Neutrosophic Number Sequences: An introductory Study

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Abstract

In this paper, Neutrosophic definitions and properties of some special number sequences which are frequently found in the science literature, called Neutrosophic Number Sequences (NNSq) via Horadam sequence are studied for the first time. Especially for Neutrosophic Fibonacci (NFNq) and Neutrosophic Lucas (NLNq) number sequences, fundamental properties and identities such as Ruggles, Honsberger, Cassini, Catalan, d'Ocagne, and Tagiuri are given. In addition, Neutrosophic definitions of the sequences of Pell (NPNq), Pell-Lucas (NPLNq), Jacobsthal (NJNq), Jacobsthal-Lucas (NJLNq), Mersenne (NMNq), Mersenne-Lucas (NMLNq), Balancing (NBNq), and Lucas-Balancing (NLBNq) numbers are introduced. Besides defining these numbers and their sequences, since fuzzy and intuitionistic fuzzy sets are restrictions of neutrosophic sets, sequences of numbers within these sets are naturally and indirectly revealed.

Keywords: Neutrosophic number sequence; Neutrosophic Fibonacci number sequence; Neutrosophic Lucas number sequence; Neutrosophic Pell number sequence; Neutrosophic Jacobsthal number sequence; Neutrosophic Jacobsthal-Lucas number sequence; Neutrosophic Mersenne number sequence; Neutrosophic Mersenne-Lucas number sequence; Neutrosophic balancing number sequence; Neutrosophic Lucas-balancing number sequence; Horadam Form

1 Introduction

Neutrosophic Logic is a nascent field of study in which each proposition is estimated to have a rate of truth (percentage) in a subset of T, an uncertainty rate in a subset of I, and a falsity rate in a subset F. We utilize a subset of truth (or indeterminacy, or falsity), instead of a number only, since in many situations we are not capable of specifying the proportions of truth and of falsity strictly but to approximate them: for instance, a proposition is between $18-57\%$ true and between $61-79\%$ false, even worst: between $32-44\%$ or $41-51\%$ true (pursuant to several observers), and 56% or between $61-77\%$ false. The subsets are not essential intervals, but any sets (open or closed or half open/half-closed interval, discrete, continuous, intersections or unions of the previous sets, etc.) in keeping with the given proposition. Zadeh initiated the landscape by provide an insight for meaning and mathematical results from uncertainty situations (fuzzy).[1] Fuzzy sets brought a new dimension to the concept of classical set theory. Atanassov defined intuitionistic fuzzy sets (IFS) including membership and non-membership degrees.[2] Neutrosophy was proposed by Florentin as a computational method to the concept of neutrality.[3] Neutrosophic sets (NS) consider membership, non-membership, and indeterminacy degrees. Intuitionistic fuzzy sets are defined by the degree of membership and non-membership and, uncertainty degrees by the 1- (membership degree plus non-membership degree), while the degree of uncertainty is evaluated independently of the degree of membership and non-membership in neutrosophic sets. Here, membership, non-membership, and degree of uncertainty (uncertainty), such as degrees of accuracy and

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falsity, can be evaluated according to the interpretation of the places to be used. It depends entirely on the subject area (the universe of discourse). This reveals a difference between neutrosophic set and intuitionistic fuzzy set. In this sense, the concept of neutrosophic is a possible solution and representation of problems in various fields. Mathematical differences between NS and IFL are as follows:

I) NS can discern absolute truth (truth in all possible worlds, according to Leibniz) from relative truth (truth in at least one world), because NS (absolute truth) = 1^+ while IFS (relative truth) $= 1$. This has practice in philosophy (see the Neutrosophy). The standard interval $[0, 1]$ used in IFS has been extended to the unitary non-standard interval $]^-0, 1^+[$ in NS. Parallel earmarks for absolute or relative falsehood, and absolute or relative indeterminacy are permitted in NS.

II) There is no limit on $\mathrm{T, I, F}$ other than they are subsets of $]^-0, 1^+[$, thus: $^-0 \leq \inf T + \inf I + \inf F \leq \sup \mathrm{T} + \sup \mathrm{I} + \sup \mathrm{F} \leq 3^+$in NL. This permissiveness allows dialetheist, para-consistent, and incomplete information to be described in NL, while these situations cannot be described in IFL since F (falsehood), T (truth), I (indeterminacy) are restricted either to $t+i+f=1$ or to $t^2+f^2 \leq 1$, if $\mathrm{T, I, F}$ are all reduced to the points $\mathrm{t, i, f}$ respectively, or to $\sup \mathrm{T} + \sup \mathrm{I} + \sup \mathrm{F} = 1$ if $\mathrm{T, I, F}$ are subsets of $[0, 1]$ in IFL.

The Fibonacci sequence has delighted mathematicians and scientists alike for centuries with its beauty and its propensity to pop up in quite unexpected places. Leonardo de Pisa has not even guessed that the number sequences would be so adventurous with the rabbit problem. However, the Fibonacci numbers are surprisingly found in Pascal's triangle, Pythagorean triples, computer algorithms, graph theory, and many other areas of mathematics. They also occur in a variety of other fields such as physics, finance, architecture, computer sciences, color image processing, geostatistics, music, and art. There have been many studies in the literature about this special number sequence because of its numerous applications. There are many generalizations on this sequence some of which can be seen in.[4–14]

Table 1: Frequently studied special number sequences in mathematics literature

n	0	1	2	3	4	5	6	7	8	9	Formula
Fibonacci	0	1	1	2	3	5	8	13	21	34	F(n)=F(n-1)+F(n-2)
Lucas	2	1	3	4	7	11	18	29	47	76	L(n)=L(n-1)+L(n-2)
Pell	0	1	2	5	12	29	70	169	408	985	P(n)=2P(n-1)+P(n-2)
Pell-Lucas	2	2	6	14	34	82	198	478	1154	2786	PL(n)=2PL(n-1)+PL(n-2)
Jacobsthal	0	1	1	3	5	11	21	43	85	171	J(n)=J(n-1)+2J(n-2)
Jacobsthal-Lucas	2	1	5	7	17	31	65	127	257	511	JL(n)=JL(n-1)+2JL(n-2)
Mersenne	0	1	3	7	15	31	63	127	255	511	M(n)=3M(n-1)-2M(n-2)
Mersenne-Lucas	2	3	5	9	17	33	65	129	257	513	ML(n)=3ML(n-1)-2ML(n-2)
Balancing	0	1	6	35	204	1189	6930	40391	235416	1372105	B(n)=6B(n-1)-B(n-2)
Lucas-Balancing	1	3	17	99	577	3363	19601	114243	665857	3880899	BL(n)=6BL(n-1)-BL(n-2)

In a sequence of numbers containing a given rule, any three consecutive terms related to each other can be defined by the second-order linear difference equation. The relation between the terms of the sequence, where H_n is a sequence of numbers. If $AH_{n-1}+BH_{n-2}$, where A and $B \in \mathbb{Q}$, it can be defined by the second-order linear difference equation. The characteristic equation of this sequence can be expressed as $\gamma^2 - A\gamma - B = 0$. The characteristic roots of the equation $\gamma^2 - A\gamma - B = 0$ are found as $\gamma_1 = \frac{A+\sqrt{A^2+4B}}{2}$ and $\gamma_2 = \frac{A-\sqrt{A^2+4B}}{2}$. The general solution of this sequence is

$$H_n = \delta_1 \gamma_1^n + \delta_2 \gamma_2^n, \quad n \geq 1.$$

It is found δ_1 and δ_2 using initial conditions $H_1 = e$ and $H_2 = f$ of the sequence.

Fibonacci and Lucas numbers are defined by the recurrence relations, respectively. $F_{n+2} = F_{n+1} + F_n, F_0 = 0, F_1 = 1, n \geq 2$ and $L_{n+2} = L_{n+1} + L_n, L_0 = 2, L_1 = 1, n \geq 2$. The nth Fibonacci and Lucas number are formalized as $F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$ and $L_n = \alpha^n + \beta^n$, where $\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2} \cdot F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$ and $L_n = \alpha^n + \beta^n$, this form of formulation is called the Binet's form.[15]

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2 Preliminaries

In this section, we give the fundamental preliminaries of fuzzy Fibonacci numbers and Neutrosophic Triangular Numbers. Also, some basic concepts that are fundamentals of this paper will be covered. Definitions of a Single Valued Triangular Neutrosophic Number (SVTrN), its α-cut, β-cut, and γ-cut sets and arithmetical operations on them are the below.

Definition 2.1. [16] Let $\{F_n\}$ be Fibonacci sequence. For $n \geq 0$ integer, Fuzzy Fibonacci numbers are defined as

$$F^{\alpha}_{rn+l} = \left[F_{r(n-1)+l} + \alpha\left(F_{rn+l} - F_{r(n-1)+l}\right), F_{r(n+1)+l} - \alpha\left(F_{r(n+1)+l} - F_{rn+l}\right)\right]$$

where $r, l \in \mathbb{Z}^+$and $\alpha \in [0,1]$. When we take $r = 1$ and $l = 0$, several Fuzzy Fibonacci numbers are following;

$$F^{\alpha}_0 = [1-\alpha, 1+\alpha], F^{\alpha}_1 = [\alpha, 1], F^{\alpha}_2 = [1, 2+\alpha], F^{\alpha}_3 = [1+\alpha, 3+\alpha], \ldots$$

Since Fuzzy Fibonacci numbers include Fibonacci numbers, then Fuzzy Fibonacci numbers satisfy the similar recurrence relation with initial conditions $F^{\alpha}_0 = [1-\alpha, 1+\alpha], F^{\alpha}_1 = [\alpha, 1]$. That is

$$F^{\alpha}_n = F^{\alpha}_{n-1} + F^{\alpha}_{n-2}$$

for $n \geq 2$.

Definition 2.2. [16] Let $\{L_n\}$ be Lucas sequence. For $n \geq 0$ integer, Fuzzy Lucas numbers are defined as

$$L^{\alpha}_{rn+l} = \left[L_{r(n-1)+l} + \alpha\left(L_{rn+l} - L_{r(n-1)+l}\right), L_{r(n+1)+l} - \alpha\left(L_{r(n+1)+l} - L_{rn+l}\right)\right]$$

where $r, l \in \mathbb{Z}^+$and $\alpha \in [0,1]$. When we take $r = 1$ and $l = 0$, several Fuzzy Lucas numbers are following:

$$L^{\alpha}_1 = [2-\alpha, 3-2\alpha], L^{\alpha}_2 = [1+2\alpha, 4-\alpha], L^{\alpha}_3 = [3+\alpha, 7-3\alpha], \ldots$$

Together with initial conditions $L^{\alpha}_0 = [-1+3\alpha, 1+\alpha], L^{\alpha}_1 = [2-\alpha, 3-2\alpha]$, Fuzzy Lucas numbers satisfy recurrence relation. That is

$$L^{\alpha}_n = L^{\alpha}_{n-1} + L^{\alpha}_{n-2}$$

for $n \geq 2$.

