Neutrosophic Triplet Inner Product

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ABSTRACT

In this paper, a notion of neutrosophic triplet inner product is given and properties of neutrosophic triplet inner product spaces are studied. Furthermore, we show that this neutrosophic triplet notion is different from the classical notion.

KEYWORDS: Neutrosophic triplet inner product, neutrosophic triplet metric spaces, neutrosophic triplet vector spaces, neutrosophic triplet normed spaces.

1. INTRODUCTION

The concept of neutrosophic logic and neutrosophic set were introduced by Smarandache (1995). In this concept, sets have truth function, falsity function and indeterminancy function. These functions defined as independent on each other. Therefore, the concept overcomes many uncertainties in our daily life. In fact Zadeh (1965) introduced the concept of fuzzy set and Atanassov (1986) introduced the concept of intuitionistic fuzzy set to overcome uncertainties. The fuzzy set has only truth (membership) function. The intuitionistic fuzzy set has truth function, falsity function and indeterminancy function. But these functions defined as dependent on each other. Therefore, neutrosophic set is the generalization of fuzzy set and intuitionistic fuzzy set. Smarandache and Kandasamy (2004 and 2006) introduced neutrosophic algebraic structures using the idea of neutrosophic theory; Smarandache and Ali (2016) introduced neutrosophic triplet theory and neutrosophic triplet groups. The neutrosophic triplet set is completely different from the classical sets, since for each element "a" in neutrosophic triplet set N together with a binary operation *; there exist a neutral of "a" called neut(a) where a*neut(a)=neut(a)*a=a and an opposite of "a" called anti(a) where a*anti(a)=anti(a)*a=neut(a). The "neut(a)" is different from the classical algebraic unitary element. A neutrosophic triplet is of the form <a, neut(a), anti(a)>. Also; Smarandache and Ali (2017) studied the neutrosophic triplet ring and the neutrosophic triplet field. Şahin and Kargın (2017) studied neutrosophic triplet metric space, neutrosophic triplet vector space and neutrosophic triplet normed space. Recently some researchers have been dealing with neutrosophic set theory. For example; Smarandache (2017) studied Neutrosophic Perspectives, Triplets, Duplets, Multisets, Hybrid Operators, Modal Logic and Hedge Algebras. Smarandache, Broumi, Bakali and Talea (2016) studied the single valued neutrosophic graphs and interval valued neutrosophic graphs. Smarandache, Broumi, Bakali, Talea and Vladareanu (2016) studied SV-Trapezoidal neutrosophic numbers. Liu and Shi (2015) studied interval neutrosophic hesitant set and neutrosophic uncertain linguistic number. Liu and Tang (2016) studied some power generalized aggregation operators based on the interval neutrosophic numbers and multi-criteria group decision-making based on interval neutrosophic uncertain linguistic variables. Liu and Wang (2016) studied interval neutrosophic prioritized OWA operator and aggregation operators based on Archimedean t-conorm and t-norm for the single valued neutrosophic numbers. Liu and Teng (2015) studied multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator. Liu, Zhang, Wang and Liu (2016) studied multi-valued neutrosophic number Bonferroni mean operators. Şahin, Olgun, Uluçay, Kargın and Smarandache (2017) studied centroid points of transformed single valued neutrosophic numbers with applications. Şahin, Ecemiş, Uluçay and Kargın (2017) studied centroid single valued triangular neutrosophic numbers and their applications.

In this paper; we introduced neutrosophic triplet inner product space. Also; we give new properties and new definitions for this structure. In this paper; in section 2; some preliminary results for neutrosophic triplet sets, neutrosophic triplet ring and field, neutrosophic triplet metric space, neutrosophic triplet vector space and neutrosophic triplet normed space are given. In section 3; neutrosophic triplet inner product space is defined and some properties of a neutrosophic triplet inner product space are given. It is show that neutrosophic triplet inner product different from the classical inner product. Also; it is show that if certain conditions are met; every neutrosophic triplet inner product space can be a neutrosophic triplet normed space and neutrosophic triplet metric space at the same time. Furthermore; the convergence of a sequence and a Cauchy sequence in a neutrosophic triplet inner product space are defined. In section 4; conclusions are given.

