal{F}_{\mathcal{A}}(x) = \mathcal{F}_{\mathcal{A}}(b_o). \end{array}$$

Now,

$$\begin{aligned} \mathcal{T}_{f(\mathcal{A})}(u \odot v) &= \sup_{x \in f^{-1}(u \odot v)} \mathcal{T}_{\mathcal{A}}(x) \\ &\geq \mathcal{T}_{\mathcal{A}}(a_o, b_o) \\ &\geq \min\{\mathcal{T}_{\mathcal{A}}(a_o), \mathcal{T}_{\mathcal{A}}(b_o)\} \\ &= \min\{\sup_{x \in f^{-1}(u)} \mathcal{T}_{\mathcal{A}}(x), \sup_{x \in f^{-1}(v)} \mathcal{T}_{\mathcal{A}}(x)\} \\ &= \min\{\mathcal{T}_{f(\mathcal{A})}(u), \mathcal{T}_{f(\mathcal{A})}(v)\}, \end{aligned}$$

$$\begin{aligned} \mathcal{I}_{f(\mathcal{A})}(u \odot v) &= \sup_{x \in f^{-1}(u \odot v)} \mathcal{I}_{\mathcal{A}}(x) \\ &\geq \mathcal{I}_{\mathcal{A}}(a_o, b_o) \\ &\geq \min\{\mathcal{I}_{\mathcal{A}}(a_o), \mathcal{I}_{\mathcal{A}}(b_o)\} \\ &= \min\{\sup_{x \in f^{-1}(u)} \mathcal{I}_{\mathcal{A}}(x), \sup_{x \in f^{-1}(v)} \mathcal{I}_{\mathcal{A}}(x)\} \\ &= \min\{\mathcal{I}_{f(\mathcal{A})}(u), \mathcal{I}_{f(\mathcal{A})}(v)\}, \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{f(\mathcal{A})}(u \odot v) &= \inf_{x \in f^{-1}(u \odot v)} \mathcal{F}_{\mathcal{A}}(x) \\ &\leq \mathcal{F}_{\mathcal{A}}(a_o, b_o) \\ &\leq \max\{\mathcal{F}_{\mathcal{A}}(a_o), \mathcal{F}_{\mathcal{A}}(b_o)\} \\ &= \max\{\inf_{x \in f^{-1}(u)} \mathcal{F}_{\mathcal{A}}(x), \inf_{x \in f^{-1}(v)} \mathcal{F}_{\mathcal{A}}(x)\} \\ &= \max\{\mathcal{F}_{f(\mathcal{A})}(u), \mathcal{F}_{f(\mathcal{A})}(v)\}. \end{aligned}$$

Hence, $f(\mathcal{A})$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}_2$. Now, we prove that the self mapping $\rho_b : (f(\mathcal{A}), \chi_f(\mathcal{A})) \to (f(\mathcal{A}), \chi_f(\mathcal{A}))$, defined by $\rho_b(v) = v \odot b$, for all $b \in \mathcal{K}_2$, is a relatively single-valued neutrosophic continuous mapping. Let $Y_{\mathcal{A}}$ be a SNS in $\chi_{\mathcal{A}}$, there exists a SNS "Y" in χ_1 such that $Y_{\mathcal{A}} = Y \cap \mathcal{A}$. We show that for a SNS in $\chi_{f(\mathcal{A})}$,

$$\rho^{-1}{}_b(Y_{f(\mathcal{A})}) \cap f(\mathcal{A}) \in \chi_{f(\mathcal{A})}$$

Since f is an injective mapping, then $f(Y_{\mathcal{A}}) = f(Y \cap \mathcal{A}) = f(Y) \cap f(\mathcal{A})$ is a SNS in $\chi_{f(\mathcal{A})}$ which shows that f is relatively single-valued neutrosophic open. In addition, f is surjective, then for all $b \in \mathcal{K}_2, a = f(b)$, where $a \in \mathcal{K}_1$.

