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## Neutrosophic Soft Bitopological Spaces

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**Abstract:** In this paper, we built bitopological space on the concept of neutrosophic soft set, we defined the basic topological concepts of this spaces which are  $N_3\text{-(bi)}^*$ -open set,  $N_3\text{-(bi)}^*$ -closed set,  $(bi)^*$ -neutrosophic soft interior,  $(bi)^*$ -neutrosophic soft closure,  $(bi)^*$ -neutrosophic soft boundary,  $(bi)^*$ -neutrosophic soft exterior and we introduced their properties. In addition, we investigated the relations of these basic topological concepts with their counterparts in neutrosophic soft topological spaces and we introduced many examples.

**Keywords :** Neutrosophic soft bitopological spaces, Star bineutrosophic soft open set, star bi neutrosophic soft closed set, fuzzy set.

### 1. Introduction

The concept of soft set is defined by Molodtsov [1] as follows: Let  $M$  be an initial universe set and  $E$  be a set of parameters. Let  $P(M)$  denotes the set of all the subsets of  $M$ . Consider  $B \neq \emptyset$ ,  $B \subseteq E$ . The collection  $(\beta, B)$  is termed to be the soft set, where  $\beta$  is a mapping by  $\beta: B \rightarrow P(M)$ , and later this concept has been redefined by Naim Cagman [20]. Smarandache [2] introduced neutrosophic set as a generalization of fuzzy set [3] and intuitionistic fuzzy set [4]. P. K. Maji [5] defended the concept of neutrosophic soft set by combining the concept of neutrosophic set and soft set. This the concept is defined as follows: let  $M$  be an initial universe set and  $E$  be a set of parameters. Let  $P(M)$  denote the set of all the neutrosophic sets of  $M$ . Consider  $B \neq \emptyset$ ,  $B \subseteq E$ . The collection  $(\beta, B)$  is termed to be the soft neutrosophic set, where  $\beta$  is a mapping by  $\beta: B \rightarrow P(M)$ . This concept has been modified by [6,7]. The concept of neutrosophic soft topological space was introduced by Bera [8]. Taha et al.[9] redefined the neutrosophic soft topological spaces differently from the study [8]. Other theoretical studies on these concepts were presented by a number of researchers, for example, Narmada, Georgiou, Cagman, Al-Nafee, Evanzalin and Salama, (see [10, 11, 22, 13, 14, 15, 16, 17, 18, 19, 20]).

Kelly, [21] introduced the concept of bitopological space. This concept is introduced as an extension of topological space. This concept has been introduced with interest in fuzzy set, soft set and neutrosophic set (see [22, 23, 24, 25]). Therefore, we find it important and necessary to build a bitopological spaces on the concept of neutrosophic soft set. In this paper, bitopological space on the concept of neutrosophic soft set is built, the basic topological concepts of this spaces which are  $N_3\text{-(bi)}^*$ -open set,  $N_3\text{-(bi)}^*$ -closed set,  $(bi)^*$ -neutrosophic soft interior,  $(bi)^*$ -neutrosophic soft closure,  $(bi)^*$ -neutrosophic soft boundary,  $(bi)^*$ -neutrosophic soft exterior are defined, the relations of these basic topological concepts with their counterparts in neutrosophic soft topological spaces are investigated and many examples on this concepts are given.

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## 2. Preliminary

In this section, we will refer to the basic definitions required in our work.

### 2.1. Definition [26]

The neutrosophic set  $N$  over  $M$  is defined as follows:

$$N = \{ \langle m, H_N(m), G_N(m), J_N(m) \rangle : m \in M \}.$$

where, the functions  $H, G, J : M \rightarrow ] - 0, +1[$  and  $- 0 \leq H_N(m) + G_N(m) + J_N(m) \leq +3$ .

From philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of  $] - 0, +1[$ . But in real life application in scientific and engineering problems it is difficult to use a neutrosophic set with value from real standard or non-standard subset of  $] - 0, +1[$ . Hence we consider the neutrosophic set which takes the value from the subset of  $[0, 1]$ .

Firstly, neutrosophic soft set defined by Maji [5] and later this concept and its operations have been redefined by [7]. Our work in this research is based on the definition below:

### 2.2. Definition [7]

Let  $M$  be an initial universe set and  $B$  be a set of parameters. Let  $P(M)$  denote the set of all the neutrosophic sets of  $M$ . Then, a neutrosophic soft set  $\beta_B$  over  $M$  is a set defined by a set valued function  $\beta$  representing a mapping from  $B$  to  $P(M)$ , where  $e \beta$  is called approximate function of the neutrosophic soft set  $\beta_B$ .

