Neutrosophic Triplet Partial v-Generalized Metric Space

Memet Şahin
Gaziantep University

Abdullah Kargın
Gaziantep University

Neutrosophic *Triplet* Structures

Volume I

Editors:

Prof. Dr. FLORENTIN SMARANDACHE

Associate Prof. Dr. MEMET ŞAHIN

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Editors:
Prof. Dr. Florentin Smarandache
Associate Prof. Dr. Memet Şahin

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PhD **Abdullah Kargın**, Department of Mathematics, Gaziantep University, Gaziantep27310-Turkey, abdullahkargin27@gmail.com.

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Chapter Two

Neutrosophic Triplet Partial v-Generalized Metric Space

Memet Şahin[1], Abdullah Kargın[2,*]
[1,2] Department of Mathematics, Gaziantep University, Gaziantep 27310, Turkey.
Email: mesahin@gantep.edu.tr, abdullahkargin27@gmail.com

Abstract

In this chapter, study the notion of neutrosophic triplet partial v-generalized metric space. Then, we give some definitions and examples for neutrosophic triplet partial v-generalized metric space and obtain some properties and prove these properties. Furthermore, we show that neutrosophic triplet partial v-generalized metric space is different from neutrosophic triplet v-generalized metric space and neutrosophic triplet partial metric space.

Keywords: neutrosophic triplet metric space, neutrosophic triplet partial metric space, neutrosophic triplet v- generalized metric spaces, neutrosophic triplet partial v- generalized metric spaces

1. Introduction

Smarandache introduced neutrosophy in 1980, which studies a lot of scientific fields. In neutrosophy, there are neutrosophic logic, set and probability in [1]. Neutrosophic logic is a generalization of a lot of logics such as fuzzy logic in [2] and intuitionistic fuzzy logic in [3]. Neutrosophic set is denoted by (t, i, f) such that "t" is degree of membership, "i" is degree of indeterminacy and "f" is degree of non-membership. Also, a lot of researchers have studied neutrosophic sets in [4-9, 35-39]. Furthermore, Smarandache and Ali obtained neutrosophic triplet (NT) in [10] and they introduced NT groups in [11]. For every element "x" in neutrosophic triplet set A, there exist a neutral of "a" and an opposite of "a". Also, neutral of "x" must different from the classical unitary element. Therefore, the NT set is different from the classical set. Furthermore, a NT "x" is denoted by <x, neut(x), anti(x)>. Also, many researchers have introduced NT structures in [12-20].

Matthew obtained partial metric spaces in [21]. The partial metric is generalization of classical metric space and it plays a significant role in fixed point theory and computer science. Also, many researchers studied partial metric space in [22-28].

Branciari obtained v-generalized metric in [29]. The v- generalized metric is a specific form of classical metric space for its triangular inequality. The most important use of v-generalized metric space is fixed point theory. Furthermore, researchers studied v- generalized metric in [30- 34].

In this chapter, we obtain NT partial v-generalized metric space. In section 2; we give definitions of NT set in [11], NT partial metric space in [17] and NT v-generalized metric space in [18]. In section 3, we introduce NT partial v-generalized metric space and give some properties and examples for NT partial v-generalized metric space. Also, we show that NT partial v-generalized metric space is different from the NT partial metric space and NT v-generalized metric space. Furthermore, we show the relationship between NT partial metric spaces, NT v-generalized metric space with NT partial v-generalized metric space. Finally, we give definition of convergent sequence, Cauchy sequence and complete space for NT partial v-generalized metric space. In section 4, we give conclusions.

2. Basic and Fundamental Concepts

Definition 2.1: [29] Let N be a nonempty set and d:NxN→ ℝ be a function. If d is satisfied the following properties, then it is called a v - generalized metric. For
n, m, c_1, c_2, … , c_v ∈ N,

i) d(n, m) ≥ 0 and d(n, m) = 0 ⇔ n = m;

ii) d(n, m)= d(m, n);

iii) d(n, m) ≤ d(n, c_1)+d(c_1, c_2)+d(c_2, c_3)+ … + d(c_{v-1}, c_v)+ d(c_v, n). Where a, c_1, c_2, … , c_v, b are all different.

Definition 2.2:[21]Let N be a nonempty set and d:NxN→ ℝ be a function. If d is satisfied the following properties, then it is a partial metric. For p, r, s ∈ N,

i) d(p, p) = d(s, s) = d(p, s) ⇔ p = s;

ii) d(p, p) ≤ d(p, s);

iii) d(p, s) = d(s, p);

iv) d(p, r) ≤d(p, s)+d(s, r)-d(s, s);

Also, (N, d) is a partial metric space.