Definition 2.3. A Single Valued Triangular Neutrosophic Number (SVTrN) $\tilde{a} = \langle (a,b,c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is a special neutrosophic set on the set of real numbers R, whose truth-membership, indeterminacy-membership and falsity-membership functions are respectively defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-a)w_{\tilde{a}}}{b-a}, & a \leq x < b \\ \frac{(c-x)w_{\tilde{a}}}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

$$\vartheta_{\tilde{a}}(x) = \begin{cases} \frac{b-x+(x-a)u_{\tilde{a}}}{b-a}, & a \leq x < b \\ \frac{x-b+(c-x)u_{\tilde{a}}}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

$$\delta_{\tilde{a}}(x) = \begin{cases} \frac{b-x+(x-a)y_{\tilde{a}}}{b-a}, & a \leq x < b \\ \frac{x-b+(c-x)y_{\tilde{a}}}{c-b}, & b \leq x \leq c \\ 0, & \text{otherwise} \end{cases}$$

The values $w_{\tilde{a}}, u_{\tilde{a}}$ and $y_{\tilde{a}}$ represent the truth-membership, indeterminacy-membership and falsity-membership, respectively, such that they satisfy the conditions $0 \leq w_{\tilde{a}} \leq 1, 0 \leq u_{\tilde{a}} \leq 1, 0 \leq y_{\tilde{a}} \leq 1$ and $0 \leq w_{\tilde{a}} + u_{\tilde{a}} + y_{\tilde{a}} \leq 3$. Additionally, if $a \geq 0$ and $c > 0$, then the SVTrN $\tilde{a} = \langle (a,b,c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a positive SVTrN, denoted by $\tilde{a} > 0$. if $c \leq 0$ and $a < 0$, then the SVTrN $\tilde{a} = \langle (a,b,c); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is called a negative SVTrN, denoted by $\tilde{a} < 0$.

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Definition 2.4. 2.2 A α-cut set of a SVTrN $\tilde{a}=\langle(a,b,c);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is a crisp subset of s set R, which is defined as $\tilde{a}_\alpha=\{x\mid\mu_{\tilde{a}}(x)\geq\alpha,0\leq\alpha\leq w_{\tilde{a}}\}$. $\tilde{a}_\alpha$ is a closed interval, denoted by $\tilde{a}_\alpha=[L_{\tilde{a}}(\alpha),R_{\tilde{a}}(\alpha)]$, α cut set of a SVTrN for truth-membership can be calculated as follows:

$$[L_{\tilde{a}}(\alpha),R_{\tilde{a}}(\alpha)]=\left[\frac{(w_{\tilde{a}}-\alpha)\,a+\alpha b}{w_{\tilde{a}}},\frac{(w_a-\alpha)\,c+\alpha b}{w_{\tilde{a}}}\right].$$

Definition 2.5. A β-cut set of a SVTrN $\tilde{a}=\langle(a,b,c);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is a crisp subset of set R, which is defined as $\tilde{a}_\beta=\{x\mid\vartheta_{\tilde{a}}(x)\leq\beta,u_{\tilde{a}}\leq\beta\leq 1\}$. $\tilde{a}_\beta$ is a closed interval, denoted by $\tilde{a}_\beta=[L_{\tilde{a}}(\beta),R_{\tilde{a}}(\beta)]$, β cut set of a SVTrN for indeterminacy-membership can be calculated as follows:

$$[L_{\tilde{a}}(\beta),R_{\tilde{a}}(\beta)]=\left[\frac{(1-\beta)b+(\beta-u_{\tilde{a}})\,a}{1-u_{\tilde{a}}},\frac{(1-\beta)b+(\beta-u_{\tilde{a}})\,c}{1-u_{\tilde{a}}}\right].$$

Definition 2.6. A γ-cut set of a SVTrN $\tilde{a}=\langle(a,b,c);w_{\tilde{a}},u_{\tilde{a}},y_{\tilde{a}}\rangle$ is a crisp subset of s set R, which is defined as $\tilde{a}_\gamma=\{x\mid\delta_{\tilde{a}}(x)\leq\gamma,y_{\tilde{a}}\leq\gamma\leq 1\}$. $\tilde{a}_\gamma$ is a closed interval, denoted by $\tilde{a}_\gamma=[L_{\tilde{a}}(\gamma),R_{\tilde{a}}(\gamma)]$, γ cut set of a SVTrN for falsity-membership can be calculated as follows:

$$[L_{\tilde{a}}(\gamma),R_{\tilde{a}}(\gamma)]=\left[\frac{(1-\gamma)b+(\gamma-y_{\tilde{a}})\,a}{1-y_{\tilde{a}}},\frac{(1-\gamma)b+(\gamma-y_{\tilde{a}})\,c}{1-y_{\tilde{a}}}\right].$$

In a similar way to the arithmetical operations of the neutrosophic numbers, the arithmetical operations over SVTrNs might be defined by Deli and Subaş.[17] Obviously, if $w_{\tilde{a}}=1,u_{\tilde{a}}=0$ and $y_{\tilde{a}}=0$; $\tilde{a}=\langle(\underline{a},a,\bar{a});1,0,0\rangle$ and $\tilde{b}=\langle(\underline{b},b,\bar{b});1,0,0\rangle$ are the single valued triangular neutrosophic numbers, then the arithmetical operations generate equalities as follows:

i. $\tilde{a}+\tilde{b}=\langle(\underline{a}+\underline{b},a+b,\bar{a}+\bar{b});1,0,0\rangle$

ii. $\tilde{a}-\tilde{b}=\langle(\underline{a}-\bar{b},a-b,\bar{a}-\underline{b});1,0,0\rangle$

iii. $\tilde{a}\tilde{b}=\begin{cases}\langle(\underline{ab},ab,\bar{a}\bar{b});1,0,0\rangle, & \text{if } \tilde{a}>0 \text{ and } \tilde{b}>0,\\ \langle(\underline{a}\bar{b},ab,\bar{a}\underline{b});1,0,0\rangle, & \text{if } \tilde{a}<0 \text{ and } \tilde{b}>0,\\ \langle(\bar{a}\bar{b},ab,\underline{ab});1,0,0\rangle, & \text{if } \tilde{a}<0 \text{ and } \tilde{b}<0,\end{cases}$

iv. $\tilde{a}/\tilde{b}=\begin{cases}\langle(\underline{a}/\bar{b},a/b,\bar{a}/\underline{b});1,0,0\rangle, & \text{if } \tilde{a}>0 \text{ and } \tilde{b}>0,\\ \langle(\bar{a}/\bar{b},a/b,\underline{a}/\underline{b});1,0,0\rangle, & \text{if } \tilde{a}<0 \text{ and } \tilde{b}>0,\\ \langle(\bar{a}/\underline{b},a/b,\underline{a}/\bar{b});1,0,0\rangle, & \text{if } \tilde{a}<0 \text{ and } \tilde{b}<0,\end{cases}$

v. $\lambda\tilde{a}=\begin{cases}\langle(\lambda\underline{a},\lambda\bar{a},\lambda\bar{a});1,0,0\rangle, & \text{if } \lambda>0,\\ \langle(\lambda\bar{a},\lambda a,\lambda\underline{a});1,0,0\rangle, & \text{if } \lambda<0,\end{cases}$

vi. $\tilde{a}^{-1}=\langle(1/\bar{a},1/a,1/\underline{a});1,0,0\rangle$

3 Neutrosophic Fibonacci and Lucas Numbers

In this section, Neutrosophic numbers including Fibonacci and Lucas number sequences with quadratic characteristic equations will be defined.

Definition 3.1. Let $\{F_n\}$ be Fibonacci sequence. For $n\geq 1$ integer, Neutrosophic Fibonacci numbers are defined as

$$\begin{aligned}F^{\alpha,\beta,\gamma}_{r(n-1)+l}=\big(&\left[F_{r(n-1)+l}+\alpha\left(F_{rn+l}-F_{r(n-1)+l}\right),F_{r(n+1)+l}-\alpha\left(F_{r(n+1)+l}-F_{rn+l}\right)\right],[(1-\beta)F_{rn+l}\\&+\beta F_{r(n-1)+l},(1-\beta)F_{rn+l}+\beta F_{r(n+1)+l}\right],\left[(1-\gamma)F_{rn+l}+\gamma F_{r(n-1)+l},(1-\gamma)F_{rn+l}\\&+\gamma F_{r(n+1)+l}\right]\big)\end{aligned}$$

where $r,l\in\mathbb{Z}^+,r(n-1)+l>1;\alpha,\beta,\gamma\in[0,1]$ and $0\leq\alpha+\beta+\gamma\leq 3$. When we take $r=1$ and $l=0$, several Neutrosophic Fibonacci numbers are following:

$$F^{\alpha,\beta,\gamma}_0=([\alpha,1],[1-\beta,1],[1-\gamma,1]);\quad F^{\alpha,\beta,\gamma}_1=([1,2-\alpha],[1,1+\beta],[1\\,1+\gamma]);\quad F^{\alpha,\beta,\gamma}_2=([1+\alpha,3-\alpha],[2-\beta,2+\beta],[2-\gamma,2+\gamma]);\ldots$$

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Remark 3.2. The first number of a Neutrosophic Fibonacci sequence for $\alpha = 0.7, \beta = 0.23, \gamma = 0.07$ is $F_0^{0.7,0.23,0.07} = ([0.7, 1], [0.8, 1], [0.93, 1])$ can be seen (bold colored) in Table 2. Moreover, if we use only α, we obtain fuzzy Fibonacci number sequence and if we use α and γ s.t. $\alpha + \gamma \leq 1$, we obtain intuitionistic fuzzy Fibonacci number sequence.

Table 2: The first 6 elements of Neutrosophic Fibonacci Sequence for α, β, and γ cuts.

