This article has been accepted for publication in IEEE Access. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2024.3399881

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.
Digital Object Identifier 10.1109/ACCESS.2024.Doi Number

# A Novel Approach to Sustainable Grain Transportation: A Neutrosophic Bi-Objective Model for Cost and Time Optimization

**KALAIVANI KASPAR[1] AND PALANIVEL KALIYAPERUMAL***

Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology (VIT), Vellore - 632014. Tamil Nadu, India.
E-mail: *drkpalanivel@gmail.com, *palanivel.k@vit.ac.in, kalaivanikaspar92@gmail.com.

**ABSTRACT** As the global economy continues to grow, the need for transportation also grows. Transportation researchers are developing new methods for integrating new technologies into existing transportation systems and for addressing the associated challenges. This paper addressed bi-objective fixed charge solid transportation problem that consider two objectives minimizing the total transportation cost, including fixed and variable costs, and minimizing the total transportation time. It is a challenging optimization problem, as it is difficult to find a solution that simultaneously minimizes both objectives. Additionally, the problem with bi-objective fixed-charge solid transportation problem (BOFCSTP) under uncertainty with neutrosophic concept is presented here. This problem constructed with all the parameters such as cost, fixed-charge, source availability and requirements as neutrosophic values. Neutrosophic sets are efficiently handling the indeterminacy and imprecise data in many fields and single-valued neutrosophic sets are extension and simpler form of NS. Further, to convert the neutrosophic values to crisp values a ranking function is used. To solve the considered BOFCSTP different approaches are employed namely, neutrosophic linear programming, neutrosophic goal programming, fuzzy goal programming to get the compromised solution to the problem. Additionally, a real-life problem is given with numerical example and the results compared with the different approaches.

**INDEX TERMS** Transportation problem (TP), Multi-objective transportation problem (MOTP), fixed- charge solid transportation problem (FCSTP), Neutrosophic set (NS), Single-valued neutrosophic set (SVNS).

## I. INTRODUCTION

The transportation problem is a mathematical optimization problem that seeks to minimize the cost of transporting goods from a set of supply points to a set of demand points, subject to capacity constraints at each supply point and demand point. It is one of the most fundamental and well-studied problems in operations research, with applications in a wide range of industries, including logistics, transportation, manufacturing, and retail. **S**olid transportation problems or models (STPs or STMs) are a type of transportation model that considers the volume or weight of the goods being transported, in addition to the distance and cost of transportation. STMs are used in a variety of industries, including mining, construction, and agriculture, to optimize the transportation of goods from origin to destination. Then researchers developed STMs that can handle uncertainty. This is important because there are many factors that can affect transportation, such as weather, traffic conditions, and equipment failures. By developing models that can handle uncertainty, businesses can make more informed decisions about transportation planning and operations. To handle the uncertainty in any real world problem fuzzy set (FS) was introduced [1] and intuitionistic fuzzy set (IFS) was developed by Atanassov [2] followed by L. A. Zadeh where FS have membership function and IFS have both membership and non-membership function of the element in the set.

The fixed charge solid transportation problem is a variation of the classic transportation problem that involves shipping a homogeneous product from multiple suppliers to multiple destinations while considering fixed charges associated with using certain suppliers or facilities. Similar to the traditional transportation problem, the goal is to minimize the total transportation cost. However, in this case, the cost involves not only the variable cost of shipping but also fixed charges associated with using particular sources or facilities. Commonly used in logistics and supply chain management to optimize the routing of goods while considering fixed charges associated with suppliers or facilities. Applicable in various industries such as manufacturing, distribution, and retail to optimize transportation costs while accounting for fixed expenses. Molla-Alizadeh-Zavardehi [3] addressed fuzzy FCSTP and used neighborhood search (VNS), simulated annealing (SA) and hybrid VNS methods to solve the problem. Bo Zhang et.al. [4] developed expected value model, chance-constrained programming model and measure-chance programming model for fixed-charge STP with uncertain values. Lin Chen et.al. [5] presented a bi-criteria STP problem with fuzzy values and solved using goal programming.

### A. LITERATURE SURVEY

Stefan Chanas [6] presented a TP with all the supplies, demands as fuzzy values and used the technique of parametric programming to solve the FTP. Palanivel [7] presented fuzzy traveler problem with trapezoidal fuzzy number. Sujeet Kumar Singh and Shiv Prasad Yadav [8] solved TP with triangular intuitionistic fuzzy numbers using accuracy function to convert the fuzzy numbers to crisp numbers. I. Deli and Y. Subas [9] proposed a method to solve SVN decision making problem employing single-valued trapezoidal and single-valued triangular neutrosophic numbers. Kalaivani and palanivel [10] employed SVTNNs to solve a transportation problem proposing new ranking function to defuzzify the SVTNNs. Jency and palanivel [11] addressed a transportation problem during disaster under uncertainty with help of fuzzy triangular numbers. STP under uncertain environment was developed by F. Jimenez and J.L. Verdegay [12] in 1998 presented two problems considering one with interval data as interval STP and other with vague data as fuzzy STP. F. Jiménez and J.L. Verdegay presented a fuzzy STP (FSTP) problem [13] and used evolutionary algorithm to solve the FSTP. Ali Ebrahimnejad and Jose Luis Verdegay [14] solved intuitionistic fuzzy transportation problem(FTP) by converting the problem to deterministic model and solved by employing fuzzy programming approach. Hosna Mollanoori et.al. [15] solved capacitated solid step fixed-charge transportation problem by employing two methods simulated Annealing (SA) and Imperialist Competitive Algorithm (ICA). Amrit Das et.al [16] proposed de-fuzzification method to a type-2 fuzzy number with application bi-objective STP. Abhijit Baidya and Uttam Kumar Bera [17] proposed a fully fuzzy STP for improved freight and cargo management for government agencies in uncertain environments through sustainable, efficient, and effective transportation operations. Sankar Kumar Roy and Sudipta Midya [18] presented a multi-objective fixed-charge STP (FCSTP) with triangular intuitionistic fuzzy number and obtained the compromise solution using intuitionistic linear programming. Sankar Kumar Roy et.al. [19] discussed multi-objective multi-item FCSTP (MOMIFCSTP) with fuzzy-rough parameters. The considered MOMIFCSTP is converted to deterministic model, solved by three different approaches namely, weighted goal programming, fuzzy programming and TOPSIS. Hamiden Abd El-Wahed et.al. [20] solved multi-objective fractional two-stage STP with all the parameters as fuzzy numbers. Divya Chhibber et.al [21] solved a intuitionistic fixed-charge STP with fuzzy programming approach. Arijit Mondal et.al.[22] presented multi-objective multi-item multi-choice step fixed-charge solid transportation problem with intuitionistic fuzzy values for all the parameters and solved by introducing new method called intuitionistic fuzzy game-theoretic method. Shubham singh et.al. [23] solved MOSTP by chance constraint programming under stochastic environment and utilized the fuzzy programming (FP) to obtain the solution to the numerical problem. Saibal Majumder et.al. [24] proposed a uncertain MOMIFCSTP models and solved using three different methods. Pravash Kumar Giri et.al. [25] developed a MIFCSTP with budget constraint problem and solved using genetic algorithm.

Later, neutrosophic set (NS) was introduced with three membership functions called the truth, the indeterminacy and the falsity membership function by Smarandache [26] which helps to analyze the problem with indeterminate, incomplete data. Rizk M. Rizk-Allah et.al. [27] addressed neutrosophic MOTP and solved the problem by neutrosophic compromise approach, solving the TP for the membership functions of the neutrosophic set where all the parameters are taken as neutrosophic numbers. A. Thamaraiselvi and R. Santhi [28] proposed defuzzification function to convert the neutrosophic

This article has been accepted for publication in IEEE Access. This is the author's version which has not been fully edited and
content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2024.3399881



values to crisp values and solved the neutrosophic TP. Firoz Ahmad and Ahmad Yusuf Adhami [29] solved a multi-objective nonlinear transportation problem (MO-NTP) using neutrosophic compromise solution approach. Further, single-valued neutrosophic set (SVNS) was introduced which is the simplest form of NS and it helps the researchers to concentrate more on the problems with incomplete, undefined data in real-life. Binoy Krishna Giri and Sankar Kumar Roy [30] presented multi-objective green 4-dimensional fixed-charge transportation problem (MG4FTP) with all the parameters as single-valued trapezoidal neutrosophic numbers and solved by utilizing neutrosophic programming. Hamiden abd el-wahed khalifa et.al. [31] proposed a new approach KKM approach to solve the TP.Anish Kumar et.al. [32] proposed MOTP with uncertain normal distribution and solved using hyperbolic programming approach. Veeramani et.al [33] discussed a multi-objective fractional TP and obtained the compromise solutions to the problem using neutrosophic goal programming. Vishwas Deep Joshi et.al. [34] presented a multi-objective linear fractional transportation problem (MOLFTP) under neutrosophic environment then solved the problem for its three membership functions. . Ovidiu Cosma et.al. [35] proposed a heuristic algorithm to solve two-stage STP. Nengmin Wang et.al. [36] addressed a problem shipment of hazardous materials during emergency situation with bi-objective vehicle routing model and solved using two-stage exact algorithm based on ε-constraint method. Shyamali Ghosh et.al. [37] addressed a problem in the waste management area with multiple objectives to develop the optimum TP model with neutrosophic numbers. Sajida Kousar et.al. [38] discussed a crop production problem with neutrosophic elements and solved using neutrosophic fuzzy linear programming method. Sarfaraz Hashemkhani Zolfani et.al. [39] presented an efficient analysis technique with input and output satisficing (EATWIOS) method based on Type-2 Neutrosophic Fuzzy Numbers. Nada A. Nabeeh [40] have been discussed about SVNs for resolving MCDM problems in many real-world situations. Soumen Kumar Das et.al. [41] addressed multi-facility location–allocation model for a two-stage solid logistics network with type-2 neutrosophic numbers. Soumen Kumar Das and Sankar Kumar Roy [42] proposed a hybrid approach based on locate-allocate heuristic and the neutrosophic compromise programming to obtain the solution to facility location to the TP. Md Samim Aktar et. al. [43] discussed a 4D-TP with multiple objectives and different methods are used to convert the MOTP to single objective TP namely weighted sum technique, max-min Zimmermann technique and neutrosophic programming technique. Further, Generalized Reduced Gradient method used to solve the problem.