2. PRELIMINARIES

Definition 2.1: Let N be a set together with a binary operation *. Then, N is called a neutrosophic triplet set if for any a∈ N, there exists a neutral of “a” called neut(a), different from the classical algebraic unitary element, and an opposite of “a” called anti(a), with neut(a) and anti(a) belonging to N, such that:

a*neut(a) = neut(a)* a = a,

and

a*anti(a)= anti(a)* a = a.

The elements a, neut(a) and anti(a) are collectively called as neutrosophic triplet, and we denote it by (a, neut(a), anti(a)). Here, we mean neutral of “a” and apparently, “a” is just the first coordinate of a neutrosophic triplet and it is not a neutrosophic triplet. For the same element “a” in N, there may be more neutrals to it neut(a) and more opposites of it anti(a). (Smarandache and Ali, 2016)

Definition 2.2: Let (N,*) be a neutrosophic triplet set. Then, N is called a neutrosophic triplet group, if the following conditions are satisfied.

1) If (N,*) is well-defined, i.e. for any a, b ∈ N, one has a*b ∈ N.

2) If (N,*) is associative, i.e. (a*b)*c = a*(b*c) for all a, b, c ∈ N.

The neutrosophic triplet group, in general, is not a group in the classical algebraic way.

One can consider that neutrosophic neutrals are replacing the classical unitary element, and the neutrosophic opposites are replacing the classical inverse elements. (Smarandache and Ali, 2016)

Definition 2.3: Let (N,*) be a neutrosophic triplet group. Then N is called a commutative neutrosophic triplet group if for all a, b ∈ N, we have a*b=b*a. (Smarandache and Ali, 2016)

Proposition 2.4: Let (N,*) be a neutrosophic triplet group with respect to * and a, b, c ∈ N, where a and anti(a) are both cancellable;

1) a*b= a*c if and only if neut(a)*b = neut(a)*c

2) b*a= c*a if and only if b*neut(a) = c*neut(a)
3) if anti(a)*b = anti(a)*c, then neut(a)*b = neut(a)*c
4) if b*anti(a) = c*anti(a), then b*neut(a) = c*neut(a) (Smarandache and Ali, 2016)

Theorem 2.5: Let (N,*) be a commutative neutrosophic triplet group with respect to * and a, b ∈ N, where a and b are both cancellable;

i) neut(a)*neut(b)= neut(a*b);
ii) anti(a)*anti(b)= anti(a*b); (Smarandache and Ali, 2016)

Theorem 2.6: Let (N,*) be a commutative neutrosophic triplet group with respect to * and a ∈ N, where a is cancellable;

i) neut(a)*neut(a) = neut(a);
ii) anti(a)*neut(a)=neut(a)* anti(a) = anti(a); (Smarandache and Ali, 2016)

Definition 2.7: Let (NTF,*,#) be a neutrosophic triplet set together with two binary operations * and #. Then (NTF,*, #) is called neutrosophic triplet field if the following conditions hold.

1. (NTF,*) is a commutative neutrosophic triplet group with respect to *.

2. (NTF, #) is a neutrosophic triplet group with respect to #.

3. a#(b*c)= (a#b)*(a#c) and (b*c)#a = (b#a)*(c#a) for all a,b,c ∈ NTF. (Smarandache and Ali, 2017)

Theorem 2.8: Let (N,*) be a neutrosophic triplet group with no zero divisors and with respect to *. For

a ∈ N;

If a = neut(a), then there exists an anti(a) such that neut(a) = anti(a) = a. (Şahin and Kargın, 2017)

Theorem 2.9: Let (N,*) be a neutrosophic triplet group with no zero divisors and with respect to *. For

a ∈ N;

i) neut(neut(a)) = neut(a)
ii) anti(neut(a)) = neut(a))
iii) anti(anti(a)) = a
iv) neut(anti(a)) = neut(a) (Şahin and Kargın, 2017)