α	β	γ	F_0	F_1	F_2	F_3	F_4	F_5	F_0	F_1	F_2	F_3	F_4	F_5
			Left Truth						Right Truth					
1	0	1	1	1	2	3	5	8	1	1	2	3	5	8
0.9	0.05	0.9	0.9	1	1.9	2.9	4.8	7.7	1	1.1	2.1	3.2	5.3	8.5
0.8	0.2	0	0.8	1	1.8	2.8	4.6	7.4	1	1.2	2.2	3.4	5.6	9
0.7	**0.23**	**0.07**	**0.7**	1	1.7	2.7	4.4	7.1	**1**	1.3	2.3	3.6	5.9	9.5
0.6	0.6	0	0.6	1	1.6	2.6	4.2	6.8	1	1.4	2.4	3.8	6.2	10
0.5	0.5	0.5	0.5	1	1.5	2.5	4	6.5	1	1.5	2.5	4	6.5	10.5
0.4	0.4	0.04	0.4	1	1.4	2.4	3.8	6.2	1	1.6	2.6	4.2	6.8	11
0.3	0.3	1	0.3	1	1.3	2.3	3.6	5.9	1	1.7	2.7	4.4	7.1	11.5
0.2	0.2	0.2	0.2	1	1.2	2.2	3.4	5.6	1	1.8	2.8	4.6	7.4	12
			Left Indeterminacy						Right Indeterminacy					
			1	1	2	3	5	8	1	1	2	3	5	8
			1	1	2	3	5	8	1	1.05	2.05	3.1	5.15	8.25
			0.95	1	1.95	2.95	4.9	7.85	1	1.2	2.2	3.4	5.6	9
			0.8	1	1.8	2.8	4.6	7.4	**1**	1.23	2.23	3.46	5.69	9.15
			0.77	1	1.77	2.77	4.54	7.31	1	1.6	2.6	4.2	6.8	11
			0.4	1	1.4	2.4	3.8	6.2	1	1.5	2.5	4	6.5	10.5
			0.5	1	1.5	2.5	4	6.5	1	1.4	2.4	3.8	6.2	10
			0.6	1	1.6	2.6	4.2	6.8	1	1.3	2.3	3.6	5.9	9.5
			0.7	1	1.7	2.7	4.4	7.1	1	1.2	2.2	3.4	5.6	9
			Left Falsity						Right Falsity					
			0	1	1	2	3	5	1	2	3	5	8	13
			0.1	1	1.1	2.1	3.2	5.3	1	1.9	2.9	4.8	7.7	12.5
			1	1	2	3	5	8	1	1	2	3	5	8
			0.93	1	1.93	2.93	4.86	7.79	**1**	1.07	2.07	3.14	5.21	8.35
			1	1	2	3	5	8	1	1	2	3	5	8
			0.5	1	1.5	2.5	4	6.5	1	1.5	2.5	4	6.5	10.5
			0.96	1	1.96	2.96	4.92	7.88	1	1.04	2.04	3.08	5.12	8.2
			0	1	1	2	3	5	1	2	3	5	8	13
			0.8	1	1.8	2.8	4.6	7.4	1	1.2	2.2	3.4	5.6	9

Remark 3.3. Neutrosophic Fibonacci numbers satisfy the similar recurrence relation with initial conditions $F_0^{\alpha,\beta,\gamma}$ and $F_1^{\alpha,\beta,\gamma}$ since Neutrosophic Fibonacci numbers include Fibonacci numbers. That is

$$F_n^{\alpha,\beta,\gamma} = F_{n-1}^{\alpha,\beta,\gamma} + F_{n-2}^{\alpha,\beta,\gamma}, n \geq 2.$$

Definition 3.4. Let $\{L_n\}$ be Lucas sequence. For $n \geq 2$ integer, Neutrosophic Lucas numbers are defined as $L_{r(n-2)+l}^{\alpha,\beta,\gamma} = ([L_{r(n-1)+l} + \alpha(L_{rn+l} - L_{r(n-1)+l}), L_{r(n+1)+l} - \alpha(L_{r(n+1)+l} - L_{rn+l})], [(1-\beta)L_{rn+l} + \beta L_{r(n-1)+l}, (1-\beta)L_{rn+l} + \beta L_{r(n+1)+l}], [(1-\gamma)L_{rn+l} + \gamma L_{r(n-1)+l}, (1-\gamma)L_{rn+l} + \gamma L_{r(n+1)+l}])$

where $r, l \in \mathbb{Z}^+, r(n-1)+l > 1; \alpha, \beta, \gamma \in [0,1]$ and $0 \leq \alpha + \beta + \gamma \leq 3$. When we take $r = 1$ and $l = 0$, several Neutrosophic Lucas numbers are following:

$L_0^{\alpha,\beta,\gamma} = ([1+2\alpha, 4-\alpha], [3-2\beta, 3+\beta], [3-2\gamma, 3+\gamma]);$ $L_1^{\alpha,\beta,\gamma} = ([3+\alpha, 7-3\alpha], [4-\beta, 4+3\beta], [4-\gamma, 4+3\gamma]);$ $L_2^{\alpha,\beta,\gamma} = ([4+3\alpha, 11-4\alpha], [7-3\beta, 7+4\beta], [7-3\gamma, 7+4\gamma]); \ldots$

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Remark 3.5. Since Neutrosophic Lucas numbers include Lucas numbers, then Neutrosophic Lucas numbers satisfy the similar recurrence relation with initial conditions $L_0^{\alpha,\beta,\gamma}$ and $L_1^{\alpha,\beta,\gamma}$. That is

$$L_n^{\alpha,\beta,\gamma} = L_{n-1}^{\alpha,\beta,\gamma} + L_{n-2}^{\alpha,\beta,\gamma}, n \geq 2$$

Remark 3.6. The numbers consisting only of the truth-membership value of the Neutrosophic Fibonacci and Lucas numbers are called the Fuzzy Fibonacci and Lucas numbers respectively, where $\alpha \in [0,1]$ should be $0 \leq \alpha \leq 1$.

Remark 3.7. The numbers consisting of only the truth-membership and falsity-membership values of the Neutrosophic Fibonacci and Lucas numbers are called the Intuitionistic Fuzzy Fibonacci and Lucas numbers respectively, where $\alpha, \beta \in [0,1]$ should be $0 \leq \alpha + \beta \leq 1$.

Proposition 3.8. *$F_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci number such that $n \geq 1$ integer. The Binet's formula of the $F_n^{\alpha,\beta,\gamma}$ numbers is*

$$\begin{aligned} F_n^{\alpha,\beta,\gamma} = \Bigg(\Bigg[\left(\frac{\gamma_1^{n+1} - \gamma_2^{n+1}}{\gamma_1 - \gamma_2} + \alpha \frac{\gamma_1^{n} - \gamma_2^{n}}{\gamma_1 - \gamma_2} \right), \left(\frac{\gamma_1^{n+3} - \gamma_2^{n+3}}{\gamma_1 - \gamma_2} + \alpha \frac{\gamma_1^{n+1} - \gamma_2^{n+1}}{\gamma_1 - \gamma_2} \right) \Bigg], \Bigg[\Bigg(\frac{\gamma_1^{n+2} - \gamma_2^{n+2}}{\gamma_1 - \gamma_2} \\ -\beta \frac{\gamma_1^{n} - \gamma_2^{n}}{\gamma_1 - \gamma_2} \Bigg), \left(\frac{\gamma_1^{n+2} - \gamma_2^{n+2}}{\gamma_1 - \gamma_2} + \beta \frac{\gamma_1^{n+1} - \gamma_2^{n+1}}{\gamma_1 - \gamma_2} \right) \Bigg], \Bigg[\Bigg(\frac{\gamma_1^{n+2} - \gamma_2^{n+2}}{\gamma_1 - \gamma_2} \\ -\gamma \frac{\gamma_1^{n} - \gamma_2^{n}}{\gamma_1 - \gamma_2} \Bigg), \left(\frac{\gamma_1^{n+2} - \gamma_2^{n+2}}{\gamma_1 - \gamma_2} + \gamma \frac{\gamma_1^{n+1} - \gamma_2^{n+1}}{\gamma_1 - \gamma_2} \right) \Bigg] \Bigg) \end{aligned}$$

where $\gamma_1 = \frac{1+\sqrt{5}}{2}$ and $\gamma_2 = \frac{1-\sqrt{5}}{2}$ are roots of the characteristic equation of the Fibonacci numbers, $\alpha, \beta, \gamma \in [0,1]$ and $0 \leq \alpha + \beta + \gamma \leq 3$.

Proof: The proof by induction is applied on n. Similarly, Binet's formula for Neutrosophic Lucas numbers is obtained as

$$\begin{aligned} L_n^{\alpha,\beta,\gamma} = \big(\big[\big(\gamma_1^{n+1} & + \gamma_2^{n+1} + \alpha \left(\gamma_1^{n} + \gamma_2^{n} \right) \big), \big(\gamma_1^{n+3} + \gamma_2^{n+3} + \alpha \left(\gamma_1^{n+1} + \gamma_2^{n+1} \right) \big) \big], \big[\big(\gamma_1^{n+2} + \gamma_2^{n+2} \\ & -\beta \left(\gamma_1^{n} + \gamma_2^{n} \right) \big), \big(\gamma_1^{n+2} + \gamma_2^{n+2} + \beta \big) \left(\gamma_1^{n+1} + \gamma_2^{n+1} \right) \big], \big[\big(\gamma_1^{n+2} + \gamma_2^{n+2} - \gamma \left(\gamma_1^{n} + \gamma_2^{n} \right) \big), \big(\gamma_1^{n+2} \\ & + \gamma_2^{n+2} + \gamma \left(\gamma_1^{n+1} + \gamma_2^{n+1} \right) \big) \big] \big). \end{aligned}$$

Proposition 3.9. *$F_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci number. The generating function formula of the $F_n^{\alpha,\beta,\gamma}$ numbers is*

$$h(t) = \frac{([2+\alpha, 5-2\alpha], [3-\beta, 3+2\beta], [3-\gamma, 3+2\gamma]) + ([1+\alpha, 3-\alpha], [2-\beta, 2+\beta], [2-\gamma, 2+\gamma])t}{1-t-t^2}$$

where $\alpha, \beta, \gamma \in [0,1]$ and $0 \leq \alpha + \beta + \gamma \leq 3$.

