

Research Article

Neutrosophic Soft Quad Structures Concerning Neutrosophic Soft Points

Arif Mehmood ¹, Farkhanda Afzal ², Muhammad Imran Khan,³ Saleem Abdullah ⁴,
and Saeed Gul ⁵

¹Department of Mathematics, University of Science and Technology Bannu, Pakistan

²Department of Humanities and Basic Sciences MCS, National University of Sciences and Technology, Islamabad, Pakistan

³Department of Pure and Applied Mathematics (Statistics), University of Haripur, Haripur 22630, Pakistan

⁴Department of Mathematics, Abdul Wali Khan University Mardan, Mardan 23200, Pakistan

⁵Faculty of Economics, Kardan University, Parwan-e- Du Square, Kabil, Afghanistan

Correspondence should be addressed to Saeed Gul; s.gul@kardan.edu.af

Received 2 October 2021; Revised 2 December 2021; Accepted 7 December 2021; Published 19 January 2022

Academic Editor: Ewa Rak

Copyright © 2022 Arif Mehmood et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this article, a new concept of neutrosophic soft quad-topological structures is introduced. Generalised neutrosophic soft $*_b$ open sets in neutrosophic soft quad-topological structures concerning soft points of the space are introduced. Different results are addressed in neutrosophic soft quad-topological structure on the basis of these new neutrosophic soft $*_b$ open sets. Neutrosophic soft separation axioms and other separation axioms are addressed in neutrosophic soft quad-topological structures. The engagement of these axioms is switched over with different results with respect to soft points. Neutrosophic soft topological properties of some results are also addressed in neutrosophic soft quad-topological structure. To secure the results, examples are constructed. The nonvalidity of some results are justified with examples. The understanding of some complicated problems is secured with simple techniques.

1. Introduction

Zadeh [1] introduced fuzzy set theory. It addresses vagueness and incomplete data used in various fields of science. Atanassov [2] addressed the short comings in fuzzy set theory with cultured way and opened a door to a new idea with new title intuitionistic fuzzy set theory. Molodtsov [3] inaugurated the concept of soft set theory to address the uncertainty. Soft set theory hugged with many applications in many fields, like smoothness of function, Riemann integration, measurement theory, and game theory. Molodtsov [4] continuously work on soft set theory. Alshehri et al. [5] applied the concept of soft set theory to K-algebra and traced some characteristics of Abelian soft K-algebras. In addition, they introduced the notion of intersection K-algebras and addressed some of their properties. Maji et al. [6] initiated first practical application of soft sets in decision-making problems. Feng et al. [7] for the first time considered

the combination of soft sets, fuzzy sets, and rough sets. Using soft sets as the granulation structures, Feng et al. [8] defined soft approximation spaces, soft rough approximations, and soft rough sets, which are generalizations of Pawlak's rough set model based on soft sets. It has been proved that, in some cases, Feng's soft rough set model could provide better approximations than classical rough sets.

Xu et al. [9] unveiled the concept of vague soft set theory which is an extension to soft set theory. Huang et al. [10] deeply studied [9] and pointed out some incorrect results. They verified the incorrect results with examples and gave some more new definitions. Chang Wang and Yaya Li [11] initiated the concept of vague soft topological structures with title topological structure of vague soft sets. The authors discussed the basic concepts related to vague soft topological space and studied the results in vague soft topology. The soft set to the hyper-soft set was generalized by Smarandache [12]. In addition to this, the author introduced the hybrids of

crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic hyper-soft set. Bera and Mahapatra [13] introduced the concept of neutrosophic soft topology and discussed some fundamental results. Ozturk in [14] introduced new concepts in neutrosophic soft topological spaces. These concepts are boundary, dense set, and neutrosophic soft basis. In addition, the concept of soft subspace on neutrosophic soft topological spaces. Some interesting results are addressed with respect to soft points. Some complicated results are secured with best examples.

Yolcu et al. redefined neutrosophic soft mapping and studied the images and inverse images of neutrosophic soft sets. The authors continued to trace the basic operations and other related properties of neutrosophic soft mapping. The authors beautifully applied neutrosophic soft mapping to application in decision-making problems.

Ozturk et al. [14] are pioneers of new operations on neutrosophic soft sets. Ozturk et al. [15] unveiled the concept of neutrosophic soft mapping, neutrosophic soft open mapping, and neutrosophic soft homeomorphism on the basis of operation defined in [14]. Some results are secured with best understandable examples. Gunduz et al. [16] introduced new concepts of neutrosophic soft sets. They defined new separation axioms in neutrosophic soft topological spaces with respect to soft points. In continuation, the relationship among these neutrosophic soft axioms has been addressed. Interior and closer of neutrosophic soft sets are also addressed. On the basis of these concepts, some other structures are also discussed. Most of the difficult results are secured with best examples.

AL-Nafee [17] introduced new family of neutrosophic soft sets. The author defined new operations on the neutrosophic set. These operations are union and intersection. On the basis of these new operations, the author defined neutrosophic soft topological space. AL-Nafee et al. [18] continued their work and extended the neutrosophic soft topological space to neutrosophic soft bitopological space on the basis of operations defined in [18]. The authors regenerated all the fundamental results of NSBTS on these basic operations. Dadas and Demiralp [19] inaugurated NSBTS on the basis of the operations defined in [13]. The authors introduced pairwise neutrosophic soft (closed) sets in NSBTS. These references [13–19] became source of motivation for my new research.

In our study, we have worked with the operations given in references [14–16] which are entirely different from references [13, 17]. In Section 2, some basic recipes are inaugurated. Originality begins in Section 3. In this section, new concepts of neutrosophic soft quad-topological spaces are addressed with examples. Some results, union and intersection are also studied in neutrosophic soft quad-topological spaces. In Section 4, some important definitions of generalized neutrosophic soft open sets in neutrosophic soft quad topological spaces are introduced. These definitions are semiopen, preopen, and $*_b$ open sets, respectively. These definitions became source of generation of different neutrosophical soft separation axioms and neutrosophical soft other separation axioms in neutrosophic soft quad topological spaces with respect to soft points of the second space.

Neutrosophical soft quad homeomorphism which is a safe carriage for different structure from one space to another is defined. Soft closer attachment with neutrosophical soft separation axioms and neutrosophical soft other separation axioms in neutrosophic soft quad topological spaces with respect to soft points of the second space are addressed. In Section 5, more main results are addressed. Among neutrosophical soft separation axioms, Hausdorff space is considered to be the most important separation axioms. Important things should be given serious attention. So, this section has almost been engaged with the study of Hausdorff space. Through neutrosophical soft function, the characteristics of one space can be migrated to another space if the neutrosophical soft function is satisfying conditions of neutrosophical soft bijections and bicontinuousness. A soft function satisfying these conditions is known as neutrosophical soft homeomorphism. Neutrosophical soft homeomorphism is giving birth to neutrosophical soft topological property. Some neutrosophical soft topological properties of Hausdorff space are addressed with respect to soft points. Sequentially, compactness and countably compact in neutrosophic soft quad topological spaces with respect to soft points of the second space are addressed. Engagement of Hausdorff space with closed sets in neutrosophic soft quad topological spaces is addressed. In Section 6, some concluding remarks and future work are given.

2. Fundamental Concepts

In this section, fundamental concepts are addressed. These fundamental concepts are neutrosophic set, soft set, neutrosophic soft set, neutrosophic soft complement, neutrosophic soft subsets, neutrosophic soft union, neutrosophic soft intersection, null neutrosophic soft, neutrosophic soft absolute set, neutrosophic soft points, and neutrosophic soft topological spaces.

Definition 1 (see [20]). A neutrosophic set (NS) symbolized by A on the key set \mathfrak{Z} is defined as: $A = \{\mathfrak{E}, \mathbb{T}_A \mathfrak{E}, \mathbb{I}_A \mathfrak{E},$

$$\mathbb{F}_A \mathfrak{E}: \mathfrak{E} \in \mathfrak{Z}\}, \text{ where } \begin{bmatrix} T: \mathfrak{Z}(0^-, 1^+) \\ I: \mathfrak{Z}(0^-, 1^+), \\ F: \mathfrak{Z}(0^-, 1^+) \\ 0^- \circ \mathbb{T}_A \langle \mathfrak{E} \rangle + \mathbb{I} \langle \mathfrak{E} \rangle \\ + \mathbb{F}_A \langle \mathfrak{E} \rangle \circ 3^+ \end{bmatrix},$$

Definition 2 (see [3]). Let \mathfrak{Z} be the key set, ∂ be a set of parameters, and $\mathcal{L}(\mathfrak{Z})$ symbolizes the power set of \mathfrak{Z} . A pair $\tilde{\eta}, \partial$ is referred to as a soft set (SS) over \mathfrak{Z} , where $\tilde{\eta}$ is a map given by $\tilde{\eta}: \partial \longrightarrow \mathcal{L}(\mathfrak{Z})$.

Definition 3 (see [21, 22]). Let \mathfrak{Z} be the key set and ∂ set of parameters. Let $\mathcal{L}(\mathfrak{Z})$ signifies power set of all neutrosophic sets on \mathfrak{Z} . Then, a neutrosophic soft set $\tilde{\eta}, \partial$ over \mathfrak{Z} is a set defined by a set valued function $\tilde{\eta}$ representing a mapping $\tilde{\eta}: \partial \longrightarrow \mathcal{L}(\mathfrak{Z})$, where $\tilde{\eta}$ is called approximate function of the neutrosophic soft set $\langle \tilde{\eta}, \partial \rangle$. It can be written as a set of

ordered pairs: $\tilde{\eta}, \partial = \{(\mathfrak{s}, \mathfrak{E}, \mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} : \mathfrak{E} \in \mathfrak{Q}) : \mathfrak{s} \in \partial\}$.

Definition 4 (see [13]). Let $\tilde{\eta}, \partial$ be a neutrosophic soft set over key set \mathfrak{Q} , the complement of $\tilde{\eta}, \partial$ is signified $\tilde{\eta}, \partial^c$ and is defined as follows:

$\tilde{\eta}, \partial^c = \{(\mathfrak{s}, \mathfrak{E}, \mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, 1 - \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} : \mathfrak{E} \in \mathfrak{Q}) : \mathfrak{s} \in \partial\}$. It is clear that

$$(\langle \tilde{\eta}, \partial^c \rangle)^c = \tilde{\eta}, \partial. \quad (1)$$

Definition 5 (see [23, 24]). Let $\langle \tilde{\eta}, \partial \rangle$ and $\langle \tilde{\rho}, \partial \rangle$ are two neutrosophic soft set over key set \mathfrak{Q} . Then, $\langle \tilde{\eta}, \partial \rangle$ is said to be neutrosophic soft subset of $\langle \tilde{\rho}, \partial \rangle$ if $\mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} \leq \mathbb{T}_{\tilde{\rho}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} \leq \mathbb{I}_{\tilde{\rho}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} \geq \mathbb{F}_{\tilde{\rho}(\mathfrak{s})}^{\mathfrak{E}}, \forall \mathfrak{s} \in \partial$, and $\forall \mathfrak{E} \in \mathfrak{Q}$. It is denoted by $\langle \tilde{\eta}, \partial \rangle \subseteq \langle \tilde{\rho}, \partial \rangle$. Neutrosophic soft $\langle \tilde{\eta}, \partial \rangle$ is said to be neutrosophic soft equal to $\langle \tilde{\rho}, \partial \rangle$, if $\langle \tilde{\eta}, \partial \rangle$ is neutrosophic soft subset of $\langle \tilde{\rho}, \partial \rangle$ and $\langle \tilde{\rho}, \partial \rangle$ neutrosophic soft subset of $\langle \tilde{\eta}, \partial \rangle$. It is denoted by $\langle \tilde{\eta}, \partial \rangle = \langle \tilde{\rho}, \partial \rangle$.