Definition 2.10: Let (N,*) be a neutrosophic triplet set and let x*y ∈ N for all x, y ∈ N. If the function

d: NxN→ $\mathbb{R}^+ \cup \{0\}$ satisfies the following conditions; d is called a neutrosophic triplet metric. For all

x, y, z ∈ N;

a) d(x, y)≥0;

b) If x=y; then d(x, y)=0

c) d(x, y)= d(y, x)

d) If there exists any element y ∈N such that d(x, z)≤ d(x, z*neut(y)), then d(x, z*neut(y))≤ d(x, y)+ d(y, z).

Furthermore; ((N,*), d) space is called neutrosophic triplet metric space. (Şahin and Kargın, 2017)

Definition 2.11: Let (NTF,$*_1$, $\#_1$) be a neutrosophic triplet field and let (NTV,$*_2$, $\#_2$) be a neutrosophic triplet set together with binary operations "$*_2$" and "$\#_2$". Then (NTV,$*_2$, $\#_2$) is called a neutrosophic triplet vector space if the following conditions hold. For all u, v ∈ NTV and for all k∈ NTF; such that u$*_2$v ∈ NTV and u $\#_2$k ∈ NTV ;

1) (u$*_2$v) $*_2$t= u$*_2$ (v$*_2$t), for every u, v, t ∈ NTV

2) u$*_2$v = v$*_2$u, for every u, v ∈ NTV

3) (v$*_2$u) $\#_2$k= (v$\#_2$k) $*_2$(u$\#_2$k), for all k∈ NTF and for all u, v ∈ NTV

4) (k$*_1$t) $\#_2$u= (k$\#_2$v) $*_1$(u$\#_2$v), for all k, t ∈ NTF and for all u ∈ NTV

5) (k$\#_1$t) $\#_2$u= k$\#_1$(t$\#_2$u), for all k, t ∈ NTF and for all u ∈ NTV

6) For all u ∈ NTV; such that u $\#_2$ neut(k)= neut(k) $\#_2$ u = u, there exists any neut(k) ∈ NTF

Here; the condition 1) and 2) indicate that the neutrosophic triplet set (NTV, $*_2$) is a commutative neutrosophic triplet group. (Şahin and Kargın, 2017)

Definition 2.12: Let (NTV,$*_2$, $\#_2$) be a neutrosophic triplet vector space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field. If $\|.\|$:NTV → $\mathbb{R}^+ \cup \{0\}$ function satisfies following condition; ‖.‖ is called neutrosophic triplet normed on (NTV,$*_2$, $\#_2$).

Where; f: NTF X NTV → $\mathbb{R}^+ \cup \{0\}$, f(α,x)= f(anti(α), anti(x)) is a function and for every x, y ∈ NTV and

α ∈ NTF;

a) ‖x‖ ≥0;

b) If x = neut(x), then ‖x‖ =0

c) ‖α$\#_2$ x‖ = f(α,x).‖x‖

d) ‖anti(x)‖= ‖x‖

e) If ‖x$*_2$ y‖ ≤ ‖x$*_2$ y$*_2$neut(k)‖; then ‖x$*_2$y$*_2$neut(k)‖≤‖x‖+‖y‖, for any k ∈ NTV.

Furthermore on (NTV, $*_2$, $\#_2$), the neutrosophic triplet vector space defined by ‖.‖ is called a neutrosophic triplet normed space and is denoted by ((NTV, $*_2$, $\#_2$), ‖.‖). (Şahin and Kargın, 2017)

Proposition 2.13: Let ((NTV, $*_2$, $\#_2$), ‖.‖) be a neutrosophic triplet normed space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field. Then, the function d: NTV x NTV→ $\mathbb{R}$ defined by d(x, y)= ‖x$*_2$ anti(y)‖ provides neutrosophic triplet metric space conditions. (Şahin and Kargın, 2017)

3. NEUTROSOPHIC TRIPLET INNER PRODUCT SPACE

Now let's define the neutrosophic triplet inner product spaces on the neutrosophic triplet vector space.