TABLE 1 LITERATURE SURVEY OF FC-STP

| Authors name | Year | No. of. Objectives | Methods |
|---|---|---|---|
| Sankar Kumar Roy and Sudipta Midya [18] | 2019 | Multi objective | Intuitionistic fuzzy programming |
| Pravash Kumar Giri et.al. [25] | 2018 | Single objective | Genetic algorithm |
| Rizk M. Rizk-Allah et.al. [27] | 2018 | Multi objective | Neutrosophic Compromise Programming Approach (NCPA) |
| Ahmad Yusuf Adhami [29] | 2019 | Multi objective | NCPA |
| Binoy Krishna Giri and Sankar Kumar Roy [30] | 2022 | Multi objective | Neutrosophic programming (NP) and Pythagorean hesitant fuzzy programming (PHFP) |
| Sajida Kousar et.al [38] | 2023 | Multi objective | Fuzzy, Neutrosophic and Intuitionistic linear programming |
| Veeramani etl.al [33] | 2021 | Multi objective Fractional TP | Neutrosophic goal programming (NGP) |
| Vishwas Deep Joshi et.al. [34] | 2022 | Multi-objective linear fractional transportation problem | NCPA |

| | | | |
|---|---|---|---|
| Shyamali Ghosh et.al. [37] | 2023 | Multi-objective | Neutrosophic linear programming and e-constraint method |
| Elsayed Badr et.al. [45] | 2022 | Multi-objective integer linear programming model | Neutrosophic goal programming (NGP) |
| Surapati Pramanik and Partha Pratim Dey [46] | 2019 | Multi-objective | NGP |

### B. MOTIVATION AND CONTRIBUTION OF THE WORK

- In the literature [44] addressed a TP with neutrosophic parameters and [18], [33] are presented MOTP and also employed various techniques to solve TP.
- [38] addressed a crop production problem with various factors as neutrosophic numbers. This motivated us to address the TP in a neutrosophic environment in the agricultural sector.
- Though advancements in exploring diverse bi-objective problems across various contexts, their application to unravel demanding agricultural transportation challenges has been covering. This study.
- This study bridges this gap by tackling a real-world transportation issue faced by farmers, demonstrating the potential of this approach in the agricultural domain.
- A BO-FCSTP problem is presented optimal solution the transportation problem of transporting different types grains from different farms to different silos using different types of conveyance.
- This BO-FCSTO under neutrosophic environment to deal with uncertainties in the processing of data due to indeterminacy.
- The neutrosophic values can be converted to crisp values while retaining the inherent neutrosophic characteristics of the data by employing the existing ranking value.
- Three different methods are used to solve the considered problem Neutrosophic linear programming problem (NLPP), Neutrosophic Goal programming (NGP) and Fuzzy Goal Programming (FGP).
- The effectiveness of the three methods are discussed with help of sensitivity analysis for different cases.

The paper is structured into eight distinct sections. Section 1 serves as the introduction, providing an overview of the research study along with an extensive review of relevant literature. In Section 2, the fundamental definition of neutrosophic sets is presented, accompanied by a discussion on a ranking function aimed at converting neutrosophic numbers into crisp values. Section 3 presents the mathematical formulation of the model FCSTP. Following this, Section 4 offers an in-depth exploration of the various solution methodologies employed within the study. Section 5 delves into a practical application scenario, analysing the transportation of different grains from farms to silos within the framework of FCSTP. This section concludes with a comprehensive examination of the results obtained through different methodological approaches. Sections 6 and 7 focus on sensitivity analysis and comparative studies, respectively, utilizing different case scenarios and ranking functions. Finally, Section 8 provides a concise conclusion, summarizing the findings and implications derived from the study.

## II. PRELIMINARY CONCEPTS

This sections includes all the basics elements to build up this article. The following definitions helps the readers to understand more about the NS.

*Definition 1* [26]:

Let $X$ be a non-empty set. Then a NS $\tilde{\xi}^N$ of $X$ is defined as

$$\tilde{\xi}^N = \{\langle x, \alpha_{\tilde{\xi}^N}(x), \beta_{\tilde{\xi}^N}, \gamma_{\tilde{\xi}^N}(x)\rangle | x \in X, \alpha_{\tilde{\xi}^N}(x), \beta_{\tilde{\xi}^N}(x), \gamma_{\tilde{\xi}^N}(x) \in ]^{-}0, 1^{+}[\},$$

Where $\alpha_{\tilde{\xi}^N}(x),\ \beta_{\tilde{\xi}^N}(x), and\ \gamma_{\tilde{\xi}^N}(x)$ are the truth membership function, an indeterminacy membership function, and a falsity function, and there is no restriction on the sum of $\alpha_{\tilde{\xi}^N}(x),\ \beta_{\tilde{\xi}^N}(x), and\ \gamma_{\tilde{\xi}^N}(x)$, so $^{-}0 \leq \alpha_{\tilde{\xi}^N}(x)+\beta_{\tilde{\xi}^N}(x) + \gamma_{\tilde{\xi}^N}(x) \leq 3^{+}$ is a nonstandard unit Interval.

*Definition 2* [29]

Let $X$ be a non-empty set. A SVNS $\tilde{\xi}_{SN}$ is defined as $\tilde{\xi}_{SN}=\{x,\alpha_{\tilde{\xi}_{SN}}(x),\beta_{\tilde{\xi}_{SN}}(x),\gamma_{\tilde{\xi}_{SN}}(x)|x\in X\}$ where $\alpha_{\tilde{\xi}_{SN}}(x),\beta_{\tilde{\xi}_{SN}}(x)$ and $\gamma_{\tilde{\xi}_{SN}}(x)\in[0,1]$ for each $x\in X$ and $0\le\alpha_{\tilde{\xi}_{SN}}(x)+\beta_{\tilde{\xi}_{SN}}(x)+\gamma_{\tilde{\xi}_{SN}}(x)\le 3$.

*Definition 3* [29]

Let $\varphi_1,\varphi_2\varphi_{3,}\varphi_4\in\mathbb{R}$ such that $\varphi_1\le\varphi_2\le\varphi_3\le\varphi_4$ and $\alpha_{\tilde{\varphi}},\beta_{\tilde{\varphi}},\gamma_{\tilde{\varphi}}\in[0,1]$. Then an SVTNNs is defined as $\tilde{\varphi}=\langle(\varphi_1,\varphi_2,\varphi_3,\varphi_4);\alpha_{\tilde{\varphi}},\beta_{\tilde{\varphi}},\gamma_{\tilde{\varphi}}\rangle$ is a special neutrosophic set on the real line set $\mathbb{R}$, whose truth membership, indeterminacy membership, and falsity membership functions are given as follows:

$$\mu_{\alpha_\varphi}(x)=\begin{cases}\alpha_{\tilde{\varphi}}\left(\dfrac{x-\varphi_1}{\varphi_2-\varphi_1}\right) & ,\varphi_1\le x\le\varphi_2\\ \alpha_{\tilde{\varphi}} & ,\varphi_2\le x\le\varphi_3\\ \alpha_{\tilde{\varphi}}\left(\dfrac{\varphi_4-x}{\varphi_4-\varphi_3}\right) & ,\varphi_3\le x\le\varphi_4\\ 0, otherwise & \end{cases}$$

$$v_{\beta_{\tilde{\varphi}}}(x)=\begin{cases}\dfrac{\varphi_2-x+\beta_{\tilde{\varphi}}(x-\varphi)}{\varphi_2-\varphi_1} & ,\varphi_1\le x\le\varphi_2\\ \beta_{\tilde{\varphi}} & ,\varphi_2\le x\le\varphi_3\\ \dfrac{x-\varphi_2+\beta_{\tilde{\varphi}}(\varphi_4-x)}{\varphi_4-\varphi_3} & ,\varphi_3\le x\le\varphi_4\\ 1\,, otherwise & \end{cases}$$

$$w_{\gamma_{\tilde{\varphi}}}(x)=\begin{cases}\dfrac{\varphi_2-x+\gamma_{\tilde{\varphi}}(x-\varphi_1)}{n_2-n_1} & ,\varphi_1\le x\le\varphi_2\\ \gamma_{\tilde{\varphi}} & ,\varphi_2\le x\le\varphi_3\\ \dfrac{x-\varphi_2+\gamma_{\tilde{\varphi}}(\varphi_4-x)}{\varphi_4-\varphi_3} & ,\varphi_3\le x\le\varphi_4\\ 1, otherwise & \end{cases}$$

Where $-0\le\mu_{\alpha_{\tilde{\varphi}}}(x)+v_{\beta_{\tilde{\varphi}}}(x)+w_{\gamma_{\tilde{\varphi}}}(x)\le 1+,x\in\tilde{\varphi}$.