Proof: Let $h(t)$ be generating function for the Neutrosophic Fibonacci numbers as $h(t) = \sum_{n=1}^{\infty} F_n^{\alpha,\beta,\gamma} t^n$. Using $h(t)$, $-t(h)t$ and $-t^2 h(t)$ we get the following equations, $-th(t) = \sum_{n=1}^{\infty} F_n^{\alpha,\beta,\gamma} t^{n+1}$ and $-t^2 h(t) = -\sum_{n=1}^{\infty} F_n^{\alpha,\beta,\gamma} t^{n+2}$. After the needed calculations, the generating function for the Intuitionistic Fuzzy Fibonacci numbers is obtained as

$$h(t) = \frac{F_2^{\alpha,\beta,\gamma} + \left(F_3^{\alpha,\beta,\gamma} - F_2^{\alpha,\beta,\gamma} \right) t}{1-t-t^2}$$

$$h(t) = \frac{([2+\alpha, 5-2\alpha], [3-\beta, 3+2\beta], [3-\gamma, 3+2\gamma]) + ([1+\alpha, 3-\alpha], [2-\beta, 2+\beta], [2-\gamma, 2+\gamma])t}{1-t-t^2}.$$

.

The proof is completed. Similarly, generation function formula for Neutrosophic Lucas numbers is obtained as

$$m(t) = \frac{([4+3\alpha, 11-4\alpha], [7-3\beta, 7+4\beta], [7-3\gamma, 7+4\gamma]) + ([3+\alpha, 7-3\alpha], [4-\beta, 4+3\beta], [4-\gamma, 4+3\gamma])t}{1-t-t^2}$$

.

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4 Some Properties of Neutrosophic Fibonacci and Lucas Numbers

In this section, we generalize some well-known identities of Fibonacci and Lucas numbers to Neutrosophic Fibonacci and Lucas numbers. We show that common identities and equalities in number theory and Koshy's book[15] are hold for Neutrosophic and Lucas Fibonacci Numbers. We notice that although some of the properties of formulas remain the same, the others change.

Proposition 4.1. *$F_n^{\alpha,\beta,\gamma}$ and $L_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci and Lucas number such that $n \geq 1$ integer, respectively. Then, the following equalities hold:*

(a) $F_{n+2}^{\alpha,\beta,\gamma} - F_{n-2}^{\alpha,\beta,\gamma} = L_n^{\alpha,\beta,\gamma}$

(b) $F_{n+1}^{\alpha,\beta,\gamma} + F_{n-1}^{\alpha,\beta,\gamma} = L_n^{\alpha,\beta,\gamma}$

(c) $L_{n+1}^{\alpha,\beta,\gamma} + L_{n-1}^{\alpha,\beta,\gamma} = 5F_n^{\alpha,\beta,\gamma}$

(d) $L_{n+2}^{\alpha,\beta,\gamma} - L_{n-2}^{\alpha,\beta,\gamma} = 5F_n^{\alpha,\beta,\gamma}$

(e) $F_n^{\alpha,\beta,\gamma} + L_n^{\alpha,\beta,\gamma} = 2F_{n+1}^{\alpha,\beta,\gamma}$

Proof:

(a) $F_{n+2}^{\alpha,\beta,\gamma} - F_{n-2}^{\alpha,\beta,\gamma} = ([F_{n+3} + \alpha F_{n+2}, F_{n+5} - \alpha F_{n+3}], [F_{n+4} - \beta F_{n+2}, F_{n+4} + \beta F_{n+3}], [F_{n+4} - \gamma F_{n+2}, F_{n+4} + \gamma F_{n+3}]) - ([F_{n-1} + \alpha F_{n-2}, F_{n+1} - \alpha F_{n-1}], [F_n - \beta F_{n-2}, F_n + \beta F_{n-1}], [F_n - \gamma F_{n-2}, F_n + \gamma F_{n-1}])$. Since $F_{n+2} - F_{n-2} = L_n$,[15] then we get together with Definition 3.1 $F_{n+2}^{\alpha,\beta,\gamma} - F_{n-2}^{\alpha,\beta,\gamma} = L_n^{\alpha,\beta,\gamma}$.

Similarly, the other proofs can be obtained easily.

Proposition 4.2. *(**Ruggles's Identity**) $F_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci number such that $m, n \geq$ 1 integer. Then, the following equality holds*

$$\begin{aligned}
&F_{m-1}^{\alpha,\beta,\gamma} F_{n-m}^{\alpha,\beta,\gamma} + F_m^{\alpha,\beta,\gamma} F_{n-m+1}^{\alpha,\beta,\gamma} \cong (F_n, F_{n+2}, F_{n+4}) \\
&= ([F_n + \alpha F_{n+1}, F_{n+4} - \alpha F_{n+3}], [F_{n+2} - \beta F_n, F_{n+2} + \beta F_{n+4}], [F_{n+2} - \gamma F_n, F_{n+2} + \gamma F_{n+4}])
\end{aligned}$$

Proof: Let's prove the truth-membership value first.

$$\begin{aligned}
F_m^\alpha F_{n-m+1}^\alpha + F_{m-1}^\alpha F_{n-m}^\alpha \\
&= [F_{m+1} + \alpha F_m, F_{m+3} - \alpha F_{m+1}] [F_{n-m+2} + \alpha F_{n-m+1}, F_{n-m+4} - \alpha F_{n-m+2}] \\
&+ [F_m + \alpha F_{m-1}, F_{m+2} - \alpha F_m] [F_{n-m+1} + \alpha F_{n-m}, F_{n-m+3} - \alpha F_{n-m+1}] \\
&= [F_{m+1} F_{n-m+2} + F_m F_{n-m+1} + \alpha (F_{m+1} F_{n-m+1} + F_m F_{n-m} + F_m F_{n-m+2} + F_{m-1} F_{n-m+1}) \\
&+ \alpha^2 (F_m F_{n-m+1} + F_{m-1} F_{n-m}), F_{m+3} F_{n-m+4} + F_{m+2} F_{n-m+3} \\
&- \alpha (F_{m+3} F_{n-m+2} + F_{m+1} F_{n-m+4} + F_{m+2} F_{n-m+1} + F_m F_{n-m+3}) \\
&+ \alpha^2 (F_{m+1} F_{n-m+2} + F_m F_{n-m+1})]
\end{aligned}$$

Since $F_{m-1} F_n + F_m F_{n+1} = F_{n+m}$,[15] then we have that

$$F_m^\alpha F_{n-m+1}^\alpha + F_{m-1}^\alpha F_{n-m}^\alpha = \left[F_n + 2\alpha F_{n-1} + \alpha^2 F_{n-2}, F_{n+4} - 2\alpha F_{n+2} + \alpha^2 F_n\right].$$

We put $\alpha = 0$ and $\alpha = 1$ in order to convert this Neutrosophic number to α-cut interval. Then we get the truth-membership value

$$F_m^\alpha F_{n-m+1}^\alpha + F_{m-1}^{\alpha,} F_{n-m}^\alpha \cong [F_n + \alpha F_{n+1}, F_{n+4} - \alpha F_{n+3}].$$

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Similarly, we find the indeterminacy-membership value

$$F_m^{\beta}F_{n-m+1}^{\beta}+F_{m-1}^{\beta}F_{n-m}^{\beta}$$
$$=\left[(1-\beta)^2F_{n+4}+2(1-\beta)\beta F_{n+3}+\beta^2F_{n+2},(1-\beta)^2F_{n+4}+2(1-\beta)\beta F_{n+5}+\beta^2F_{n+6}\right].$$

We put $\beta=0$ and $\beta=1$ in order to convert this Neutrosophic number to β-cut interval. Then we get the indeterminacy-membership value

$$F_m^{\beta}F_{n-m+1}^{\beta}+F_{m-1}^{\beta}F_{n-m}^{\beta}\cong\left[F_{n+2}-\beta F_n,F_{n+2}+\beta F_{n+4}\right].$$

Finally, we find the falsity-membership value

$$F_m^{\gamma}F_{n-m+1}^{\gamma}+F_{m-1}^{\gamma}F_{n-m}^{\gamma}$$
$$=\left[(1-\gamma)^2F_{n+4}+2(1-\gamma)\gamma F_{n+3}+\gamma^2F_{n+2},(1-\gamma)^2F_{n+4}+2(1-\gamma)\gamma F_{n+5}+\gamma^2F_{n+6}\right].$$

We put $\gamma=0$ and $\gamma=1$ in order to convert this Neutrosophic number to γ-cut interval. Then we get the falsity-membership value

$$F_m^{\gamma}F_{n-m+1}^{\gamma}+F_{m-1}^{\gamma}F_{n-m}^{\gamma}\cong\left[F_{n+2}-\gamma F_n,F_{n+2}+\gamma F_{n+4}\right].$$

The same identity is obtained as

$$L_{m-1}^{\alpha,\beta,\gamma}L_{n-m}^{\alpha,\beta,\gamma}+L_m^{\alpha,\beta,\gamma}L_{n-m+1}^{\alpha,\beta,\gamma}\cong 5\left(F_n,F_{n+2},F_{n+4}\right)$$
$$=5\left(\left[F_n+\alpha F_{n+1},F_{n+4}-\alpha F_{n+3}\right],\left[F_{n+2}-\beta F_n,F_{n+2}+\beta F_{n+4}\right],\left[F_{n+2}-\gamma F_n,F_{n+2}+\gamma F_{n+4}\right]\right)$$

for Neutrosophic Lucas number.

Proposition 4.3. ***(Honsberger's Identity*** *) $F_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci number such that $m,n\geq 1$ integer. Then, the following equality holds*

$$F_{m-1}^{\alpha,\beta,\gamma}F_n^{\alpha,\beta,\gamma}+F_m^{\alpha,\beta,\gamma}F_{n+1}^{\alpha,\beta,\gamma}\cong\left(F_{n+m},F_{n+m+2},F_{n+m+4}\right)$$
$$=\left(\left[F_{n+m}+\alpha F_{n+m+1},F_{n+m+4}-\alpha F_{n+m+3}\right],\left[F_{n+m+2}-\beta F_{n+m},F_{n+m+2}+\beta F_{n+m+4}\right],\left[F_{n+m+2}\right.\right.$$
$$\left.\left.-\gamma F_{n+m},F_{n+m+2}+\gamma F_{n+m+4}\right]\right).$$

Proof: Honsberger's identity is obtained by taking $n=n+m$ in the Ruggles's identity. The same identity is obtained as

$$L_{m-1}^{\alpha,\beta,\gamma}L_n^{\alpha,\beta,\gamma}+L_m^{\alpha,\beta,\gamma}L_{n+1}^{\alpha,\beta,\gamma}\cong 5\left(F_{n+m},F_{n+m+2},F_{n+m+4}\right)$$
$$=5\left(\left[F_{n+m}+\alpha F_{n+m+1},F_{n+m+4}-\alpha F_{n+m+3}\right],\left[F_{n+m+2}-\beta F_{n+m},F_{n+m+2}\right.\right.$$
$$\left.\left.+\beta F_{n+m+4}\right],\left[F_{n+m+2}-\gamma F_{n+m},F_{n+m+2}+\gamma F_{n+m+4}\right]\right)$$

for Neutrosophic Lucas number.