Definition 6 (see [16]). Let $\langle \tilde{\eta}_1, \partial \rangle, \langle \tilde{\eta}_2, \partial \rangle$ be two neutrosophic soft subsets over key set \mathfrak{Q} so that $\langle \tilde{\eta}_1, \partial \rangle \neq \langle \tilde{\eta}_2, \partial \rangle$, then their union is denoted by $\langle \tilde{\eta}_1, \partial \rangle \cup \langle \tilde{\eta}_2, \partial \rangle = \langle \tilde{\eta}_3, \partial \rangle$ and is defined as $\langle \tilde{\eta}_3, \partial \rangle = \{(\mathfrak{s}, \mathfrak{E}, \mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} : \mathfrak{E} \in \mathfrak{Q}) : \mathfrak{s} \in \partial\}$, where

$$\begin{cases} \mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = \max[\mathbb{T}_{\tilde{\mathfrak{f}}_1(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{T}_{\tilde{\mathfrak{f}}_2(\mathfrak{s})}^{\mathfrak{E}}], \\ \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = \max[\mathbb{I}_{\tilde{\mathfrak{f}}_1(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{I}_{\tilde{\mathfrak{f}}_2(\mathfrak{s})}^{\mathfrak{E}}], \\ \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = \min[\mathbb{F}_{\tilde{\mathfrak{f}}_1(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{F}_{\tilde{\mathfrak{f}}_2(\mathfrak{s})}^{\mathfrak{E}}]. \end{cases} \quad (2)$$

Definition 7 (see [16]). Let $\langle \tilde{\eta}_1, \partial \rangle$ and $\langle \tilde{\eta}_2, \partial \rangle$ be two neutrosophic soft subsets over key set \mathfrak{Q} such that $\langle \tilde{\eta}_1, \partial \rangle \neq \langle \tilde{\eta}_2, \partial \rangle$, then their intersection is defined as $\langle \tilde{\eta}_1, \partial \rangle \cap \langle \tilde{\eta}_2, \partial \rangle = \langle \tilde{\eta}_3, \partial \rangle$ is given as $[\langle \tilde{\eta}_3, \partial \rangle = \{(\mathfrak{s}, \mathfrak{E}, \mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} : \mathfrak{E} \in \mathfrak{Q}) : \mathfrak{s} \in \partial\}]$ where

$$\begin{cases} \mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = \min[\mathbb{T}_{\tilde{\mathfrak{f}}_1(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{T}_{\tilde{\mathfrak{f}}_2(\mathfrak{s})}^{\mathfrak{E}}], \\ \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = \min[\mathbb{I}_{\tilde{\mathfrak{f}}_1(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{I}_{\tilde{\mathfrak{f}}_2(\mathfrak{s})}^{\mathfrak{E}}], \\ \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = \max[\mathbb{F}_{\tilde{\mathfrak{f}}_1(\mathfrak{s})}^{\mathfrak{E}}, \mathbb{F}_{\tilde{\mathfrak{f}}_2(\mathfrak{s})}^{\mathfrak{E}}]. \end{cases} \quad (3)$$

Definition 8 (see [14]). Let $\tilde{\eta}, \partial$ be a neutrosophic soft set over a key set \mathfrak{Q} , then it is said to be a null neutrosophic soft set if $\mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = 0, \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = 0, \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = 1; \forall \mathfrak{s} \in \partial, \forall \mathfrak{E} \in \mathfrak{Q}$.

It is signified as $0_{(\mathfrak{Q}, \partial)}$.

Definition 9 (see [14]). Let $\tilde{\eta}, \partial$ be a neutrosophic soft set over a key set \mathfrak{Q} , then it is said to be an absolute soft set if $\mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = 1, \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = 1, \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}} = 0; \forall \mathfrak{s} \in \partial$ and for all $\mathfrak{E} \in \mathfrak{Q}$.

It is signified as $1_{(\mathfrak{Q}, \partial)}$; clearly, $0_{(\mathfrak{Q}, \partial)}^c = 1_{(\mathfrak{Q}, \partial)}, 1_{(\mathfrak{Q}, \partial)}^c = 0_{(\mathfrak{Q}, \partial)}$.

Definition 10 (see [14]). Let $NSS(\tilde{\mathfrak{Q}}, \partial)$ be the family of all neutrosophic soft sets and $\tau \subseteq NSS(\tilde{\mathfrak{Q}}, \partial)$, then τ is said to be a neutrosophic soft topology on $\tilde{\mathfrak{Q}}$ if (1): $0_{(\mathfrak{Q}, \partial)}, 1_{(\mathfrak{Q}, \partial)} \in \tau$, (2): the union of any number of neutrosophic soft sets in $\tau \in \tau$, (3): the intersection of a finite number of neutrosophic soft sets in $\tau \in \tau$; then, $(\tilde{\mathfrak{Q}}, \tau, \partial)$ is said to be an NSTS over $\tilde{\mathfrak{Q}}$.

Definition 11 (see [14]). Let NS be the family of all neutrosophic sets over $\tilde{\mathfrak{Q}}, \mathfrak{E} \in \tilde{\mathfrak{Q}}$, then neutrosophic set $\mathfrak{E}_{(\gamma_1, \gamma_2, \gamma_3)}^{(\gamma)}$ is neutrosophic point, for $0 < \gamma_1, \gamma_2, \gamma_3 \leq 1$ and is defined as follows:

$$\mathfrak{E}_{(\gamma_1, \gamma_2, \gamma_3)}^{(\gamma)} = \left[\begin{cases} (\gamma_1, \gamma_2, \gamma_3) \text{ if } y = \mathfrak{E} \\ (0, 0, 1) \text{ if } y \neq \mathfrak{E} \end{cases} \right]. \quad (4)$$

Definition 12 (see [14]). Let $NSS(\tilde{\mathfrak{Q}}, \partial)$ be the family of all neutrosophic soft sets over KS \mathfrak{Q} . Then, neutrosophic soft set $(\mathfrak{E}_{(\gamma_1, \gamma_2, \gamma_3)}^{\mathfrak{s}})^{\mathfrak{s}}$ is called a neutrosophic soft point if for every $\mathfrak{E} \in \tilde{\mathfrak{Q}}, 0 < \gamma_1, \gamma_2, \gamma_3 \leq 1, \mathfrak{s} \in \partial$ and is defined as follows:

$$\mathfrak{E}^{\mathfrak{s}} = \left[\begin{cases} (\gamma_1, \gamma_2, \gamma_3) \text{ if } \mathfrak{s} = \mathfrak{s}, \text{ then } y = \mathfrak{E} \\ (0, 0, 1) \text{ if } \mathfrak{s} \neq \mathfrak{s}, \text{ then } y \neq \mathfrak{E} \end{cases} \right]. \quad (5)$$

Definition 13 (see [14]). Let $\tilde{\eta}, \partial$ be a neutrosophic soft sets over KS \mathfrak{Q} , then $\mathfrak{E}^{\mathfrak{s}} \in NSS(\tilde{\eta}, \partial)$ if

$$\gamma_1 \leq \mathbb{T}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \gamma_2 \leq \mathbb{I}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}, \gamma_3 \geq \mathbb{F}_{\tilde{\mathfrak{f}}(\mathfrak{s})}^{\mathfrak{E}}. \quad (6)$$

Definition 14 (see [14]). Let $(\tilde{\mathfrak{Q}}, \tau, \partial)$ be a soft neutrosophic topological space over \mathfrak{Q} . Let $\tilde{\eta}, \partial$ be a neutrosophic soft set. Then, $\tilde{\eta}, \partial$ is called a neutrosophic soft neighborhood of the neutrosophic soft point $\mathfrak{E}^{\mathfrak{s}}$, if there exists a neutrosophic soft open set (\mathcal{U}, ∂) such that $\mathfrak{E}^{\mathfrak{s}} \in (\mathcal{U}, \partial) \subseteq \tilde{\eta}, \partial$.

3. Characterisation of Neutrosophic Soft Quad-Topological Structures

In this section, the concept of neutrosophic soft quad topological space (NSQTS) is defined. Furthermore, new types of open and closed sets have been introduced in neutrosophic soft quad topological spaces.

3.1. Definition. If $(\mathfrak{Q}, \tau_1, \partial), (\mathfrak{Q}, \tau_2, \partial), (\mathfrak{Q}, \tau_3, \partial)$, and $(\mathfrak{Q}, \tau_4, \partial)$ are four neutrosophic soft topological space (NSTS), then $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is called neutrosophic soft quad topological space. A neutrosophic soft subset $\langle \tilde{\eta}, \partial \rangle$ is said to be neutrosophic soft quad open in $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ if there exists a neutrosophic soft open set $\langle \tilde{\eta}_1, \partial \rangle$ in τ_1 , neutrosophic soft open set $\langle \tilde{\eta}_2, \partial \rangle$ in τ_2 , neutrosophic soft open set $\langle \tilde{\eta}_3, \partial \rangle$ in τ_3 and neutrosophic soft open $\langle \tilde{\eta}_4, \partial \rangle$ in τ_4 such that $\langle \tilde{\eta}, \partial \rangle = \langle \tilde{\eta}_1, \partial \rangle \cup \langle \tilde{\eta}_2, \partial \rangle \cup \langle \tilde{\eta}_3, \partial \rangle \cup \langle \tilde{\eta}_4, \partial \rangle$.

3.2. *Example.* Let $\mathfrak{X} = \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}$, $\partial = \{\mathfrak{s}_1, \mathfrak{s}_2\}$ and $\tau_1 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}, (p_1, \partial), (p_2, \partial)\}$, $\tau_2 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}, (\mathbb{J}_1, \partial), (\mathbb{J}_2, \partial)\}$, $\tau_3 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}, (\mathfrak{Q}_1, \partial), (\mathfrak{Q}_2, \partial)\}$, and $\tau_4 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}\}$ where $(p_1, \partial), (p_2, \partial), (\mathbb{J}_1, \partial), (\mathbb{J}_2, \partial)$ and $(\mathfrak{Q}_1, \partial), (\mathfrak{Q}_2, \partial)$ being NSSs as follows:

$$f_{(p_1, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.2 \times 10^{-1}, 0.3 \times 10^{-1}, 0.8 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.2 \times 10^{-4}, 0.3 \times 10^{-4}, 0.8 \times 10^{-4} \rangle, \\ \langle \mathfrak{E}_3, 0.2 \times 10^{-1}, 0.4 \times 10^{-1}, 0.3 \times 10^{-1} \rangle \end{bmatrix} \quad (7)$$

$$f_{(p_1, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.3 \times 10^{-1}, 0.2 \times 10^{-1}, 0.6 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.1 \times 10^{-1}, 0.5 \times 10^{-1}, 0.5 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.4 \times 10^{-1}, 0.3 \times 10^{-1}, 0.4 \times 10^{-1} \rangle \end{bmatrix} \quad (8)$$

$$f_{(p_2, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.4 \times 10^{-1}, 0.3 \times 10^{-1}, 0.6 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.4 \times 10^{-1}, 0.5 \times 10^{-1}, 0.3 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.3 \times 10^{-1}, 0.5 \times 10^{-1}, 0.2 \times 10^{-1} \rangle \end{bmatrix} \quad (9)$$

$$f_{(p_2, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.3 \times 10^{-1}, 0.4 \times 10^{-1}, 0.5 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.2 \times 10^{-1}, 0.6 \times 10^{-1}, 0.4 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.4 \times 10^{-1}, 0.6 \times 10^{-1}, 0.3 \times 10^{-1} \rangle \end{bmatrix} \quad (10)$$

$$f_{(\mathbb{J}_1, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.5 \times 10^{-1}, 0.4 \times 10^{-1}, 0.4 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.6 \times 10^{-1}, 0.6 \times 10^{-1}, 0.2 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.4 \times 10^{-1}, 0.6 \times 10^{-1}, 0.1 \times 10^{-1} \rangle \end{bmatrix} \quad (11)$$

$$f_{(\mathbb{J}_1, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.4 \times 10^{-1}, 0.6 \times 10^{-1}, 0.3 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.3 \times 10^{-1}, 0.7 \times 10^{-1}, 0.3 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.5 \times 10^{-1}, 0.7 \times 10^{-1}, 0.1 \times 10^{-1} \rangle \end{bmatrix} \quad (12)$$