*Definition 4* [47]

Let $\tilde{\theta}=\langle(\theta_1,\theta_2,\theta_3,\theta_4);\alpha_{\tilde{\theta}},\beta_{\tilde{\theta}},\gamma_{\tilde{\theta}}\rangle$ and $\tilde{\eta}=\langle(\eta_1,\eta_2,\eta_3,\eta_4);\alpha_{\tilde{\eta}},\beta_{\tilde{\eta}},\gamma_{\tilde{\eta}}\rangle$ be two single valued trapezoidal neutrosophic numbers and $k\ne o$, then

Addition:

$$\tilde{\theta}+\tilde{\eta}=\langle(\theta_1+\eta_1,\theta_2+\eta_2,\theta_3+\eta_3,\theta_4+\eta_4);\alpha_{\tilde{\theta}}\wedge\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\vee\beta_{\tilde{\eta}},\gamma_\theta\vee\gamma_{\tilde{\eta}}\rangle$$

Difference:

$$\tilde{\theta}-\tilde{\eta}=\langle(\theta_1-\eta_4,\theta_2-\eta_2,\theta_3-\eta_2,\theta_4-\eta_1);\alpha_{\tilde{\theta}}\vee\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\wedge\beta_{\tilde{\eta}},\gamma_{\tilde{\theta}}\wedge\gamma_{\tilde{\eta}}\rangle$$

Product:

$$\tilde{\theta}\tilde{\eta}=\begin{Bmatrix}\langle(\theta_1\eta_1,\theta_2\eta_2,\theta_3\eta_3,\theta_4\eta_4);\alpha_{\tilde{\theta}}\wedge\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\vee\beta_{\tilde{\eta}},\gamma_\theta\vee\gamma_{\tilde{\eta}}\rangle\\(d_1>0,d_2>0)\\\langle(\theta_1\eta_4,\theta_2\eta_3,\theta_3\eta_2,\theta_4\eta_1);\alpha_{\tilde{\theta}}\wedge\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\vee\beta_{\tilde{\eta}},\gamma_\theta\vee\gamma_{\tilde{\eta}}\rangle\\(d_1<0,d_2>0)\\\langle(\theta_4\eta_4,\theta_3\eta_3,\theta_2\eta_2,\theta_1\eta_1);\alpha_{\tilde{\theta}}\wedge\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\vee\beta_{\tilde{\eta}},\gamma_\theta\vee\gamma_{\tilde{\eta}}\rangle\\(d_1<0,d_2<0)\end{Bmatrix}$$

Division:

$$\tilde{\theta}/\tilde{\eta}=\begin{Bmatrix}\left\langle\begin{matrix}(\theta_1/\eta_4,\theta_2/\eta_3,\theta_3/\eta_2,\theta_4/\eta_1);\\\alpha_{\tilde{\theta}}\wedge\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\vee\beta_{\tilde{\eta}},\gamma_\theta\vee\gamma_{\tilde{\eta}}\end{matrix}\right\rangle(d_1>0,d_2>0)\\\left\langle\begin{matrix}(\theta_4/\eta_4,\theta_3/\eta_3,\theta_2/\eta_2,\theta_1/\eta_1);\\\alpha_{\tilde{\theta}}\wedge\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\vee\beta_{\tilde{\eta}},\gamma_\theta\vee\gamma_{\tilde{\eta}}\end{matrix}\right\rangle(d_1<0,d_2>0)\\\left\langle\begin{matrix}(\theta_4/\eta_1,\theta_3/\eta_2,\theta_2/\eta_3,\theta_1/\eta_4);\\\alpha_{\tilde{\theta}}\wedge\alpha_{\tilde{\eta}},\beta_{\tilde{\theta}}\vee\beta_{\tilde{\eta}},\gamma_\theta\vee\gamma_{\tilde{\eta}}\end{matrix}\right\rangle(d_1<0,d_2<0)\end{Bmatrix}$$

Scalar product:

$$k\tilde{\theta}=\begin{Bmatrix}\langle(k\theta_1,k\theta_2,k\theta_3,k\theta_4);\alpha_{\tilde{\theta}},\beta_{\tilde{\theta},}\gamma_{\tilde{\theta}}\rangle(k>0)\\\langle(k\theta_4,k\theta_3,k\theta_2,k\theta_1);\alpha_{\tilde{\theta}},\beta_{\tilde{\theta},}\gamma_{\tilde{\theta}}\rangle(k<o)\end{Bmatrix}$$

*Definition 5* [10]

Let $\tilde{A}_{SN}$ and $\tilde{B}_{SN}$ be two SVNNs. The ranking of $\tilde{A}_{SVN}$ and $\tilde{B}_{SVN}$ by the R (.) on the set of SVNNs is defined as follows.

I. $R(\tilde{A}_{SVN})>R(\tilde{B}_{SVN})$ iff $\tilde{A}_{SVN}>\tilde{B}_{SVN}$
II. $(\tilde{A}_{SVN})<R(\tilde{B}_{SVN})$ iff $\tilde{A}_{SVN}<\tilde{B}_{SVN}$
III. $R(\tilde{A}_{SVN})=R(\tilde{B}_{SVN})$ iff $\tilde{A}_{SVN}=\tilde{B}_{SVN}$

*Definition 6* [10]

` The ordering $\ge$ and $\le$ between any two SVNNs $\tilde{A}_{SVN}$ and $\tilde{B}_{SVN}$ are defined as follows.

I. $\tilde{A}_{SN}\ge\tilde{B}_{SVN}$ iff $\tilde{A}_{SVN}>\tilde{B}_{SVN}$ or $\tilde{A}_{SVN}=\tilde{B}_{SVN}$
II. $\tilde{A}_{SN}\le\tilde{B}_{SVN}$ iff $\tilde{A}_{SVN}<\tilde{B}_{SVN}$ or $\tilde{A}_{SVN}=\tilde{B}_{SVN}$

*Definition 7* [10]

Let $\{\tilde{A}_{SVN_i}, i = 1,2,\dots,n\}$ be a set of SVNNs. If $R(\tilde{A}_{SVN}) \le R(\tilde{B}_{SVN})$ for all $i$, then the SVNNs $\tilde{A}_{SVN_k}$ is the minimum of $\{\tilde{A}_{SVN_i}, i = 1,2,\dots,n\}$

*Definition 8* [10]

Let $\{\tilde{A}_{SVN_i}, i = 1,2,\dots,n\}$ be a set of SVNNs. If $R(\tilde{A}_{SVN}) \ge R(\tilde{B}_{SVN})$ for all $i$, then the SVNNs $\tilde{A}_{SVN_t}$ is the minimum of $\{\tilde{A}_{SVN_i}, i = 1,2,\dots,n\}$

*Definition 9* [48]

De-fuzzification methods are used to convert the fuzzy values to crisp value to reduce the problem as a crisp problem to solve the problems. Here, to convert the neutrosophic values to crisp value the ranking function called score function is used. Let $\tilde{A} = \langle(a_1, a_2, a_3, a_4); T, I, F\rangle$ be the SVTNNs. Then the score function $S(\tilde{A})$ [48] is given as,

$$S(\tilde{A}) = \frac{1}{12}(a_1 + a_2 + a_3 + a_4) \times [2 + T - I - F]$$

## III. MATHEMATICAL MODEL FOR BI-OBJECTIVE FCSTP

To transport non-homogenous products from different sources to different destinations with certain condition in TP, we consider a multi- objective (two) fixed – charge solid transportation problem. However, there are constraints on the quantities of products available for transport from the sources, the requirements at the destinations and the vehicles have limited capacities to transport a particular products. Two objective functions are taken, first objective function is to minimize the total transportation cost and second objective is to minimize the total transportation time with respect to the set of constraints. The aim is to obtain optimal solution for transporting non-homogenous $l$ products from $m$ sources to $n$ destinations using the $k$ conveyances at shipping cost $\tilde{C}^N_{ijkl}$ per unit product in such a way that all objectives are optimized simultaneously. Fixed-charge $\tilde{F}^N_{ijkl}$ is fixed for shipping $l$ products from source $i$ to destination $j$ means of $k$ conveyance and each source has $\tilde{S}^N_{il}$ units of $l$ products as supply and at each destination has $\tilde{R}^N_{il}$ units of $l$ products as requirement and each conveyance transport $l$ product from source $i$ to destination $j$ has $\tilde{E}^N_{kl}$ units pf capacity.

*Notations*:

The following notations are used to formulate the proposed model.

$\tilde{x}^N_{ijkl}$- amount of $l^{th}$ product to be transported from $i^{th}$ source to $j^{th}$ destination through $k^{th}$ conveyance

$\tilde{C}^N_{ijkl}$- cost per unit of $l^{th}$ product to be transported from $i^{th}$ source to $j^{th}$ destination through $k^{th}$ conveyance

$\tilde{F}^N_{ijkl}$- fixed cost of $l^{th}$ product to be transported from $i^{th}$ source to $j^{th}$ destination through $k^{th}$ conveyance

$\tilde{t}^N_{ijkl}$- time to transport $l^{th}$ product from $i^{th}$ source to $j^{th}$ destination through $k^{th}$ conveyance

$\tilde{a}^N_{il}$- total supply at $i^{th}$ source

$\tilde{b}^N_{jl}$- total demand at $j^{th}$ destination

$\tilde{e}^N_{kl}$- capacity of $k^{th}$ conveyance

$L$- total quantity of $l^{th}$ product

**Model 1:**

$$\tilde{\zeta}^N_1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{q}(\tilde{C}^N_{ijkl} + \tilde{F}^N_{ijkl})\,\tilde{x}^N_{ijkl} \quad (1)$$
$$\tilde{\zeta}^N_2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{k=1}^{q}\tilde{t}^N_{ijkl}\,\tau(\tilde{x}^N_{ijkl}) \quad (2)$$

Subject to the constraints,

$$\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{q}\tilde{x}^N_{ijkl} \le \tilde{a}^N_{il}, i = 1,2,3,\dots,m \quad (3)$$
$$\sum_{i=1}^{m}\sum_{k=1}^{p}\sum_{l=1}^{q}\tilde{x}^N_{ijkl} \ge \tilde{b}^N_{jl}, j = 1,2,3,\dots,n \quad (4)$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{l=1}^{q}\tilde{x}^N_{ijkl} \ge \tilde{e}^N_{kl}, k = 1,2,3,\dots,p \quad (5)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{p}\sum_{l=1}^{q}\tilde{x}^N_{ijkl} \le L, l = 1,2,3,\dots,q \quad (6)$$

$$\tilde{x}^N_{ijkl} \ge 0, \forall i,j,k,l \quad (7)$$

$$\tau(\tilde{x}_{ijkl}) = \begin{cases} 1, if\ \tilde{x}_{ijkl} > 0 \\ 0, otherwise \end{cases} \quad (8)$$

With the feasibility condition of this TP,

$$\sum_{i=1}^{m}\sum_{l=1}^{q}\tilde{a}^N_{il} \ge \sum_{j=1}^{n}\sum_{l=1}^{q}\tilde{b}^N_{jl}$$

This article has been accepted for publication in IEEE Access. This is the author's version which has not been fully edited and
content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2024.3399881



$$\sum_{k=1}^{p}\sum_{l=1}^{q}\tilde{e}_{kl}^{N} \geq \sum_{j=1}^{n}\sum_{l=1}^{q}\tilde{b}_{jl}^{N}$$

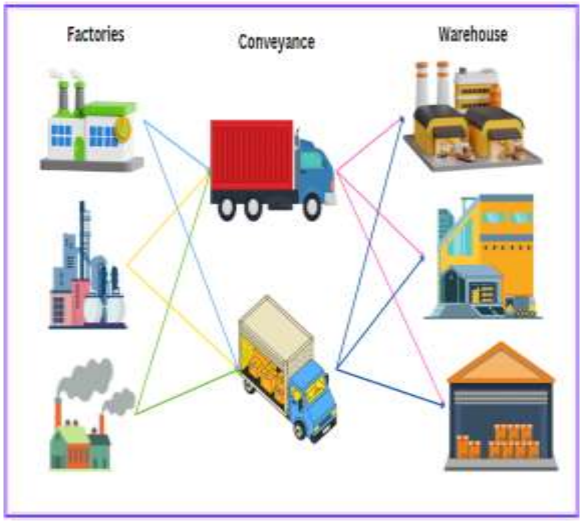



FIGURE 1. **Transportation model for STP**

## IV. SOLUTION METHODOLOGY:

In multi-objective optimization problem, there does not always exist a solution which is the best for all the objective functions. That is the solution will be the best for one objective function and that may be worst for another objective function. The objective functions are conflicting to each other and hence the solutions cannot simply compare to each other. For this cause, we discuss three methods for solving neutrosophic MFSTP as follows:

- Neutrosophic linear programming problem (NLPP),
- Neutrosophic goal programming (NGP)
- Fuzzy goal programming (FGP)

### 4.1 Neutrosophic linear programming problem (NLPP)

NLPP to derive the compromise solution of multi-objective decision making problem. To solve the proposed model 1 using NLPP, we describe the following steps as:

Step 1: Transform the neutrosophic optimization problem into crisp problem using the definition.