Definition 3.1: Let (NTV,$*_2$, $\#_2$) be a neutrosophic triplet vector space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field. If <., .> :NTV x NTV → $\mathbb{R}^+ \cup \{0\}$ function satisfies following condition; <., .> is called neutrosophic triplet inner product on (NTV,$*_2$, $\#_2$).

Where; f: NTF X NTV X NTV → $\mathbb{R}^+ \cup \{0\}$, f(α, x, y)= f(anti(α), anti(x), anti(y)) and f(α, x, y)= f(α,y, x), is a function and for every x, y ∈ NTV and α, β ∈ NTF;

a) <x, x> ≥0;

b) If x= neut(x), then <x, x> =0

c) <(α$\#_2$ x)$*_2$ (β$\#_2$ y)$, z$> = f(α, x, z). <x, z>+ f(β,y,z). <y, z>

d) <anti(x), anti(x)> = <x, x>

e) <x, y>=<y , x>

Furthermore on (NTV, $*_2$, $\#_2$), the neutrosophic triplet vector space defined by <., .> is called a neutrosophic triplet inner product space and is denoted by ((NTV, $*_2$, $\#_2$), <., .>).

Corollary 3.2: It is clear by definition 3.1 that neutrosophic triplet inner product spaces are generally different from classical inner product spaces, since for there is not any "f" function in classical inner product space.

Example 3.3: Let X = {1, 2}, and P(X) be power set of X. From definition 2.11; (P(X), *, ∩) is a neutrosophic triplet vector space on the (P(X), *, ∩) neutrosophic triplet field. Where;

$$A*B=\begin{cases} B\backslash A, & s(A)<s(B) \wedge B\supset A \wedge A'=B \\ A\backslash B, & s(A)>s(B) \wedge A\supset B \wedge B'=A \\ (A\backslash B)', & s(A)>s(B) \wedge A\supset B \wedge B'\neq A \\ (B\backslash A)', & s(A)<s(B) \wedge B\supset A \wedge A'\neq B \\ X, & s(A)=s(B) \wedge A\neq B \\ \emptyset, & A=B \end{cases}$$

The neutrosophic triplets with respect to *;

neut(∅)=∅, anti(∅)=∅; neut({1})= {1, 2}, anti({1})={2}; neut({2})= {1, 2}, anti({2})={1}; neut({1, 2})= ∅, anti({1,2})={1, 2};

The neutrosophic triplets with respect to ∩;

neut(A) =A and anti(A) = B, where; B⊃A;

Then taking f: P(X) × P(X) × P(X) → $\mathbb{R}^+ \cup \{0\}$, f(A,B,C)= s((A∩B)\C)/s(B\C), ‖.‖:P(X) → $\mathbb{R}^+ \cup \{0\}$. Now we show that, <A, B> = s((A\B)∪(B\A)) is a neutrosophic triplet inner product and ((P(X),*, ∩), ‖.‖) is a neutrosophic triplet normed space. Where; s(A) is number of elements in A ∈ P(X) and A' is complement of A ∈P(X). Now we show that <., .> is a neutrosophic triplet inner product.

a) <A, B>= s((A\B)∪(B\A))≥0.

b) If A=neut(A)= ∅, then <A , A>= 0.

c) It is clear that <(A∩B)$*$ (C ∩ D)$, E$> = f(A, B, E). <B, E> + f(C, D, E). <D, E>

d) it is clear that <anti(A), anti(A)>= <A , A>.

e) It is clear that <A, B>= s((A\B)∪(B\A))= s((B\A)∪(A\B))= <B , A>.