Proposition 4.4. *$F_n^{\alpha,\beta,\gamma}$ and $L_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci and Lucas number such that $n\geq 1$ integer, respectively. Then, the following equalities hold:*

(a) $\left(F_n^{\alpha,\beta,\gamma}\right)^2+\left(F_{n+1}^{\alpha,\beta,\gamma}\right)^2\cong F_{2n+1}^{\alpha,\beta,\gamma}$

(b) $\left(L_n^{\alpha,\beta,\gamma}\right)^2+\left(L_{n+1}^{\alpha,\beta,\gamma}\right)^2\cong 5F_{2n+1}^{\alpha,\beta,\gamma}$

(c) $\left(F_{n+1}^{\alpha,\beta,\gamma}\right)^2-\left(F_{n-1}^{\alpha,\beta,\gamma}\right)^2\cong F_{2n}^{\alpha,\beta,\gamma}$

(d) $\left(L_{n+1}^{\alpha,\beta,\gamma}\right)^2-\left(L_{n-1}^{\alpha,\beta,\gamma}\right)^2\cong 5F_{2n}^{\alpha,\beta,\gamma}$

(e) $F_n^{\alpha,\beta,\gamma}L_n^{\alpha,\beta,\gamma}\cong F_{2n}^{\alpha,\beta,\gamma}$

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Proof (a): Let's prove the truth-membership value first.

$$\begin{aligned}(F_n^{\alpha})^2 &= [F_{n+1}+\ \alpha F_n, F_{n+3}-\alpha F_{n+1}]\,[F_{n+1}+\alpha F_n, F_{n+3}-\alpha F_{n+1}]\\ &= \left[F_{n+1}^2+2\alpha F_{n+1}F_n+\alpha^2 F_n^2, F_{n+3}^2-2\alpha F_{n+3}F_{n+1}+\alpha^2 F_{n+1}^2\right]\end{aligned}$$

We put $\alpha = 0$ and $\alpha = 1$ in order to convert this Neutrosophic number to α-cut interval. Then we get the truth-membership value and

$$(F_n^{\alpha})^2 \cong \left(F_{n+1}^2, F_{n+2}^2, F_{n+3}^2\right) = F_{n+2}^2$$

$$\left(F_{n+1}^{\alpha}\right)^2 \cong \left(F_{n+2}^2, F_{n+3}^2, F_{n+4}^2\right) = F_{n+3}^2$$

Since $F_n^2 + F_{n+1}^2 = F_{2n+1}$,[15] then we get that

$$\left(F_n^{\alpha,\beta,\gamma}\right)^2 + \left(F_{n+1}^{\alpha,\beta,\gamma}\right)^2 \cong F_{2n+1}^{\alpha,\beta,\gamma}.$$

Similarly, we find the indeterminacy-membership and falsity-membership value. The other steps of proofs are easily obtained with the same way.

Proposition 4.5. *$F_n^{\alpha,\beta,\gamma}$ and $L_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci and Lucas number such that $n \geq 1$ integer, respectively. Then, the following equalities hold:*

(a) $\sum_{i=1}^{n} F_i^{\alpha,\beta,\gamma} = F_{n+2}^{\alpha,\beta,\gamma} - F_2^{\alpha,\beta,\gamma}$

(b) $\sum_{i=1}^{n} F_{2i}^{\alpha,\beta,\gamma} = F_{2n+1}^{\alpha,\beta,\gamma} - F_1^{\alpha,\beta,\gamma}$

(c) $\sum_{i=2}^{n} F_{2i-1}^{\alpha,\beta,\gamma} = F_{2n}^{\alpha,\beta,\gamma} - F_1^{\alpha,\beta,\gamma}$

(d) $\sum_{i=1}^{n} \left(F_i^{\alpha,\beta,\gamma}\right)^2 \cong F_n^{\alpha,\beta,\gamma} F_{n+1}^{\alpha,\beta,\gamma}$

(e) $\sum_{i=1}^{n} L_i^{\alpha,\beta,\gamma} = L_{n+2}^{\alpha,\beta,\gamma} - L_2^{\alpha,\beta,\gamma}$

(f) $\sum_{i=1}^{n} L_{2i}^{\alpha,\beta,\gamma} = L_{2n+1}^{\alpha,\beta,\gamma} - L_1^{\alpha,\beta,\gamma}$

(g) $\sum_{i=2}^{n} L_{2i-1}^{\alpha,\beta,\gamma} = L_{2n}^{\alpha,\beta,\gamma} - L_2^{\alpha,\beta,\gamma}$

(h) $\sum_{i=1}^{n} \left(L_i^{\alpha,\beta,\gamma}\right)^2 \cong L_n^{\alpha,\beta,\gamma} L_{n+1}^{\alpha,\beta,\gamma} - L_2^{\alpha,\beta,\gamma} + L_1^{\alpha,\beta,\gamma}$

Proof (a): Let's prove the truth-membership value first.

$$\begin{aligned}\sum_{i=1}^{n} F_i^{\alpha} &= \left[\sum_{i=2}^{n+1} F_i + \alpha \sum_{i=1}^{n} F_i, \sum_{i=4}^{n+3} F_i - \alpha \sum_{i=2}^{n+1} F_i\right]\\ &= \left[(F_{n+3}-F_2-F_1)+\alpha\left(F_{n+2}-F_2\right), (F_{n+5}-F_5)-\alpha\left(F_{n+3}-F_2-F_1\right)\right]\\ &= [F_{n+3}+\alpha F_{n+2}, F_{n+5}-\alpha F_{n+3}] - [F_3+\alpha F_2, F_5-\alpha F_3] = F_{n+2}^{\alpha} - F_2^{\alpha}.\end{aligned}$$

Similarly, we find the indeterminacy-membership and falsity-membership value. The other steps of proof are easily obtained by following the same way.

Remark 4.6. Some special sequences well-known for the Fibonacci and Lucas sequences have also been calculated for the neutrosophic Fibonacci and Lucas numbers. The proofs of these equations are omitted. $F_n^{\alpha,\beta,\gamma}$ and $L_n^{\alpha,\beta,\gamma}$ be nth Neutrosophic Fibonacci and Lucas number such that $n \geq 1$ integer, respectively. Then, the following equalities hold:

(a) ***Tagiuri's Identity*** :

$$F_{m+k}^{\alpha,\beta,\gamma} F_{n-k}^{\alpha,\beta,\gamma} - F_m^{\alpha,\beta,\gamma} F_n^{\alpha,\beta,\gamma} \cong (-1)^{n-k}\left(-F_k F_{m-n+k}, 0, F_k F_{m-n+k}\right)$$

$$L_{m+k}^{\alpha,\beta,\gamma} L_{n-k}^{\alpha,\beta,\gamma} - L_m^{\alpha,\beta,\gamma} L_n^{\alpha,\beta,\gamma} \cong 5(-1)^{n-k+1}\left(-F_k F_{m-n+k}, 0, F_k F_{m-n+k}\right)$$

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(b) ***d'Ocagne's Identity*** :

$$F_{m+1}^{\alpha,\beta,\gamma}F_{n-1}^{\alpha,\beta,\gamma} - F_{m}^{\alpha,\beta,\gamma}F_{n}^{\alpha,\beta,\gamma} \cong (-1)^{n-1}\left(-F_{m-n+1},0,F_{m-n+1}\right)$$
$$L_{m+1}^{\alpha,\beta,\gamma}L_{n-1}^{\alpha,\beta,\gamma} - L_{m}^{\alpha,\beta,\gamma}L_{n}^{\alpha,\beta,\gamma} \cong 5(-1)^{n} - \left(F_{m-n+1},0,F_{m-n+1}\right)$$

(c) ***Catalan's Identity*** :

$$F_{n+k}^{\alpha,\beta,\gamma}F_{n-k}^{\alpha,\beta,\gamma} - F_{n}^{\alpha,\beta,\gamma}F_{n}^{\alpha,\beta,\gamma} \cong (-1)^{n-k}\left(-F_{k}^{2},0,F_{k}^{2}\right)$$
$$L_{n+k}^{\alpha,\beta,\gamma}L_{n-k}^{\alpha,\beta,\gamma} - L_{n}^{\alpha,\beta,\gamma}L_{n}^{\alpha,\beta,\gamma} \cong 5(-1)^{n-k+1}\left(-F_{k}^{2},0,F_{k}^{2}\right)$$

(d) ***Cassini's Identity*** :

$$F_{n+1}^{\alpha,\beta,\gamma}F_{n-1}^{\alpha,\beta,\gamma} - F_{n}^{\alpha,\beta,\gamma}F_{n}^{\alpha,\beta,\gamma} \cong (-1)^{n-1}(-1,0,1)$$
$$L_{n+1}^{\alpha,\beta,\gamma}L_{n-1}^{\alpha,\beta,\gamma} - L_{n}^{\alpha,\beta,\gamma}L_{n}^{\alpha,\beta,\gamma} \cong 5(-1)^{n}(-1,0,1).$$

5 Other Neutrosophic Number Sequences

In this section, we focus to define numbers and their sequences of Neutrosophic and Neutrosophic-Lucas versions, and the recurrence relations with various initial conditions in the second book of Koshy.[18] Sequences of neutrosophic numbers whose characteristic equation of the second order can be defined within number sequences such as Pell,[18] Pell-Lucas,[18] Jacobsthal,[19] Jacobsthal-Lucas,[19] Mersenne,[20] Mersenne-Lucas,[21] Balancing,[22] and Lucas-Balancing.[22] Definitions of these neutrosophic number sequences are given below.