Step 2: Each objective is solved individually with subject to all the constraints.

Step 3: Determine the positive and negative solution to find the upper and lower bound for each objective function by pay-off matrix.

TABLE 2. PAY-OFF MATRIX

| | $\zeta_1$ | $\zeta_2$ |
|---|---|---|
| $X_1^*$ | $\zeta_1(X_1^*)$ | $\zeta_2(X_1^*)$ |
| $X_2^*$ | $\zeta_1(X_1^*)$ | $\zeta_2(X_2^*)$ |

Step 4: Determine the truth, indeterminacy and falsity membership functions by setting the tolerance and constructing the according to bounds.

$$T_{\psi}\left(\zeta_{\psi}'(x)\right) = \begin{cases} 1, if\ \zeta_{\psi}' \leq L_{\psi}^{T} \\ \dfrac{U_{\psi}^{T} - \zeta_{\psi}'(x)}{U_{\psi}^{T} - L_{\psi}^{T}}, if\ L_{\psi}^{T} \leq \zeta_{\psi}'(x) \leq U_{\psi}^{T} \\ 0, if\ \zeta_{\psi}'(x) \geq U_{\psi}^{T} \end{cases}$$

$$I_{\psi}\left(\zeta_{\psi}'(x)\right) = \begin{cases} 1, if\ \zeta_{\psi}' \leq L_{\psi}^{I} \\ \dfrac{U_{\psi}^{I} - \zeta_{\psi}'(x)}{U_{\psi}^{I} - L_{\psi}^{I}}, if\ L_{\psi}^{I} \leq \zeta_{\psi}'(x) \leq U_{\psi}^{I} \\ 0, if\ \zeta_{\psi}'(x) \geq U_{\psi}^{I} \end{cases}$$

$$F_{\psi}\left(\zeta_{\psi}'(x)\right) = \begin{cases} 1, if\ \zeta_{\psi}' \leq L_{\psi}^{F} \\ \dfrac{U_{\psi}^{F} - \zeta_{\psi}'(x)}{U_{\psi}^{F} - L_{\psi}^{F}}, if\ L_{\psi}^{F} \leq \zeta_{\psi}'(x) \leq U_{\psi}^{F} \\ 0, if\ \zeta_{\psi}'(x) \geq U_{\psi}^{F} \end{cases}$$

Here $U_{\psi}^{T} = U_{\psi}$ and $L_{\psi}^{T} = L_{\psi}$ the upper and lower bounds for truth membership function, $U_{\psi}^{I} = L_{\psi}^{T} + (U_{\psi}^{T} - L_{\psi}^{T})\, e_{\psi}$, $L_{\psi}^{I} = L_{\psi}^{T}$ the upper and lower bound for indeterminacy membership function, $U_{\psi}^{F} = U_{\psi}^{T}$, $L_{\psi}^{F} = L_{\psi}^{T} + (U_{\psi}^{T} - L_{\psi}^{T})\, f_{\psi}$, the upper and lower bounds for falsity membership function where $e_{\psi} and\ f_{\psi}$ are tolerance.

Step 5: Choose the values for $a, b\ and\ c$ in [0,1] for each neutrosophic number as the truth, indeterminacy and falsity degrees respectively.

Step 6: Constitute the NLPP from Model 1.

Using the above steps for NLPP the given Model 1 is transformed into Model 2. From Model 2 the Model 3 is obtained.

**Model 2**:

$$Maximize\ T_\psi\left(\zeta'_\psi(x)\right)(\psi = 1,2)$$

$$Maximize\ I_\psi\left(\zeta'_\psi(x)\right)(\psi = 1,2)$$

$$Minimize\ F_\psi\left(\zeta'_\psi(x)\right)(\psi = 1,2)$$

Subject to (3)-(8)

**Model 3:**

$Maximize\ a$

$Maximize\ b$

$Minimize\ c$

Subject to

$$T_\psi\left(\zeta'_\psi(x)\right) \geq a$$

$$I_\psi\left(\zeta'_\psi(x)\right) \geq b$$

$$F_\psi\left(\zeta'_\psi(x)\right) \leq c$$

$a + b + c \leq 3, a + b + c \geq 0, a \geq b, a \geq c, and\ a, b, c \in [0,1]$, ($\psi$=1,2),

Constraints (3)-(8).

The simplified model 3 derives the compromise solution to the considered problem by transforming into Model 4.

**Model 4:**

$maximize\ a + b - c$

Subject to $\zeta'_\psi(x) + (U^T_\psi - L^T_\psi)a \leq U^T_\psi$,

$\zeta'_\psi(x) + (U^I_\psi - L^I_\psi)b \leq U^I_\psi$,

$\zeta'_\psi(x) - (U^T_\psi - L^T_\psi)c \leq U^F_\psi$,

$a + b + c \leq 3, a + b + c \geq 0, a \geq b, a \geq c, and\ a, b, c \in [0,1]$, ($\psi$=1, 2),

Constraints (3)-(8).

Step 7: Solve the model 4 using LINGO 19.0 iterative solver.

### 4.2 Neutrosophic goal programming (NGP)

For neutrosophic goal programming problem the neutrosophic decision set is formulated as,

$$D_N = \left(\bigcap_{k=1}^{K} G_k\right)\left(\bigcap_{j=1}^{J} C_j\right) = (y, T_D(y), I_D(y), F_D(y))$$

Further the formulate the goal programming [46] and solve using LINGO19.0.

### 4.3 Fuzzy goal programming (FGP)

All minimizing type of a multi objective linear programming problem is of the form

$$\min[\zeta_1(x), \zeta_2(x), \dots, \zeta_k(x)]$$

$$h_j(x) \leq b_j, j = 1\ to\ m$$

Then the fuzzy decision set can be defined as, $D = G \cap C$ where $D$ is fuzzy decision, $G$ fuzzy goal and $C$ fuzzy constraints. The fuzzy goal programming model [45] formulated and solved using LINGO 19.0 .

## V. REAL-LIFE EXPERIMENT

In this section we include a real-life problem of transporting grain from multiple farms to multiple silos. The fixed charge in this case could be the cost of loading and unloading the grain at each farm and silo. FCSTPs are often complex and difficult to solve. This is because the number of possible transportation routes and schedules can be very large, especially for problems with many origins and destinations. One of the biggest challenges in transporting grain from multiple farms to multiple silos is finding the most efficient and cost-effective transportation route. This is because there are many possible routes, and each route has its own pros and cons. For example, a route that is shorter may be more expensive, while a route that is longer may be less expensive. Another

This article has been accepted for publication in IEEE Access. This is the author's version which has not been fully edited and
content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2024.3399881



challenge is ensuring that the grain is transported in a timely manner. This is important because grain is a perishable commodity and can spoil if it is not stored properly. It is also important to ensure that the grain is transported without damage. Transporting grain from multiple farms to multiple silos is a complex problem that involves a number of factors, including:

- The distance between the farms and silos
- The volume of grain to be transported
- The type of transportation mode to be used
- The cost of transportation
- The time it takes to transport the grain

A number of different transportation modes can be used to transport grain from multiple farms to multiple silos. The cost of transporting grain from multiple farms to multiple silos varies depending on a number of factors, including the distance to be traveled, the volume of grain to be transported, and the type of transportation mode used. Overall, the problem of transporting grain from multiple farms to multiple silos is a complex one that involves a number of factors.


**FIGURE 2. Transporting grains to silos from farms**

In this problem two different type of grains are transported from two different farms to four different silos through two different vehicles. The objectives are to minimize the total transportation cost and time with subject to set of constraints. The transportation cost, availability and demand of grains at the farms and silos, the total amount of different grains, capacity of the conveyance are taken as single-valued trapezoidal neutrosophic numbers. The total time to transport the grains from farms to silos taken as crisp values. Total amount of grains at farm $\{a_1^N$= (9550, 10800, 12000, 20870; 0.9, 0.4, 0.2) and $a_2^N$= (4500, 4900, 5330, 6150; 0.6, 0.2, 0.1$\}$m, the amount of grains required at the silos $\{b_1^N$= (3420, 5100, 6000, 6350; 0.7, 0.2, 0.2), $b_2^N$= (5700, 7600, 8510, 9500; 0.8, 0.4, 0.1), $b_3^N$ = (3200, 4500, 5480, 6460; 0.7, 0.4, 0.1), $b_4^N$= (2450, 3020, 3900, 4200; 0.5, 0.1, 0.1)$\}$, the total capacity of the conveyance $\{ e_1^N$= (8200, 9550, 10060, 10800; 0.6, 0.2, 0.2) and $e_2^N$ = (7390, 8500, 9600, 10000; 0.9, 0.4, 0.2)$\}$ and the total amount of grains $\{ L_1^N$ =( 9880, 10400, 10900, 21000; 0.8, 0.4, 0.1) and $L_2^N$=( 11100, 13800, 15720, 22000; 0.7, 0.2, 0.2)$\}$.