In other words,  $\beta_B$  is a parameterized family of some elements of the set  $P(M)$  and therefore it can be written as a set of ordered pairs,

$$\beta_B = \{ (r, \{ \langle m^{(H_{\beta(r)}(m), G_{\beta(r)}(m), J_{\beta(r)}(m)) \rangle : m \in M \}) : r \in B \}.$$

Where,

$H_{\beta(r)}(m), G_{\beta(r)}(m), J_{\beta(r)}(m) \in [0, 1]$ , respectively called the truth-membership, indeterminacy-membership, falsity-membership function of  $\beta(r)$ . Since supremum of each  $H, G, J$  is 1 so the inequality,

$$0 \leq H_{\beta(r)}(m) + G_{\beta(r)}(m) + J_{\beta(r)}(m) \leq 3 \text{ is obvious.}$$

From now on, the set of all neutrosophic sets over  $M$  is denoted by  $N_3(M)$ .

### 2.3. Definition [9,5]

Let  $\beta_B$  and  $\mu_B \in N_3(M)$  such that;

$$\beta_B = \{ (r, \{ \langle m^{(H_{\beta(r)}(m), G_{\beta(r)}(m), J_{\beta(r)}(m)) \rangle : m \in M \}) : r \in B \}.$$

$$\mu_B = \{ (r, \{ \langle m^{(H_{\mu(r)}(m), G_{\mu(r)}(m), J_{\mu(r)}(m)) \rangle : m \in M \}) : r \in B \}. \text{ Then:}$$

- ❖  $\tilde{M}_B = \{ (r, \{ \langle m^{(1,1,0)} \rangle : m \in M \}) : r \in B \}$  [ Absolute neutrosophic soft set ].
- ❖  $\tilde{\emptyset}_B = \{ (r, \{ \langle m^{(0,0,1)} \rangle : m \in M \}) : r \in B \}$  [ Null neutrosophic soft set ].
- ❖  $\beta_B \sqsubseteq \mu_B \leftrightarrow \{ (r, \{ \langle m^{(H_{\beta(r)}(m) \leq H_{\mu(r)}(m), G_{\beta(r)}(m) \leq G_{\mu(r)}(m), J_{\beta(r)}(m) \leq J_{\mu(r)}(m)) \rangle : m \in M \}) : r \in B \}.$
- ❖  $\beta_B \sqcup \mu_B = \{ (r, \{ \langle m^{(H_{\beta(r)}(m) \vee H_{\mu(r)}(m), G_{\beta(r)}(m) \vee G_{\mu(r)}(m), J_{\beta(r)}(m) \vee J_{\mu(r)}(m)) \rangle : m \in M \}) : r \in B \}.$
- ❖  $\beta_B \sqcap \mu_B = \{ (r, \{ \langle m^{(H_{\beta(r)}(m) \wedge H_{\mu(r)}(m), G_{\beta(r)}(m) \wedge G_{\mu(r)}(m), J_{\beta(r)}(m) \wedge J_{\mu(r)}(m)) \rangle : m \in M \}) : r \in B \}.$

### 2.4. Definition

Let  $\beta_B \in N_3(M)$ , The complement of  $\beta_B$  is denoted by  $(\beta_B)^c$  and is defined as:

$$(\beta_B)^c = \{ (r, \{ \langle m^{(1-H_{\beta(r)}(m), 1-G_{\beta(r)}(m), 1-J_{\beta(r)}(m)) \rangle : m \in M \}) : r \in B \}.$$

### 2.5. Definition [9]

Let  $T \subseteq (N_3(M))$ . The collection  $T$  is called a neutrosophic soft topology on  $M$ , if the following conditions are true:

- 1)  $\tilde{M}_B, \tilde{\emptyset}_B$  belong to  $T$ .
- 2) If  $\beta_{j_B} \in T ; j \in J$ , then  $\cup_{j \in J} \beta_{j_B} \in T \forall j \in J$ .
- 3) If  $\beta_B, \mu_B \in T$ , then  $\beta_B \cap \mu_B \in T$ .

Then the triplet  $(M, B, T)$  is a neutrosophic soft topological space or  $(N_3\text{-Top for short})$ .

Members of  $T$  are called a neutrosophic soft open sets ( $N_3\text{-T-open for short}$ ) and their complements are a neutrosophic soft open sets ( $N_3\text{-T-closed for short}$ ).

The neutrosophic soft interior of  $\beta_B \in N_3(M)$  ( $(\beta_B)^0$  for short) is defined as:

$$(\beta_B)^0 = \cup \{(\omega_B) : \omega_B \text{ is a } N_3\text{-T-open set, } \omega_B \subseteq \beta_B\}.$$

The neutrosophic soft closure of  $\beta_B \in N_3(M)$  ( $\overline{(\beta_B)}$  for short) is defined as:

$$\overline{(\beta_B)} = \cap \{(\omega_B) : \omega_B \text{ is a } N_3\text{-T-closed set, } \beta_B \subseteq \omega_B\}.$$

### 2.6. Example

Let  $M = \{m_1, m_2, m_3\}$ ,  $B = \{r\}$  and  $\beta_B, \mu_B, \gamma_B \in N_3(M)$ .

Such that

$$\beta_B = \{(r, \{< m_1(1, 1, 0) >, < m_2(0, 0, 1) >, < m_3(0, 0, 1) >\})\},$$

$$\mu_B = \{(r, \{< m_1(1, 1, 0) >, < m_2(0, 0, 1) >, < m_3(1, 1, 0) >\})\},$$

$$\gamma_B = \{(r, \{< m_1(0, 0, 1) >, < m_2(0, 0, 1) >, < m_3(1, 1, 0) >\})\}.$$

Then,  $T_2 = \{\tilde{\emptyset}_B, \tilde{M}_B, \beta_B, \mu_B\}$  is a neutrosophic soft topology on  $M$ .