Now,

$$\begin{aligned}
\mathcal{T}_{f^{-1}(\rho^{-1}{}_b(Y_{f(\mathcal{A})}))}(u) &= \mathcal{T}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})}))}(u)\\
&= \mathcal{T}_{\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})})}(f(u))\\
&= \mathcal{T}_{(Y_{f(\mathcal{A})})}(\rho_{f(a)}(f(u)))\\
&= \mathcal{T}_{(Y_{f(\mathcal{A})})}(f(u)\odot f(a))\\
&= \mathcal{T}_{f^{-1}(Y_{f(\mathcal{A})})}(u\odot a)\\
&= \mathcal{T}_{f^{-1}(Y_{f(\mathcal{A})})}(\rho_a(u))\\
&= \mathcal{T}_{\rho^{-1}{}_{(a)}}(f^{-1}(Y_{f(\mathcal{A})}))(u),
\end{aligned}$$

$$\begin{aligned}
\mathcal{I}_{f^{-1}(\rho^{-1}{}_b(Y_{f(\mathcal{A})}))}(u) &= \mathcal{I}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})}))}(u)\\
&= \mathcal{I}_{\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})})}(f(u))\\
&= \mathcal{I}_{(Y_{f(\mathcal{A})})}(\rho_{f(a)}(f(u)))\\
&= \mathcal{I}_{(Y_{f(\mathcal{A})})}(f(u)\odot f(a))\\
&= \mathcal{I}_{f^{-1}(Y_{f(\mathcal{A})})}(u\odot a)\\
&= \mathcal{I}_{f^{-1}(Y_{f(\mathcal{A})})}(\rho_a(u))\\
&= \mathcal{I}_{\rho^{-1}{}_{(a)}}(f^{-1}(Y_{f(\mathcal{A})}))(u),
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{f^{-1}(\rho^{-1}{}_b(Y_{f(\mathcal{A})}))}(u) &= \mathcal{F}_{f^{-1}(\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})}))}(u)\\
&= \mathcal{F}_{\rho^{-1}{}_{f(a)}(Y_{f(\mathcal{A})})}(f(u))\\
&= \mathcal{F}_{(Y_{f(\mathcal{A})})}(\rho_{f(a)}(f(u)))\\
&= \mathcal{F}_{(Y_{f(\mathcal{A})})}(f(u)\odot f(a))\\
&= \mathcal{F}_{f^{-1}(Y_{f(\mathcal{A})})}(u\odot a)\\
&= \mathcal{F}_{f^{-1}(Y_{f(\mathcal{A})})}(\rho_a(u))\\
&= \mathcal{F}_{\rho^{-1}{}_{(a)}}(f^{-1}(Y_{f(\mathcal{A})}))(u).
\end{aligned}$$

This implies that $f^{-1}(\rho^{-1}_{(b)}((Y_{f(\mathcal{A})}))) = \rho^{-1}_{(a)}(f^{-1}(Y_{(\mathcal{A})}))$. Since $\rho_a : (\mathcal{A}, \chi_{\mathcal{A}}) \to (\mathcal{A}, \chi_{\mathcal{A}})$ is relatively single-valued neutrosophic continuous mapping and f is relatively single-valued neutrosophic continues mapping from $(\mathcal{A}, \chi_{\mathcal{A}})$ into $(f(\mathcal{A}), \chi_{f(\mathcal{A})})$, $f^{-1}(\rho^{-1}_{(b)}((Y_{f(\mathcal{A})}))) \cap \mathcal{A} = \rho^{-1}_{(a)}(f^{-1}(Y_{(\mathcal{A})})) \cap \mathcal{A}$ is a SNS in $\chi_{\mathcal{A}}$. Hence, $f(f^{-1}(\rho_{(b)}((Y_{f(\mathcal{A})}))) \cap \mathcal{A}) = \rho^{-1}_{(b)}(Y_{f(\mathcal{A})}) \cap f(\mathcal{A})$ is a SNS in $\chi_{\mathcal{A}}$, which completes the proof. □

Example 2. *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given in Example 1. We define a SNS as:*

$$\mathcal{A} = \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.6)\},$$
$$\mathcal{B} = \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\},$$

for all $s \neq e \in G$, where $\mathcal{A}, \mathcal{B} \in [0, 1]$. The collection $\chi_{\mathcal{K}} = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}, \mathcal{B}\}$ of SNSs of $\mathcal{K}$ is a SNT on $\mathcal{K}$ and $(\mathcal{K}, \chi_{\mathcal{K}})$ is a SNTS. Let $\mathcal{C}$ be a SNS in $\mathcal{K}$, defined as:

$$\mathcal{C} = \{(e, 0.7, 0.5, 0.2), (s, 0.5, 0.4, 0.6)\}, \forall s \neq e \in G.$$