Using the de-neutrsophication function all the neutrosophic values are converted to crisp value and the objective functions and constraints are given below. The considered problem is solved by three different approaches NLPP, NGP and FGP using the LINGO 19.0 iterative software.

$$\zeta_1^N = 121.82 * x_{11111} + 56 * x_{1112} + 122.05 * x_{1121} + 111 * x_{1122} + 98.43 * x_{1211} + 112 * x_{1212} + 130.07 * x_{1221} + 140.44 * x_{1222} + 55 * x_{1311} + 108 * x_{1312} + 110 * x_{1321} + 113 * x_{1322} + 134.55 * x_{1411} + 90.87 * x_{1412} + 100.02 * x_{1421} + 87.56 * x_{1422} + 98.45 * x_{2111} + 133 * x_{2112} + 150 * x_{2121} + 105.49 * x_{2122} + 122.82 * x_{2211} + 125.06 * x_{2212} + 142 * x_{2221} + 103.03 * x_{2222} + 158.85 * x_{2311} + 189.99 * x_{2312} + 122.22 * x_{2321} + 109.34 * x_{2322} + 140.50 * x_{2411} + 105.20 * x_{2412} + 98.43 * x_{2421} + 134.55 * x_{2422}$$

$$\zeta_2^N = 3 * x_{11111} + 2 * x_{1112} + 1 * x_{1121} + 1.5 * x_{1122} + 3 * x_{1211} + 2 * x_{1212} + 2 * x_{1221} + 4 * x_{1222} + 3.5 * x_{1311} + 1 * x_{1312} + 2 * x_{1321} + 3 * x_{1322} + 3 * x_{1411} + 1.5 * x_{1412} + 2 * x_{1421} + 4 * x_{1422} + 4 * x_{2111} + 3 * x_{2112} + 1 * x_{2121} + 1.5 * x_{2122} + 1 * x_{2211} + 2 * x_{2212} + 2 * x_{2221} + 3 * x_{2222} + 2.5 * x_{2311} + 2.5 * x_{2312} + 2.5 * x_{2321} + 1 * x_{2322} + 3 * x_{2411} + 4 * x_{2412} + 2 * x_{2421} + 2 * x_{2422}$$

Subject to,

$$x_{1111} + x_{1112} + x_{1211} + x_{1222} + x_{1311} + x_{1322} + x_{1411} + x_{1422} \geq 2550$$

$$x_{2111} + x_{2112} + x_{2211} + x_{2222} + x_{2311} + x_{2322} + x_{2411} + x_{2422} \geq 1000$$

$$x_{1111} + x_{1121} + x_{1122} + x_{1122} + x_{2111} + x_{2121} + x_{2122} \leq 1500$$

$$x_{1211} + x_{1221} + x_{1222} + x_{2211} + x_{2221} + x_{2222} \leq 500$$

$$x_{1311} + x_{1321} + x_{1322} + x_{2311} + x_{2321} + x_{2322} \leq 900$$

$$x_{1411} + x_{1421} + x_{1422} + x_{2411} + x_{2421} + x_{2422} \leq 650$$

$$x_{1111} + x_{1112} + x_{1211} + x_{1212} + x_{1311} + x_{1312} + x_{1411} + x_{1412} + x_{2111} + x_{2112} + x_{2211} + x_{2212} + x_{2311} + x_{2312} + x_{2411} + x_{2412} \leq 1850$$

$$x_{1121} + x_{1122} + x_{1221} + x_{1222} + x_{1321} + x_{1322} + x_{1421} + x_{1422} + x_{2121} + x_{2122} + x_{2221} + x_{2222} + x_{2321} + x_{2322} + x_{2421} + x_{2422} \leq 1700$$

$$x_{1111} + x_{1121} + x_{1211} + x_{1221} + x_{1311} + x_{1321} + x_{1411} + x_{1421} + x_{2111} + x_{2121} + x_{2211} + x_{2221} + x_{2311} + x_{2321} + x_{2411} + x_{2421} \leq 2500$$

$$x_{1112} + x_{1122} + x_{1212} + x_{1222} + x_{1312} + x_{1322} + x_{1412} + x_{1422} + x_{2112} + x_{2122} + x_{2212} + x_{2222} + x_{2312} + x_{2322} + x_{2412} + x_{2422} \leq 2500$$

$$x_{ijkl}, \forall\, i, j, k, l$$

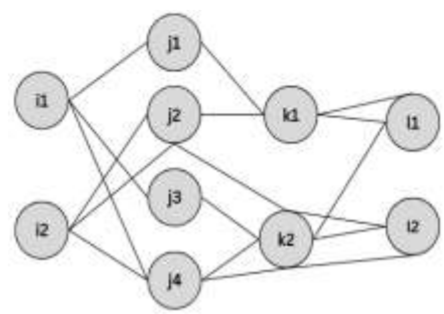



**FIGURE 3a. Optimal distribution using NLPP method**

TABLE 3 NEUTROSOPHIC COST PER UNIT

| | | $l = 1$ | |
|---|---|---|---|
| | | $k = 1$ | $k = 2$ |
| | $j = 1$ | (58, 77, 89, 120; 0.9, 0.4, 0.2) | (57,68, 80,100; 0.7, 0.2, 0.2) |
| | $j = 2$ | (49, 67, 76, 90; 0.8, 0.3, 0.2) | (88, 100, 108, 115; 0.5, 0.1, 0.1) |
| | $j = 3$ | (29, 38, 40, 44; 0.9, 0.4, 0.2) | (100, 110, 115, 126; 0.7, 0.3, 0.1) |

| | | | |
|---|---|---|---|
| $i = 1$ | $j = 4$ | (68, 78, 92, 102; 0.6, 0.2, 0.1) | (56, 66, 71, 85; 0.6, 0.2, 0.1) |
| $i = 2$ | $j = 1$ | (53, 62, 77, 105;0.8, 0.4, 0.1) | (71, 82, 91, 105; 0.9, 0.3, 0.3) |
| | $j = 2$ | (66, 79, 89, 114; 0.7, 0.2, 0.2) | (77, 81, 94, 105; 0.9, 0.3, 0.3) |
| | $j = 3$ | (99, 110, 118, 136; 0.6, 0.2, 0.1) | (60, 73, 88, 98; 0.7, 0.2, 0.2) |
| | $j = 4$ | (80, 90, 115, 120; 0.8, 0.3, 0.2) | (69, 77, 87, 98; 0.8, 0.3, 0.2) |

| | | $l = 2$ | |
|---|---|---|---|
| | | $k = 1$ | $k = 2$ |
| $i = 1$ | $j = 1$ | (35, 42, 50, 55; 0.5, 0.1, 0.1) | (66, 78, 90, 120; 0.6, 0.2, 0.1) |
| | $j = 2$ | (66, 78, 88, 99; 0.7, 0.3, 0.1) | (97,105,118,130; 0.6, 0.2, 0.2) |
| | $j = 3$ | (50, 70, 81, 102; 0.8, 0.3, 0.2) | (79, 90, 102, 112; 0.7, 0.2, 0.2) |
| | $j = 4$ | (55, 60, 73, 86; 0.8, 0.3, 0.2) | (56, 66, 80, 98; 0.5, 0.1, 0.1) |
| $i = 2$ | $j = 1$ | (81,92, 100, 111; 0.5, 0.1, 0.1) | (71, 82, 91, 100; 0.9, 0.3, 0.3) |
| | $j = 2$ | (80, 90, 101, 110; 0.7, 0.3, 0.1) | (69, 75, 88, 98; 0.8, 0.3, 0.2) |
| | $j = 3$ | (120, 130, 138, 148; 0.5, 0.1, 0.1) | (79, 86, 99, 102; 0.6, 0.2, 0.1) |
| | $j = 4$ | (60, 72, 89, 100; 0.7, 0.2, 0.2) | (98, 105, 110, 117; 0.9, 0.4, 0.2) |

TABLE 4 NEUTROSOPHIC FIXED CHARGE COST

| | | $l = 1$ | |
|---|---|---|---|
| | | $k = 1$ | $k = 2$ |
| $i = 1$ | $j = 1$ | (59, 69, 76, 96; 0.9, 0.4, 0.2) | (55, 65, 74, 98; 0.7, 0.2, 0.2) |
| | $j = 2$ | (30, 44, 58, 70; 0.8, 0.3, 0.2) | (60, 74, 85, 110; 0.6, 0.2, 0.1) |
| | $j = 3$ | (29, 30, 40,50; 0.9, 0.4, 0.2) | (22,30, 43,51; 0.7, 0.3, 0.1) |
| | $j = 4$ | (77, 82, 90, 102; 0.6, 0.2, 0.1) | (44, 50, 64, 80; 0.6, 0.2, 0.1) |
| $i = 2$ | $j = 1$ | (39, 45, 58, 73; 0.8, 0.4, 0.1) | (90, 100, 110, 120; 0.6, 0.2, 0.1) |
| | $j = 2$ | (56, 62, 76, 87; 0.7, 0.2, 0.2) | (83, 97, 101, 118; 0.9, 0.3, 0.3) |
| | $j = 3$ | (66, 74, 86, 110; 0.6, 0.2, 0.1) | (66, 75, 82, 103; 0.5, 0.1, 0.1) |
| | $j = 4$ | (63, 80, 86, 97; 0.8, 0.3, 0.2) | (39, 42, 49, 57; 0.8, 0.3, 0.2) |

This article has been accepted for publication in IEEE Access. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/ACCESS.2024.3399881



| | | $l = 2$ | |
|---|---|---|---|
| | | $k = 1$ | $k = 2$ |
| $i = 1$ | $j = 1$ | (20, 32, 40, 44; 0.5, 0.1, 0.1) | (35, 46, 62, 88; 0.6, 0.2, 0.1) |
| | $j = 2$ | (44, 54, 66, 98; 0.7, 0.3, 0.1) | (67, 74, 82, 108; 0.6, 0.2, 0.1) |
| | $j = 3$ | (40, 55, 69, 84; 0.8, 0.3, 0.2) | (38, 50, 62, 73; 0.7, 0.2, 0.2) |
| | $j = 4$ | (42, 46, 54, 61; 0.8, 0.3, 0.2) | (30, 36, 41, 50; 0.5, 0.1, 0.1) |
| $i = 2$ | $j = 1$ | (59, 79, 87, 92; 0.5, 0.1, 0.1) | (39, 52, 61, 69; 0.9, 0.3, 0.3) |
| | $j = 2$ | (47, 56, 79, 82; 0.7, 0.3, 0.1) | (39, 50, 56, 66; 0.8, 0.3, 0.2) |
| | $j = 3$ | (97, 104, 123, 130; 0.7, 0.2, 0.2) | (40, 44, 50, 70; 0.6, 0.2, 0.1) |
| | $j = 4$ | (44, 56, 60, 70; 0.7, 0.3, 0.1) | (54, 64, 76, 85; 0.4, 0.2) |