Theorem 3.4: Let $(NTV, *_2, \#_2)$ be a neutrosophic triplet vector space on $(NTF, *_1, \#_1)$ neutrosophic triplet field and let $((NTV, *_2, \#_2), <.,.>)$ be a neutrosophic triplet inner product space on $(NTV, *_2, \#_2)$ and f: NTF X NTV X NTV $\rightarrow \mathbb{R}^+ \cup \{0\}$, f(α, x, y)= f(anti(α), anti(x), anti(y)) is a function and for every x, y ∈ NTV and α, β ∈ NTF; Then;

$<(\alpha \#_2 x) *_2 (\beta \#_2 y), (\alpha \#_2 x) *_2 (\beta \#_2 y)> =$

$f(\alpha, (\alpha \#_2 x) *_2 (\beta \#_2 y), x,).f(\alpha, x, x,).<x, x> +$

$[f(\alpha, (\alpha \#_2 x) *_2 (\beta \#_2 y), x,). f(\beta, x, y,)+ f(\beta, (\alpha \#_2 x) *_2 (\beta \#_2 y), y).f(\alpha, x, y,)].<x, y> +$

$f(\beta, (\alpha \#_2 x) *_2 (\beta \#_2 y), y). f(\beta, y, y,).<y, y>$

Proof:

$<(\alpha \#_2 x) *_2 (\beta \#_2 y), (\alpha \#_2 x) *_2 (\beta \#_2 y)> = f(\alpha,x, (\alpha \#_2 x) *_2 (\beta \#_2 y)).<x, (\alpha \#_2 x) *_2 (\beta \#_2 y) >+$

$f(\beta,y, (\alpha \#_2 x) *_2 (\beta \#_2 y)).<y, (\alpha \#_2 x) *_2 (\beta \#_2 y) >$. From the definition 3.1; since $<x, y>= <x, y>$ and

$f(\alpha, x, y)= f(\alpha,y, x)$;

$f(\alpha,x, (\alpha \#_2 x) *_2 (\beta \#_2 y)).<x, (\alpha \#_2 x) *_2 (\beta \#_2 y) > +$

$f(\beta,y, (\alpha \#_2 x) *_2 (\beta \#_2 y)).<y, (\alpha \#_2 x) *_2 (\beta \#_2 y) > =$

$f(\alpha, (\alpha \#_2 x) *_2 (\beta \#_2 y), x,).< (\alpha \#_2 x) *_2 (\beta \#_2 y), x > +$

$f(\beta, (\alpha \#_2 x) *_2 (\beta \#_2 y), y).< (\alpha \#_2 x) *_2 (\beta \#_2 y), y > =$

$f(\alpha, (\alpha \#_2 x) *_2 (\beta \#_2 y), x,).[f(\alpha, x, x,).<x, x>+ f(\beta, x, y,).<x, y>] +$

$f(\beta, (\alpha \#_2 x) *_2 (\beta \#_2 y), y).[f(\alpha, x, y,).<x, y>+ f(\beta, y, y,).<y, y>] =$

$f(\alpha, (\alpha \#_2 x) *_2 (\beta \#_2 y), x).f(\alpha, x, x,).<x, x> +$

$[f(\alpha, (\alpha \#_2 x) *_2 (\beta \#_2 y), x,). f(\beta, x, y,)+ f(\beta, (\alpha \#_2 x) *_2 (\beta \#_2 y), y).f(\alpha, x, y,)].<x, y> +$

$f(\beta, (\alpha \#_2 x) *_2 (\beta \#_2 y), y).f(\beta, y, y,).<y, y>$

Theorem 3.5: Let $(NTV, *_2, \#_2)$ be a neutrosophic triplet vector space on $(NTF, *_1, \#_1)$ neutrosophic triplet field and let $((NTV, *_2, \#_2), <.,.>)$ be a neutrosophic triplet inner product space on $(NTV, *_2, \#_2)$ and f: NTF X NTV X NTV $\rightarrow \mathbb{R}^+ \cup \{0\}$, f(α, x, y)= f(anti(α), anti(x), anti(y)) is a function and for every x, y ∈ NTV and α ∈ NTF. If neut(x) = neut(y) then;