Clearly, $\mathcal{C}$ is a single-valued neutrosophic K-subalgebra of $\mathcal{K}$. By direct calculations relative topology $\chi_{\mathcal{C}}$ is obtained as $\chi_{\mathcal{C}} = \{\emptyset_{\mathcal{A}}, 1_{\mathcal{A}}, \mathcal{A}\}$. Then, the pair $(\mathcal{C}, \chi_{\mathcal{C}})$ is a single-valued neutrosophic subspace of $(\mathcal{K}, \chi_{\mathcal{K}})$. We show that $\mathcal{C}$ is a single-valued neutrosophic topological K-subalgebra of $\mathcal{K}$, i.e., the self mapping $\rho_a : (\mathcal{C}, \chi_{\mathcal{C}}) \to (\mathcal{C}, \chi_{\mathcal{C}})$ defined by $\rho_a(u) = u \odot a, \forall a \in \mathcal{K}$ is relatively single-valued neutrosophic continuous mapping, i.e., for a SNOS $\mathcal{A}$ in $(\mathcal{C}, \chi_{\mathcal{C}})$, $\rho_a^{-1}(\mathcal{A}) \cap \mathcal{C} \in \chi_{\mathcal{C}}$. Since ρ_a is homomorphism, then $\rho_a^{-1}(\mathcal{A}) \cap \mathcal{C} = \mathcal{A} \in \chi_{\mathcal{C}}$. Therefore, $\rho_a : (\mathcal{C}, \chi_{\mathcal{C}}) \to (\mathcal{C}, \chi_{\mathcal{C}})$ is relatively single-valued neutrosophic continuous mapping. Hence, $\mathcal{C}$ is a single-valued neutrosophic topological K-algebra of $\mathcal{K}$.

Example 3. *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given in Example 3.1. We define a SNS as:*

$$\begin{aligned}
\mathcal{A} &= \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.6)\}, \\
\mathcal{B} &= \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\}, \\
\mathcal{D} &= \{(e, 0.2, 0.1, 0.3), (s, 0.1, 0.1, 0.5)\},
\end{aligned}$$

for all $s \neq e \in G$, where $\mathcal{A}, \mathcal{B} \in [0, 1]$. The collection $\chi_1 = \{\emptyset_{SN}, 1_{SN}, \mathcal{D}\}$ and $\chi_2 = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}, \mathcal{B}\}$ of SNSs of $\mathcal{K}$ are SNTs on $\mathcal{K}$ and $(\mathcal{K}, \chi_1)$, $(\mathcal{K}, \chi_2)$ be two SNTSs. Let $\mathcal{C}$ be a SNS in $(\mathcal{K}, \chi_2)$, defined as:

$$\mathcal{C} = \{(e, 0.7, 0.5, 0.2), (s, 0.5, 0.4, 0.6)\}, \forall s \neq e \in G.$$

Now, Let $f : (\mathcal{K}, \chi_1) \to (\mathcal{K}, \chi_2)$ be a homomorphism such that $f^{-1}(\chi_2) = \chi_1$ (we have not consider $\mathcal{K}$ to be distinct), then, by Proposition 3, f is a single-valued neutrosophic continuous function and f is also relatively single-valued neutrosophic continues mapping from $(\mathcal{K}, \chi_1)$ into $(\mathcal{K}, \chi_2)$. Since $\mathcal{C}$ is a SNS in $(\mathcal{K}, \chi_2)$ and with relative topology $\chi_{\mathcal{C}} = \{\emptyset_{\mathcal{A}}, 1_{\mathcal{A}}, \mathcal{A}\}$ is also a single-valued neutrosophic topological K-algebra of $(\mathcal{K}, \chi_2)$. We prove that $f^{-1}(\mathcal{C})$ is a single-valued neutrosophic topological K-algebra in $(\mathcal{K}, \chi_1)$. Since f is a continuous function, then, by Definition 8, $f^{-1}(\mathcal{C})$ is a single-valued neutrosophic K-subalgebra in $(\mathcal{K}, \chi_1)$. To prove that $f^{-1}(c)$ is a single-valued neutrosophic topological K-algebra, then for $b \in \mathcal{K}_1$ take

$$\rho_b : (f^{-1}(\mathcal{C}), \chi_{f^{-1}(\mathcal{C})}) \to (f^{-1}(\mathcal{C}), \chi_{f^{-1}(\mathcal{C})}),$$

for $\mathcal{A} \in \chi_{f^{-1}(C)}, \rho_b^{-1}(\mathcal{A}) \cap f^{-1}(\mathcal{C}) \in \chi_{f^{-1}(C)}$ which shows that $f^{-1}(C)$ is a single-valued neutrosophic topological K-algebra in $(\mathcal{K}, \chi_1)$. Similarly, we can show that $f(\mathcal{C})$ is a a single-valued neutrosophic topological K-algebra in $(\mathcal{K}, \chi_2)$ by considering a bijective homomorphism.