TABLE 5 TIME AS CRISP VALUE

| | | $l = 1$ | | $l = 2$ | |
|---|---|---|---|---|---|
| | | $k = 1$ | $k = 2$ | $k = 1$ | $k = 2$ |
| $i = 1$ | $j = 1$ | 3 | 1 | 2 | 1.5 |
| | $j = 2$ | 3 | 2 | 2 | 4 |
| | $j = 3$ | 4 | 1 | 3.5 | 2 |
| | $j = 4$ | 3 | 1.5 | 3 | 2 |
| $i = 2$ | $j = 1$ | 4 | 3 | 4 | 1 |
| | $j = 2$ | 1.5 | 2 | 1 | 2 |
| | $j = 3$ | 3 | 2.5 | 2.5 | 2.5 |
| | $j = 4$ | 3 | 2 | 4 | 2 |

TABLE 7 OPTIMAL RESULTS USING ALL THREE DIFFERENT METHODS

| S.No. | Methods | Optimal solution | Objectives value |
|---|---|---|---|
| 1 | NLPP | $x_{1111} = 1500$; $x_{1322} = 900$; $x_{1421} = 150$; $x_{2211} = 350$; $x_{2222} = 150$; $x_{2422} = 500$ | $\zeta_1^N = 424960$; $\zeta_2^N = 9300$ |
| 2 | NGP | $x_{1112} = 1300$; $x_{1222} = 500$; $x_{1322} = 550$; $x_{1421} = 200$; $x_{2112} = 200$; $x_{2311} = 350$; $x_{2422} = 450$ | $\zeta_1^N = 397918$; $\zeta_2^N = 9025$ |
| 3 | FGP | $x_{1111} = 1500$; $x_{1322} = 900$; $x_{1421} = 150$; $x_{2211} = 350$; $x_{2222} = 150$; $x_{2422} = 500$ | $\zeta_1^N = 424960$; $\zeta_2^N = 9300$ |

### A. RESULT AND DISCUSSION

Solving the problem with help of three different approaches NLPP, NGP and FGP the compromise solution of the problem are depicted in the above section. Therefore the values of the two objective functions obtained using by three methods are displayed below in the table 5. Comparing the results of the method NLPP, NGP and FGP, we conclude that the results obtained by NGP gives the better results than NLPP and FGP methods. The detailed distribution of the optimal solution to the problem by the three different methods are given below table 6. and in Fig. 3a, 3b, 3c and Fig. 4 shows the overall results of the problem obtained by using the different methods.

TABLE 6 COMPROMISE SOLUTIONS OF NLPP, NGP AND FGP

| S. No. | Methods | $\zeta_1^N$ | $\zeta_2^N$ |
|---|---|---|---|
| 1. | NLPP | 424960 | 9300 |
| 2. | NGP | 367918 | 9025 |
| 3. | FGP | 424960 | 9300 |

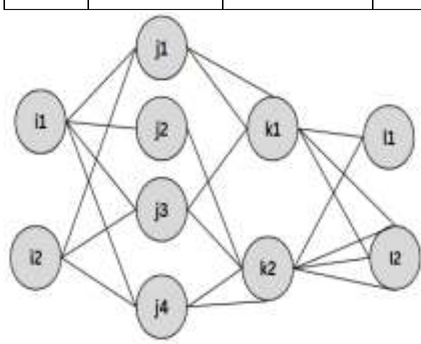



FIGURE 3b. **Optimal distribution using NGP method**

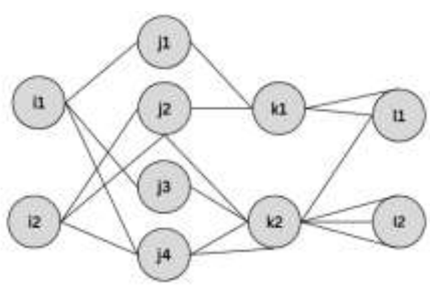



FIGURE 3c. **Optimal distribution using FGP method**

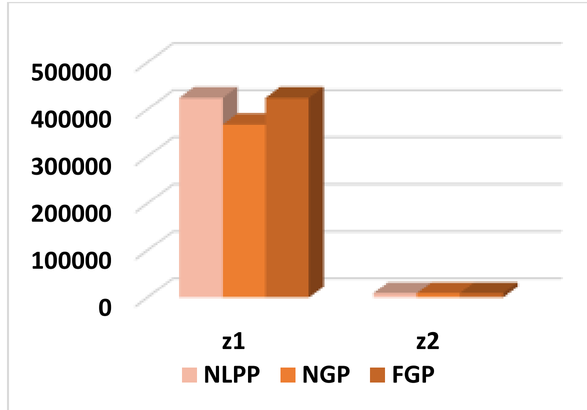



FIGURE 4. **Results using different methods**

## VI. SENSITIVITY ANALYSIS

Sensitivity analysis is a powerful tool that can be used to improve the understanding of models and systems. It is a technique used to identify how changes in the input parameters of a model or system affect its output. It can be used to identify important input parameters, assess the uncertainty in model outputs, and make informed decisions about model development and use. It is a widely used tool in a variety of fields, including engineering, finance, economics, and science.

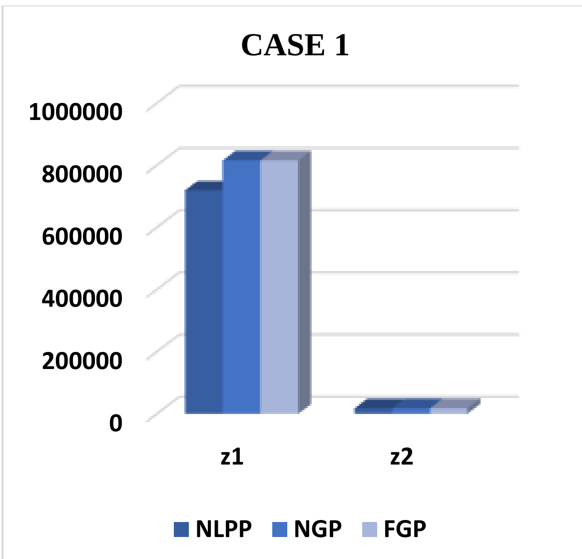
Sensitivity analysis was used for this problem in order to examine the results based on three different situations with fixed parameters for time, cost, and transportation. For each of the three situations, each item's total quantity, demand, and availability are adjusted; the results are presented in the graph below. Among all the three the case 3 gives the optimal result.

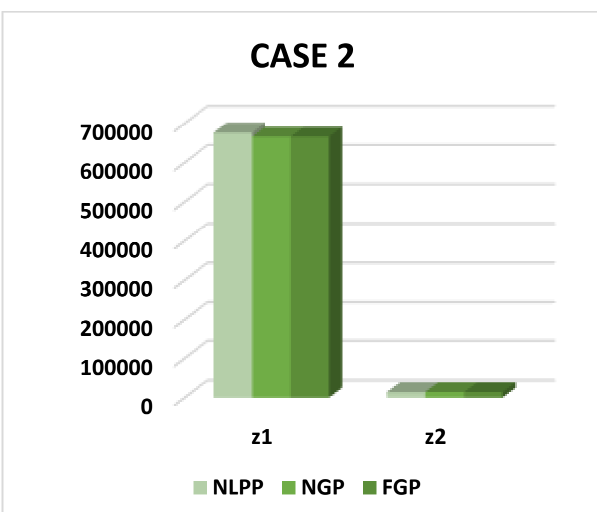
TABLE 8 SENSITIVITY ANALYSIS

| Different cases | Availability | Demand | Capacity of the vehicles | Total amount of items transported |
|---|---|---|---|---|
| Case 1 | a1=2500, a2=3500 | b1=2000, b2=1500, b3=1500, b4=1000 | e1=3500, e2=2500 | l1=2500, l2=3500 |
| Case 2 | a1=3000, a2=2500 | b1=1500, b2=1450, b3=1050, b4=1500 | e1=2900, e2=2600 | l1=2500, l2=3000 |
| Case 3 | a1=3000, a2=1000 | b1=500, b2=1500, b3=1200, b4=800 | e1=2100, e2=1900 | l1=2000, l2=2500 |

TABLE 9 RESULTS FOR THE THREE DIFFERENT CASES

| Different cases | NLPP | | NGP | | FGP | |
|---|---|---|---|---|---|---|
| | Z1 | Z2 | Z1 | Z2 | Z1 | Z2 |
| Case 1 | 718460 | 17000 | 676183 | 13850 | 433475 | 9400 |
| Case 2 | 814954 | 17749 | 665815 | 14074 | 430073 | 10024 |
| Case 3 | 814954 | 17749 | 665815 | 14074 | 430073 | 10024 |



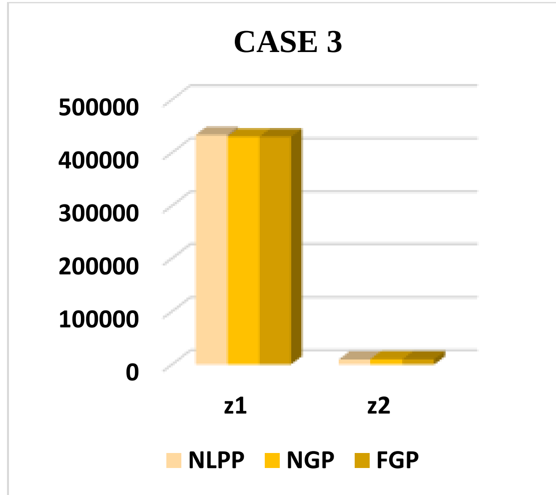




**FIGURE 5.** **Results using different methods for three different cases**

## VII. COMPARATIVE STUDY

This section presents comparative study analysis of different ranking techniques which helps in de-fuzzification of the neutrosophic values to crisp value. Some of the following ranking functions accuracy function [48], Uma and Uthra's ranking function [49] and Kalai and palanivel ranking function [10] are used to de-neutrosify the neutrosophic values in the considered problem and the obtained optimal results using the different methods are presented in the table 10 and Fig. 7. Each of the three ranking functions yields distinct optimal solutions for the TP under consideration, but the score function provides the best optimal solution overall. This comparative analysis demonstrates that all optimal results from the different ranking functions exceed the optimal result obtained by the score function alone (refer to Table 6). Specifically, for the considered BIFCSTP, the score function offers the optimal solution across all three methods: NLPP, NGP, and FGP.