Definition 2.3: [11] Let # be a binary operation. (X, #) is a NT set (NTS) such

i) There must be neutral of “x” such x#neut(x) = neut(x)#x = x, x ∈ X.

ii) There must be anti of “x” such x#anti(x) = anti(x)#x = neut(x), x ∈ X.

Furthermore, a NT “x” is showed with (x, neut(x), anti(x)).

Also, neut(x) must different from classical unitary element.

Definition 2.4: [17] Let (M, *) be a NTS and m*n ∈ M for ∀ m, n ∈ M. NT partial metric (NTPM) is a function p: M x M → $\mathbb{R}^{+}\cup\{0\}$ such for every s, p, r ∈ M,

i) p(m, n) ≥p(n, n)≥0

ii) If p_N(m, m) = p_N(m, n) = p_N(n, n) = 0, then there exits at least one m, n pair such that m = n.

iii) p(m, n) = p(n, m)

iv) If there exists at least an element n∈A for each m, k ∈ M pair such that

p(m, k)≤ p(m, k*neut(n)), then p(m, k*neut(n))≤ p(m, n) + p(n, k) - p(n, n)

Also, ((N, *), p) is called NTPM space (NTPMS).

Definition 2.5: [13] A NT metric on a NTS (N, *) is a function d:NxN→ ℝ such that for every n, m, s ∈ N,

i) n * m ∈ N

ii) d(n, m) ≥ 0

iii) If n = m, then d(n, m) = 0

iv) d(n, m)= d(n, m)

v) If there exists at least an element s ∈ N for each n, m ∈ N pair such that

d(n, m) ≤ d(n, m*neut(s)), then d(n, m*neut(s)) ≤ d(n, s)+ d_T(s, n).

Definiton 2.6: [18] Let (N, *) be a NTS. A NT v- generalized metric on N is a function d_v:NxN→ ℝ such that for every n, m, k_1, k_2, … , k_v ∈ N;

i) n*m ∈ N

ii) 0≤ d_v(n, m)

iii) if n = m, then d_v(n, m) = 0

iv) d_v(n, m) = d_v(m, n)

v) If there exists elements n, m, $k_1, \dots, k_v$ ∈ N such that

d_v(n, m)≤ d_v(n, m#neut(k_v)),

d_v(n, k_2)≤ d_v(n, k_2#neut(k_1)),

d_v(k_1, k_3)≤ d_v(k_1, k_3#neut(k_2)),

… ,

d_v(k_{v-1}, m)≤ d_v(k_{v-1}, m#neut(k_v));

then d_v(n, m*neut(k_v))≤ d_v(n, k_1)+ d_v(k_1, k_2) + … + d_v(k_{v-1}, k_v) + d_v(k_v, m).

Where n, $k_1, \dots, k_v$, m are all different.

Furthermore, ((N, *), d_v) is called NTVGM space (NTVGMS).

3. Neutrosophic Triplet Partial v – Generalized Metric Space

Definition 3.1: Let (N, *) be a NTS. A NT partial v- generalized metric on N is a function d_{pv}:NxN→ ℝ such every n, m, $k_1, k_2, \dots, k_v$ ∈ N;

i) n*m ∈ N

ii) d_{pv}(n, m)≥ d_{pv}(n, n)≥0

iii) If d_{pv}(n, n) = d_{pv}(n, m) = d_{pv}(m, m) = 0, then n = m.

iv) d_{pv}(n, m) = d_{pv}(m, n)

v) If there exists elements n, m, $k_1, \dots, k_v$ ∈ N such that

d_{pv}(n, m)≤ d_{pv}(n, m*neut(k_v)),

d_{pv}(n, k_2)≤ d_{pv}(n, k_2*neut(k_1)),

d_{pv}(k_1, k_3)≤ d_{pv}(k_1, k_3*neut(k_2)),

… ,

d_{pv}(k_{v-1}, m)≤ d_{pv}(k_{v-1}, m*neut(k_v));

then

d_{pv}(n, m*neut(k_v))≤ d_{pv}(n, k_1)+ d_{pv}(k_1, k_2) + … + d_{pv}(k_{v-1}, k_v) + d_{pv}(k_v, m)-

[d_{pv}(k_1, k_1) + d_{pv}(k_2, k_2) + … + d_{pv}(k_v, k_v)].