### 2.7. Example

Let  $M = \{m_1, m_2, m_3\}$ ,  $B = \{r\}$  and  $\beta_B, \mu_B, \gamma_B, \delta_B \in N_3(M)$ . Such that

$$\beta_B = \{(r, \{< m_1(1, 1, 0) >, < m_2(0, 0, 1) >, < m_3(0, 0, 1) >\})\}.$$

$$\mu_B = \{(r, \{< m_1(1, 1, 0) >, < m_2(0, 0, 1) >, < m_3(1, 1, 0) >\})\}.$$

$$\gamma_B = \{(r, \{< m_1(1, 1, 0) >, < m_2(1, 1, 0) >, < m_3(0, 0, 1) >\})\}.$$

$$\delta_B = \{(r, \{< m_1(0, 0, 1) >, < m_2(1, 1, 0) >, < m_3(1, 1, 0) >\})\}.$$

Then,  $T_2 = \{\tilde{\emptyset}_B, \tilde{M}_B, \beta_B, \mu_B, \gamma_B\}$  is a neutrosophic soft topology on  $M$ .

## 3. Neutrosophic soft bitopological space

In this section, we defined the neutrosophic soft bitopological space or ( $N_3\text{-Bi-Top for short}$ ) on the concept of neutrosophic soft set and the basic topological concepts of this spaces which are  $N_3\text{-biopen}$  and  $N_3\text{-biclosed}$ .

### 3.1. Definition

Let  $(M, B, T_1)$  and  $(M, B, T_2)$  be two  $N_3\text{-Top}$  spaces defined on  $M$ . Then  $(M, B, T_1, T_2)$  is called a neutrosophic soft bitopological space or ( $N_3\text{-Bi-Top for short}$ ).

### 3.2. Example

Let  $M = \{m_1, m_2\}$ ,  $B = \{r\}$  and  $\beta_B, \mu_B \in N_3(M)$  such that

$$\beta_B = \{(r, \{< m_1^{(0.6, 0.2, 0.5)} >, < m_2^{(0.5, 0.4, 0.9)} > \})\}, \mu_B = \{(r, \{< m_1^{(0.6, 0.2, 0.4)} >, < m_2^{(0.6, 0.4, 0.7)} > \})\}.$$

Then,  $T_1 = \{\tilde{\mathcal{O}}_B, \tilde{M}_B, \beta_B\}$  is an  $N_3$ -Top on  $M$  and  $T_2 = \{\tilde{\mathcal{O}}_B, \tilde{M}_B, \mu_B\}$  is an  $N_3$ -Top on  $M$ .

Therefore,  $(M, B, T_1, T_2)$  is an  $N_3$ -Bi-Top space.

### 3.3. Definition

Let  $(M, B, T_1, T_2)$  be an  $N_3$ -Bi-Top space. The members of  $(M, B, T_1, T_2)$  are called bineutrosophic soft open sets ( $N_3$ -biopen for short) and their complements are bineutrosophic soft closed sets ( $N_3$ -biclosed for short).

### 3.4. Remark

- Every neutrosophic soft open (closed) set in  $(M, B, T_1)$  or  $(M, B, T_2)$  is an  $N_3$ -biopen ( $N_3$ -biclosed) set.
- Every  $N_3$ -Bi-Top space  $(M, B, T_1, T_2)$  induces two  $N_3$ -Top spaces as  $(M, B, T_1)$  and  $(M, B, T_2)$ .
- If  $(M, B, T_1)$  is an  $N_3$ -Top space then  $(M, B, T_1, T_1)$  is an  $N_3$ -Bi-Top space.

### 3.5. Theorem

If  $(M, B, T_1, T_2)$  is an  $N_3$ -Bi-Top space, then  $(M, B, T_1 \cap T_2)$  is an  $N_3$ -Top space.

#### Proof

Let  $(M, B, T_1, T_2)$  be an  $N_3$ -Bi-Top space.

(1) Clearly that  $\tilde{\mathcal{O}}_B, \tilde{M}_B \in (T_1 \cap T_2)$ .

(2) Let  $\beta_B, \mu_B \in (T_1 \cap T_2)$ , then  $\beta_B, \mu_B \in T_1$  and  $\beta_B, \mu_B \in T_2$ . This implies that,  $\beta_B \cap \mu_B \in T_1$  and  $\beta_B \cap \mu_B \in T_2$ . Therefore,  $\beta_B \cap \mu_B \in (T_1 \cap T_2)$ .

(3) Let  $\beta_{j_B} \in (T_1 \cap T_2); j \in J$ . Then  $\beta_{j_B} \in T_1$  and  $\beta_{j_B} \in T_2; j \in J$ . Therefore  $\cup_{j \in J} \beta_{j_B} \in T_1$  and  $\cup_{j \in J} \beta_{j_B} \in T_2 \forall j \in J$ . Thus, we have  $\cup_{j \in J} \beta_{j_B} \in (T_1 \cap T_2)$ .