$(< x, y >)^2 \leq <x, x>.<y, y>$

Proof: It is clear that if x = neut(x) or y = neut(y) then; $(< x, y >)^2 \leq <x, x>.<y, y>$. We suppose that x ≠ neut(x). From the theorem 3.4; if

$f(\alpha, (\alpha \#_2 x) *_2 (\beta \#_2 y), x,) = f(\alpha, x, x)= f(\alpha, x, x)= \frac{-<x,y>}{<x,x>}$,

$f(\beta, (\alpha \#_2 x) *_2 (\beta \#_2 y), y)= f(\beta, y, y,)= f(\beta, x, y,) =1$ are taken;

$0 \leq <(\alpha \#_2 x) *_2 (\beta \#_2 y), (\alpha \#_2 x) *_2 (\beta \#_2 y) > =$

f(α, (α$\#_2$ x)$*_2$ (β$\#_2$ y), x).f(α, x, x,).<x, x> +

[f(α, (α$\#_2$ x)$*_2$ (β$\#_2$ y), x,). f(β, x, y,)+ f(β, (α$\#_2$ x)$*_2$ (β$\#_2$ y), y).f(α, x, y,)].<x, y> +

f(β, (α$\#_2$ x)$*_2$ (β$\#_2$ y), y). f(β, y, y,).<y, y>=

$(\frac{<x,y>}{<x,x>})^2$. <x, x>- $\frac{<x,y>.<x,y>}{<x,x>}$ - $\frac{<x,y>.<x,y>}{<x,x>}$+ <y, y>=

$\frac{(<x,y>)^2}{<x,x>}$ - $(\frac{<x,y>)^2}{<x,x>}$ - $\frac{(<x,y>)^2}{<x,x>}$ +<y, y>= <y, y> - $\frac{(<x,y>)^2}{<x,x>}$. Thus; we have 0≤<y, y> - $\frac{(<x,y>)^2}{<x,x>}$ and

$0 \leq < x,x >$.<y, y> - $< x,x >.\frac{(<x,y>)^2}{<x,x>}$

$(< x,y >)^2 \leq < x,x >$.<y, y>.

Theorem 3.6: Let (NTV,$*_2$, $\#_2$) be a neutrosophic triplet vector space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field and let ((NTV, $*_2$, $\#_2$), <. , .>) be a neutrosophic triplet inner product space on (NTV,$*_2$, $\#_2$) and f: NTF X NTV X NTV $\rightarrow \mathbb{R}^+ \cup \{0\}$, f(α,x,y)= f(anti(α), anti(x), anti(y)) is a function and for every x, y ∈ NTV and α ∈ NTF. If f(α, x, x) = f(α,x) and $\|x\| = < \mathrm{x},\mathrm{x} >^{1/2}$. Then; ((NTV, $*_2$, $\#_2$), $\|.\|$) is a neutrosophic triplet normed space on (NTV,$*_2$, $\#_2$).

Proof: As ((NTV, $*_2$, $\#_2$), <. , .>) is a neutrosophic triplet inner product space and f(α, x, x) = f(α,x), we have;