Definition 14. *Let χ be a SNT on $\mathcal{K}$ and $(\mathcal{K}, \chi)$ be a SNTS. Then, $(\mathcal{K}, \chi)$ is called single-valued neutrosophic C_5-disconnected topological space if there exist a SNOS and SNCS $\mathcal{H}$ such that $\mathcal{H} = (\mathcal{T}_{\mathcal{H}}, \mathcal{I}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}},) \neq 1_{SN}$ and $\mathcal{H} = (\mathcal{T}_{\mathcal{H}}, \mathcal{I}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}},) \neq \emptyset_{SN}$, otherwise $(\mathcal{K}, \chi)$ is called single-valued neutrosophic C_5-connected.*

Example 4. *Every indiscrete SNT space on $\mathcal{K}$ is C_5-connected.*

Proposition 4. *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two SNTSs and $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ be a surjective single-valued neutrosophic continuous mapping. If $(\mathcal{K}_1, \chi_1)$ is a single-valued neutrosophic C_5-connected space, then $(\mathcal{K}_2, \chi_2)$ is also a single-valued neutrosophic C_5-connected space.*

Proof. Suppose on contrary that $(\mathcal{K}_2, \chi_2)$ is a single-valued neutrosophic C_5-disconnected space. Then, by Definition 14, there exist both SNOS and SNCS $\mathcal{H}$ be such that $\mathcal{H} \neq 1_{SN}$ and $\mathcal{H} \neq \emptyset_{SN}$. Since f is a single-valued neutrosophic continuous and onto function, so $f^{-1}(\mathcal{H}) = 1_{SN}$ or $f^{-1}(\mathcal{H}) = \emptyset_{SN}$, where $f^{-1}(\mathcal{H})$ is both SNOS and SNCS. Therefore,

$$\mathcal{H} = f(f^{-1}(\mathcal{H})) = f(1_{SN}) = 1_{SN} \tag{2}$$

and

$$\mathcal{H} = f(f^{-1}(\mathcal{H})) = f(\emptyset_{SN}) = \emptyset_{SN}, \tag{3}$$

a contradiction. Hence, $(\mathcal{K}_2, \chi_2)$ is a single-valued neutrosophic C_5-connected space. □

Corollary 1. *Let χ be a SNT on $\mathcal{K}$. Then, $(\mathcal{K}, \chi)$ is called a single-valued neutrosophic C_5-connected space if and only if there does not exist a single-valued neutrosophic continuous map $f : (\mathcal{K}, \chi) \to (\mathcal{F}_T, \chi_T)$ such that $f \neq 1_{SN}$ and $f \neq \emptyset_{SN}$*

Definition 15. *Let $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$ be a SNS in $\mathcal{K}$. Let χ be a SNT on $\mathcal{K}$. The interior and closure of $\mathcal{A}$ in $\mathcal{K}$ is defined as:*

$\mathcal{A}^{Int}$: The union of SNOSs which contained in $\mathcal{A}$.
$\mathcal{A}^{Clo}$: The intersection of SNCSs for which $\mathcal{A}$ is a subset of these SNCSs.

Remark 1. *Being union of SNOS $\mathcal{A}^{Int}$ is a SNO and $\mathcal{A}^{Clo}$ being intersection of SNCS is SNC.*

Theorem 4. *Let $\mathcal{A}$ be a SNS in a SNTS $(\mathcal{K}, \chi)$. Then, $\mathcal{A}^{Int}$ is such an open set which is the largest open set of $\mathcal{K}$ contained in $\mathcal{A}$.*

Corollary 2. *$\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a SNOS in $\mathcal{K}$ if and only if $\mathcal{A}^{Int} = \mathcal{A}$ and $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ is a SNCS in $\mathcal{K}$ if and only if $\mathcal{A}^{Clo} = \mathcal{A}$.*

Proposition 5. *Let $\mathcal{A}$ be a SNS in $\mathcal{K}$. Then, following results hold for $\mathcal{A}$:*

(i) $(1_{SN})^{Int} = 1_{SN}$.
(ii) $(\emptyset_{SN})^{Clo} = \emptyset_{SN}$.
(iii) $\overline{(\mathcal{A})}^{Int} = \overline{(\mathcal{A})^{Clo}}$.
(iv) $\overline{(\mathcal{A})}^{Clo} = \overline{(\mathcal{A})^{Int}}$.