$$\mathbb{J}f_{(\mathbb{J}_2, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.1 \times 10^{-1}, 0.2 \times 10^{-1}, 0.7 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.3 \times 10^{-1}, 0.3 \times 10^{-1}, 0.3 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.1 \times 10^{-1}, 0.2 \times 10^{-1}, 0.2 \times 10^{-1} \rangle \end{bmatrix} \quad (13)$$

$$f_{(\mathbb{J}_2, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.1 \times 10^{-1}, 0.2 \times 10^{-1}, 0.7 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.3 \times 10^{-1}, 0.3 \times 10^{-1}, 0.3 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.1 \times 10^{-1}, 0.2 \times 10^{-1}, 0.2 \times 10^{-1} \rangle \end{bmatrix} \quad (14)$$

$$f_{(\mathfrak{Q}_1, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.6 \times 10^{-1}, 0.5 \times 10^{-1}, 0.2 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.4 \times 10^{-1}, 0.5 \times 10^{-1}, 0.2 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.4 \times 10^{-1}, 0.5 \times 10^{-1}, 0.1 \times 10^{-1} \rangle \end{bmatrix} \quad (15)$$

$$f_{(\mathfrak{Q}_1, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.3 \times 10^{-1}, 0.4 \times 10^{-1}, 0.4 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.3 \times 10^{-1}, 0.6 \times 10^{-1}, 0.2 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.4 \times 10^{-1}, 0.6 \times 10^{-1}, 0.1 \times 10^{-1} \rangle \end{bmatrix} \quad (16)$$

$$f_{(\mathfrak{Q}_2, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.5 \times 10^{-1}, 0.4 \times 10^{-1}, 0.2 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.4 \times 10^{-1}, 0.5 \times 10^{-1}, 0.2 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.3 \times 10^{-1}, 0.5 \times 10^{-1}, 0.1 \times 10^{-1} \rangle \end{bmatrix} \quad (17)$$

$$f_{(\mathfrak{Q}_2, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.3 \times 10^{-1}, 0.4 \times 10^{-1}, 0.5 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.2 \times 10^{-1}, 0.6 \times 10^{-1}, 0.2 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.4 \times 10^{-1}, 0.5 \times 10^{-1}, 0.1 \times 10^{-1} \rangle \end{bmatrix} \quad (18)$$

Therefore, τ_1, τ_2, τ_3 , and τ_4 are NSQTS on \mathfrak{X} and so $(\mathfrak{X}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is a NSQTS.

3.3. *Theorem.* Let $(\mathfrak{X}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS. Then, $\tau_1 \widetilde{\cap} \tau_2 \widetilde{\cap} \tau_3 \widetilde{\cap} \tau_4$ is a NSQTS on \mathfrak{X} .

Proof. For this, we have to verify all the three conditions of neutrosophic soft quad topological space. Conditions (1) and (3) are obvious; for condition (2), let $\{(p_i, \partial); i \in I\} \in \tau_1 \widetilde{\cap} \tau_2 \widetilde{\cap} \tau_3 \widetilde{\cap} \tau_4$, then $(p_i, \partial) \in \tau_1, (p_i, \partial) \in \tau_2, (p_i, \partial) \in \tau_3$, and $(p_i, \partial) \in \tau_4$. As τ_1, τ_3 , and τ_4 are NSTS on \mathfrak{X} , then $\bigcup_i (p_i, \partial) \in \tau_1, \bigcup_i (p_i, \partial) \in \tau_2, \bigcup_i (p_i, \partial) \in \tau_3$, and $\bigcup_i (p_i, \partial) \in \tau_4$. Therefore, $\bigcup_i (p_i, \partial) \in \tau_1 \widetilde{\cap} \tau_2 \widetilde{\cap} \tau_3 \widetilde{\cap} \tau_4$. \square

Remark 1. Let $(\mathfrak{X}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS, then $\tau_1 \widetilde{\cup} \tau_2 \widetilde{\cup} \tau_3 \widetilde{\cup} \tau_4$ need not be a NSQTS on \mathfrak{X} .

3.4. *Example.* Let $\mathfrak{X} = \{\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3\}$, $\partial = \{\mathfrak{s}_1, \mathfrak{s}_2\}$ and $\tau_1 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}, (p_1, \partial), (p_2, \partial), (p_3, \partial)\}$, $\tau_2 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}, (\mathbb{J}_1, \partial), (\mathbb{J}_2, \partial)\}$, $\tau_3 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}, (\mathfrak{Q}_1, \partial), (\mathfrak{Q}_2, \partial)\}$, and $\tau_4 = \{0_{(\mathfrak{X}, \partial)}, 1_{(\mathfrak{X}, \partial)}\}$, where $(p_1, \partial), (p_2, \partial), (\mathbb{J}_1, \partial), (\mathbb{J}_2, \partial)$ and $(\mathfrak{Q}_1, \partial), (\mathfrak{Q}_2, \partial)$ being NSSs as follows:

$$f_{(p_1, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.2 \times 10^{-1}, 0.3 \times 10^{-1}, 0.8 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.4 \times 10^{-1}, 0.4 \times 10^{-1}, 0.4 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.2 \times 10^{-1}, 0.4 \times 10^{-1}, 0.3 \times 10^{-1} \rangle \end{bmatrix} \quad (19)$$

$$f_{(p_1, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 0.3 \times 10^{-1}, 0.2 \times 10^{-1}, 0.6 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 0.1 \times 10^{-1}, 0.5 \times 10^{-1}, 0.5 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 0.4 \times 10^{-1}, 0.3 \times 10^{-1}, 0.5 \times 10^{-1} \rangle \end{bmatrix} \quad (20)$$

$$f_{(p_2, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 04 \times 10^{-1}, 03 \times 10^{-1}, 06 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 04 \times 10^{-1}, 05 \times 10^{-1}, 03 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 03 \times 10^{-1}, 05 \times 10^{-1}, 02 \times 10^{-1} \rangle \end{bmatrix} \quad (21)$$

$$f_{(p_2, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 03 \times 10^{-1}, 04 \times 10^{-1}, 05 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 02 \times 10^{-1}, 06 \times 10^{-1}, 04 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} \rangle \end{bmatrix} \quad (22)$$

$$f_{(p_3, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 05 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 06 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 00 \times 10^{-1} \rangle \end{bmatrix} \quad (23)$$

$$f_{(p_3, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 03 \times 10^{-1}, 07 \times 10^{-1}, 03 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 05 \times 10^{-1}, 07 \times 10^{-1}, 01 \times 10^{-1} \rangle \end{bmatrix} \quad (24)$$

$$f_{(\mathbb{J}_1, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 05 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 06 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 01 \times 10^{-1} \rangle \end{bmatrix} \quad (25)$$

$$f_{(\mathbb{J}_1, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 03 \times 10^{-1}, 07 \times 10^{-1}, 03 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 05 \times 10^{-1}, 07 \times 10^{-1}, 01 \times 10^{-1} \rangle \end{bmatrix} \quad (26)$$

$$f_{(\mathbb{J}_2, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 01 \times 10^{-1}, 02 \times 10^{-1}, 07 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 03 \times 10^{-1}, 03 \times 10^{-1}, 03 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 01 \times 10^{-1}, 02 \times 10^{-1}, 02 \times 10^{-1} \rangle \end{bmatrix} \quad (27)$$

$$f_{(\mathbb{J}_2, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 01 \times 10^{-1}, 02 \times 10^{-1}, 07 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 03 \times 10^{-1}, 03 \times 10^{-1}, 03 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 01 \times 10^{-1}, 02 \times 10^{-1}, 02 \times 10^{-1} \rangle \end{bmatrix} \quad (28)$$

$$f_{(\mathfrak{Q}_1, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 06 \times 10^{-1}, 05 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 04 \times 10^{-1}, 05 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 04 \times 10^{-1}, 05 \times 10^{-1}, 01 \times 10^{-1} \rangle \end{bmatrix} \quad (29)$$

$$f_{(\mathfrak{Q}_1, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 03 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 03 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 01 \times 10^{-1} \rangle \end{bmatrix} \quad (30)$$

$$f_{(\mathfrak{Q}_2, \partial)}(\mathfrak{s}_1) = \begin{bmatrix} \langle \mathfrak{E}_1, 03 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 03 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 01 \times 10^{-1} \rangle \end{bmatrix} \quad (31)$$

$$f_{(\mathfrak{Q}_2, \partial)}(\mathfrak{s}_2) = \begin{bmatrix} \langle \mathfrak{E}_1, 04 \times 10^{-1}, 05 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_2, 04 \times 10^{-1}, 05 \times 10^{-1}, 02 \times 10^{-1} \rangle, \\ \langle \mathfrak{E}_3, 03 \times 10^{-1}, 05 \times 10^{-1}, 01 \times 10^{-1} \rangle \end{bmatrix} \quad (32)$$

Here, $\tau_1 \tilde{\cup} \tau_2 \tilde{\cup} \tau_3 \tilde{\cup} \tau_4 = \{0_{(\mathfrak{Q}, \partial)}, 1_{(\mathfrak{Q}, \partial)}, (p_1, \partial), (p_2, \partial), (p_3, \partial), (\mathbb{J}_1, \partial), (\mathbb{J}_2, \partial), (\mathfrak{Q}_1, \partial), (\mathfrak{Q}_2, \partial)\}$ is not a NSQTS on \mathfrak{Q} .

3.5. Definition. Let $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS. Then, an NSS

$$(\mathbb{J}, \partial) = \{\mathfrak{s}, \{\langle \mathfrak{E}, T_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}), I_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}), F_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}) \rangle\} : \mathfrak{E} \in \mathfrak{Q}, \mathfrak{s} \in \partial\}, \quad (33)$$

is called as a pairwise NSQ open set (PNSOS) if there exists a NSOS (\mathbb{J}_1, ∂) in τ_1 , NSOS (\mathbb{J}_2, ∂) in τ_2 , NSOS (\mathbb{J}_3, ∂) in τ_3 and NSOS (\mathbb{J}_4, ∂) in τ_4 such that for all $\mathfrak{E} \in \mathfrak{Q}$ such that

$$\begin{aligned} (\mathbb{J}, \partial) &= (\mathbb{J}_1, \partial) \tilde{\cup} (\mathbb{J}_2, \partial) \tilde{\cup} (\mathbb{J}_3, \partial) \tilde{\cup} (\mathbb{J}_4, \partial) \\ &= \left\{ \left(\mathfrak{s}, \left\{ \begin{array}{l} \langle \mathfrak{E}, \max\{T_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}), T_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E})\}, \\ \max\{I_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}), I_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E})\}, \min\{F_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}), F_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E})\} \rangle \end{array} \right\} : \mathfrak{s} \in \partial \right\}. \end{aligned} \quad (34)$$

3.6. Definition. Let $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS. Then, an NSS

$$(\mathbb{J}, \partial) = \left\{ \left(\mathfrak{s}, \{\langle \mathfrak{E}, T_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}), I_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}), F_{\mathbb{J}(\mathfrak{s})}(\mathfrak{E}) \rangle\} \right) : \mathfrak{E} \in \mathfrak{Q}, \mathfrak{s} \in \partial \right\} \quad (35)$$

is called as a PNSOS if there exist a NSOS (\mathbb{J}_1, ∂) in τ_1 and a NSOS (\mathbb{J}_2, ∂) in τ_2 , NSOS (\mathbb{J}_3, ∂) in τ_3 , and NSOS (\mathbb{J}_4, ∂) in τ_4 such that, for all $\mathbb{E} \in \mathfrak{Z}$,

$$(\mathbb{J}, \partial) = (\mathbb{J}_1, \partial) \widetilde{\cup} (\mathbb{J}_2, \partial) \widetilde{\cup} (\mathbb{J}_3, \partial) \widetilde{\cup} (\mathbb{J}_4, \partial) = \left\{ \left(\mathfrak{s}, \left\{ \begin{array}{l} < \mathbb{E}, \max\{T_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), T_{\mathbb{J}(\mathfrak{s})}(\mathbb{E})\}, \\ \max\{I_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), I_{\mathbb{J}(\mathfrak{s})}(\mathbb{E})\}, \min\{F_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), F_{\mathbb{J}(\mathfrak{s})}(\mathbb{E})\} > \end{array} \right\} : \mathfrak{s} \in \partial \right\}. \quad (36)$$

The set of all pairwise neutrosophic soft quad open sets in $(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is denoted by PNSO $(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$.