Where n, k_1, … , k_v, m are all different.

Furthermore, ((N, *), d_{pv}) is called NTPVGM space (NTPVGMS).

Furthermore, if v = k, (k∈ ℕ), then NTPVGMS is showed that NTPkGMS. For example, if v = 2, then NTPVGMS is showed that NTP2GMS.

Example 3.2: Let N = {∅, {k}, {l}, {k, l}} be a set and s(M) be number of elements in M ∈ N. Also, we can take neut(M)= M, anti(M)=M for all M ∈ N since M∪M = M, for M ∈ N. Furthermore, (N, ∪) is a NTS. Then, we take that d_{pv}: NxN→N is a function such that d_{pv}(S, K)= $2^{\max\{s(S),s(K)\}}$.

Now we show that d_{pv} is a NTPVGM.

i) S ∪ K ∈ N for S, K ∈ N.

ii) d_{pv}(S, K)= $2^{\max\{s(S),s(K)\}} \geq 2^{\max\{s(S),s(S)\}} \geq 0$

iii) There are not any elements S, K such that d_{pv}(S, K) = d_{pv}(S, S) = d_{pv}(K, K) = 0

iv) d_{Pv}(S, K) = $2^{\max\{s(S),s(K)\}}$ = $2^{\max\{s(K),s(S)\}}$ = d_{pv}(K, S)

v)

a) d_{pv}({l}, {k, l})≤ d_{pv}({l}, {k, l}∪ {k}) and d_{pv}({l, k}, ∅) ≤ d_{pv}({l, k}, ∅ ∪ {1}). Also,

d_{pv}({l}, {k, l}) = 2^2 = 4, d_{pv}({l}, {k}) = 2^1 = 2, d_{pv}({k}, ∅) = 2^1 = 2,

d_{pv}(∅, {1, k}) = 2^2 = 4. Thus,

d_{pv}({l}, {k, l}∪{k})≤

d_{pv}({l}, {k})+d_{pv}({k}, ∅)+d_{pv}(∅, {1, k})- d_{pv}({k}, {k})-d_{pv}({1}, {1}).

b) d_{pv}({l}, {k})≤ d_{pv}({l}, {k}∪ {k, 1}) and d_{pv}({ k}, ∅) ≤ d_{pv}({k}, ∅ ∪ {1}). Also,

d_{pv}({l}, {k}) = 2^1 = 2, d_{pv}({l}, {k, 1}) = 2^2 = 4, d_{pv}({k, 1}, ∅) = 2^2 = 4,

d_{pv}(∅, {k})= 2^1 = 2. Thus,

d_{pv}({l}, {k}∪ {k,1})≤

d_{pv}({l}, {k,1})+d_{pv}({k,1}, ∅)+d_{pv}(∅, {k})- d_{pv}({k, 1}, {k,1})-d_{pv}({1}, {1}).

c) d_{pv}({l}, ∅)≤ d_{pv}({l}, ∅ ∪ {k, 1}) and d_{pv}(∅, { k}) ≤ d_{pv}(∅, { k}∪ {1}). Also,

d_{pv}({l}, ∅ ∪ {k, 1}) = 2^2 = 4, d_{pv}({l}, {k, 1}) = 2^2 = 4, d_{pv}({k, 1}, {k}) = 2^2 = 4, d_{pv}({k}, {1})= 2^1 = 2. Thus,

d_{pv}({l}, ∅ ∪ {k, 1}) ≤

d_{pv}({l}, {k,1})+d_{pv}({k,1}, ∅)+d_{pv}(∅, {k})- d_{pv}({k, 1}, {k,1})-d_{pv}({1}, {1}).

d) d_{pv}({k}, ∅)≤ d_{pv}({k}, ∅ ∪ {k, 1}) and d_{pv}(∅, {l}) ≤ d_{pv}(∅, { l}∪ {k}). Also,

d_{pv}({k}, ∅ ∪ {k, 1}) = 2^2 = 4, d_{pv}({k}, {k, 1}) = 2^2 = 4, d_{pv}({k, 1}, ∅) = 2^2 = 4,

d_{pv}(∅, {k})= 2^1 = 2. Thus,

d_{pv}({k}, ∅ ∪ {k, 1}) ≤

d_{pv}({k}, {k, 1})+d_{pv}({k,1}, ∅)+d_{pv}(∅, {k})- d_{pv}({k, 1}, {k,1})-d_{pv}({k}, {k}).