Definition 5.1. ***Neutrosophic Pell numbers*** are defined as follows:

$P_0^{\alpha,\beta,\gamma} = ([\alpha, 2-\alpha],[1-\beta,1+\beta],[1-\gamma,1+\gamma]);\quad P_1^{\alpha,\beta,\gamma} = ([1+\alpha,5-3\alpha],[2-\beta,2+3\beta],[2-\gamma,2+3\gamma]); P_2^{\alpha,\beta,\gamma} = ([2+3\alpha,12-7\alpha],[5-3\beta,5+7\beta],[5-3\gamma,5+7\gamma]);\ldots.$

Remark 5.2. Neutrosophic Pell numbers satisfy the recurrence relation with initial conditions $P_0^{\alpha,\beta,\gamma}$ and $P_1^{\alpha,\beta,\gamma}$. That is $P_n^{\alpha,\beta,\gamma} = 2P_{n-1}^{\alpha,\beta,\gamma} + P_{n-\lambda,\gamma}^{\alpha,\beta}, n \geq 2.$

Definition 5.3. ***Neutrosophic Pell-Lucas numbers*** are defined as follows:

$PL_0^{\alpha,\beta,\gamma} = ([2,6-4\alpha],[2,2+4\beta],[2,2+4\gamma]);\quad PL_1^{\alpha,\beta,\gamma} = ([2+4\alpha,14-8\alpha],[6-4\beta,6+8\beta],[6-4\gamma,6+8\gamma]);\quad PL_2^{\alpha,\beta,\gamma} = ([6+8\alpha,34-20\alpha],[14-8\beta,14+20\beta],[14-8\gamma,14+20\gamma]);.$

Remark 5.4. Neutrosophic Pell-Lucas numbers satisfy the recurrence relation with initial conditions $PL_0^{\alpha,\beta,\gamma}$ and $PL_1^{\alpha,\beta,\gamma}$. That is $PL_n^{\alpha,\beta,\gamma} = 2PL_{n-1}^{\alpha,\beta,\gamma} + PL_{n-2}^{\alpha,\beta,\gamma}, n \geq 2.$

Definition 5.5. ***Neutrosophic Jacobsthal numbers*** are defined as follows:

$J_0^{\alpha,\beta,\gamma} = ([\alpha,1],[1-\beta,1],[1-\gamma,1]);\quad J_1^{\alpha,\beta,\gamma} = ([1,3-2\alpha],[1,1+2\beta],[1,1+2\gamma]);\quad J_2^{\alpha,\beta,\gamma} = ([1+2\alpha,5-2\alpha],[3-2\beta,3+2\beta],[3-2\gamma,3+2\gamma]);$

.

Remark 5.6. Neutrosophic Jacobsthal numbers satisfy the recurrence relation with initial conditions $J_0^{\alpha,\beta,\gamma}$ and $J_1^{\alpha,\beta,\gamma}$. That is $J_n^{\alpha,\beta,\gamma} = J_{n-1}^{\alpha,\beta,\gamma} + 2J_{n-2}^{\alpha,\beta,\gamma}, n \geq 2.$

Definition 5.7. ***Neutrosophic Jacobsthal-Lucas numbers*** are defined as follows:

$JL_0^{\alpha,\beta,\gamma} = ([1+4\alpha,7-2\alpha],[5-4\beta,5+2\beta],[5-4\gamma,5+2\gamma]);\quad JL_1^{\alpha,\beta,\gamma} = ([5+2\alpha,17-10\alpha],[7-2\beta,7+10\beta],[7-2\gamma,7+10\gamma]); JL_2^{\alpha,\beta,\gamma} = ([7+10\alpha,31-14\alpha],[17-10\beta,17+14\beta],[17-10\gamma,17+14\gamma]);\ldots$

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Remark 5.8. Neutrosophic Jacobsthal-Lucas numbers satisfy the recurrence relation with initial conditions $JL_0^{\alpha,\beta,\gamma}$ and $JL_1^{\alpha,\beta,\gamma}$. That is $JL_n^{\alpha,\beta,\gamma} = JL_{n-1}^{\alpha,\beta,\gamma} + 2JL_{n-2}^{\alpha,\beta,}, n \geq 2$.

Definition 5.9. ***Neutrosophic Balancing numbers*** are defined as follows:

$B_0^{\alpha,\beta,\gamma} = ([\alpha, 6-5\alpha], [1-\beta, 1+5\beta], [1-\gamma, 1+5\gamma]); B_1^{\alpha,\beta,\gamma} = ([1+5\alpha, 35-29\alpha], [6-5\beta, 6+29\beta], [6-5\gamma, 6+29\gamma]);\quad B_2^{\alpha,\beta,\gamma} = ([6+29\alpha, 204-169\alpha], [35-29\beta, 35+169\beta], [35-29\gamma, 35+169\gamma]); \ldots$

Remark 5.10. Neutrosophic Balancing numbers satisfy the recurrence relation with initial conditions $B_0^{\alpha,\beta,\gamma}$ and $B_1^{\alpha,\beta,\gamma}$. That is $B_n^{\alpha,\beta,\gamma} = 6B_{n-1}^{\alpha,\beta,\gamma} - B_{n-2}^{\alpha,\beta,\gamma}, n \geq 2$.

Definition 5.11. ***Neutrosophic Lucas-Balancing numbers*** are defined as follows:

$BL_0^{\alpha,\beta,\gamma} = ([1+2\alpha, 17-14\alpha], [3-2\beta, 3+14\beta], [3-2\gamma, 3+14\gamma]);\quad BL_1^{\alpha,\beta,\gamma} = ([3+14\alpha, 99-82\alpha], [17-14\beta, 17+82\beta], [17-14\gamma, 17+82\gamma]);\quad BL_2^{\alpha,\beta,\gamma} = ([17+82\alpha, 577-478\alpha], [99-82\beta, 99+478\beta], [99-82\gamma, 99+478\gamma]); \ldots$

Remark 5.12. Neutrosophic Lucas-Balancing numbers satisfy the recurrence relation with initial conditions $BL_0^{\alpha,\beta,\gamma}$ and $BL_1^{\alpha,\beta,\gamma}$. That is $BL_n^{\alpha,\beta,\gamma} = 6BL_{n-1}^{\alpha,\beta,\gamma} - BL_{n-2}^{\alpha,\beta,\gamma}, n \geq 2$.

Definition 5.13. ***Neutrosophic Mersenne numbers*** are defined as follows:

$M_0^{\alpha,\beta,\gamma} = ([\alpha, 3-2\alpha], [1-\beta, 1+2\beta], [1-\gamma, 1+2\gamma]);\quad M_1^{\alpha,\beta,\gamma} = ([1+2\alpha, 7-4\alpha], [3-2\beta, 3+4\beta], [3-2\gamma, 3+4\gamma]); M_2^{\alpha,\beta,\gamma} = ([3+4\alpha, 15-8\alpha], [7-4\beta, 7+8\beta], [7-4\gamma, 7+8\gamma]); \ldots$

Remark 5.14. Neutrosophic Mersenne numbers satisfy the recurrence relation with initial conditions $M_0^{\alpha,\beta,\gamma}$ and $M_1^{\alpha,\beta,\gamma}$. That is $M_n^{\alpha,\beta,\gamma} = 3M_{n-1}^{\alpha,\beta} - 2M_{n-2}^{\alpha,\beta}, n \geq 2$.

Definition 5.15. ***Neutrosophic Mersenne-Lucas numbers*** are defined as follows:

$ML_0^{\alpha,\beta,\gamma} = ([2+\alpha, 5-2\alpha], [3-\beta, 3+2\beta], [3-\gamma, 3+2\gamma]);\quad ML_1^{\alpha,\beta,\gamma} = ([3+2\alpha, 9-4\alpha], [5-2\beta, 5+4\beta], [5-2\gamma, 5+4\gamma]); ML_2^{\alpha,\beta,\gamma} = ([5+4\alpha, 17-8\alpha], [9-4\beta, 9+8\beta], [9-4\gamma, 9+8\gamma]); \ldots$
.

Remark 5.16. Neutrosophic Mersenne-Lucas numbers satisfy the recurrence relation with initial conditions $ML_0^{\alpha,\beta,\gamma}$ and $ML_1^{\alpha,\beta,\gamma}$. That is $ML_n^{\alpha,\beta,\gamma} = 3ML_{n-1}^{\alpha,\beta} - 2ML_{n-2}^{\alpha,\beta}, n \geq 2$.

6 A Generalization of Neutrosophic Number Sequences via Horadam

In this section, we introduce a generalization approach of Horadam[23] interpretation version of Neutrosophic number sequences. The well-known sequences, A and $B \in \mathbb{Q}$, defined by the second-order linear difference equation in the form of $H_n = AH_{n-1} + BH_{n-2}$, can be grouped according to their initial conditions as follows:

$H_n = AH_{n-1} + BH_{n-2}$ and $H_0 = 0, H_1 = 1$,

$H_n = AH_{n-1} + BH_{n-2}$ and $H_0 = 2, H_1 = A$,

$H_n = AH_{n-1} + BH_{n-2}$ and $H_0 = 1, H_1 = \frac{A}{2}$,

$H_n = AH_{n-1} + BH_{n-2}$ and $H_0 = 0, H_1 = \frac{A}{2}$.

Definition 6.1. Let $\{H_n\}$ be Horadam sequence. For $n \geq 0$ integer, *fuzzy number* sequences via Horadam are defined as

$H_n^{\alpha,\beta,\gamma} = (H_{n-1}, H_n, H_{n+1}) = [H_{n-1} + \alpha(H_n - H_{n-1}), H_{n+1} - \alpha(H_{n+1} - H_n)]$

where $\alpha \in [0, 1]$.

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Table 3: Some well-known number sequences via Horadam Interpretation

A	B	H_n	Sequence Name	Initial Conditions H_0 H_1
1	1	$H_{n-1}+H_{n-2}$	Fibonacci	0 1
1	1	$H_{n-1}+H_{n-2}$	Lucas	2 1
2	1	$2H_{n-1}+H_{n-2}$	Pell	0 1
2	1	$2H_{n-1}+H_{n-2}$	Pell-Lucas	2 2
1	2	$H_{n-1}+2H_{n-2}$	Jacobsthal	0 1
1	2	$H_{n-1}+2H_{n-2}$	Jacobsthal-Lucas	2 1
3	-2	$3H_{n-1}-2H_{n-2}$	Mersenne	0 1
3	-2	$3H_{n-1}-2H_{n-2}$	Mersenne-Lucas	2 3
6	-1	$6H_{n-1}-H_{n-2}$	Balancing	0 1
6	-1	$6H_{n-1}-H_{n-2}$	Lucas-Balancing	1 3
2	1	$2H_{n-1}+H_{n-2}$	Modified-Lucas	1 1
p	q	$pH_{n-1}+qH_{n-2}$	Horadam	r s

Definition 6.2. Let $\{H_n\}$ be Horadam sequence. For $n \geq 0$ integer, *intuitionistic fuzzy* number sequences via Horadam are defined as

$H_n^{\alpha,\beta,\gamma} = (H_{n-1}, H_n, H_{n+1}) = ([H_{n-1} + \alpha(H_n - H_{n-1}), H_{n+1} - \alpha(H_{n+1} - H_n)], [H_n - \gamma(H_n - H_{n-1}), H_n + \gamma(H_{n+1} - H_n)])$ where $\alpha, \gamma \in [0, 1]$ and $0 \leq \alpha + \gamma \leq 1$.