Hence,  $(M, B, T_1 \cap T_2)$  is an  $N_3$ -Top space.

### 3.6. Remark

If we take the operation of union instead of the operation of intersection, then the above theorem is not generally correct.

### 3.7. Example

Let  $M = \{m_1, m_2\}$ ,  $B = \{r\}$  and  $\beta_B, \mu_B \in N_3(M)$  such that

$$\beta_B = \{(r, \{< m_1^{(0.3, 0.5, 0.7)} >, < m_2^{(0.2, 0.4, 0.6)} > \})\}, \mu_B = \{(r, \{< m_1^{(0.5, 0.7, 0.8)} >, < m_2^{(0.3, 0.6, 0.8)} > \})\}.$$

Then,  $T_1 = \{\tilde{\mathcal{O}}_B, \tilde{M}_B, \mu_B\}$  is an  $N_3$ -Top on  $M$  and  $T_2 = \{\tilde{\mathcal{O}}_B, \tilde{M}_B, \beta_B\}$  is an  $N_3$ -Top on  $M$ . Thus,  $(M, B, T_1, T_2)$  is an  $N_3$ -Bi-Top space. But,  $(M, B, T_1 \cup T_2)$  is not an  $N_3$ -Top space. Because,  $\beta_B \cup \mu_B$  does not belong to  $(T_1 \cup T_2)$ .

## 4. $N_3$ -(bi)\*-open set in neutrosophic soft bitopological space

In this section,  $N_3$ -(bi)\*-open set,  $N_3$ -(bi)\*-closed set, (bi)\*-neutrosophic soft interior, (bi)\*-neutrosophic soft closure, (bi)\*-neutrosophic soft boundary, (bi)\*-neutrosophic soft exterior are defined based on the idea of  $\delta$ -open set which was defined in [27].

### 4.1. Definition

A subset  $\beta_B \in N_3(M)$  of an  $N_3$ -Bi-Top space  $(M, B, T_1, T_2)$  is called star bineutrosophic soft open ( $N_3$ -(bi)\*-open, for short) in  $(M, B, T_1, T_2)$  if and only if  $\beta_B \subseteq \overline{(\beta_B)^{\circ T}}_{(T_1)^{\circ T_2}}$  and their complement is an  $N_3$ -(bi)\*-closed set. The

set of all  $N_3$ -(bi)\*-open [ $N_3$ -(bi)\*-closed] sets in  $(M, B, T_1, T_2)$  is denoted by  $M^{(Bi)*-N}$  [ $M^{(Bi)*-NSC}$ ] respectively.

#### 4.2. Example

Let  $M = \{m_1, m_2, m_3\}$ ,  $B = \{r\}$  and  $\beta_B, \mu_B \in N_3(M)$  such that

$$\beta_B = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(0, 0, 1)} >, < m_3^{(0, 0, 1)} >\})\},$$

$$\mu_B = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(1, 1, 0)} >, < m_3^{(0, 0, 1)} >\})\}.$$

$T_1 = \{\tilde{\emptyset}_B, \tilde{M}_B\}$  is an  $N_3$ -Top on  $M$  and  $T_2 = \{\tilde{\emptyset}_B, \tilde{M}_B, \beta_B, \mu_B\}$  is an  $N_3$ -Top on  $M$ . Thus,  $(M, B, T_1, T_2)$  is an  $N_3$ -Bi-Top space.

Note that:

$$\beta_B = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(0, 0, 1)} >, < m_3^{(0, 0, 1)} >\})\} \subseteq \overline{(\beta_B)^{oT_2}}^{(T_1)^{oT_2}} = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 1, 0)} >\})\}.$$

$$\therefore \beta_B \subseteq \overline{(\beta_B)^{oT_2}}^{(T_1)^{oT_2}}.$$

$$\mu_B = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(1, 1, 0)} >, < m_3^{(0, 0, 1)} >\})\} \subseteq \overline{(\mu_B)^{oT_2}}^{(T_1)^{oT_2}} = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 1, 0)} >\})\}.$$

$$\therefore \mu_B \subseteq \overline{(\mu_B)^{oT_2}}^{(T_1)^{oT_2}}.$$

$$\gamma_B = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(0, 0, 1)} >, < m_3^{(1, 1, 0)} >\})\}.$$

$$\{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(0, 0, 1)} >, < m_3^{(1, 1, 0)} >\})\} \subseteq \overline{(\gamma_B)^{oT_2}}^{(T_1)^{oT_2}} = \{(r, \{< m_1^{(1, 1, 0)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 1, 0)} >\})\}.$$

$$\therefore \gamma_B \subseteq \overline{(\gamma_B)^{oT_2}}^{(T_1)^{oT_2}}.$$

$$\delta_B = \{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 1, 0)} >\})\}.$$