e) d_{pv}({1, k}, ∅)≤ d_{pv}({1, k}, ∅ ∪ {k}) and d_{pv}(∅, {l, k}) ≤ d_{pv}(∅, {l, k}∪ {l}). Also,

d_{pv}({1, k}, ∅ ∪ {k}) = 2^2 = 4, d_{pv}({1, k}, {k}) = 2^2 = 4, d_{pv}({k}, {1}) = 2^1 = 2,

d_{pv}({1}, ∅) = 2^1 = 2. Thus,

d_{pv}({1, k}, ∅ ∪ {k}) ≤

d_{pv}({1, k}, {k})+d_{pv}({k}, {1})+d_{pv}({1}, ∅)- d_{pv}({k}, {k})-d_{pv}({1}, {1}).

f) d_{pv}({k}, {1, k})≤ d_{pv}({k}, {1, k} ∪ {1}) and d_{pv}({l, k}, ∅) ≤ d_{pv}({l, k}, ∅ ∪ {k}). Also,

d_{pv}({k}, {1, k} ∪ {1}) = 2^2 = 4, d_{pv}({ k}, {1}) = 2^1 = 2, d_{pv}({1}, ∅) = 2^1 = 2,

d_{pv}({∅, {1,k}) = 2^2 = 4. Thus,

d_{pv}({k}, {1, k} ∪ {1}) ≤

d_{pv}({ k}, {1}) +d_{pv}({1}, ∅)+d_{pv}({∅, {1,k}) - d_{pv}({1}, {1})-d_{pv}({k}, {k}).

Hence, (X), ∪), d_{pv}) is a NT2GMS.

Corollary 3.3: NTPVGMS is different from the partial metric space and NTPMS, since for triangle inequality and * binary operation.

Corollary 3.4: The NTPVGMS is different from NTVMS since for triangle inequality and condition iii in Definition 3.1.

Theorem 3.5: In Definition 3.1, if it is taken such that $k_1 = k_2 = \ldots = k_v$, and d_{pv}(n, n) = 0, then each NTPVGMS is a NTVGMS.

Proof: Let ((X, #), d_{pv}) be a NTPVGMS.

i) a#b ∈ X since ((X, #), d_{pv}) is a NTPVGMS.

ii) 0≤ d_{pv}(n, m) since ((X, #), d_{pv}) is a NTPVGMS.

iii) If n = m, then d_{pv}(n, m) = 0; because d_{pv}(n, n) = 0

iv) d_v(n, m) = d_v(m, n) since ((X, #), d_{pv}) is a NTPVGMS.

v) From Definition 3.1,

If there exists elements n, m, $k_1, \ldots, k_v$ ∈ N such that

d_{pv}(n, m)≤ d_{pv}(n, m#neut(k_v)),

d_{pv}(n, k_2)≤ d_{pv}(n, k_2#neut(k_1)),

$d_{pv}(k_1, k_3)$≤ $d_{pv}(k_1, k_3$#neut$(k_2))$,

… ,

$d_{pv}(k_{v-1}$, m)≤ $d_{pv}(k_{v-1}$, m#neut$(k_v))$;

Then,

d_{pv}(n, m*neut(k_v)) ≤

d_{pv}(n, k_1)+ $d_{pv}(k_1, k_2) + \ldots + d_{pv}(k_{v-1}, k_v) + d_{pv}(k_v$, m) -

$[d_{pv}(k_1, k_1) + d_{pv}(k_2, k_2) + \ldots + d_{pv}(k_v, k_v)]$ (1)

Then we take $k_1 = k_2 = \ldots = k_v$, and d_{pv}(n, n) = 0 in (1). Thus, for a, b, u_1∈ X,

d_{pv}(n, m) ≤ d_{pv}(n, m#neut(k_1))

d_{pv}(n, k_1)≤ d_{pv}(n, k_1#neut(k_1)),

d_{pv}(k_1, k_1)≤ d_{pv}(k_1, k_1#neut(k_1)),

… ,

d_{pv}(k_1, m)≤ d_{pv}(k_1, m#neut(k_1));

then

d_{pv}(n, m*neut(k_v)) = d_{pv}(n, m*neut(k_1)) ≤

d_{pv}(n, k_1)+ d_{pv}(k_1, k_1) + … + d_{pv}(k_1, k_1) + d_{pv}(k_1,m) –

[d_{pv}(k_1, k_1) + d_{pv}(k_1, k_1) + … + d_{pv}(k_1, k_1)] = d_{pv}(n, k_1)+ 0 + d_{pv}(k_1,m) – 0 =

d_{pv}(n, k_1) + d_{pv}(k_1,n).