3.7. Definition. Let $(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS. Then, a NSS

$$(\mathbb{J}, \partial) = \left\{ \left(\mathfrak{s}, \left\{ \langle \mathbb{E}, T_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), I_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), F_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}) \rangle \right\} : \mathbb{E} \in \mathfrak{Z}, \mathfrak{s} \in \partial \right\}, \quad (37)$$

is called as a pairwise neutrosophic soft quad closed set (PNSC) if $(\mathbb{J}, \partial)^c$ is a PNSO. It is clear that (\mathbb{J}, ∂) is a PNSC set if there exists a NSOC (\mathbb{J}_1, ∂) in τ_1 , NSOC (\mathbb{J}_2, ∂) in τ_2 ,

NSOC (\mathbb{J}_3, ∂) in τ_3 , and NSOC (\mathbb{J}_4, ∂) in τ_4 such that, for all $\mathbb{E} \in \mathfrak{Z}$,

$$(\mathbb{J}, \partial) = (\mathbb{J}_1, \partial) \widetilde{\cup} (\mathbb{J}_2, \partial) \widetilde{\cup} (\mathbb{J}_3, \partial) \widetilde{\cup} (\mathbb{J}_4, \partial) = \left\{ \left(\mathfrak{s}, \left\{ \begin{array}{l} < \mathbb{E}, \max\{T_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), T_{\mathbb{J}(\mathfrak{s})}(\mathbb{E})\}, \\ \max\{I_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), I_{\mathbb{J}(\mathfrak{s})}(\mathbb{E})\}, \min\{F_{\mathbb{J}(\mathfrak{s})}(\mathbb{E}), F_{\mathbb{J}(\mathfrak{s})}(\mathbb{E})\} > \end{array} \right\} : \mathfrak{s} \in \partial \right\}. \quad (38)$$

The set of all PNSC in $(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is denoted by PNCS $(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$.

3.8. Theorem. Let $(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSBTS. In this case,

- (1) $0_{(\mathfrak{Z}, \partial)}, 1_{(\mathfrak{Z}, \partial)} \in \text{PNSO}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$
- (2) If $\{(\mathbb{J}_i, \partial) | i \in I\} \in \text{PNSO}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$, then $\bigcup_{i \in I} (\mathbb{J}_i, \partial) \in \text{PNSO}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$
- (3) If $\{(\mathbb{U}_i, \partial) | i \in I\} \in \text{PNCS}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ then $\bigcap_{i \in I} (\mathbb{U}_i, \partial) \in \text{PNCS}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$

Proof

- (1) Since $0_{(\mathfrak{Z}, \partial)} \cup 0_{(\mathfrak{Z}, \partial)} = 0_{(\mathfrak{Z}, \partial)}$ and $1_{(\mathfrak{Z}, \partial)} \cup 1_{(\mathfrak{Z}, \partial)} = 1_{(\mathfrak{Z}, \partial)}$, then $0_{(\mathfrak{Z}, \partial)}$ and $1_{(\mathfrak{Z}, \partial)}$ are PNSO.
- (2) Since $(\mathbb{J}_i, \partial) \in \text{PNSO}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$, there exist $(\mathbb{J}_i^1, \partial) \in \tau_1$, $(\mathbb{J}_i^2, \partial) \in \tau_2$, $(\mathbb{J}_i^3, \partial) \in \tau_3$, and $(\mathbb{J}_i^4, \partial) \in \tau_4$ such that $(\mathbb{J}_i, \partial) = (\mathbb{J}_i^1, \partial) \widetilde{\cup} (\mathbb{J}_i^2, \partial) \widetilde{\cup} (\mathbb{J}_i^3, \partial) \widetilde{\cup} (\mathbb{J}_i^4, \partial)$ for all $i \in I$. Then,

$$\bigcup_{i \in I} (\mathbb{J}_i, \partial) = \bigcup_{i \in I} ((\mathbb{J}_i, \partial) = (\mathbb{J}_i^1, \partial) \widetilde{\cup} (\mathbb{J}_i^2, \partial) \widetilde{\cup} (\mathbb{J}_i^3, \partial) \widetilde{\cup} (\mathbb{J}_i^4, \partial)) = \left(\bigcup_{i \in I} (\mathbb{J}_i^1, \partial) \right) \widetilde{\cup} \left(\bigcup_{i \in I} (\mathbb{J}_i^2, \partial) \right) \widetilde{\cup} \left(\bigcup_{i \in I} (\mathbb{J}_i^3, \partial) \right) \widetilde{\cup} \left(\bigcup_{i \in I} (\mathbb{J}_i^4, \partial) \right). \quad (39)$$

As τ_1, τ_2, τ_3 and τ_4 are NSTS on $U_{i \in I} (\mathbb{J}_i^1, \partial) \in \tau_1$, $U_{i \in I} (\mathbb{J}_i^2, \partial) \in \tau_2$, $U_{i \in I} (\mathbb{J}_i^3, \partial) \in \tau_3$, and $U_{i \in I} (\mathbb{J}_i^4, \partial) \in \tau_4$. Therefore, $\bigcup_{i \in I} (\mathbb{J}_i, \partial) \in \text{PNSO}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$.

- (3) Since $(\mathbb{U}_i^1, \partial) \in \text{PNCS}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$, there exist $(\mathbb{U}_i^1, \partial)^c \in \tau_1$, $(\mathbb{U}_i^2, \partial)^c \in \tau_2$, $(\mathbb{U}_i^3, \partial)^c \in \tau_3$, and $(\mathbb{U}_i^4, \partial)^c \in \tau_4$ such that $(\mathbb{U}_i, \partial) = (\mathbb{U}_i^1, \partial) \widetilde{\cap} (\mathbb{U}_i^2, \partial) \widetilde{\cap} (\mathbb{U}_i^3, \partial) \widetilde{\cap} (\mathbb{U}_i^4, \partial)$, for all $i \in I$. Then,

$$\begin{aligned} \bigcap_{i \in I} (\mathbb{U}_i, \partial) &= \bigcap_{i \in I} ((\mathbb{U}_i^1, \partial) \widetilde{\cap} (\mathbb{U}_i^2, \partial) \widetilde{\cap} (\mathbb{U}_i^3, \partial) \widetilde{\cap} (\mathbb{U}_i^4, \partial)) \\ &= \left(\bigcap_{i \in I} (\mathbb{U}_i^1, \partial) \right) \widetilde{\cap} \left(\bigcap_{i \in I} (\mathbb{U}_i^2, \partial) \right) \widetilde{\cap} \left(\bigcap_{i \in I} (\mathbb{U}_i^3, \partial) \right) \widetilde{\cap} \left(\bigcap_{i \in I} (\mathbb{U}_i^4, \partial) \right). \end{aligned} \quad (40)$$

Then, $\bigcap_{i \in I} (\mathbb{U}_i, \partial) \in \text{PNCS}(\mathfrak{Z}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ as $(\bigcap_{i \in I} (\mathbb{U}_i^1, \partial))^c \in \tau_1$, $(\bigcap_{i \in I} (\mathbb{U}_i^2, \partial))^c \in \tau_2$, $(\bigcap_{i \in I} (\mathbb{U}_i^3, \partial))^c \in \tau_3$, and $(\bigcap_{i \in I} (\mathbb{U}_i^4, \partial))^c \in \tau_4$. \square

3.9. Definition. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS, $(\mathbb{J}, \partial) \in \text{NSS}(\mathfrak{L})$. The PNS closure of (\mathbb{J}, ∂) , denoted by $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$, is the intersection of all PNSC containing (\mathbb{J}, ∂) , i.e.,

$$\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial) = \{(\omega, \partial) \in \text{PNSC}(\mathfrak{L}) \mid (\mathbb{J}, \partial) \in (\omega, \partial)\}. \quad (41)$$

It is clear that $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$ is the smallest PNSCS containing (\mathbb{J}, ∂) .

3.10. Theorem. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS and $(\mathbb{J}, \partial), (\mathfrak{Q}, \partial) \in \text{NSS}(\mathfrak{L})$. Then,

- (1) $\text{cl}_p^{\text{NSS}}(0_{(\mathfrak{L}, \partial)}) = 0_{(\mathfrak{L}, \partial)}$ and $\text{cl}_p^{\text{NSS}}(1_{(\mathfrak{L}, \partial)}) = 1_{(\mathfrak{L}, \partial)}$
- (2) $(\mathbb{J}, \partial) \in \text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$
- (3) (\mathbb{J}, ∂) is a PNSCS if $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial) = (\mathbb{J}, \partial)$
- (4) $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial) \in \text{cl}_p^{\text{NSS}}(\mathfrak{Q}, \partial)$ if $(\mathbb{J}, \partial) \in (\mathfrak{Q}, \partial)$
- (5) $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial) \cup \text{cl}_p^{\text{NSS}}(\mathfrak{Q}, \partial) \in \text{cl}_p^{\text{NSS}}((\mathbb{J}, \partial) \cup (\mathfrak{Q}, \partial))$
- (6) $\text{cl}_p^{\text{NSS}}(\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)) = \text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$, i.e., $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$ is a PNSCS

Proof. It is obvious. \square

3.11. Theorem. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS, $(\mathbb{J}, \partial) \in \text{NSS}(\mathfrak{L})$. Then, $\mathbb{E}^\sharp \in \text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$ if and only if for all $U_{\mathbb{E}^\sharp} \in \tau_{1234}$, where $U_{\mathbb{E}^\sharp}$ any is PNSOS contains \mathbb{E}^\sharp and $\tau_{1234}(\mathbb{E}^\sharp)$ is the family of all PNSOS contains \mathbb{E}^\sharp , $U_{\mathbb{E}^\sharp} \cap (\mathbb{J}, \partial) \neq 0_{(\mathfrak{L}, \partial)}$.

Proof. Let $\mathbb{E}^\sharp \in \text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$ and suppose that there exists $U_{\mathbb{E}^\sharp} \in \tau_{1234}(\mathbb{E}^\sharp)$ such that $U_{\mathbb{E}^\sharp} \cap (\mathbb{J}, \partial) = 0_{(\mathfrak{L}, \partial)}$. Then, $(\mathbb{J}, \partial) \in (U_{\mathbb{E}^\sharp})^c$. Thus, $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial) \in \text{cl}_p^{\text{NSS}}(U_{\mathbb{E}^\sharp})^c = (U_{\mathbb{E}^\sharp})^c$ which implies $\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial) \cap U_{\mathbb{E}^\sharp} = 0_{(\mathfrak{L}, \partial)}$, a contradiction.

Conversely, that $\mathbb{E}^\sharp \notin \text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial)$, then $\mathbb{E}^\sharp \in (\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial))^c \in \tau_{1234}(\mathbb{E}^\sharp)$. Therefore, by hypothesis $(\text{cl}_p^{\text{NSS}}(\mathbb{J}, \partial))^c \cap (\mathbb{J}, \partial) \neq 0_{(\mathfrak{L}, \partial)}$, a contradiction. \square

4. Exhibition of Some Definitions and Main Results

In this section, some important definitions of generalized neutrosophic soft open sets in neutrosophic soft quad topological spaces are introduced which pave the way to our new results. These definitions are semiopen, preopen, and $*_b$ open sets, respectively. These definitions became source of generation of different neutrosophical soft separation axioms and neutrosophical soft other separation axioms in neutrosophic soft quad topological spaces with respect to soft points of the second space. Neutrosophical soft quad homeomorphism which is a safe carriage for different structure from one space to another is defined. Soft closer attachment with neutrosophical soft separation axioms and neutrosophical soft other separation axioms in neutrosophic soft quad topological spaces with respect to soft points of the second space are addressed.