a) $\|x\| = < \mathrm{x},\mathrm{x} >^{1/2} \geq 0$.
b) If x= neut(x) then; $< \mathrm{x},\mathrm{x} >^{1/2} = \|x\| = 0$.
c) $\|\alpha\#_2\mathrm{x}\| = < \alpha\#_2\mathrm{x}, \alpha\#_2\mathrm{x} >^{1/2} = \mathrm{f}(\alpha,\mathrm{x},\mathrm{x})^{1/2}.\mathrm{f}(\alpha,\mathrm{x},\mathrm{x})^{1/2}. < \mathrm{x},\mathrm{x} >^{1/2} = \mathrm{f}(\alpha,\mathrm{x},\mathrm{x}). < \mathrm{x},\mathrm{x} >^{1/2} =$ $\mathrm{f}(\alpha,\mathrm{x}). \|x\|$
d) $\|\mathrm{anti}(\mathrm{x})\| = < \mathrm{anti}(\mathrm{x}),\mathrm{anti}(\mathrm{x}) >^{1/2} = < \mathrm{x},\mathrm{x} >^{1/2} = \|\mathrm{x}\|$
e) From the theorem 3.4; if
f(α, (α$\#_2$ x)$*_2$ (β$\#_2$ y), x,) = f(α, x, x) = f(α, x, x)= f(β, (α$\#_2$ x)$*_2$ (β$\#_2$ y), y)= f(β, y, y,)= f(β, x, y,) =1 and <(α$\#_2$ x)$*_2$ (β$\#_2$ y), (α$\#_2$ x)$*_2$ (β$\#_2$ y)> = <x$*_2$ y, x$*_2$ y > are taken;
$\|x\#_2y\|^2$ = <(α$\#_2$ x)$*_2$ (β$\#_2$ y), (α$\#_2$ x)$*_2$ (β$\#_2$ y)> = <x$*_2$ y, x$*_2$ y > =
f(α, (α$\#_2$ x)$*_2$ (β$\#_2$ y), x).f(α, x, x,).<x, x> +

[f(α, (α$\#_2$ x)$*_2$ (β$\#_2$ y), x,). f(β, x, y,)+ f(β, (α$\#_2$ x)$*_2$ (β$\#_2$ y), y).f(α, x, y,)].<x, y> +

f(β, (α$\#_2$ x)$*_2$ (β$\#_2$ y), y). f(β, y, y,).<y, y>= <x,x> + 2.<x,y> + <y,y>=
$\|x\|^2$+ 2.<x,y> + $\|y\|^2$. From the theorem 3.5; if neut(x)= neut(y) then;

$(< \mathrm{x},\mathrm{y} >)^2 \leq$ <x, x>.<y, y> = $\|x\|^2$ + 2.<x,y>+ $\|y\|^2 \leq$

$\|x\|^2 + 2\|x\|\|y\| + \|y\|^2 = (\|x\| + \|y\|)^2$. Since neut(x) = neut(y); it is clear that
$\|x*_2 y\| \leq \|x*_2 y*_2\mathrm{neut}(k)\|$. Where we can take neut(k) = neut(x).Thus; $\|x*_2y*_2\mathrm{neut}(k)\| \leq \|x\| + \|y\|$

Corollary 3.7: Let ((NTV, $*_2$, $\#_2$), $\|.\|$) be a neutrosophic triplet normed space on (NTF,$*_1$, $\#_1$)neutrosophic triplet field and let ((NTV, $*_2$, $\#_2$), <., .>) be a neutrosophic triplet inner product space on (NTF,$*_1$, $\#_1$)neutrosophic triplet field such that $\|x\| = < \mathrm{x},\mathrm{x} >^{1/2}$. Then, the function d: NTV x NTV→ $\mathbb{R}$ defined by

$$d(x, y) = \|x*_2 \mathrm{anti}(y)\| = < \mathrm{x} *_2 \mathrm{anti}(\mathrm{y}), \mathrm{x} *_2 \mathrm{anti}(\mathrm{y}) >^{1/2}$$

provides neutrosophic triplet metric space conditions.

Proof: It is clear that from proposition 2.14.

Corollary 3.8: Every neutrosophic triplet metric space is reduced by a neutrosophic triplet inner product space. But the opposite is not always true. Similarly; every neutrosophic triplet normed space is reduced by a neutrosophic triplet inner product space. But the opposite is not always true.

Definition 3.9: Let ((NTV, $*_2$, $\#_2$), ‖.‖) be a ((NTV, $*_2$, $\#_2$), <. , .>) normed space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field and ((NTV, $*_2$, $\#_2$), <. , .>) be a neutrosophic triplet inner product space such that $\|x\|= < x, x >^{1/2}$. d: NTVx NTV→ $\mathbb{R}$ neutrosophic triplet metric define by

$$d(x, y)= \| x *_2 \text{anti}(y)\| = < x *_2 \text{anti}(y), x *_2 \text{anti}(y) >^{1/2}$$

is called the neutrosophic triplet inner product space reduced by (NTV, $*_2$, $\#_2$).