Therefore, $(X, \#), d_v$) is a NTVGMS.

Corollary 3.6: In Theorem 3.5, we can define a NTP1GMS with each NTVGMS. Also, from Theorem 3.5, each NTVGMS is a NTP1GMS.

Theorem 3.7: Let ((X, #), d) be a NTVGMS and d_v be a function such that

d_{pv}(n, m) = d(n, m) + k (k ∈ $\mathbb{R}^+$). Then ((X, #), d_{pv}) is a NTPVGMS.

Proof:

We take d_{pv}(n, m) = d(n, m) + k. Where, d_{pv}(n, n) = d(n, n) + k = k since ((X, #), d) is a NTVGMS.

i) It clear that a#b ∈ X since ((X, #), d) is a NTVGMS.

ii) d_{pv}(n, m) = d(n, m) + k≥ d_{pv}(n, n) = k ≥ 0 since ((X, #), d) is a NTVGMS.

iii) There is not any pair of element n, m such that

d_{pv}(n, m) = d(n, m) + k = d_{pv}(n, n) = k = d_{pv}(m, m) = k = 0. Because, k ∈ $\mathbb{R}^+$.

iv) d_{pv}(n, m) = d(n, m) + k = d(m, n) + k = d_{pv}(m, n)

v) In Definition 2.6, If there exists elements n, m, k_1, … , k_v ∈ X such that

d(n, m)≤ d(n, m#neut(k_v)),

d(n, k_2)≤ d(n, k_2#neut(k_1)),

d(k_1, k_3)≤ d(k_1, k_3#neut(k_2)),

… ,

d(k_{v-1}, m)≤ d(k_{v-1}, m#neut(k_v));

then

d(n, m*neut(k_v))≤

d(n, k_1)+d(k_1, k_2)+ … +d(k_{v-1}, k_v)+d(k_v, m) (2)

As d_{pv}(n, m) = d(n, m) + k, we can take in (2),

d_{pv}(n, m) = d(n, m) + k ≤ d_{pv}(n, m#neut(k_v)) = d(n, m#neut(k_v)) + k,

d_{pv}(n, k_2) = d(n, k_2) + k ≤ d_{pv}(n, k_2#neut(k_1)) = d(n, k_2#neut(k_1)) + k

d_{pv}(k_1, k_3) = d(k_1, k_3) + k ≤ d_{pv}(k_1, k_3#neut(k_2)) = d(k_1, k_3#neut(k_2)) + k,

… ,

d_{pv}(k_{v-1}, m) = d(k_{v-1}, m) + k ≤ d_{pv}(k_{v-1}, m#neut(k_v)) = d(k_{v-1}, m#neut(k_v)) + k;

then

d_{pv}(n, m*neut(k_v)) = d(n, m*neut(k_v)) + k ≤ d(n, k_1)+ k + d(k_1, u_2) +k + … + d(k_{v-1}, k_v) +k + d(k_v, m) +k. Thus,

d_{pv}(n, m*neut(k_v)) = d(n, m*neut(k_v)) + k ≤

d(n, k_1)+ d(k_1, k_2) + … + d(k_{v-1}, k_v) + d(k_v, m)-v.k. Therefore,

d_{pv}(n, m*neut(k_v))≤ d_{pv}(n, k_1)+ d_{pv}(k_1, k_2) + … + d_{pv}(k_{v-1}, k_v) + d_{pv}(k_v, m)-

[d_{pv}(k_1, k_1) + d_{pv}(k_2, k_2) + … + d_{pv}(k_v, k_v)] since d_v(n, n) = k for all n ∈ X.

Corollary 3.8: In Theorem 3.7, we can define a NTPVGMS with each NTVGMS.

Definition 3.9: Let ((X, #), d_v) be a NTPVGMS and $\{x_n\}$ be a sequence in NTPVGMS and m ∊ X. If there exist N ∊ ℕ for every ε>0 such that

d_v(m, $\{x_n\}$) < ε + d_v(m, m),

then $\{x_n\}$ converges to m. where n ≥M. Also, it is showed that

$\lim_{n\to\infty} x_n$= m or $x_n\to$ m.