Definition 15. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS over \mathfrak{L} , $(\tilde{\eta}, \partial)$ be a NS set over \mathfrak{L} . Then, $(\tilde{\eta}, \partial)$ is

- (1) Neutrosophic soft quad semiopen if $(\tilde{\eta}, \partial) \in \text{VScl}(\text{VSint}(\tilde{\eta}, \partial))$
- (2) : Neutrosophic soft quad preopen if $(\tilde{\eta}, \partial) \in \text{VSint}(\text{VScl}(\tilde{\eta}, \partial))$
- (3) : Neutrosophic soft quad $*_b$ open if $(\tilde{\eta}, \partial) \in \text{VScl}(\text{VSint}(\tilde{\eta}, \partial)) \cup \text{VSint}(\text{VScl}(\tilde{\eta}, \partial))$ and neutrosophic soft quad $*_b$ close if $(\tilde{\eta}, \partial) \supseteq \text{VScl}(\text{VSint}(\tilde{\eta}, \partial)) \cap \text{VSint}(\text{VScl}(\tilde{\eta}, \partial))$

Definition 16. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS over \mathfrak{L} , $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \neq \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ are NSQ points if there exist NSQ $*_b$ -open sets $(\tilde{\eta}, \partial)$ and (\mathbb{U}, ∂) such that $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \in (\tilde{\eta}, \partial)$, $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \cap (\mathbb{U}, \partial) = 0_{(\mathfrak{L}, \partial)}$ or $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \in (\mathbb{U}, \partial)$, $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \cap (\tilde{\eta}, \partial) = 0_{(\mathfrak{L}, \partial)}$. Then, $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is called a NSQ $*_b$ space.

Definition 17. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS over \mathfrak{L} , $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \neq \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ are NSQ points. If there exist NSQ $*_b$ -open sets $(\tilde{\eta}, \partial)$, (\mathbb{U}, ∂) such that

$$\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \in (\tilde{\eta}, \partial), \mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \cap (\mathbb{U}, \partial) = 0_{(\mathfrak{L}, \partial)}, \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \in (\mathbb{U}, \partial), \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \cap (\tilde{\eta}, \partial) = 0_{(\mathfrak{L}, \partial)}, \text{ then } (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \text{ is called a NSQ }*_b \text{ space.}$$

Definition 18. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS over \mathfrak{L} , $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \neq \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ are NSQ points. If there exists NSQ $*_b$ open set $(\tilde{\eta}, \partial, nUq, h\partial)$ such that $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \in \tilde{\eta}, \partial$, $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \in (\mathbb{U}, \partial)$ and $(\tilde{\eta}, \partial) \cap (\mathbb{U}, \partial) = 0_{(\mathfrak{L}, \partial)}$, then $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \theta)$ is called a NSQ $*_b$ space.

Definition 19. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \theta)$ and $(\tilde{\mathfrak{Y}}, \tilde{\mathfrak{T}}_1, \tilde{\mathfrak{T}}_2, \tilde{\mathfrak{T}}_3, \tilde{\mathfrak{T}}_4, \partial)$ be two NSQTS. A NS function $\tilde{\eta}, \partial: (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \rightarrow (\tilde{\mathfrak{Y}}, \tilde{\mathfrak{T}}_1, \tilde{\mathfrak{T}}_2, \tilde{\mathfrak{T}}_3, \tilde{\mathfrak{T}}_4, \partial)$ is said to NSQ homeomorphism if (i) $(\tilde{\eta}, \partial)$ is NSQ bijective; (ii) $(\tilde{\eta}, \partial)$ is NSQ continuous; (iii) $((\tilde{\eta}, \partial))^{-1}$ is NSQ continuous or $(\tilde{\eta}, \partial)$ is NSO or $(\tilde{\eta}, \partial)$ is NSC.

Theorem 1. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \theta)$ be a NSTQS over the father set \mathfrak{L} , then is NSQ $*_b$ space if and only if for distinct NS points $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3)$ and $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$, there exists an NSQ $*_b$ -open set $(\tilde{\eta}, \partial)$ containing but not $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ such that $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \notin (\tilde{\eta}, \partial)$.

Proof. Let $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \neq \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ be two NS points in NSQ $*_b$ space, then there exists disjoint NSQ $*_b$ open sets $(\tilde{\eta}, \partial)$, (\mathbb{U}, ∂) such that $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \in (\tilde{\eta}, \partial)$ and $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \in (\mathbb{U}, \partial)$, since $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \cap \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) = 0_{(\mathfrak{L}, \partial)}$ and $(\tilde{\eta}, \partial) \cap (\mathbb{U}, \partial) = 0_{(\mathfrak{L}, \partial)}$. $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \notin (\tilde{\eta}, \partial) \Rightarrow \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \notin \overline{(\tilde{\eta}, \partial)}$. Next suppose that, $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3) \neq \mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ then there exists a NSQ $*_b$ open set $(\tilde{\eta}, \partial)$ containing $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3)$ but not $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ such that $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3) \notin \overline{(\tilde{\eta}, \partial)}$ that is and $(\tilde{\eta}, \partial)^c$ are mutually exclusive NSQ $*_b$ open sets supposing $\mathbb{E}_1(\gamma_1, \gamma_2, \gamma_3)$ and $\mathbb{E}_2(\gamma_1, \gamma_2, \gamma_3)$ in turn. \square

Theorem 2. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS. Then, $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \theta)$ is NSQ $*_b$ space if every NSQ point $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in \tilde{\eta}, \partial \in (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ if there exists an NSQ $*_b$ open set (\mathfrak{U}, ∂) such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{U}, \partial) \subseteq \overline{(\mathfrak{U}, \partial)} \subseteq \tilde{\eta}, \partial$, then $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \theta)$ an NSQ $*_b$ space.

Proof. Suppose $\xi_1(\gamma_1, \gamma_2, \gamma_3) \cap \xi_2(\gamma_1, \gamma_2, \gamma_3) = 0_{(\mathfrak{L}, \partial)}$. Since $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ $*_b$ space. $\xi_1(\gamma_1, \gamma_2, \gamma_3)$ and $\xi_2(\gamma_1, \gamma_2, \gamma_3)$ are NSQ $*_b$ closed sets in $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$; then, $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in \xi_2(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$. Thus, there exists a NSQ $*_b$ open set $(\mathfrak{U}, \partial) \in (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{U}, \partial) \subseteq \overline{(\mathfrak{U}, \partial)} \subseteq (\xi_2(\gamma_1, \gamma_2, \gamma_3))^c$. So, we have $\xi_2(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{U}, \partial)$ and $(\mathfrak{U}, \partial) \cap (\mathfrak{U}, \partial)^c = 0_{(\mathfrak{L}, \partial)}$, that is, $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is a NSQ $*_{b2}$ space. \square

Definition 20. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS. $\tilde{\eta}, \partial$ be a NSQ $*_b$ closed set and $\xi_1(\gamma_1, \gamma_2, \gamma_3) \cap \tilde{\eta}, \partial = 0_{(\mathfrak{L}, \partial)}$. If there exists NSQ $*_b$ open sets $(\mathfrak{U}_1, \partial)$ and $(\mathfrak{U}_2, \partial)$ such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{U}_1, \partial), \tilde{\eta}, \partial \in (\mathfrak{U}_2, \partial)$ and $\xi_1(\gamma_1, \gamma_2, \gamma_3) \cap (\mathfrak{U}_1, \partial) = 0_{(\mathfrak{L}, \partial)}$, then $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is called a NSQ $*_b$ -regular space. $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is said to be NSQ $*_{b3}$ space, if is both a NSQ regular and NSQ $*_b$ space.

Theorem 3. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQTS. $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ $*_{b3}$ space iff for every $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in \tilde{\eta}, \partial$, that is, $(\mathfrak{U}, \partial) \in ((\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{U}, \partial) \subseteq \overline{(\mathfrak{U}, \partial)} \subseteq \tilde{\eta}, \partial$.

Proof. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ $*_{b3}$ space and $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in (\eta, \partial) \in (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$. Since $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ $*_{b3}$ space for the NSQ point $\xi_1(\gamma_1, \gamma_2, \gamma_3)$ and NSQ $*_b$ closed set $\tilde{\eta}, \partial^c$, there exists $(\mathfrak{U}_1, \partial)$ and $(\mathfrak{U}_2, \partial)$ such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{U}_1, \partial), (\eta, \partial)^c \in (\mathfrak{U}_2, \partial)$ and $(\mathfrak{U}_1, \partial) \cap (\mathfrak{U}_2, \partial) = 0_{(\mathfrak{L}, \partial)}$. Then, we have $\xi_1^{\mathfrak{S}} \in (\mathfrak{U}_1, \partial) \subseteq (\mathfrak{U}_2, \partial)^c \subseteq \tilde{\eta}, \partial$. Since $(\mathfrak{U}_2, \partial)^c$ NSQ $*_b$ closed set. $(\mathfrak{U}_1, \partial) \subseteq (\mathfrak{U}_2, \partial)^c$. Conversely, let $\xi_1^{\mathfrak{S}} \cap (h, \partial) = 0_{(\mathfrak{L}, \partial)}$ and (h, ∂) be a NSQ $*_b$ closed set. $\xi_1^{\mathfrak{S}} \in (h, \partial)^c$ and we have $\xi_1^{\mathfrak{S}} \in (\mathfrak{U}, \partial) \subseteq \overline{(\mathfrak{U}, \partial)} \subseteq (h, \partial)^c$. Thus, $\xi_1^{\mathfrak{S}} \in (\mathfrak{U}, \partial), (h, \partial) \subseteq \overline{(\mathfrak{U}, \partial)}$ and $(\mathfrak{U}, \partial) \cap (\mathfrak{U}, \partial)^c = 0_{(\mathfrak{L}, \partial)}$. So, $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ $*_{b3}$ space. \square

5. Characterization of More Results Concerning New Definition

In this section, more main results are addressed. Among neutrosophical soft quad separation axioms, Hausdorff space is considered to be the most important separation axioms. Important things should be given serious attention. So, this section has almost been engaged with the study of Hausdorff space. Through neutrosophical soft quad function, the characteristics of one space can be migrated to another space if the neutrosophical soft quad function is satisfying conditions of neutrosophical soft quad bijections and bicontinuousness. A soft quad function satisfying these conditions is known as neutrosophical soft quad homeomorphism. Neutrosophical soft quad homeomorphism is giving birth to neutrosophical soft quad topological property. Some neutrosophical soft quad topological properties

of Hausdorff space are addressed with respect to soft points. Sequentially compactness and countably compact in neutrosophic soft quad topological spaces with respect to soft points of the second space are addressed. Engagement of Hausdorff space with closed sets in neutrosophic soft quad topological spaces is addressed.

Theorem 4. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be NSQTS such that it is NSQ $*_b$ Hausdorff space and $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be another NSQTS such that let $f, \theta: (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \rightarrow (\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be NSQ homeomorphism. Then, $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ is also of the same behavior of NSQ $*_b$ Hausdorffness.

Proof. Let $\sqcup_1(\gamma_1, \gamma_2, \gamma_3), \sqcup_2(\gamma_1, \gamma_2, \gamma_3) \in \tilde{Y}$ such that $\sqcup_1(\gamma_1, \gamma_2, \gamma_3) \neq \sqcup_2(\gamma_1, \gamma_2, \gamma_3)$. Since $\langle f, \partial \rangle: (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \rightarrow (\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be NSQ homeomorphism. So, there exists $\xi_1(\gamma_1, \gamma_2, \gamma_3), \xi_2(\gamma_1, \gamma_2, \gamma_3) \in \mathfrak{L}$ such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \neq \xi_2(\gamma_1, \gamma_2, \gamma_3)$. Since $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ $*_b$ Hausdorff space, so there exist (h, ∂) and (\mathfrak{U}, ∂) in NSQ $*_b$ Hausdorff space such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \in (h, \partial)$ and $\xi_2(\gamma_1, \gamma_2, \gamma_3) \in (\mathfrak{U}, \partial)$ such that $\xi_1(\gamma_1, \gamma_2, \gamma_3) \cap (\mathfrak{U}, \partial) = 0_{(\mathfrak{L}, \partial)}$, $\xi_2(\gamma_1, \gamma_2, \gamma_3) \cap (h, \partial) = 0_{(\mathfrak{L}, \partial)}$. Hence, $f(\xi_1(\gamma_1, \gamma_2, \gamma_3)) \in f((h, \partial)), f(\xi_2(\gamma_1, \gamma_2, \gamma_3)) \in f((\mathfrak{U}, \partial))$. Since $\langle f, \partial \rangle: (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \rightarrow (\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ is NSQ homeomorphism, so f, θ is NSQ $*_b$ open. Also, $f((h, \partial)) \cap f((\mathfrak{U}, \partial)) = f((h, \partial) \cap (\mathfrak{U}, \partial)) = f(0_{(\mathfrak{L}, \partial)}) = 0_{(\tilde{Y}, \partial)}$. This proves $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ is also of characteristics of NSQ $*_b$ Hausdorffness. \square

Theorem 5. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be NSQTS and $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be another NSQTS which satisfies one more condition of NSQ $*_b$ Hausdorffness. Let $f, \theta: (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \rightarrow (\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ a NSQ function such that it is NSQ homeomorphism, then $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is also of characteristics of NSQ $*_b$ Hausdorffness.