Definition 16. *Let $\mathcal{K}$ be a K-algebra and χ be a SNT on $\mathcal{K}$. A SNOS $\mathcal{A}$ in $\mathcal{K}$ is said to be single-valued neutrosophic regular open if*

$$\mathcal{A} = (\mathcal{A}^{Clo})^{Int}. \tag{4}$$

Remark 2. *Every SNOS which is regular is single-valued neutrosophic open and every single-valued neutrosophic closed and open set is a single-valued neutrosophic regular open.*

Definition 17. *A single-valued neutrosophic super connected K-algebra is such a K-algebra in which there does not exist a single-valued neutrosophic regular open set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ such that $\mathcal{A} \neq \emptyset_{SN}$ and $\mathcal{A} \neq 1_{SN}$. If there exists such a single-valued neutrosophic regular open set $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$ such that $\mathcal{A} \neq \emptyset_{SN}$ and $\mathcal{A} \neq 1_{SN}$, then K-algebra is said to be a single-valued neutrosophic super disconnected.*

Example 5. *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given in Example 1 We define a SNS as:*

$$\mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}.$$

Let $\chi_{\mathcal{K}} = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$ be a SNT on $\mathcal{K}$ and let $\mathcal{B} = \{(e, 0.3, 0.3, 0.8), (s, 0.2, 0.2, 0.6)\}$ be a SNS in $\mathcal{K}$. here

$SNOSs : \emptyset_{SN} = \{0, 0, 1\}, 1_{SN} = \{1, 1, 0\}, \mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}$.
$SNCSs : (\emptyset_{SN})^c = (\{0, 0, 1\})^c = (\{1, 1, 0\}) = 1_{SN}, (1_{SN})^c = (\{1, 1, 0\})^c = (\{0, 0, 1\}) = \emptyset_{SN}$,
$(\mathcal{A})^c = (\{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\})^c = (\{(e, 0.8, 0.3, 0.2), (s, 0.6, 0.2, 0.1)\}) = \mathcal{A}'(say)$.

Then, closure of $\mathcal{B}$ is the intersection of closed sets which contain $\mathcal{B}$. Therefore,

$$\mathcal{A}^{'} = \mathcal{B}^{Clo}. \tag{5}$$

Now, interior of $\mathcal{B}$ is the union of open sets which contain in $\mathcal{B}$. Therefore,

$$\emptyset_{SN} \bigcup \mathcal{A} = \mathcal{A}$$

$$\mathcal{A} = \mathcal{B}^{Int}. \tag{6}$$

Note that $(\mathcal{B}^{Clo})^{Clo} = \mathcal{B}^{Clo}$. Now, if we consider a SNS $\mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}$ in a K-algebra $\mathcal{K}$ and if $\chi_{\mathcal{K}} = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$ is a SNT on $\mathcal{K}$. Then, $(\mathcal{A})^{Clo} = \mathcal{A}$ and $(\mathcal{A})^{Int} = \mathcal{A}$. Consequently,

$$\mathcal{A} = (\mathcal{A}^{Clo})^{Int}, \tag{7}$$

which shows that $\mathcal{A}$ is a SN regular open set in K-algebra $\mathcal{K}$. Since $\mathcal{A}$ is a SN regular open set in $\mathcal{K}$ and $\mathcal{A} \neq \emptyset_{SN}, \mathcal{A} \neq 1_{SN}$, then, by Definition 17, K-algebra $\mathcal{K}$ is a single-valued neutrosophic supper disconnected K-algebra.

Proposition 6. *Let $\mathcal{K}$ be a K-algebra and let $\mathcal{A}$ be a SNOS. Then, the following statements are equivalent:*

(i) *A K-algebra is single-valued neutrosophic super connected.*
(ii) $(\mathcal{A})^{Clo} = 1_{SN}$*, for each SNOS* $\mathcal{A} \neq \emptyset_{SN}$.
(iii) $(\mathcal{A})^{Int} = \emptyset_{SN}$*, for each SNCS* $\mathcal{A} \neq 1_{SN}$.
(iv) *There do not exist SNOSs $\mathcal{A}, \mathcal{F}$ such that $\mathcal{A} \subseteq \overline{\mathcal{F}}$ and $\mathcal{A} \neq \emptyset_{SN} \neq \mathcal{F}$ in K-algebra $\mathcal{K}$.*

Definition 18. *Let $(\mathcal{K}, \chi)$ be a SNTS, where $\mathcal{K}$ is a K-algebra. Let S be a collection of SNOSs in $\mathcal{K}$ denoted by $S = \{(\mathcal{T}_{\mathcal{A}_j}, \mathcal{I}_{\mathcal{A}_j}, \mathcal{F}_{\mathcal{A}_j}) : j \in J\}$. Let $\mathcal{A}$ be a SNOS in $\mathcal{K}$. Then, S is called a single-valued neutrosophic open covering of $\mathcal{A}$ if $\mathcal{A} \subseteq \bigcup S$.*