$$\{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 1, 0)} >\})\} \not\subseteq \overline{(\delta_B)^{oT_2}}^{(T_1)^{oT_2}} = \{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(0, 0, 1)} >, < m_3^{(0, 0, 1)} >\})\}.$$

$$\therefore \delta_B \not\subseteq \overline{(\delta_B)^{oT_2}}^{(T_1)^{oT_2}}.$$

$$\epsilon_B = \{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 1, 0)} >\})\}.$$

$$\{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 0, 0)} >\})\} \not\subseteq \overline{(\epsilon_B)^{oT_2}}^{(T_1)^{oT_2}} = \{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(0, 0, 1)} >, < m_3^{(0, 0, 1)} >\})\}.$$

$$\therefore \epsilon_B \not\subseteq \overline{(\epsilon_B)^{oT_2}}^{(T_1)^{oT_2}}.$$

$$\theta_B = \{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(0, 0, 1)} >, < m_3^{(1, 1, 0)} >\})\}.$$

$$\{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(0, 0, 1)} >, < m_3^{(1, 1, 0)} >\})\} \not\subseteq \overline{(\theta_B)^{oT_2}}^{(T_1)^{oT_2}} = \{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(0, 0, 1)} >, < m_3^{(0, 0, 1)} >\})\}.$$

$$\therefore \theta_B \not\subseteq \overline{(\theta_B)^{oT_2}}^{(T_1)^{oT_2}}.$$

In general in any  $N_3$ -Bi-Top space,  $\tilde{\emptyset}_B, \tilde{M}_B$  are clearly  $N_3$ -(bi)\*-open sets.

Hence:

$$M^{(Bi)*-NSO} = \{\tilde{\emptyset}_B, \tilde{M}_B, \beta_B, \mu_B, \gamma_B\}.$$

$$M^{(Bi)*-NSC} = \{ \{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(1, 1, 0)} >, < m_3^{(1, 1, 0)} >\})\},$$

$$\{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(0, 0, 1)} >, < m_3^{(1, 1, 0)} >\})\},$$

$$\{(r, \{< m_1^{(0, 0, 1)} >, < m_2^{(1, 1, 0)} >, < m_3^{(0, 0, 1)} >\})\},$$

$$\tilde{\emptyset}_B,$$

$$\tilde{M}_B \}.$$

### 4.3. Remark

Let  $\beta_B$  and  $\mu_B$  be an  $N_3$ -(bi)\*-open sets, then  $\beta_B \cap \mu_B$  is not necessary an  $N_3$ -(bi)\*-open set.

### 4.4. Example

Let  $M = \{m_1, m_2, m_3, m_4, m_5\}$ ,  $B = \{r\}$  and  $\beta_B, \mu_B, \gamma_B, \varepsilon_B, \vartheta_B, \alpha_B \in N_3(M)$ .

Such that

$$\beta_B = \{(r, \{< m_1^{(1,1,0)} >, < m_2^{(0,0,1)} >, < m_3^{(0,0,1)} >, < m_4^{(0,0,1)} > \})\}.$$

$$\mu_B = \{(r, \{< m_1^{(0,0,1)} >, < m_2^{(0,0,1)} >, < m_3^{(0,0,1)} >, < m_4^{(1,1,0)} > \})\}.$$

$$\gamma_B = \{(r, \{< m_1^{(1,1,0)} >, < m_2^{(0,0,1)} >, < m_3^{(0,0,1)} >, < m_4^{(1,1,0)} > \})\}.$$

$$\varepsilon_B = \{(r, \{< m_1^{(0,0,1)} >, < m_2^{(1,1,0)} >, < m_3^{(1,1,0)} >, < m_4^{(0,0,1)} > \})\}.$$

$$\vartheta_B = \{(r, \{< m_1^{(1,1,0)} >, < m_2^{(1,1,0)} >, < m_3^{(1,1,0)} >, < m_4^{(0,0,1)} > \})\}.$$

$$\alpha_B = \{(r, \{< m_1^{(0,0,1)} >, < m_2^{(1,1,0)} >, < m_3^{(1,1,0)} >, < m_4^{(1,1,0)} > \})\}.$$

$T_1 = \{\tilde{\theta}_B, \tilde{M}_B, \beta_B, \mu_B, \gamma_B\}$  is an  $N_3$ -Top on  $M$  and  $T_2 = \{\tilde{\theta}_B, \tilde{M}_B, \beta_B, \mu_B, \gamma_B, \varepsilon_B, \vartheta_B, \alpha_B\}$  is an  $N_3$ -Top on  $M$ . Thus,  $(M, B, T_1, T_2)$  is an  $N_3$ -Bi-Top space. Then

$\varepsilon_B$  and  $\{(r, \{< m_1^{(1,1,0)} >, < m_2^{(0,0,1)} >, < m_3^{(1,1,0)} >, < m_4^{(0,0,1)} > \})\}$  are an  $N_3$ -(bi)\*-open sets, but the intersection of them  $\{(r, \{< m_1^{(0,0,1)} >, < m_2^{(0,0,1)} >, < m_3^{(1,1,0)} >, < m_4^{(0,0,1)} > \})\}$  is not an  $N_3$ -(bi)\*-open set.