Proof. Suppose $\xi_1^{\mathfrak{S}}, \xi_2^{\mathfrak{S}} \in \tilde{\mathfrak{L}}$ such that $\xi_1^{\mathfrak{S}} \neq \xi_2^{\mathfrak{S}}$. Since f, ∂ NSQ homeomorphism, so $f\xi_1^{\mathfrak{S}} \neq f\xi_2^{\mathfrak{S}}$. Let $\sqcup_1^{\mathfrak{S}}, \sqcup_2^{\mathfrak{S}} \in \tilde{Y}$ such that $\sqcup_1^{\mathfrak{S}} \neq \sqcup_2^{\mathfrak{S}}$.

Let $f(\xi_1^{\mathfrak{S}}) = \sqcup_1^{\mathfrak{S}}, f(\xi_2^{\mathfrak{S}}) = \sqcup_2^{\mathfrak{S}} \Rightarrow f \Rightarrow \xi_1(\gamma_1, \gamma_2, \gamma_3) = f^{-1}(\sqcup_1^{\mathfrak{S}})$ and $\xi_2(\gamma_1, \gamma_2, \gamma_3) = f^{-1}(\sqcup_2^{\mathfrak{S}})$

Let $\sqcup_1^{\mathfrak{S}}, \sqcup_2^{\mathfrak{S}} \in \tilde{Y}$ such that $\sqcup_1^{\mathfrak{S}} \neq \sqcup_2^{\mathfrak{S}}$. Since $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ is NSQ $*_b$ Hausdorff space. There definitely exists NSQ $*_b$ open sets $\langle h_1, \theta \rangle$ and $\langle h_2, \partial \rangle$ such that $\sqcup_1^{\mathfrak{S}} \in h_1, \partial$ and $\sqcup_2^{\mathfrak{S}} \in h_2, \partial$ with $h_1, \partial \neq h_2, \partial$.

$f^{-1}(\sqcup_1^{\mathfrak{S}}), f^{-1}(\sqcup_2^{\mathfrak{S}})$ are NSQ $*_b$ open in $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ Now, $f^{-1}(\sqcup_1^{\mathfrak{S}}) \cap f^{-1}(\sqcup_2^{\mathfrak{S}}) = f^{-1}(\sqcup_1^{\mathfrak{S}} \cap \sqcup_2^{\mathfrak{S}}) \Rightarrow f^{-1}(0_{(\tilde{Y}, \partial)}) = 0_{(\mathfrak{L}, \partial)}$. So is NSQTS is $*_b$ Hausdorff space. \square

Theorem 6. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be NSQTS and $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be another NSQTS. Let $f, \partial: (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \theta) \rightarrow (\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be a NSQ mapping. Let $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ is NSQ $*_b$ Hausdorff space, then it is guaranteed that $\mathcal{M}, \theta = \{(\xi_1(\gamma_1, \gamma_2, \gamma_3), \xi_2(\gamma_1, \gamma_2, \gamma_3)): f$

$(\mathfrak{E}_1(\gamma_1, \gamma_2, \gamma_3)) = f(\mathfrak{E}_2(\gamma_1, \gamma_2, \gamma_3))$ is a NSQ^*_b closed subset of $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Y}}$.

Proof. Given that $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \theta)$ be NSQTS and $(\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ be another NSQTS. Let $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \theta) \longrightarrow (\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ be a NSQ mapping such that it is NSQ continuous mapping $(\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ is NSQ^*_b Hausdorff space. Then, we will prove that \mathcal{M}, ∂ is NSQ close subset of $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Q}}$. Equivalently, we will prove that \mathcal{M}, ∂^c is NSQ^*_b an open subset of $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Q}}$. Let $(\mathfrak{E}_1^s, \mathfrak{E}_2^s) \in \mathcal{M}, \partial^c$. Then, $\mathfrak{E}_1^s \neq \mathfrak{E}_2^s \Rightarrow f(\mathfrak{E}_1^s) > f(\mathfrak{E}_2^s)$ or $< f(\mathfrak{E}_1^s) < f(\mathfrak{E}_2^s)$ accordingly. Since, $(\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ is NSQ *_b Hausdorff space. Certainly, $f(\mathfrak{E}_1^s \neq \mathfrak{E}_2^s), f(\mathfrak{E}_2^s)$ are NS points of $\tilde{\mathfrak{Y}}$, so there exists NSQ^*_b open sets $\langle \mathcal{G}, \partial \rangle, \langle h, \partial \rangle \in \langle \tilde{\mathfrak{Y}} \rangle$ such that $f(\mathfrak{E}_1^s) \in \mathcal{G}, \partial, f(\mathfrak{E}_2^s) \in h, \partial$ provided $\langle \mathcal{G}, \partial \rangle \cap \langle h, \partial \rangle = \emptyset_{(\mathfrak{Q}, \partial)}$. Since, $\langle f, \partial \rangle$ is soft continuous, $f^{-1}(\langle \mathcal{G}, \theta \rangle) \times f^{-1}(\langle h, \partial \rangle)$ are NSQ^*_b open sets in $\tilde{\mathfrak{Q}}$. So, $f^{-1}(\langle \mathcal{G}, \theta \rangle) \times f^{-1}(\langle h, \partial \rangle)$ is NSQ^*_b open in $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Q}}$. Now, we show that $f^{-1}(\langle \mathcal{G}, \theta \rangle) \times f^{-1}(\langle h, \partial \rangle)$ For this, let $(\mathfrak{E}_1(\gamma_1, \gamma_2, \gamma_3), \mathfrak{E}_2(\gamma_1, \gamma_2, \gamma_3)) \in \mathcal{M}, \partial^c \Rightarrow f^{-1}(\mathcal{G}, \partial \times f^{-1}f, \partial) \longrightarrow \mathfrak{E}_1(\gamma_1, \gamma_2, \gamma_3) \in f^{-1}(\mathcal{G}, \partial)$ and $\mathfrak{E}_2(\gamma_1, \gamma_2, \gamma_3) \in f^{-1}(\langle h, \partial \rangle) \Rightarrow f(\mathfrak{E}_1^s) = \mathcal{G}, \partial$ and $(\mathfrak{E}_2^s) = \langle h, \partial \rangle$. Since $\mathcal{G}, \partial \cap h, \partial = \emptyset_{(\pi, \partial)} \Rightarrow \mathfrak{E}_1^s \neq \mathfrak{E}_2^s \Rightarrow \mathfrak{E}_1^s \in \mathcal{M}, \partial^c \Rightarrow f^{-1}(\langle \mathcal{G}, \partial \rangle) \times f^{-1}(h, \partial) \subseteq \mathcal{M}, \partial^c$. Thus, $f^{-1}(\langle \mathcal{G}, \partial \rangle) \times f^{-1}(\langle h, \partial \rangle)$ is NSQ^*_b open in $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Q}}$. So, this implies that $(\mathfrak{E}_1^s, \mathfrak{E}_2^s)$ is NSQ interior point of \mathcal{M}, ∂^c . So, every point of \mathcal{M}, ∂^c is NSQ interior point of \mathcal{M}, ∂^c . So \mathcal{M}, ∂^c is *_b open in $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Q}}$. \square

Theorem 7. Let $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be NSQTS and $(\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ be another NSQTS. Let $\langle h, \partial \rangle: (\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \longrightarrow (\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ be an NSQ^*_b open mapping such that it is onto. If the soft set $\mathcal{M}, \partial = \{(\mathfrak{E}_1^s, \mathfrak{E}_2^s): f(\mathfrak{E}_1^s) = f(\mathfrak{E}_2^s)\}$ is NSQ^*_b closed in $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Q}}$, then $\tilde{\mathfrak{Y}}$ is NSQ^*_b Hausdorff space.

Proof. Suppose $f(\mathfrak{E}_1^s), f(\mathfrak{E}_2^s)$ be two NSQ points of $\tilde{\mathfrak{Y}}$ such $f(\mathfrak{E}_1^s) \neq f(\mathfrak{E}_2^s) \longrightarrow f(\mathfrak{E}_1^s) > f(\mathfrak{E}_2^s)$ or $< \mathfrak{E}_1^s < f(\mathfrak{E}_2^s)$. Then, $(\mathfrak{E}_1(\gamma_1, \gamma_2, \gamma_3), \mathfrak{E}_2(\gamma_1, \gamma_2, \gamma_3)) \notin \mathcal{M}, \partial$, that is, $(\mathfrak{E}_1(\gamma_1, \gamma_2, \gamma_3), \mathfrak{E}_2(\gamma_1, \gamma_2, \gamma_3)) \in \mathcal{M}, \partial^c$. Since \mathcal{M}, ∂^c NSQ^*_b open in $\tilde{\mathfrak{Q}} \times \tilde{\mathfrak{Q}}$, there exist NSQ^*_b open sets \mathcal{G}, ∂ and $\langle h, \partial \rangle$ in $\tilde{\mathfrak{Y}}$ such that $(\mathfrak{E}_1^s, \mathfrak{E}_2^s) \in \langle \mathcal{G}, \partial \rangle \times \langle h, \partial \rangle \in \mathcal{M}, \partial^c$. Then, since NSQ^*_b is open, $\langle \mathcal{G}, \partial \rangle, f(h, \partial)$ are NSQ^*_b open in $\tilde{\mathfrak{Y}}$ containing $f(\mathfrak{E}_1^s), f(\mathfrak{E}_2^s)$, and $\langle \mathcal{G}, \partial \rangle$ as $\langle h, \partial \rangle$ disjoint. It follows that $\tilde{\mathfrak{Y}}$ is NSQ^*_b Hausdorff space. \square

Theorem 8. Let $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ be a NSQ second countable space and let $\langle h, \partial \rangle$ be NSQ uncountable subset of $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$, then at least one point of $\langle h, \partial \rangle$ is a NSQ limit point of $\langle h, \partial \rangle$.

Proof. Let $\mathfrak{B} = \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \dots, \mathcal{B}_n: n \in \mathbb{N}$ for $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$.