Definition 19. *Let $\mathcal{K}$ be a K-algebra and $(\mathcal{K}, \chi)$ be a SNTS. Let L be a finite sub-collection of S. If L is also a single-valued neutrosophic open covering of $\mathcal{A}$, then it is called a finite sub-covering of S and $\mathcal{A}$ is called single-valued neutrosophic compact if each single-valued neutrosophic open covering S of $\mathcal{A}$ has a finite sub-cover. Then, $(\mathcal{K}, \chi)$ is called compact K-algebra.*

Remark 3. *If either $\mathcal{K}$ is a finite K-algebra or χ is a finite topology on $\mathcal{K}$, i.e., consists of finite number of single-valued neutrosophic subsets of $\mathcal{K}$, then the SNT $(\mathcal{K}, \chi)$ is a single-valued neutrosophic compact topological space.*

Proposition 7. *Let $(\mathcal{K}_1, \chi_1)$ and $(\mathcal{K}_2, \chi_2)$ be two SNTSs and f be a single-valued neutrosophic continuous mapping from $\mathcal{K}_1$ into $\mathcal{K}_2$. Let $\mathcal{A}$ be a SNS in $(\mathcal{K}_1, \chi_1)$. If $\mathcal{A}$ is single-valued neutrosophic compact in $(\mathcal{K}_1, \chi_1)$, then $f(\mathcal{A})$ is single-valued neutrosophic compact in $(\mathcal{K}_2, \chi_2)$.*

Proof. Let $f : (\mathcal{K}_1, \chi_1) \to (\mathcal{K}_2, \chi_2)$ be a single-valued neutrosophic continuous function. Let $\acute{S} = (f^{-1}(\mathcal{A}_j : j \in J))$ be a single-valued neutrosophic open covering of $\mathcal{A}$ since $\mathcal{A}$ be a SNS in $(\mathcal{K}_1, \chi_1)$. Let $\acute{L} = (\mathcal{A}_j : j \in J)$ be a single-valued neutrosophic open covering of $f(\mathcal{A})$. Since $\mathcal{A}$ is compact, then there exists a single-valued neutrosophic finite sub-cover $\bigcup\limits_{j=1}^{n} f^{-1}(\mathcal{A}_j)$ such that

$$\mathcal{A} \subseteq \bigcup_{j=1}^{n} f^{-1}(\mathcal{A}_j)$$

We have to prove that there also exists a finite sub-cover of $\acute{L}$ for $f(\mathcal{A})$ such that

$$f(\mathcal{A}) \subseteq \bigcup_{j=1}^{n} (\mathcal{A}_j)$$

Now,

$$\mathcal{A} \subseteq \bigcup_{j=1}^{n} f^{-1}(\mathcal{A}_j)$$

$$f(\mathcal{A}) \subseteq f(\bigcup_{j=1}^{n} f^{-1}(\mathcal{A}_j))$$

$$f(\mathcal{A}) \subseteq \bigcup_{j=1}^{n} (f(f^{-1}(\mathcal{A}_j)))$$

$$f(\mathcal{A}) \subseteq \bigcup_{j=1}^{n} (\mathcal{A}_j).$$

Hence, $f(\mathcal{A})$ is single-valued neutrosophic compact in $(\mathcal{K}_2, \chi_2)$. □

Definition 20. *A single-valued neutrosophic set $\mathcal{A}$ in a K-algebra $\mathcal{K}$ is called a single-valued neutrosophic point if*

$$\mathcal{T}_{\mathcal{A}}(v) = \begin{cases} \alpha \in (0,1], & \text{if } v{=}u \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathcal{I}_{\mathcal{A}}(v) = \begin{cases} \beta \in (0,1], & \text{if } v{=}u \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathcal{F}_{\mathcal{A}}(v) = \begin{cases} \gamma \in [0,1), & \text{if } v{=}u \\ 0, & \text{otherwise}, \end{cases}$$

with support u and value (α, β, γ), denoted by $u(\alpha, \beta, \gamma)$. This single-valued neutrosophic point is said to "belong to" a SNS $\mathcal{A}$, written as $u(\alpha, \beta, \gamma) \in \mathcal{A}$ if $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u) \geq \beta, \mathcal{F}_{\mathcal{A}}(u) \leq \gamma$ and said to be "quasi-coincident with" a SNS $\mathcal{A}$, written as $u(\alpha, \beta, \gamma)\ q\ \mathcal{A}$ if $\mathcal{T}_{\mathcal{A}}(u) + \alpha > 1, \mathcal{I}_{\mathcal{A}}(u) + \beta > 1, \mathcal{F}_{\mathcal{A}}(u) + \gamma < 1$.