### 4.5. Theorem

Let  $(M, B, T_1, T_2)$  be an  $N_3$ -Bi-Top space, then every neutrosophic soft open set in  $(M, B, T_2)$  is an  $N_3$ -(bi)\*-open set in  $(M, B, T_1, T_2)$ .

#### Proof

Let  $\beta_B$  be a neutrosophic soft open set in  $(M, B, T_2)$ . Then  $(\beta_B)^{oT_2} = \beta_B$ . Since  $\beta_B \subseteq \overline{(\beta_B)^{T_1}}$ ,  $\beta_B \subseteq \overline{(\beta_B)^{oT_2}}^{T_1}$ ,  $(\beta_B)^{oT} \subseteq \overline{(\beta_B)^{oT}}^{T_1 oT_2}$ . Therefore  $\beta_B \subseteq \overline{(\beta_B)^{oT}}^{T_1 oT_2}$  and thus  $\beta_B$  is an  $N_3$ -(bi)\*-open set in  $(M, B, T_1, T_2)$ .

### 4.6. Remark

The converse of above remark is not true in general. In Example 3.4 note that,  $\{(r, \{< m_1^{(1,1,0)} >, < m_2^{(0,0,1)} >, < m_3^{(1,1,0)} >, < m_4^{(0,0,1)} > \})\}$  is an  $N_3$ -(bi)\*-open set in  $(M, B, T_1, T_2)$ , but not a neutrosophic soft open set in  $(M, B, T_2)$ .

### 4.7. Definition

If  $(M, B, T_1, T_2)$  is an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ , then the largest  $N_3$ -(bi)\*-open set contained in  $\beta_B$  is called (bi)\*-neutrosophic soft interior of  $\beta_B$ ,  $((\beta_B)^{0(bi)*})$  for short). i.e.

$$(\beta_B)^{0(bi)*} = \sqcup \{(\omega_B) : \omega_B \text{ is a } N_3\text{-(bi)*-open set, } \omega_B \subseteq \beta_B\}.$$

### 4.8. Theorem

Let  $(M, B, T_1, T_2)$  be an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ . Then  $\beta_B$  is an  $N_3$ -(bi)\*-open set if and only if  $\beta_B = (\beta_B)^{0(bi)*}$ .

#### Proof

Let  $\beta_B$  be an  $N_3$ -(bi)\*-open set. Then  $\beta_B$  is itself an  $N_3$ -(bi)\*-open set which contains  $\beta_B$ . Therefore,  $\beta_B$  is the largest  $N_3$ -(bi)\*-open set contained in  $\beta_B$  and  $\beta_B = (\beta_B)^{0(bi)*}$ . Conversely, suppose that  $\beta_B = (\beta_B)^{0(bi)*}$ , then  $\beta_B$  is the largest  $N_3$ -(bi)\*-open set contained in  $\beta_B$ . Thus,  $\beta_B$  is an  $N_3$ -(bi)\*-open set.

**4.9. Theorem**

Let  $\beta_B, \mu_B \in N_3(M)$ .

- a)  $(\beta_B)^{0(bi)*} \subseteq \beta_B$ .
- b)  $((\beta_B)^{0(bi)*})^{0(bi)*} = (\beta_B)^{0(bi)*}$ .
- c)  $(\beta_B)^{0(bi)*} \subseteq (\mu_B)^{0(bi)*}$ ; whenever  $\beta_B \subseteq \mu_B$ .
- d)  $(\beta_B \cap \mu_B)^{0(bi)*} = (\beta_B)^{0(bi)*} \cap (\mu_B)^{0(bi)*}$ .
- e)  $(\beta_B \sqcup \mu_B)^{0(bi)*} \supseteq (\beta_B)^{0(bi)*} \sqcup (\mu_B)^{0(bi)*}$ .
- f)  $(\tilde{M}_B)^{0(bi)*} = \tilde{M}_B$ .
- g)  $(\tilde{\emptyset}_B)^{0(bi)*} = \tilde{\emptyset}_B$ .

**Proof**

(a), (f), (g), (c) (Straightforward).

(b) Let  $\mu_B = (\beta_B)^{0(bi)*}$ . Then  $\mu_B = (\mu_B)^{0(bi)*}$  (from Theorem 4.8). Thus  $((\beta_B)^{0(bi)*})^{0(bi)*} = (\beta_B)^{0(bi)*}$ .

(d) Since,  $(\beta_B \cap \mu_B)^{0(bi)*} \subseteq (\beta_B)^{0(bi)*}$  and  $(\beta_B \cap \mu_B)^{0(bi)*} \subseteq (\mu_B)^{0(bi)*}$ . Then,  $(\beta_B \cap \mu_B)^{0(bi)*} \subseteq (\beta_B)^{0(bi)*} \cap (\mu_B)^{0(bi)*} \dots (1)$ .