Let, if possible, no point of $\langle f, \partial \rangle$ be a NSQ limit point of $\langle f, \partial \rangle$. Then, for each $\mathfrak{E}_1^s \in \langle f, \partial \rangle$, there exists NSQ^*_b open set $\rho, \partial_{(\mathfrak{E}_1^s)}$ such that $\mathfrak{E}_1^s \in \rho, \partial_{(\mathfrak{E}_1^s)}$ and $\rho, \partial_{(\mathfrak{E}_1^s)} \cap f, \partial = \{\mathfrak{E}_1^s\}$. Since \mathfrak{B} is soft base, there exists $\mathcal{B}_n \in \mathfrak{B}$ such that $(\mathfrak{E}_1^s) \in \mathcal{B}_n \in \rho, \partial_{(\mathfrak{E}_1^s)}$. Therefore, $\mathcal{B}_n \cap f, \partial \in \rho, \partial_{(\mathfrak{E}_1^s)} \cap f, \partial = \{\mathfrak{E}_1^s\}$. Moreover, if \mathfrak{E}_1^s and \mathfrak{E}_2^s be any two NSQ points such that $\mathfrak{E}_1^s \neq \mathfrak{E}_2^s$ which means either $\mathfrak{E}_1^s > \mathfrak{E}_2^s$ or $\mathfrak{E}_1^s < \mathfrak{E}_2^s$, then there exists \mathcal{B}_n and \mathcal{B}_m in \mathfrak{B} such that $\mathcal{B}_n \cap f, \partial = \{\mathfrak{E}_1^s\}$ and $\mathcal{B}_m \cap f, \partial = \{\mathfrak{E}_2^s\}$. Now, $\mathfrak{E}_1^s \neq \mathfrak{E}_2^s$ which guarantees that $\{\mathfrak{E}_1^s\} \neq \{\mathfrak{E}_2^s\}$ which implies that $\mathcal{B}_n \cap f, \partial \neq \mathcal{B}_m \cap f, \partial$ which implies $\mathcal{B}_n \neq \mathcal{B}_m$. Thus, there exists a one to one NSQ correspondence of $\langle f, \partial \rangle$ on to $\{\mathcal{B}_n: (\mathfrak{E}_1^s) \in f, \partial\}$. Now, $\langle f, \partial \rangle$ being NSQ uncountable, it follows that $\{\mathcal{B}_n: \mathfrak{E}_1^s \in \langle f, \partial \rangle\}$ is NSQ uncountable. This is a contradiction. \square

Theorem 9. Let $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ and $(\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ be two NSQTS and suppose \mathfrak{f}, ∂ be a NSQ continuous function such that $\mathfrak{f}, \partial: (\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial) \longrightarrow (\tilde{\mathfrak{Y}}, \mathfrak{F}_1, \mathfrak{F}_2, \mathfrak{F}_3, \mathfrak{F}_4, \partial)$ is NSQ continuous function and let $\mathcal{L}, \partial \in (\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ supposes the B.V.P. Then, safely, $\mathfrak{f}(\mathcal{L}, \partial)$ has the B.V.P.

Proof. Suppose \mathcal{L}, ∂ be an infinite NSQ subset of \mathfrak{f}, ∂ , so that \mathcal{L}, ∂ contains an enumerable NS set $(\mathfrak{E}_1^s)_n: n \in \mathbb{N}$, then there exists enumerable NSQ set $(\mathfrak{E}_2^s)_n: n \in \mathbb{N} \in \mathcal{L}, \partial$ such that $\mathfrak{f}((\mathfrak{E}_2^s)_n) = (\mathfrak{E}_1^s)_n$. \mathcal{L}, ∂ has B.V.P implying that every infinite soft subset \mathcal{L}, ∂ supposes NSQ accumulation point belonging to \mathcal{L}, ∂ , and this implies that $(\mathfrak{E}_2^s)_n: n \in \mathbb{N}$ has NSQ limit point, say, $(\mathfrak{E}_2^s)_0 \in \mathcal{L}, \partial$ implies that the limit of NSQ sequence $(\mathfrak{E}_2^s)_n: n \in \mathbb{N}$ is $(\mathfrak{E}_2^s)_0 \in \mathcal{L}, \partial \longrightarrow (\mathfrak{E}_2^s)_n \longrightarrow (\mathfrak{E}_2^s)_0 \in \mathcal{L}, \partial$. \mathfrak{f} is NSQ continuous. Furthermore, $(\mathfrak{E}_2^s)_n (\mathfrak{E}_2^s)_0 \in \mathcal{L}, \partial \longrightarrow \mathfrak{f}((\mathfrak{E}_2(\gamma_1, \gamma_2, \gamma_3))_n) \mathfrak{f}((\mathfrak{E}_2^s)_0) \in \mathfrak{f}(\mathcal{L}, \partial) \longrightarrow (\mathfrak{E}_1^s)_n \mathfrak{f}(\mathfrak{E}_2^s)_0 \in \mathfrak{f}(\mathcal{L}, \partial)$ implies that NSQ limit of a NSQ sequence $(\mathfrak{E}_1^s)_n: n \in \mathbb{N}$ is $\mathfrak{f}(\mathfrak{E}_2^s)_0 \in \mathfrak{f}(\mathcal{L}, \partial)$ implying that NSQ limit of a NSQ sequence $(\mathfrak{E}_1^s)_n: n \in \mathbb{N}$ is $\mathfrak{f}(\mathfrak{E}_2^s)_0 \in \mathfrak{f}, \partial(\mathcal{L}, \partial)$. Finally, we have shown that there exists infinite soft subset $(\mathfrak{E}_1^s)_n: n \in \mathbb{N}$ of $\mathfrak{f}(\mathcal{L}, \partial)$ containing a limit point $\mathfrak{f}((\mathfrak{E}_2^s)_0) \in \mathfrak{f}(\mathcal{L}, \partial)$; this guarantees that $\mathfrak{f}(\mathcal{L}, \partial)$ has B.V.P. \square

Theorem 10. Let $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS and let $\widetilde{\mathfrak{E}_1^s}_n$ be a NSQ sequence in $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ such that it converges to NSQ a point $(\mathfrak{E}_1^s)_0$, then the NSQ set $\langle f, \partial \rangle$ consisting of NSQ points $(\mathfrak{E}_1^s)_{n_0}$ and $(\mathfrak{E}_1^s)_n (n = 1, 2, 3, \dots)$ is soft NSQ compact.

Proof. Given $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS and let $\widetilde{\mathfrak{E}_1^s}_n$ be an NSQ sequence in $(\mathfrak{Q}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ such that it converges to a point $(\mathfrak{E}_1^s)_{n_0}$ that is $(\mathfrak{E}_1^s)_n \longrightarrow (\mathfrak{E}_1^s)_{n_0} \in \tilde{\mathfrak{Q}}$. Let $\langle g, \partial \rangle = \left[\begin{array}{c} (\mathfrak{E}_1^s)_1, (\mathfrak{E}_1^s)_2, (\mathfrak{E}_1^s)_3, \\ (\mathfrak{E}_1^s)_4, (\mathfrak{E}_1^s)_5, \\ (\mathfrak{E}_1^s)_6, (\mathfrak{E}_1^s)_7, \dots \end{array} \right]$. Let $\{\mathfrak{G}, \partial_\alpha: \alpha \in \Delta\}$ be NSQ^*_b open covering of $\langle g, \partial \rangle$ so that $\langle g, \partial \rangle \in \bigcup \{\mathfrak{G}, \partial_\alpha:$

$\alpha \in \Delta$, $(\xi_1^s)_{n_0} \in \langle g, \partial \rangle$ implies that there exists $\alpha_0 \in \Delta$ such that $(\xi_1^s)_{n_0} \in \mathfrak{S}, \partial_{\alpha_0}$. By NSQ convergence, $(\xi_1^s)_{n_0} \in \mathfrak{S}, \partial_{\alpha_0} \in (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ implies that there exists $n_0 \in \mathbb{N}$ s.t. $n \geq n_0$ and $(\xi_1^s)_n \in \mathfrak{S}, \partial_{\alpha_0}$. Evidently, $\mathfrak{S}, \partial_{\alpha_0}$ contains the NSQ points $(\xi_1^s)_{n_0}, (\xi_1^s)_{n_0+1}, (\xi_1^s)_{n_0+2}, (\xi_1^s)_{n_0+3}, (\xi_1^s), \dots, (\xi_1^s)_{n_0+n}, \dots$. Look carefully at the points and train them in a way as $(\xi_1^s)_1, (\xi_1^s)_2, (\xi_1^s)_3, (\xi_1^s)_4, \dots, (\xi_1^s)_n$ generating a finite soft set. Let $1 \leq n_{0-1}$. Then, $(\xi_1^{(y_1, y_2, y_3)})_i \in \langle g, \partial \rangle$. For this $i, (\xi_1^s)_i \in \langle g, \partial \rangle$. Hence, there exists $\alpha_i \in \Delta$ such that $(\xi_1^s)_i \in \mathfrak{S}, \partial_{\alpha_i}$. Evidently, $\langle g, \partial \rangle \in \bigcup_{r=0}^{n_{0-1}} \mathfrak{S}, \partial_{\alpha_i}$. This shows that $\{\mathfrak{S}, \partial_{\alpha_i}: 0 \leq n_{0-1}\}$ is NSQ*_b open cover of $\langle g, \partial \rangle$. Thus, an arbitrary NSQ*_b open cover $\{\mathfrak{S}, \partial_{\alpha}: \alpha \in \Delta\}$ of $\langle g, \partial \rangle$ is reducible to a finite NSQ subcover $\{\mathfrak{S}, \partial_{\alpha_i}: i = 0, 1, 2, 3, \dots, n_{0-1}\}$, and it follows that $\langle g, \partial \rangle$ is NSQ*_b compact. \square

Theorem 11. If $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS such that it has the characteristics of NSQ*_b sequentially compactness. Then, $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ*_b countably compact.

Proof. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS and let ρ, ∂ be finite

soft subset of $(L, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$. Let $\left[\begin{array}{c} (\xi_1^s)_1, (\xi_1^s)_2, (\xi_1^s)_3, \\ (\xi_1^s)_4, (\xi_1^s)_5, \\ (\xi_1^s)_6, (\xi_1^s)_7, \dots \end{array} \right]$

be an NSQ sequence of NSQ points of ρ, ∂ . Then, ρ, ∂ being finite, at least one of the elements in ρ, ∂ say $(\xi_1^s)_0$ must be duplicated an infinite number of times in the NSQ sequence.

Hence, $\left[\begin{array}{c} (\xi_1^s)_0, (\xi_1^s)_0, \\ (\xi_1^s)_0, (\xi_1^s)_0, (\xi_1^s)_0, \\ (\xi_1^s)_0, (\xi_1^s)_0, \dots \end{array} \right]$ is soft NSQ subsequence of

$(\xi_1^s)_n$ such that it is NSQ constant sequence and repeatedly constructed by single soft number $(\xi_1^s)_0$, and we know that a soft constant sequence converges on its self. So, it converges to $(\xi_1^s)_0$ which belongs to ρ, ∂ . Hence, ρ, ∂ is soft sequentially NSQ*_b compact. \square

Theorem 12. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS and $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be another NSQTS. Let $\langle f, \partial \rangle$ be a soft continuous mapping of an NSQ sequentially compact NSQ*_b space $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ into $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$, then $\langle f, \partial \rangle (\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ is NSQ*_b sequentially compact.

Proof. Given $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS and $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ be another NSQTS. Let $\langle f, \partial \rangle$ be a NSQ continuous mapping of a NSB sequentially compact space $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ into $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$, then we have to prove $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQ sequentially. For this, we

proceed as follows. Let $\left[\begin{array}{c} (\xi_2^s)_1, (\xi_2^s)_2, \\ (\xi_2^s)_3, (\xi_2^s)_4, \dots \\ (\xi_2^s)_7, \dots, (\xi_2^s)_n, \dots \end{array} \right]$ be a NSQ

sequence of NSQ points. Then, for each $n \in \mathbb{N}$, there exists $(\xi_1^s)_1, (\xi_1^s)_2, (\xi_1^s)_4, (\xi_1^s)_5, \dots, (\xi_1^s)_7, \dots, (\xi_1^s)_n, \dots \in (L,$

$\tau_1, \tau_2, \tau_3, \tau_4, \partial)$. Thus, we obtain an NSQ sequence

$\left[\begin{array}{c} (\xi_1^s)_1, (\xi_1^s)_2, \\ (\xi_1^s)_3, (\xi_1^s)_4, \dots \\ (\xi_1^s)_5, (\xi_1^s)_6, \\ (\xi_1^s)_7, \dots, (\xi_1^s)_n, \dots \end{array} \right]$ in $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$. But $(\mathfrak{L}, \tau_1, \tau_2,$

$\tau_3, \tau_4, \partial)$ being soft sequentially NSQ*_b compact, there is an NSQ subsequence $(\xi_1^s)_{n_i}$ of

$(\xi_1^s)_n$ such that $(\xi_1^s)_{n_i} \rightarrow (\xi_1^s) \in (\tilde{\mathfrak{L}}, \tau_1, \tau_2, \tau_3, \partial)$. So, by NSQ*_b continuity of $f, (\xi_1^s)_{n_i} \rightarrow (\xi_1^s)$ implies that $f((\xi_1^s)_{n_i}) \rightarrow f((\xi_1^s)_n) \in f(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$. Thus,

$f((\xi_2^{(y_1, y_2, y_3)})_{n_i})$ is a soft subsequence of

$\left[\begin{array}{c} (\xi_2^s)_1, (\xi_2^s)_2, \\ (\xi_2^s)_3, (\xi_2^s)_5, \\ (\xi_2^s)_5, (\xi_2^s)_6, \\ (\xi_2^s)_7, \dots, (\xi_2^s)_n, \dots \end{array} \right]$ converges to $f(\kappa_1)$ in $(\tilde{\mathfrak{L}}, \tau_1,$

$\tau_2, \tau_3, \tau_4, \partial)$. Hence, $((\tilde{\mathfrak{L}}, \tau_1, \tau_2, \tau_3, \tau_4, \partial))$ NSQ*_b sequentially compact. \square

Theorem 13. Let $(\tilde{\mathfrak{L}}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS and suppose $\langle f, \partial \rangle, \langle g, \partial \rangle$ be two NSQ continuous function on an NSQTS $(\tilde{\mathfrak{L}}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ in to an NSQTS $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ which is NSQ*_b Hausdorff. Then, soft set $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ is NSQ*_b closed of $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$.