Definition 6.3. Let $\{H_n\}$ be Horadam sequence. For $n \geq 0$ integer, neutrosophic number sequences via Horadam are defined as

$H_n^{\alpha,\beta,\gamma} = (H_{n-1}, H_n, H_{n+1}) = ([H_{n-1} + \alpha(H_n - H_{n-1}), H_{n+1} - \alpha(H_{n+1} - H_n)], [H_n - \beta(H_n - H_{n-1}), H_n + \beta(H_{n+1} - H_n)], [H_n - \gamma(H_n - H_{n-1}), H_n + \gamma(H_{n+1} - H_n)])$ where $\alpha, \beta, \gamma \in [0, 1]$ and $0 \leq \alpha + \beta + \gamma \leq 3$.

7 Applications

In this paper, we have studied interval valued neutrosophic sequence versions of many number sequences. Due to the usefulness and nature of number sequences, we have provided many alternatives for application to real-life problems. On the other hand, since each of the series we studied can correspond to a moment or situation, it can be easily applied to many fields such as forecasting, statistical methods, decision-making systems, mathematical economics. On the other hand, in terms of core mathematical research, many applications and features can be easily studied in areas such as mathematical analysis, matrix theory, theory of functions, graph theory, and sequence spaces on number sequences. In summary, we believe that this study opens a door for novel work, both theoretically and in terms of its application to real life problems.

8 Conclusions

In this paper, we have given a comprehensive introductory study of Neutrosophic number sequences as a guide, by exemplifying tables (in Appendix A) and studying basic number sequence properties. As a future direction, we plan to study other identities and properties of number sequences by increasing the diversity of these number sequences. Researchers, who are far from neutrosophic studies, might think that the scope of this paper is limited to neutrosophic at first glance, but since Neutrosophy is a nice fuzzy generalization, we offer a field of study for all fuzzy varieties and will continue to work with picture fuzzy, type fuzzy sets (like type-2 etc.) and extensions.

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9 Appendix A

In Appendix A, in order to prevent the reader from drowning in the formulas, the calculated formulas for all the number sequences mentioned in the paper are given as tables.

Table 4: The first 6 elements of Neutrosophic Lucas Sequence for α, β, and γ cuts.

α	β	γ	L_0	L_1	L_2	L_3	L_4	L_5	L_0	L_1	L_2	L_3	L_4	L_5
			Left Truth						Right Truth					
1	0	1	3	4	7	11	18	29	3	4	7	11	18	29
0.9	0.05	0.9	2.8	3.9	6.7	10.6	17.3	27.9	3.1	4.3	7.4	11.7	19.1	30.8
0.8	0.2	0	2.6	3.8	6.4	10.2	16.6	26.8	3.2	4.6	7.8	12.4	20.2	32.6
0.7	0.23	0.07	2.4	3.7	6.1	9.8	15.9	25.7	3.3	4.9	8.2	13.1	21.3	34.4
0.6	0.6	0	2.2	3.6	5.8	9.4	15.2	24.6	3.4	5.2	8.6	13.8	22.4	36.2
0.5	0.5	0.5	2	3.5	5.5	9	14.5	23.5	3.5	5.5	9	14.5	23.5	38
0.4	0.4	0.04	1.8	3.4	5.2	8.6	13.8	22.4	3.6	5.8	9.4	15.2	24.6	39.8
0.3	0.3	1	1.6	3.3	4.9	8.2	13.1	21.3	3.7	6.1	9.8	15.9	25.7	41.6
0.2	0.2	0.2	1.4	3.2	4.6	7.8	12.4	20.2	3.8	6.4	10.2	16.6	26.8	43.4
			Left Indeterminacy						Right Indeterminacy					
			3	4	7	11	18	29	3	4	7	11	18	29
			3	4	7	11	18	29	3.05	4.15	7.2	11.4	18.6	29.9
			2.9	3.95	6.85	10.8	17.7	28.5	3.2	4.6	7.8	12.4	20.2	32.6
			2.6	3.8	6.4	10.2	16.6	26.8	3.23	4.69	7.92	12.6	20.5	33.1
			2.54	3.77	6.31	10.1	16.4	26.5	3.6	5.8	9.4	15.2	24.6	39.8
			1.8	3.4	5.2	8.6	13.8	22.4	3.5	5.5	9	14.5	23.5	38
			2	3.5	5.5	9	14.5	23.5	3.4	5.2	8.6	13.8	22.4	36.2
			2.2	3.6	5.8	9.4	15.2	24.6	3.3	4.9	8.2	13.1	21.3	34.4
			2.4	3.7	6.1	9.8	15.9	25.7	3.2	4.6	7.8	12.4	20.2	32.6
			Left Falsity						Right Falsity					
			1	3	4	7	11	18	4	7	11	18	29	47
			1.2	3.1	4.3	7.4	11.7	19.1	3.9	6.7	10.6	17.3	27.9	45.2
			3	4	7	11	18	29	3	4	7	11	18	29
			2.86	3.93	6.79	10.7	17.5	28.2	3.07	4.21	7.28	11.5	18.8	30.3
			3	4	7	11	18	29	3	4	7	11	18	29
			2	3.5	5.5	9	14.5	23.5	3.5	5.5	9	14.5	23.5	38
			2.92	3.96	6.88	10.8	17.7	28.6	3.04	4.12	7.16	11.3	18.4	29.7
			1	3	4	7	11	18	4	7	11	18	29	47
			2.6	3.8	6.4	10.2	16.6	26.8	3.2	4.6	7.8	12.4	20.2	32.6

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Table 5: The first 6 elements of Neutrosophic Pell Sequence for α, β, and γ cuts.

α	β	γ	P_0	P_1	P_2	P_3	P_4	P_5	P_0	P_1	P_2	P_3	P_4	P_5
			Left Truth						Right Truth					
1	0	1	1	2	5	12	29	70	1	2	5	12	29	70
0.9	0.05	0.9	0.9	1.9	4.7	11.3	27.3	65.9	1.1	2.3	5.7	13.7	33.1	79.9
0.8	0.2	0	0.8	1.8	4.4	10.6	25.6	61.8	1.2	2.6	6.4	15.4	37.2	89.8
0.7	0.23	0.07	0.7	1.7	4.1	9.9	23.9	57.7	1.3	2.9	7.1	17.1	41.3	99.7
0.6	0.6	0	0.6	1.6	3.8	9.2	22.2	53.6	1.4	3.2	7.8	18.8	45.4	109.6
0.5	0.5	0.5	0.5	1.5	3.5	8.5	20.5	49.5	1.5	3.5	8.5	20.5	49.5	119.5
0.4	0.4	0.04	0.4	1.4	3.2	7.8	18.8	45.4	1.6	3.8	9.2	22.2	53.6	129.4
0.3	0.3	1	0.3	1.3	2.9	7.1	17.1	41.3	1.7	4.1	9.9	23.9	57.7	139.3
			Left Indeterminacy						Right Indeterminacy					
			1	2	5	12	29	70	1	2	5	12	29	70
			0.95	1.95	4.85	11.65	28.15	67.95	1.05	2.15	5.35	12.85	31.05	74.95
			0.8	1.8	4.4	10.6	25.6	61.8	1.2	2.6	6.4	15.4	37.2	89.8
			0.77	1.77	4.31	10.39	25.09	60.57	1.23	2.69	6.61	15.91	38.43	92.77
			0.4	1.4	3.2	7.8	18.8	45.4	1.6	3.8	9.2	22.2	53.6	129.4
			0.5	1.5	3.5	8.5	20.5	49.5	1.5	3.5	8.5	20.5	49.5	119.5
			0.6	1.6	3.8	9.2	22.2	53.6	1.4	3.2	7.8	18.8	45.4	109.6
			0.7	1.7	4.1	9.9	23.9	57.7	1.3	2.9	7.1	17.1	41.3	99.7
			Left Falsity						Right Falsity					
			0	1	2	5	12	29	2	5	12	29	70	169
			0.1	1.1	2.3	5.7	13.7	33.1	1.9	4.7	11.3	27.3	65.9	159.1
			1	2	5	12	29	70	1	2	5	12	29	70
			0.93	1.93	4.79	11.51	27.81	67.13	1.07	2.21	5.49	13.19	31.87	76.93
			1	2	5	12	29	70	1	2	5	12	29	70
			0.5	1.5	3.5	8.5	20.5	49.5	1.5	3.5	8.5	20.5	49.5	119.5
			0.96	1.96	4.88	11.72	28.32	68.36	1.04	2.12	5.28	12.68	30.64	73.96
			0	1	2	5	12	29	2	5	12	29	70	169

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Table 6: The first 6 elements of Neutrosophic Pell-Lucas Sequence for α, β, and γ cuts.

α	β	γ	PL_0	PL_1	PL_2	PL_3	PL_4	PL_5	PL_0	PL_1	PL_2	PL_3	PL_4	PL_5
			Left Truth						Right Truth					
1	0	1	2	6	14	34	82	198	2	6	14	34	82	198
0.9	0.05	0.9	2	5.6	13.2	32	77.2	186.4	2.4	6.8	16	38.8	93.6	226
0.8	0.2	0	2	5.2	12.4	30	72.4	174.8	2.8	7.6	18	43.6	105.2	254
0.7	0.23	0.07	2	4.8	11.6	28	67.6	163.2	3.2	8.4	20	48.4	116.8	282
0.6	0.6	0	2	4.4	10.8	26	62.8	151.6	3.6	9.2	22	53.2	128.4	310
0.5	0.5	0.5	2	4	10	24	58	140	4	10	24	58	140	338
0.4	0.4	0.04	2	3.6	9.2	22	53.2	128.4	4.4	10.8	26	62.8	151.6	366
0.3	0.3	1	2	3.2	8.4	20	48.4	116.8	4.8	11.6	28	67.6	163.2	394
			Left Indeterminacy						Right Indeterminacy					
			2	6	14	34	82	198	2	6	14	34	82	198
			2	5.8	13.6	33	79.6	192.2	2.2	6.4	15	36.4	87.8	212
			2	5.2	12.4	30	72.4	174.8	2.8	7.6	18	43.6	105.2	254
			2	5.08	12.16	29.4	70.96	171.3	2.92	7.84	18.6	45.04	108.7	262.4
			2	3.6	9.2	22	53.2	128.4	4.4	10.8	26	62.8	151.6	366
			2	4	10	24	58	140	4	10	24	58	140	338
			2	4.4	10.8	26	62.8	151.6	3.6	9.2	22	53.2	128.4	310
			2	4.8	11.6	28	67.6	163.2	3.2	8.4	20	48.4	116.8	282
			Left Falsity						Right Falsity					
			2	2	6	14	34	82	6	14	34	82	198	478
			2	2.4	6.8	16	38.8	93.6	5.6	13.2	32	77.2	186.4	450
			2	6	14	34	82	198	2	6	14	34	82	198
			2	5.72	13.44	32.6	78.64	189.9	2.28	6.56	15.4	37.36	90.12	217.6
			2	6	14	34	82	198	2	6	14	34	82	198
			2	4	10	24	58	140	4	10	24	58	140	338
			2	5.84	13.68	33.2	80.08	193.4	2.16	6.32	14.8	35.92	86.64	209.2
			2	2	6	14	34	82	6	14	34	82	198	478

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Table 7: The first 6 elements of Neutrosophic Jacobsthal Sequence for α, β, and γ cuts.