Since,  $(\beta_B)^{0(bi)*} \subseteq \beta_B$  and  $(\mu_B)^{0(bi)*} \subseteq \mu_B$ , then  $(\beta_B)^{0(bi)*} \cap (\mu_B)^{0(bi)*} \subseteq \beta_B \cap \mu_B$ . But  $(\beta_B)^{0(bi)*} \cap (\mu_B)^{0(bi)*}$  is a  $N_3$ -(bi)\*-open subset of  $\beta_B \cap \mu_B$ . Therefore, from the detention, we have that  $(\beta_B \cap \mu_B)^{0(bi)*} \supseteq (\beta_B)^{0(bi)*} \cap (\mu_B)^{0(bi)*} \dots (2)$ .

Hence,  $(\beta_B \cap \mu_B)^{0(bi)*} = (\beta_B)^{0(bi)*} \cap (\mu_B)^{0(bi)*}$ .

(e) Since,  $\beta_B \subseteq (\beta_B \sqcup \mu_B)$  and  $\mu_B \subseteq (\beta_B \sqcup \mu_B)$ , therefore  $(\beta_B \sqcup \mu_B)^{0(bi)*} \supseteq (\beta_B)^{0(bi)*}$  and  $(\beta_B \sqcup \mu_B)^{0(bi)*} \supseteq (\mu_B)^{0(bi)*}$ . So,  $(\beta_B \sqcup \mu_B)^{0(bi)*} \supseteq (\beta_B)^{0(bi)*} \sqcup (\mu_B)^{0(bi)*}$ .

**4.10. Example**

Let us consider  $\beta_B, \mu_B, \gamma_B \in N_3(M)$  in Example 2.6. Such that,  $T_2 = \{\tilde{\emptyset}_B, \tilde{M}_B, \beta_B, \mu_B\}$  is an  $N_3$ -Top on  $M$  and  $T_1 = \{\tilde{\emptyset}_B, \tilde{M}_B, \beta_B\}$  is an  $N_3$ -Top on  $M$ . Thus,  $(M, B, T_1, T_2)$  is an  $N_3$ -Bi-Top space.

Note that: 1)  $(\beta_B \sqcup \gamma_B)^{0(bi)*} \not\subseteq (\beta_B)^{0(bi)*} \sqcup (\gamma_B)^{0(bi)*}$ . 2)  $\gamma_B \not\subseteq (\gamma_B)^{0(bi)*}$ .

**4.11. Definition**

If  $(M, B, T_1, T_2)$  is an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ , then the intersection of all  $N_3$ -(bi)\*-closed sets containing  $\beta_B$  is called a (bi)\*-neutrosophic soft closure of  $\beta_B$ ,  $\overline{(\beta_B)}^{(bi)*}$  for short). i.e.

$$\overline{(\beta_B)}^{(bi)*} = \cap \{(\omega_B) : \omega_B \text{ is an } N_3\text{-(bi)*-closed set, } \beta_B \subseteq \omega_B\}.$$

**4.12. Theorem**

Let  $\beta_B, \mu_B \in N_3(M)$ .

- a)  $\beta_B \subseteq \overline{(\beta_B)}^{(bi)*}$ .
- b)  $\overline{((\beta_B)^{0(bi)*})}^{(bi)*} = \overline{(\beta_B)}^{(bi)*}$ .
- c)  $\overline{(\beta_B)}^{(bi)*} \subseteq \overline{(\mu_B)}^{(bi)*}$ ; whenever  $\beta_B \subseteq \mu_B$ .
- d)  $\overline{(\beta_B \cap \mu_B)}^{(bi)*} \subseteq \overline{(\beta_B)}^{(bi)*} \cap \overline{(\mu_B)}^{(bi)*}$ .
- e)  $\overline{(\beta_B \sqcup \mu_B)}^{(bi)*} = \overline{(\beta_B)}^{(bi)*} \sqcup \overline{(\mu_B)}^{(bi)*}$ .



- f)  $\overline{(\tilde{M}_B)^{(bi)*}} = \tilde{M}_B$ .  
 g)  $\overline{(\tilde{\emptyset}_B)^{(bi)*}} = \tilde{\emptyset}_B$ .

**Proof** Straightforward.

#### 4.13. Remark

In above theorem, it is not necessary the converse of (a) and (d) be true.

#### 4.14. Example

Let us take,  $\beta_B, \mu_B, \gamma_B, \delta_B \in N_3(M)$  in Example 2.7.

$T_2 = \{\tilde{\emptyset}_B, \tilde{M}_B, \beta_B, \mu_B, \gamma_B\}$  is an  $N_3$ -Top on  $M$  and  $T_1 = \{\tilde{\emptyset}_B, \tilde{M}_B\}$  is an  $N_3$ -Top on  $M$ . Thus,  $(M, B, T_1, T_2)$  is an  $N_3$ -Bi-Top space.

Note that:

- 1)  $(\beta_B \sqcup \gamma_B)^{0(bi)*} \not\subseteq (\beta_B)^{0(bi)*} \sqcup (\gamma_B)^{0(bi)*}$ . 2)  $\gamma_B \not\subseteq (\gamma_B)^{0(bi)*}$ .

#### 4.15. Theorem

Let  $(M, B, T_1, T_2)$  be an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ .

- a)  $((\beta_B)^C)^{0(bi)*} = \left(\overline{(\beta_B)^{(bi)*}}\right)^C$ .  
 b)  $\overline{(\beta_B)^C}^{(bi)*} = ((\beta_B)^{0(bi)*})^C$ .

**Proof**

(a) We know that,  $\overline{(\beta_B)^{(bi)*}} = \cap\{\omega_B : (\omega_B)^C \text{ is a } N_3\text{-(bi)*-open set, } \beta_B \sqsubseteq \omega_B\}$ . So, we have that,

$$\left(\overline{(\beta_B)^{(bi)*}}\right)^C = \cup\{(\omega_B)^C : (\omega_B)^C \text{ is an } N_3\text{-(bi)*-open set, } (\omega_B)^C \sqsubseteq (\beta_B)^C\} = ((\beta_B)^C)^{0(bi)*}. \text{ Thus, } \left((\beta_B)^C\right)^{0(bi)*} = \left(\overline{(\beta_B)^{(bi)*}}\right)^C.$$

(b) If we take,  $(\beta_B)^C$  instead of  $\beta_B$  in (a), we get that,

$$\left(\overline{((\beta_B)^C)^{(bi)*}}\right)^C = (((\beta_B)^C)^{0(bi)*})^C = ((\beta_B)^{0(bi)*})^C. \text{ So, } \overline{(\beta_B)^C}^{(bi)*} = ((\beta_B)^{0(bi)*})^C.$$

#### 4.16. Theorem

If  $(M, T_1, T_2)$  is an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ , then  $\beta_B$  is an  $N_3$ -(bi)\*-closed set if and only if  $\beta_B = \overline{(\beta_B)^{(bi)*}}$ .

**Proof**

Let  $\beta_B$  be an  $N_3$ -(bi)\*-closed set, then  $\beta_B$  is itself an  $N_3$ -(bi)\*-closed set which contains  $\beta_B$ . Therefore,  $\beta_B$  is the intersection of all  $N_3$ -(bi)\*-closed sets containing  $\beta_B$  and  $\beta_B = \overline{(\beta_B)^{(bi)*}}$ .

Conversely, suppose that  $\beta_B = \overline{(\beta_B)^{(bi)*}}$ , then  $\beta_B$  is the intersection of all  $N_3$ -(bi)\*-closed sets containing  $\beta_B$ . Thus,  $\beta_B$  is an  $N_3$ -(bi)\*-closed set.

#### 4.17. Definition

If  $(M, T_1, T_2)$  is an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ , then the (bi)\*-neutrosophic soft exterior of  $\beta_B$ , (bi)\*-ext( $\beta_B$ ) for short) is defined as, (bi)\*-ext( $\beta_B$ ) =  $((\beta_B)^C)^{0(bi)*}$ .

#### 4.18. Definition

If  $(M, B, T_1, T_2)$  is an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ , then the (bi)\*-neutrosophic soft boundary of  $\beta_B$ , (bi)\*-br( $\beta_B$ ) for short) is defined as, (bi)\*-br( $\beta_B$ ) =  $\overline{(\beta_B)^C}^{(bi)*} \cap \overline{(\beta_B)^{(bi)*}}$ .

**4.19. Theorem**

Assume that  $(M, B, T_1, T_2)$  is an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ .

- $(bi)^*-br((\beta_B)^C) = (bi)^*-ext(\beta_B) \sqcup (\beta_B)^{0(bi)*}$ .
- $\overline{(\beta_B)}^{(bi)*} = (bi)^*-br(\beta_B) \sqcup (\beta_B)^{0(bi)*}$ .
- $(bi)^*-br(\beta_B) \cap (\beta_B)^{0(bi)*} = \tilde{\emptyset}_B$ .
- $(bi)^*-br(\beta_B)^{0(bi)*} \sqsubseteq (bi)^*-br(\beta_B)$ .

**Proof** Straightforward.

**4.20. Theorem**

Assume that  $(M, B, T_1, T_2)$  is an  $N_3$ -(Bi)\*-Top space and  $\beta_B \in N_3(M)$ .

- $\beta_B \in M^{(Bi)*-NSO}$  if and only if  $(bi)^*-br(\beta_B) \cap \beta_B = \tilde{\emptyset}_B$ .
- $\beta_B \in M^{(Bi)*-NSC}$  if and only if  $(bi)^*-br(\beta_B) \sqsubseteq \beta_B$ .

**Proof** Straightforward.

**Conclusion**

In this research, bitopological space on the concept of neutrosophic soft set is built, the basic topological concepts of these spaces which are  $N_3$ -(bi)\*-open set,  $N_3$ -(bi)\*-closed set, (bi)\*-neutrosophic soft interior, (bi)\*-neutrosophic soft closure, (bi)\*-neutrosophic soft boundary, (bi)\*-neutrosophic soft exterior are defined and many examples on these concepts are given.

This paper is just a beginning of a new structure and we have studied a few ideas only, it will be necessary to carry out more theoretical research to establish a general framework for the practical application.

We hope that the findings in this paper will help researchers enhance and promote the further study on neutrosophic soft bitopological space.

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