Definition 21. *Let $\mathcal{K}$ be a K-algebra and let $(\mathcal{K}, \chi)$ be a SNTS. Then, $(\mathcal{K}, \chi)$ is called a single-valued neutrosophic Hausdorff space if and only if, for any two distinct single-valued neutrosophic points $u_1, u_2 \in \mathcal{K}$, there exist SNOSs $\mathcal{B}_1 = (\mathcal{T}_{\mathcal{B}_1}, \mathcal{I}_{\mathcal{B}_1}, \mathcal{F}_{\mathcal{B}_1}), \mathcal{B}_2 = (\mathcal{T}_{\mathcal{B}_2}, \mathcal{I}_{\mathcal{B}_2}, \mathcal{F}_{\mathcal{B}_2})$ such that $u_1 \in \mathcal{B}_1, u_2 \in \mathcal{B}_2$, i.e.,*

$$\mathcal{T}_{\mathcal{B}_1}(u_1) = 1, \mathcal{I}_{\mathcal{B}_1}(u_1) = 1, \mathcal{F}_{\mathcal{B}_1}(u_1) = 0,$$
$$\mathcal{T}_{\mathcal{B}_2}(u_2) = 1, \mathcal{I}_{\mathcal{B}_2}(u_2) = 1, \mathcal{F}_{\mathcal{B}_2}(u_2) = 0$$

and satisfy the condition that $\mathcal{B}_1 \cap \mathcal{B}_2 = \emptyset_{SN}$. Then, $(\mathcal{K}, \chi)$ is called single-valued neutrosophic Hausdorff space and K-algebra is said to ba a Hausdorff K-algebra. In fact, $(\mathcal{K}, \chi)$ is a Hausdorff K-algebra.

Example 6. *Let $\mathcal{K} = (G, \cdot, \odot, e)$ be a K-algebra and let $(\mathcal{K}, \chi_{\mathcal{K}})$ be a SNTS on $\mathcal{K}$, where $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$ is the cyclic group of order 9 and Caley's table for $\odot$ is given in Example 1. We define two SNSs as $\mathcal{A} = \{(e, 1, 1, 0), (s, 0, 0, 1)\}$. $\mathcal{B} = \{(e, 0, 0, 1), (s, 1, 1, 0)\}$. Consider a single-valued neutrosophic point for $e \in \mathcal{K}$ such that*

$$\mathcal{T}_{\mathcal{A}}(e) = \begin{cases} 0.3, & \text{if } e{=}u \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathcal{I}_{\mathcal{A}}(e) = \begin{cases} 0.2, & \text{if } e{=}u \\ 0, & \text{otherwise}, \end{cases}$$

$$\mathcal{F}_{\mathcal{A}}(e)=\begin{cases} 0.4, & \textit{if } e{=}u \\ 0, & \textit{otherwise.} \end{cases}$$

Then, $e(0.3, 0.2, 0.4)$ is a single-valued neutrosophic point with support e and value $(0.3, 0.2, 0.4)$. This single-valued neutrosophic point belongs to SNS "A" but not SNS "B".

Now, for all $s \neq e \in \mathcal{K}$

$$\mathcal{T}_{\mathcal{B}}(s)=\begin{cases} 0.5, & \textit{if } s{=}u \\ 0, & \textit{otherwise,} \end{cases}$$

$$\mathcal{I}_{\mathcal{B}}(s)=\begin{cases} 0.4, & \textit{if } s{=}u \\ 0, & \textit{otherwise,} \end{cases}$$

$$\mathcal{F}_{\mathcal{B}}(s)=\begin{cases} 0.3, & \textit{if } s{=}u \\ 0, & \textit{otherwise.} \end{cases}$$

Then, $s(0.5, 0.4, 0.3)$ is a single-valued neutrosophic point with support s and value $(0.5, 0.4, 0.3)$. This single-valued neutrosophic point belongs to SNS "B" but not SNS "A". Thus, $e(0.3, 0.2, 0.4) \in \mathcal{A}$ and $e(0.3, 0.2, 0.4) \notin \mathcal{B}$, $s(0.5, 0.4, 0.3) \in \mathcal{B}$ and $s(0.5, 0.4, 0.3) \notin \mathcal{A}$ and $\mathcal{A} \bigcap \mathcal{B} = \emptyset_{SN}$. Thus, K-algebra is a Hausdorff K-algebra and $(\mathcal{K}, \chi_{\mathcal{K}})$ is a Hausdorff topological space.