### *A. MANAGERIAL IMPLICATIONS*

It is crucial to acknowledge the significant potential and consequences outlined in this study. The focus of this paper is a specific Transportation Problem (TP) within the agricultural sector, addressing various uncertain parameters associated with the transportation of grains from farms to silos. Different types of grains are conveyed using various transportation methods, each with a fixed charge for transporting a unit of grain from farms to silos. Traditional approaches have proven ineffective in finding the optimal solution for this particular problem, known as BOFCSTP. The uncertainties related to cost, supply, and demand are effectively managed through the utilization of neutrosophic numbers, particularly in the form of SVTNNs. The paper concludes by discussing optimal solutions for each method, comparing them with alternative approaches, and presenting a sensitivity analysis of the problem. Different scenarios are explored by varying one parameter while keeping others constant, providing a comprehensive understanding of the problem's distinctions. Moreover the comparative study with other few ranking functions is performed which ensures the optimal solution obtained by Score function. Finally the contribution and the future scope was discussed in the conclusion section.

TABLE 10 COMPARATIVE STUDY WITH DIFFERENT RANKING FUNCTIONS

| Ranking Functions | Methods | Optimal solutions | Objective Values |
|---|---|---|---|
| Accuracy function [48] $A(\tilde{A}) = \frac{1}{12}(a_1 + a_2 + a_3 + a_4) \times [2 + T - I + F]$ | NLPP | $x_{1111} = 3620;\ x_{1222} = 1620; x_{1322} = 3900; x_{1421} = 2800;\ x_{2112} = 770; x_{2211} = 3580$ | $\zeta_1^N = 22650034$; $\zeta_2^N = 38280$ |
| | NGP | $x_{1111} = 3300;\ x_{1222} = 5520; x_{1311} = 350; x_{1421} = 2800;\ x_{2112} = 800; x_{2311} = 3550$ | $\zeta_1^N = 2442494$; $\zeta_2^N = 50080$ |
| | FGP | $x_{1111} = 3300;\ x_{1222} = 5520; x_{1311} = 350; x_{1421} = 2800;\ x_{2112} = 800; x_{2311} = 3550$ | $\zeta_1^N = 2442494$; $\zeta_2^N = 50080$ |
| Uma and Uthras ranking function [49] $R(\tilde{A}) = \frac{(a+b+c+d)}{4} \times \frac{(W+1-u+1-V)}{3}$ | NLPP | $x_{1111} = 4000;\ x_{1222} = 800; x_{1322} = 5000; x_{1421} = 400;\ x_{2112} = 600; x_{2211} = 2600$ | $\zeta_1^N = 1476299$; $\zeta_2^N = 37000$ |
| | NGP | $x_{1111} = 4000;\ x_{1112} = 300; x_{1222} = 800; x_{1322} = 5000;\ x_{1421} = 100; x_{2211} = 2900; x_{2422} = 1100$ | $\zeta_1^N = 1381944$; $\zeta_2^N = 36000$ |
| | FGP | $x_{1112} = 1200;\ x_{1211} = 3200; x_{1222} = 800; x_{1322} = 5000;\ x_{2112} = 2800; x_{2422} = 1200$ | $\zeta_1^N = 1471634$; $\zeta_2^N = 41000$ |
| Kalaivani & Palanivel [10] $A(\tilde{A}) = \frac{1}{2}((a_3 + a_4) - ((a_2 + a_1))$*(2*T-I+F) | NLPP | $x_{1111} = 6000;\ x_{1222} = 2000; x_{1322} = 5000; x_{1421} = 1400;\ x_{2112} = 1300; x_{2211} = 700$ | $\zeta_1^N = 3006211$; $\zeta_2^N = 48400$ |
| | NGP | $x_{1111} = 4000;\ x_{1222} = 4060; x_{1311} = 2000; x_{1322} = 3000;\ x_{1421} = 1400; x_{2111} = 700; x_{2121} = 1300$ | $\zeta_1^N = 2713754$; $\zeta_2^N = 53500$ |
| | FGP | $x_{1111} = 4000;\ x_{1222} = 4060; x_{1311} = 2000; x_{1322} = 3000;\ x_{1421} = 1400; x_{2111} = 700; x_{2121} = 1300$ | $\zeta_1^N = 2713754$; $\zeta_2^N = 53500$; |

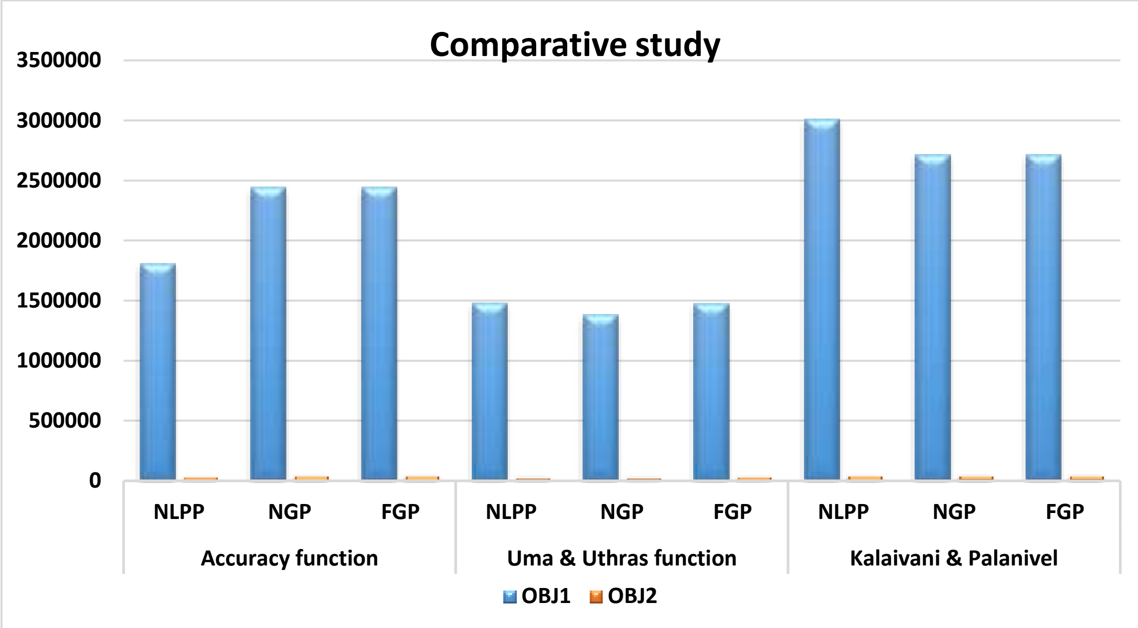



**FIGURE 6.** Results for comparative study using different ranking functions

## VIII. CONCLUSION

Transportation systems have become increasingly complex in recent years due to globalization, urbanization, and technological advancements. This complexity has led to a growing need for efficient and effective transportation planning and optimization algorithms. So we need transportation to be more efficient and sustainable, and because of e-commerce and new transportation technologies. MOTP, MOSTP and MOFCSTP are special cases of TP in real-life transportations and this paper presented a BOFCSTP of transporting grains from farms to different silos. To find the efficient optimal solution the parameters such as cost, fixed-charge, availability, demand are treated as neutrosophic numbers and the transportation time is taken as real number. Since fuzzy and intuitionistic sets cannot handle the indeterminacy part of the data, the neutrosophic set is employed to overcome the challenge. The main aim of the paper is developing neutrosophic mathematical model under uncertainty with two objectives to minimize the transportation cost and time with a numerical illustration of transporting grains from farms to silos. Moreover, to obtain the neutrosophic compromise solution to the neutrosophic BOFCSTP three different approaches are used to solve the BOFCSTP and the obtained results are discussed with each other. This research can be further expanded by applying it to different scenarios within the framework of MOFCSTP, where parameters can be adjusted to match specific situations. Additionally, the model can be adapted to various problem types, including interval-valued models, multi-stage models, and routing problems, all while maintaining the neutrosophic environment for uncertainty handling.

## REFERNCES

[1] L. A. Zadeh, "Fuzzy logic," *Computer,* vol. 21, pp. 83-93, 1988.
[2] K. Atanassov, "Intuitionistic fuzzy sets," *International journal bioautomation,* vol. 20, p. 1, 2016.
[3] S. Molla-Alizadeh-Zavardehi, S. S. Nezhad, R. Tavakkoli-Moghaddam, and M. Yazdani, "Solving a fuzzy fixed charge solid transportation problem by metaheuristics," *Mathematical and computer modelling,* vol. 57, pp. 1543-1558, 2013.
[4] B. Zhang, J. Peng, S. Li, and L. Chen, "Fixed charge solid transportation problem in uncertain environment and its algorithm," *Computers & Industrial Engineering,* vol. 102, pp. 186-197, 2016.
[5] L. Chen, J. Peng, and B. Zhang, "Uncertain goal programming models for bicriteria solid transportation problem," *Applied Soft Computing,* vol. 51, pp. 49-59, 2017.
[6] S. Chanas, W. Kołodziejczyk, and A. Machaj, "A fuzzy approach to the transportation problem," *Fuzzy sets and Systems,* vol. 13, pp. 211-221, 1984.
[7] K. Palanivel, "Fuzzy commercial traveler problem of trapezoidal membership functions within the sort of α optimum solution using ranking technique," *Afrika Matematika,* vol. 27, pp. 263-277, 2016.
[8] S. K. Singh and S. P. Yadav, "A new approach for solving intuitionistic fuzzy transportation problem of type-2," *Annals of operations research,* vol. 243, pp. 349-363, 2016.