Proof. Let if $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ is an NSQ set of function. If $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\} = 0_{(\mathfrak{L}, \partial)}$, it is clearly NSQ*_b open, and therefore, $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ is NSQ*_b closed, that is, nothing is proved in this case. Let us consider the case when $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ and let $\rho \in \{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$. Then, ρ does not belong $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)(k_1)\}$. Result in $(f)(\rho) \neq (g)(\rho)$. Now, $(\tilde{Y}, \tilde{\mathfrak{F}}_1, \tilde{\mathfrak{F}}_2, \tilde{\mathfrak{F}}_3, \tilde{\mathfrak{F}}_4, \partial)$ being NSQ*_b Hausdorff space, so there exists NSQ*_b open sets $(f, \partial), (\mathfrak{H}, \partial)$ of $(f)(\rho)$ and $(g)(\rho)$, respectively, such that (f, ∂) and (\mathfrak{H}, ∂) such that these NSQ sets such that the possibility of one rules out the possibility of other. By soft continuity of $(f, \partial), (g, \partial), (f, \partial)^{-1}$ as well as is NSQ*_b open nhd of ρ , and therefore, $(f, \partial)^{-1} \cap (g, \partial)^{-1}$ is contained in $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ for, $\{(\xi_1^s) \in ((f, \partial)^{-1} \cap (g, \partial)^{-1}) (g)((\xi_1^{(y_1, y_2, y_3)})) \in (g, \partial)\}$ and $(g)(f)((\xi_1^s)) \neq (g)((\xi_1^s))$ because (f, ∂) and \mathfrak{H}, ∂ are mutually exclusive. This implies that κ_1 does not belong to $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)(\xi_1^s) = (h)(\xi_1^s)\}$. Therefore, this shows that $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ is nhd of each of its points. So, $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ NSQ*_b open, and hence $\{(\xi_1^s) \in \tilde{\mathfrak{L}}: (f)((\xi_1^s)) = (g)((\xi_1^s))\}$ is NSQ*_b closed. \square

Theorem 14. Let $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ NSQTS such that it is $NSQ*_b$ Hausdorff space and let (f) be soft continuous function of $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$ into itself. Then, the NSQ set of fixed points under (g) is a $NSQ*_b$ closed set.

Proof. Let $\mathfrak{S} = \{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}$. If $\mathfrak{S}^c = 0_{(\mathfrak{L}, \partial)}$, Then, $NSQ*_b$ is open, and therefore, $\mathfrak{S} = \{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}$ closed. So, let $\mathfrak{S} = \{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}$ and let $\mathfrak{E}_2^{\mathfrak{g}} \in \{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}^c$. Then, $\mathfrak{E}_2^{\mathfrak{g}}$ does not belong to $\{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}$, and therefore, $(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})$. Now, $\mathfrak{E}_2^{\mathfrak{g}}, (g)(\mathfrak{E}_2^{\mathfrak{g}})$ being two distinct points of the $NSQ*_b$ Hausdorff space $(\mathfrak{L}, \tau_1, \tau_2, \tau_3, \tau_4, \partial)$, so there exist $NSQ*_b$ open sets $(g, \partial), (\mathfrak{H}, \partial)$ such that $\mathfrak{E}_2^{\mathfrak{g}} \in (g, \partial), (h)(\mathfrak{E}_2^{\mathfrak{g}}) \in (\mathfrak{H}, \partial)$ and $(g, \partial), (\mathfrak{H}, \partial)$ are disjoint. Also, by the NSQ continuity, $NSQ*_b$ is an open set containing $\mathfrak{E}_2^{\mathfrak{g}}$, we pretend that $(g, \partial) \cap (f)^{-1}(\mathfrak{H}, \partial) \in \{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}^c$. Since $\mu \in (g, \partial) \cap (f)^{-1}(\mathfrak{H}, \partial) \Rightarrow \mu \in (g, \partial), \mu \in (h)^{-1} \Rightarrow \mu \in (g, \partial), (h)(\mu) \in (\mathfrak{H}, \partial) \Rightarrow \mu \neq (g)(\mu)$. As $(g, \partial) \cap (\mathfrak{H}, \partial) = 0_{(\mathfrak{L}, \partial)}$ implies that μ does not belong to $\{(f)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\} \Rightarrow \mu \in \{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}^c$. Therefore, $\mathfrak{E}_1^{\mathfrak{g}} \in (g, \partial) \cap (g)^{-1}(\mathfrak{H}, 0_{(\mathfrak{L}, \partial)}) \in \{(f)(\kappa_{1 \vee_1, \vee_2, \vee_3}) = (\kappa_{1 \vee_1, \vee_2, \vee_3})\}^c$. Thus, $\{(f)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}^c$ is the NSQ neighborhood of each of its points. So, $\{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}^c$ is $NSQ*_b$ open, and hence, $\{(g)((\mathfrak{E}_1^{\mathfrak{g}})) = (\mathfrak{E}_1^{\mathfrak{g}})\}$ is $NSQ*_b$ closed. \square

6. Conclusion

Neutrosophic soft topology (NST) is extension of vague soft topology (VST) and VST is extension of fuzzy soft topology (FST). VST gives two type of informations. One is true, and second one is false. It does not give informations about the indeterminacy (doubtful) case. NST is dominant over VST because it supposes all the three informations that is true, false, and indeterminacy at the same time. NST has narrow domain as compared to neutrosophic soft bitopology (NSBT). Some problems are very hard to discuss in NST, so need of NSBT is felt. Still NSTS is unable to afford problems of large number of domain. So, extension is needed to neutrosophic soft tritopology (NSTT) and neutrosophic soft quad-topology (NSQT). In our work, we regenerated some structures in NSQTS with new definition that is $*_b$ open sets relative to soft points. We worked with the operations given in references [14–16] which are entirely different from references [13, 17]. In future, we will develop neutrosophic soft penta-topology and neutrosophic soft hexa-topology relative to soft points of the space under more generalized neutrosophic soft open sets. We will try to develop neutrosophic soft separation axioms and their engagement with each other. In addition to this, neutrosophic soft other separation axioms will also be given special attention. After doing this, on the basis of reference [12], we will try to build the same structures with respect to hyper-soft sets and then with respect to plithogenic hyper-soft sets.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

Authors' Contributions

All authors read and approved the final manuscript.

References

- [1] L. A. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, no. 3, pp. 338–353, 1965.
- [2] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.
- [3] D. Molodtsov, "Soft set theory—first results," *Computers & Mathematics with Applications*, vol. 37, no. 4/5, p. 19, 1999.
- [4] D. Molodtsov, *The Theory of Soft Sets*, URSS Publishers, Moscow, (in Russian), 2004.
- [5] N. O. Alshehri, M. Akram, and R. S. Al-ghamdi, "Applications of soft sets in -algebras," *Advances in Fuzzy Systems*, vol. 2013, pp. 1–8, Article ID 319542, 2013.
- [6] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications*, vol. 45, no. 4–5, pp. 555–562, 2003.
- [7] F. Feng, C. Li, B. Davvaz, and M. I. Ali, "Soft sets combined with fuzzy sets and rough sets: a tentative approach," *Soft Computing*, vol. 14, no. 9, pp. 899–911, 2010.
- [8] F. Feng, X. Liu, V. Leoreanu-Fotea, and Y. B. Jun, "Soft sets and soft rough sets," *Information Sciences*, vol. 181, no. 6, pp. 1125–1137, 2011.
- [9] X. Wei, M. Jian, S. Wang, and G. Hao, "Vague soft sets and their properties," *Computers and Mathematics with Applications*, vol. 59, pp. 787–794, 2010.
- [10] X. Huang, H. Li, and Y. Yin, "Notes on 'Vague soft sets and their properties'," *Computers & Mathematics with Applications*, vol. 64, no. 6, pp. 2153–2157, 2012.
- [11] C. Wang and Y. Li, "Topological structure of vague soft sets," *Abstract and Applied Analysis*, vol. 2014, pp. 1–8, Article ID 504021, 2014.
- [12] F. Smarandache, "Extension of soft set to hyper-soft set, and then to plithogenic hyper-soft set," *Neutrosophic Sets and Systems*, vol. 22, pp. 168–170, 2018.
- [13] T. Bera and N. K. Mahapatra, "Introduction to neutrosophic soft topological space," *Opsearch*, vol. 54, no. 4, pp. 841–867, 2017.
- [14] T. Y. Ozturk, "Some structures on neutrosophic topological spaces," *Applied Mathematics and Nonlinear Sciences*, vol. 6, no. 1, pp. 467–478, 2021.
- [15] T. Y. Öztürk, E. Karataş, and A. Yolcu, "On neutrosophic soft continuous mappings," *Turkish Journal of Mathematics*, vol. 45, no. 1, pp. 81–95, 2021.
- [16] Ç. Gunduz, T. Y. Ozturk, and S. Bayramov, "Separation axioms on neutrosophic soft topological spaces," *Turkish Journal of Mathematics*, vol. 43, no. 1, pp. 498–510, 2019.
- [17] A. B. AL-Nafee, "New family of neutrosophic soft sets," *Neutrosophic Sets and Systems*, vol. 38, pp. 482–496, 2020.
- [18] A. B. AL-Nafee, S. Broumi, and F. Smarandache, "Neutrosophic soft bitopological spaces," *International Journal of Neutrosophic Science*, vol. 14, no. 1, pp. 47–56, 2021.
- [19] H. Dadas and Sibel Demiralp, "Introduction to neutrosophic soft bitopological spaces," *International Journal of Neutrosophic Science*, vol. 14, no. 2, pp. 78–81, 2021.

- [20] F. Smarandache, "Neutrosophic set - a generalization of the intuitionistic fuzzy set," *2006 IEEE International Conference on Granular Computing*, vol. 24, no. 3, pp. 287–297, 2005.
- [21] I. Deli and S. Broumi, "Neutrosophic soft relations and some properties," *Annals of Fuzzy Mathematics and Informatics*, vol. 9, no. 1, pp. 169–182, 2015.
- [22] P. K. Maji, "Neutrosophic soft set," *Annals of Fuzzy Mathematics and Informatics*, vol. 5, no. 1, pp. 157–168, 2013.
- [23] T. Y. Ozturk, C. G. Aras, and S. Bayramov, "A new approach to operations on neutrosophic soft sets and to neutrosophic soft topological spaces," *Communications in Mathematics and Applications*, vol. 10, no. 3, pp. 481–493, 2019.
- [24] A. Yolcu, E. Karatas, and T. Y. Ozturk, "A new approach to neutrosophic soft mappings and application in decision making," in *Neutrosophic Operational Research*, F. Smarandache and M. Abdel-Basset, Eds., Springer, Cham, pp. 291–313, 2021.