Now let's define the convergence of a sequence and a Cauchy sequence in the neutrosophic triplet inner space with respect to neutrosophic triplet metric which is reduced by neutrosophic triplet inner space.

Definition 3.10: Let ((NTV,$*_2$, $\#_2$), <., .>) be a neutrosophic triplet inner product space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ be a sequence in this space and *d* be a neutrosophic triplet metric reduced by ((NTV,$*_2$, $\#_2$), <. , .>). For all ε>0, x ∊ NTV such that for all n ≥M

$$d(x, \{x_n\})= < x *_2 \text{anti}(\{x_n\}), x *_2 \text{anti}(\{x_n\}) >^{1/2} < ε,$$

if there exists a M∊ $\mathbb{N}$; $\{x_n\}$ sequence converges to x. It is denoted by

$$\lim_{n\to\infty} x_n = x \text{ or } x_n \to x.$$

Definition 3.11: Let ((NTV,$*_2$, $\#_2$), <. , .>) be a neutrosophic triplet inner product space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ be a sequence in this space and *d* be a neutrosophic triplet metric reduced by ((NTV,$*_2$, $\#_2$), <. , .>). For all ε>0, x ∊ NTV such that for all n ≥M

$$d((\{x_m\}, \{x_n\})= \|x *_2 \text{anti}(\{x_n\})\| < (\{x_m\} *_2 \text{anti}(\{x_n\}), (\{x_m\} *_2 \text{anti}(\{x_n\}) >^{1/2} < ε$$

if there exists a M∊ $\mathbb{N}$; $\{x_n\}$ sequence is called Cauchy sequence.

Definition 3.12: Let ((NTV,$*_2$, $\#_2$), <. , .>) be a neutrosophic triplet inner product space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field, $\{x_n\}$ be a sequence in this space and *d* be a neutrosophic triplet metric reduced by ((NTV,$*_2$, $\#_2$), <. , .>). If each $\{x_n\}$ Cauchy sequence in this space is convergent to d reduced neutrosophic triplet metric; ((NTV,$*_2$, $\#_2$), <. , .>) is called neutrosophic triplet Hilbert space.

Theorem 3.13: Let ((NTV,$*_2$, $\#_2$), <. , .>) be a neutrosophic triplet inner product space on (NTF,$*_1$, $\#_1$) neutrosophic triplet field, and $\{x_n\}$ and $\{y_n\}$ be sequences in ((NTV,$*_2$, $\#_2$), <. , .>) such that

$\{x_n\}$ → x ∊ NTV and $\{y_n\}$ → y ∊NTV, then;

$$\lim_{n\to\infty} < x_n, y_n >= <x,y>$$

Proof: $|< x_n, y_n > - < x, y >|=|< x_n, y_n > - < x_n, y > + < x_n, y > - < x, y >|$

$\leq |< x_n, y_n > - < x_n, y >| + |< x_n, y > - < x, y >|$
$= |< x_n, y_n - y >| + |< x_n - x, y >|$. From theorem 3.5; as

$(< x, y >)^2 \leq$ <x, x>.<y, y> and from definition 3.9; as

d(x, y)= $\| x *_2 \text{anti}(y) \| = < x *_2 \text{ anti}(y), x *_2 \text{ anti}(y) >^{1/2}$,

$|< x_n, y_n - y >| + |< x_n - x, y >| \leq \|x_n\| \|y_n - y\| + \|x_n - x\| \|y\|$.

As $\{x_n\} \rightarrow$ x and $\{y_n\} \rightarrow$ y; $\lim_{n \to \infty} < x_n, y_n >=$ <x, y>

4. CONCLUSION

In this paper; we introduced neutrosophic triplet inner product space. We also show that this neutrosophic triplet notion different from the classical notion. This neutrosophic triplet notion has several extraordinary properties compared to the classical notion. We also studied some interesting properties of this newly born structure. We give rise to a new field or research called neutrosophic triplet inner product space.

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