Theorem 5. *Let $(\mathcal{K}_1, \chi_1)$, $(\mathcal{K}_2, \chi_2)$ be two SNTSs. Let f be a single-valued neutrosophic homomorphism from $(\mathcal{K}_1, \chi_1)$ into $(\mathcal{K}_2, \chi_2)$. Then, $(\mathcal{K}_1, \chi_1)$ is a single-valued neutrosophic Hausdorff space if and only if $(\mathcal{K}_2, \chi_2)$ is a single-valued neutrosophic Hausdorff K-algebra.*

Proof. Let $(\mathcal{K}_1, \chi_1)$, $(\mathcal{K}_2, \chi_2)$ be two SNTSs. Let $\mathcal{K}_1$ be a single-valued neutrosophic Hausdorff space, then, according to the Definition 21, there exist two SNOSs X and Y for two distinct single-valued neutrosophic points $u_1, u_2 \in \chi_2$ also $a, b \in \mathcal{K}_1 (a \neq b)$ such that $X \bigcap Y = \emptyset_{SN}$.
Now, for $w \in \mathcal{K}_1$, consider $(f^{-1}(u_1))(w) = u_1(f^{-1}(w))$, where $u_1(f^{-1}(w)) = s \in (0, 1]$ if $w = f^{-1}(a)$, otherwise 0. That is, $(f^{-1}(u_1))(w) = ((f^{-1}(u))_1(w))$. Therefore, we have $f^{-1}(u_1) = (f^{-1}(u))_1$. Similarly, $f^{-1}(u_2) = (f^{-1}(u))_2$. Now, since f^{-1} is a single-valued neutrosophic continuous mapping from $\mathcal{K}_2$ into $\mathcal{K}_1$, there exist two SNOSs $f(X)$ and $f(Y)$ of u_1 and u_2, respectively, such that $f(X) \bigcap f(Y) = f(\emptyset_{SN}) = \emptyset_{SN}$. This implies that $\mathcal{K}_2$ is a single-valued neutrosophic Hausdorff K-algebra. The converse part can be proved similarly. □

Theorem 6. *Let f be a single-valued neutrosophic continuous function which is both one-one and onto, where f is a mapping from a single-valued neutrosophic compact K-algebra $\mathcal{K}_1$ into a single-valued neutrosophic Hausdorff K-algebra $\mathcal{K}_2$. Then, f is a homomorphism.*

Proof. Let $f : \mathcal{K}_1 \rightarrow \mathcal{K}_2$ be a single-valued neutrosophic continuous bijective function from single-valued neutrosophic compact K-algebra $\mathcal{K}_1$ into a single-valued neutrosophic Hausdorff K-algebra $\mathcal{K}_2$. Since f is a single-valued neutrosophic continuous mapping from $\mathcal{K}_1$ into $\mathcal{K}_2$, f is a homomorphism. Since f is bijective, we only prove that f is single-valued neutrosophic closed. Let $\mathcal{D} = (\mathcal{T}_{\mathcal{D}}, \mathcal{I}_{\mathcal{D}}, \mathcal{F}_{\mathcal{D}})$ be a single-valued neutrosophic closed in $\mathcal{K}_1$. If $\mathcal{D} = \emptyset_{SN}$ is single-valued neutrosophic closed in $\mathcal{K}_1$, then $f(\mathcal{D}) = \emptyset_{SN}$ is single-valued neutrosophic closed in $\mathcal{K}_2$. However, if $\mathcal{D} \neq \emptyset_{SN}$, then $\mathcal{D}$ will be a single-valued neutrosophic compact, being subset of a single-valued neutrosophic compact K-algebra. Then, $f(\mathcal{D})$, being single-valued neutrosophic continuous image of a single-valued neutrosophic compact K-algebra, is also single-valued neutrosophic compact. Therefore, $\mathcal{K}_2$ is closed, which implies that mapping f is closed. Thus, f is a homomorphism. □

4. Conclusions

Non-classical logic is considered as a powerful tool for inspecting uncertainty and indeterminacy found in real world problems. Being a great extension of classical logic, neutrosophic set theory is considered as a useful mathematical tool to cope up with uncertainties in science, technology, and computer science. We have used this mathematical model with a topological structure to investigate the uncertainty in K-algebras. We have introduced the notion of single-valued neutrosophic topological K-algebras and presented certain concepts, including continuous function between two topological on K-algebras, relatively continuous function and homomorphism. We have investigated the image and pre-image of single-valued neutrosophic topological K-algebras under this homomorphism. We have proposed some conclusive concepts, including single-valued neutrosophic compact K-algebras and single-valued neutrosophic Hausdorff K-algebras. We plan to extend our study to: (i) single-valued neutrosophic soft topological K-algebras; and (ii) bipolar neutrosophic soft topological K-algebras.

For other notations and terminologies, readers are referred to [21–26].

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