[9] I. Deli and Y. Şubaş, "A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems," *International Journal of Machine Learning and Cybernetics,* vol. 8, pp. 1309-1322, 2017.

[10] P. Kaliyaperumal and K. Kalaivani, "A neutrosophic approach to the transportation problem using single-valued trapezoidal neutrosophic numbers," *Proyecciones (Antofagasta, On line),* vol. 42, pp. 533-547, 2023.

[11] J. L. Edward and K. Palanivel, "Two-Stage Transportation Model for Distributing Relief Aids to the Affected Regions in an Emergency Response Under Uncertainty," in *Lecture Notes in Networks and Systems*. vol. 758, ed Turkey: Springer Nature Switzerland, 2023, pp. 426-439.

[12] F. Jiménez and J. L. Verdegay, "Uncertain solid transportation problems," *Fuzzy sets and systems,* vol. 100, pp. 45-57, 1998.

[13] F. Jiménez and J. L. Verdegay, "Solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach," *European journal of operational research,* vol. 117, pp. 485-510, 1999.

[14] A. Ebrahimnejad and J. L. Verdegay, "A new approach for solving fully intuitionistic fuzzy transportation problems," *Fuzzy optimization and decision making,* vol. 17, pp. 447-474, 2018.

[15] H. Mollanoori, R. Tavakkoli-Moghaddam, C. Triki, M. Hajiaghaei-Keshteli, and F. Sabouhi, "Extending the solid step fixed-charge transportation problem to consider two-stage networks and multi-item shipments," *Computers & Industrial Engineering,* vol. 137, p. 106008, 2019.

[16] A. Das, U. K. Bera, and M. Maiti, "Defuzzification and application of trapezoidal type-2 fuzzy variables to green solid transportation problem," *Soft computing,* vol. 22, pp. 2275-2297, 2018.

[17] A. Baidya and U. K. Bera, "Solid transportation problem under fully fuzzy environment," *International Journal of Mathematics in Operational Research,* vol. 15, pp. 498-539, 2019.

[18] S. K. Roy and S. Midya, "Multi-objective fixed-charge solid transportation problem with product blending under intuitionistic fuzzy environment," *Applied Intelligence,* vol. 49, pp. 3524-3538, 2019.

[19] S. K. Roy, S. Midya, and G.-W. Weber, "Multi-objective multi-item fixed-charge solid transportation problem under twofold uncertainty," *Neural Computing and Applications,* vol. 31, pp. 8593-8613, 2019.

[20] H. A. E.-W. Khalifa, P. Kumar, and M. G. Alharbi, "On characterizing solution for multi-objective fractional two-stage solid transportation problem under fuzzy environment," *Journal of intelligent systems,* vol. 30, pp. 620-635, 2021.

[21] D. Chhibber, D. C. Bisht, and P. K. Srivastava, "Pareto-optimal solution for fixed-charge solid transportation problem under intuitionistic fuzzy environment," *Applied Soft Computing,* vol. 107, p. 107368, 2021.

[22] A. Mondal, S. K. Roy, and S. Midya, "Intuitionistic fuzzy sustainable multi-objective multi-item multi-choice step fixed-charge solid transportation problem," *Journal of Ambient Intelligence and Humanized Computing,* vol. 14, pp. 6975-6999, 2023.

[23] S. Singh, A. Pradhan, and M. Biswal, "Multi-objective solid transportation problem under stochastic environment," *Sādhanā,* vol. 44, p. 105, 2019.

[24] S. Majumder, P. Kundu, S. Kar, and T. Pal, "Uncertain multi-objective multi-item fixed charge solid transportation problem with budget constraint," *Soft Computing,* vol. 23, pp. 3279-3301, 2019.

[25] P. K. Giri, M. K. Maiti, and M. Maiti, "Simulation approach to solve fuzzy fixed charge multi-item solid transportation problems under budget constraint," *International Journal of Operational Research,* vol. 32, pp. 56-91, 2018.

[26] F. Smarandache, "Neutrosophic set - a generalization of the intuitionistic fuzzy set," pp. 38-42, 2006.

[27] R. M. Rizk-Allah, A. E. Hassanien, and M. Elhoseny, "A multi-objective transportation model under neutrosophic environment," *Computers & Electrical Engineering,* vol. 69, pp. 705-719, 2018.

[28] A. Thamaraiselvi and R. Santhi, "A new approach for optimization of real life transportation problem in neutrosophic environment," *Mathematical Problems in Engineering,* vol. 2016, pp. 1-9, 2016.

[29] F. Ahmad and A. Y. Adhami, "Neutrosophic programming approach to multiobjective nonlinear transportation problem with fuzzy parameters," *International journal of management science and engineering management,* vol. 14, pp. 218-229, 2019.

[30] B. K. Giri and S. K. Roy, "Neutrosophic multi-objective green four-dimensional fixed-charge transportation problem," *International Journal of Machine Learning and Cybernetics,* vol. 13, pp. 3089-3112, 2022.

[31] H. A. E.-W. Khalifa, P. Kumar, and S. Mirjalili, "A KKM approach for inverse capacitated transportation problem in neutrosophic environment," *Sādhanā,* vol. 46, p. 166, 2021.

[32] A. Kumar, P. Singh, and Y. Kacher, "Neutrosophic hyperbolic programming strategy for uncertain multi-objective transportation problem," *Applied Soft Computing,* p. 110949, 2023.

[33] C. Veeramani, S. Edalatpanah, and S. Sharanya, "Solving the multiobjective fractional transportation problem through the neutrosophic goal programming approach," *Discrete Dynamics in Nature and Society,* vol. 2021, pp. 1-17, 2021.

[34] V. D. Joshi, J. Singh, R. Saini, and K. S. Nisar, "Solving multi-objective linear fractional transportation problem under neutrosophic environment," *Journal of Interdisciplinary Mathematics,* vol. 25, pp. 123-136, 2022.

[35] O. Cosma, D. Dănciulescu, and P. C. Pop, "On the two-stage transportation problem with fixed charge for opening the distribution centers," *IEEE Access,* vol. 7, pp. 113684-113698, 2019.

[36] N. Wang, M. Zhang, A. Che, and B. Jiang, "Bi-objective vehicle routing for hazardous materials transportation with no vehicles travelling in echelon," *IEEE Transactions on Intelligent Transportation Systems,* vol. 19, pp. 1867-1879, 2017.

[37] S. Ghosh, S. K. Roy, and G.-W. Weber, "Interactive strategy of carbon cap-and-trade policy on sustainable multi-objective solid transportation problem with twofold uncertain waste management," *Annals of Operations Research,* pp. 1-41, 2023.

[38] S. Kousar, M. N. Sangi, N. Kausar, D. Pamucar, E. Ozbilge, and T. Cagin, "Multi-objective optimization model for uncertain crop production under neutrosophic fuzzy environment: A case study," *AIMS Mathematics,* vol. 8, pp. 7584-7605, 2023.

[39] S. H. Zolfani, Ö. F. Görçün, M. Çanakçıoğlu, and E. B. Tirkolaee, "Efficiency analysis technique with input and output

satisficing approach based on Type-2 Neutrosophic Fuzzy Sets: A case study of container shipping companies," *Expert Systems with Applications,* vol. 218, p. 119596, 2023.

[40] M. A.-B. NADA A. NABEEH, HAITHAM A. EL-GHAREEB and A. A. ABOELFETOUH, "Neutrosophic Multi-Criteria Decision Making Approach for IoT-Based Enterprises," *IEEE Access,* 2019.

[41] S. K. Das, F. Y. Vincent, S. K. Roy, and G. W. Weber, "Location–allocation problem for green efficient two-stage vehicle-based logistics system: A type-2 neutrosophic multi-objective modeling approach," *Expert Systems with Applications,* vol. 238, p. 122174, 2024.

[42] S. K. Das and S. K. Roy, "Effect of variable carbon emission in a multi-objective transportation-p-facility location problem under neutrosophic environment," *Computers & Industrial Engineering,* vol. 132, pp. 311-324, 2019.

[43] H. Gholizadeh, M. Goh, H. Fazlollahtabar, and Z. Mamashli, "Modelling uncertainty in sustainable-green integrated reverse logistics network using metaheuristics optimization," *Computers & Industrial Engineering,* vol. 163, p. 107828, 2022.

[44] N. Qiuping, T. Yuanxiang, S. Broumi, and V. Uluçay, "A parametric neutrosophic model for the solid transportation problem," *Management Decision,* vol. 61, pp. 421-442, 2023.

[45] E. Badr, E. E. Seidy, A. Elrayes, and A. Rabie, "Analyzing the application of the sustainable development goals for Egypt using a neutrosophic goal programming approach," *Process Integration and Optimization for Sustainability,* vol. 7, pp. 489-500, 2023.

[46] S. Pramanik and P. P. Dey, *Multi-level linear programming problem with neutrosophic numbers: A goal programming strategy*: Infinite Study, 2019.

[47] İ. Delİ and E. Kirmizi ÖZtÜRk, "A Defuzzification Method on Single Valued Trapezoidal Neutrosophic Number and Multiple Criteria Decision Making," *Cumhuriyet Science Journal,* vol. 41, pp. 22-37, 2020.

[48] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and L. Vladareanu, "Computation of shortest path problem in a network with SV-trapezoidal neutrosophic numbers," in *2016 International Conference on Advanced Mechatronic Systems (ICAMechS)*, 2016, pp. 417-422.

[49] R. Umamageswari and G. Uthra, *Generalized Single Valued Neutrosophic Trapezoidal Numbers and their Application to Solve Transportation Problem*: Infinite Study, 2020.

**KALAIVANI K** received M.Phil. in the department of mathematics, Auxilium College, Vellore affiliated to Thiruvalluvar University, Vellore. Currently doing research in the field of fuzzy optimization techniques under the supervision of Dr. K. Palanivel in the Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore – 632014, Tamil Nadu.

**Dr. PALANIVEL K** received Ph.D. in the department of mathematics, TBML College, Porayar affiliated to Bharathidasan University, Trichy. Having 16 years of experience in the expertise field of mathematics about teaching and research. Currently working as an Associate Professor in the Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore – 632014, Tamil Nadu, since July 2010. At present researching, the field of fuzzy optimization, fuzzy decision making, fuzzy algebra, and soft computing.