Definition 3.10: Let ((X, #), d_v) be a NTPVGMS and $\{x_n\}$ be a sequence in NTPVGMS. If there exist a N ∈ ℕ for every ε>0 such that

$d_v(\{x_m\}, \{x_n\})$< ε + d_v(m, m),

then $\{x_n\}$ is a Cauchy sequence in NTPVGMS. Where, n ≥m≥M and m ∈ X.

Definition 3.11: Let ((X, #), d_v) be a NTPVGMS and $\{x_n\}$ be a sequence in NTPVGMS. If there exist N ∈ ℕ for every ε>0 such that

$d_v(\{x_n\}, \{x_{n+1+jk}\})$< ε + d_v(m, m), (j = 0, 1, 2, …)

then { x_n} is a k - Cauchy sequence in NTPVGMS. Where, k ∈ ℕ and m ∈ X.

Definition 3.12: Let ((X, #), d_v) be a NTPVGMS and $\{x_n\}$ be Cauchy sequence in NTPVGMS. NTPVGMS is complete ⇔ every $\{x_n\}$ converges in NTPVGMS.

Definition 3.13: Let ((X, #), d_v) be a NTPVGMS and $\{x_n\}$ be k - Cauchy sequence in NTPVGMS. NTPVGMS is k - complete ⇔ every $\{x_n\}$ converges in NTPVGMS.

Conclusions

In this chapter, we obtained NTPVGMS. We also show that NTPVGMS is different from the NTVGMS and NTPMS. Also, we defined complete space and k-complete space for NTPIPS. Thus, we have added a new structure to NT structure and we gave rise to a new field or research called NTPIPS. Also, thanks to NTPIPS researcher can obtain new structure and properties. For example, NT partial v – generalized normed space, NT partial v – generalized inner product space and fixed point theorems for NTVGMS.

Abbreviations

NT: Neutrosophic triplet

NTS: Neutrosophic triplet set

NTM: Neutrosophic triplet metric

NTMS: Neutrosophic triplet metric space

NTPM: Neutrosophic triplet partial metric

NTPMS: Neutrosophic triplet partial metric space

NTVGM: Neutrosophic triplet v-generalized metric

NTVGMS: Neutrosophic triplet v-generalized metric space

NTPVGM: Neutrosophic triplet partial v-generalized metric

NTPVGMS: Neutrosophic triplet partial v-generalized metric space

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The **Neutrosophic Triplets** were introduced by F. Smarandache & M. Ali in 2014 – 2016, and consequently the neutrosophic triplet group, ring, field - in general the neutrosophic triplet structures; while the **Neutrosophic Extended Triplets** were introduced by F. Smarandache in 2016 and consequently the neutrosophic extended triplet structures:

http://fs.unm.edu/NeutrosophicTriplets.htm

Definition of Neutrosophic Triplet (NT).

A neutrosophic triplet is an object of the form $<x, neut(x), anti(x)>$, for $x \in N$, where

$neut(x) \in N$ is the neutral of x, different from the classical algebraic unitary element if any, such that:

$$x*neut(x) = neut(x)*x = x$$

and $anti(x) \in N$ is the opposite of x such that:

$$x*anti(x) = anti(x)*x = neut(x).$$

In general, an element x may have more *anti*'s.

Definition of Neutrosophic Extended Triplet (NET).

A neutrosophic extended triplet is a neutrosophic triplet, defined as above, but where the *neutral* of x {denoted by $_eneut(x)$ and called "extended neutral"} is allowed to also be equal to the classical algebraic unitary element (if any). Therefore, the restriction "different from the classical algebraic unitary element if any" is released.

As a consequence, the "extended opposite" of x, denoted by $_eanti(x)$, is also allowed to be equal to the classical inverse element from a classical group.

Thus, a neutrosophic extended triplet is an object of the form $<x, {}_eneut(x), {}_eanti(x)>$, for $x \in N$, where $_eneut(x) \in N$ is the *extended neutral* of x, which can be equal or different from the classical algebraic unitary element if any, such that:

$$x*{}_eneut(x) = {}_eneut(x)*x = x$$

and $_eanti(x) \in N$ is the *extended opposite* of x such that:

$$x*{}_eanti(x) = {}_eanti(x)*x = {}_eneut(x).$$

In general, for each $x \in N$ there are may exist many $_eneut$'s and $_eanti$'s.

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