α	β	γ	J_0	J_1	J_2	J_3	J_4	J_5	J_0	J_1	J_2	J_3	J_4	J_5
			Left Truth						Right Truth					
1	0	1	1	1	3	5	11	21	1	1	3	5	11	21
0.9	0.05	0.9	0.9	1	2.8	4.8	10.4	20	1	1.2	3.2	5.6	12	23
0.8	0.2	0	0.8	1	2.6	4.6	9.8	19	1	1.4	3.4	6.2	13	25
0.7	0.23	0.07	0.7	1	2.4	4.4	9.2	18	1	1.6	3.6	6.8	14	28
0.6	0.6	0	0.6	1	2.2	4.2	8.6	17	1	1.8	3.8	7.4	15	30
0.5	0.5	0.5	0.5	1	2	4	8	16	1	2	4	8	16	32
0.4	0.4	0.04	0.4	1	1.8	3.8	7.4	15	1	2.2	4.2	8.6	17	34
0.3	0.3	1	0.3	1	1.6	3.6	6.8	14	1	2.4	4.4	9.2	18	36
			Left Indeterminacy						Right Indeterminacy					
			1	1	3	5	11	21	1	1	3	5	11	21
			0.95	1	2.9	4.9	10.7	20.5	1	1.1	3.1	5.3	11.5	22.1
			0.8	1	2.6	4.6	9.8	19	1	1.4	3.4	6.2	13	25.4
			0.77	1	2.54	4.54	9.62	18.7	1	1.46	3.46	6.38	13.3	26.1
			0.4	1	1.8	3.8	7.4	15	1	2.2	4.2	8.6	17	34.2
			0.5	1	2	4	8	16	1	2	4	8	16	32
			0.6	1	2.2	4.2	8.6	17	1	1.8	3.8	7.4	15	29.8
			0.7	1	2.4	4.4	9.2	18	1	1.6	3.6	6.8	14	27.6
			Left Falsity						Right Falsity					
			0	1	1	3	5	11	1	3	5	11	21	43
			0.1	1	1.2	3.2	5.6	12	1	2.8	4.8	10.4	20	41
			1	1	3	5	11	21	1	1	3	5	11	21
			0.93	1	2.86	4.86	10.58	20.3	1	1.14	3.14	5.42	11.7	23
			1	1	3	5	11	21	1	1	3	5	11	21
			0.5	1	2	4	8	16	1	2	4	8	16	32
			0.96	1	2.92	4.92	10.76	20.6	1	1.08	3.08	5.24	11.4	22
			0	1	1	3	5	11	1	3	5	11	21	43

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Table 8: The first 6 elements of Neutrosophic Jacobsthal-Lucas Sequence for α, β, and γ cuts.

α	β	γ	JL_0	JL_1	JL_2	JL_3	JL_4	JL_5	JL_0	JL_1	JL_2	JL_3	JL_4	JL_5
			Left Truth						Right Truth					
1	0	1	5	7	17	31	65	127	5	7	17	31	65	127
0.9	0.05	0.9	4.6	6.8	16	29.6	61.6	120.8	5.2	8	18.4	34.4	71.2	140
0.8	0.2	0	4.2	6.6	15	28.2	58.2	114.6	5.4	9	19.8	37.8	77.4	153
0.7	0.23	0.07	3.8	6.4	14	26.8	54.8	108.4	5.6	10	21.2	41.2	83.6	166
0.6	0.6	0	3.4	6.2	13	25.4	51.4	102.2	5.8	11	22.6	44.6	89.8	179
0.5	0.5	0.5	3	6	12	24	48	96	6	12	24	48	96	192
0.4	0.4	0.04	2.6	5.8	11	22.6	44.6	89.8	6.2	13	25.4	51.4	102.2	205
0.3	0.3	1	2.2	5.6	10	21.2	41.2	83.6	6.4	14	26.8	54.8	108.4	218
			Left Indeterminacy						Right Indeterminacy					
			5	7	17	31	65	127	5	7	17	31	65	127
			4.8	6.9	16.5	30.3	63.3	123.9	5.1	7.5	17.7	32.7	68.1	133.5
			4.2	6.6	15	28.2	58.2	114.6	5.4	9	19.8	37.8	77.4	153
			4.08	6.54	14.7	27.78	57.18	112.74	5.46	9.3	20.22	38.82	79.26	156.9
			2.6	5.8	11	22.6	44.6	89.8	6.2	13	25.4	51.4	102.2	205
			3	6	12	24	48	96	6	12	24	48	96	192
			3.4	6.2	13	25.4	51.4	102.2	5.8	11	22.6	44.6	89.8	179
			3.8	6.4	14	26.8	54.8	108.4	5.6	10	21.2	41.2	83.6	166
			Left Falsity						Right Falsity					
			1	5	7	17	31	65	7	17	31	65	127	257
			1.4	5.2	8	18.4	34.4	71.2	6.8	16	29.6	61.6	120.8	244
			5	7	17	31	65	127	5	7	17	31	65	127
			4.72	6.86	16.3	30	62.62	122.7	5.14	7.7	18	33.38	69.34	136.1
			5	7	17	31	65	127	5	7	17	31	65	127
			3	6	12	24	48	96	6	12	24	48	96	192
			4.84	6.92	16.6	30.4	63.64	124.5	5.08	7.4	17.6	32.36	67.48	132.2
			1	5	7	17	31	65	7	17	31	65	127	257

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Table 9: The first 6 elements of Neutrosophic Balancing Sequence for α, β, and γ cuts.

α	β	γ	B_0	B_1	B_2	B_3	B_4	B_5	B_0	B_1	B_2	B_3	B_4	B_5
			Left Truth						Right Truth					
1	0	1	1	6	35	204	1189	6930	1	6	35	204	1189	6930
0.9	0.05	0.9	0.9	5.5	32.1	187	1091	6356	1.5	8.9	51.9	302.5	1763	10276
0.8	0.2	0	0.8	5	29.2	170	992	5782	2	11.8	68.8	401	2337	13622
0.7	0.23	0.07	0.7	4.5	26.3	153	893.5	5208	2.5	14.7	85.7	499.5	2911	16968
0.6	0.6	0	0.6	4	23.4	136	795	4634	3	17.6	103	598	3485	20314
0.5	0.5	0.5	0.5	3.5	20.5	120	696.5	4060	3.5	20.5	120	696.5	4060	23661
0.4	0.4	0.04	0.4	3	17.6	103	598	3485	4	23.4	136	795	4634	27007
0.3	0.3	1	0.3	2.5	14.7	85.7	499.5	2911	4.5	26.3	153	893.5	5208	30353
			Left Indeterminacy						Right Indeterminacy					
			1	6	35	204	1189	6930	1	6	35	204	1189	6930
			0.95	5.75	33.55	195.6	1139.8	6643	1.25	7.45	43.45	253.25	1476.1	8603.05
			0.8	5	29.2	170.2	992	5781.8	2	11.8	68.8	401	2337.2	13622.2
			0.77	4.85	28.33	165.1	962.45	5609.6	2.15	12.67	73.87	430.55	2509.4	14626
			0.4	3	17.6	102.6	598	3485.4	4	23.4	136.4	795	4633.6	27006.6
			0.5	3.5	20.5	119.5	696.5	4059.5	3.5	20.5	119.5	696.5	4059.5	23660.5
			0.6	4	23.4	136.4	795	4633.6	3	17.6	102.6	598	3485.4	20314.4
			0.7	4.5	26.3	153.3	893.5	5207.7	2.5	14.7	85.7	499.5	2911.3	16968.3
			Left Falsity						Right Falsity					
			0	1	6	35	204	1189	6	35	204	1189	6930	40391
			0.1	1.5	8.9	51.9	302.5	1763	5.5	32.1	187	1091	6356	37045
			1	6	35	204	1189	6930	1	6	35	204	1189	6930
			0.93	5.65	32.97	192	1120	6528	1.35	8.03	46.8	273	1591	9272.3
			1	6	35	204	1189	6930	1	6	35	204	1189	6930
			0.5	3.5	20.5	120	696.5	4060	3.5	20.5	120	696.5	4060	23661
			0.96	5.8	33.84	197	1150	6700	1.2	7.16	41.8	243.4	1419	8268.4
			0	1	6	35	204	1189	6	35	204	1189	6930	40391

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Table 10: The first 6 elements of Neutrosophic Lucas-Balancing Sequence for α, β, and γ cuts.

α	β	γ	LB_0	LB_1	LB_2	LB_3	LB_4	LB_5	LB_0	LB_1	LB_2	LB_3	LB_4	LB_5
			Left Truth						Right Truth					
1	0	1	3	17	99	577	3363	19601	3	17	99	577	3363	19601
0.9	0.05	0.9	2.8	15.6	90.8	529	3084	17977	4.4	25.2	147	855.6	4987	29065
0.8	0.2	0	2.6	14.2	82.6	481	2806	16353	5.8	33.4	195	1134	6611	38529
0.7	0.23	0.07	2.4	12.8	74.4	434	2527	14730	7.2	41.6	242	1413	8234	47994
0.6	0.6	0	2.2	11.4	66.2	386	2249	13106	8.6	49.8	290	1691	9858	57458
0.5	0.5	0.5	2	10	58	338	1970	11482	10	58	338	1970	11482	66922
0.4	0.4	0.04	1.8	8.6	49.8	290	1691	9858	11.4	66.2	386	2249	13106	76386
0.3	0.3	1	1.6	7.2	41.6	242	1413	8234	12.8	74.4	434	2527	14730	85850
			Left Indeterminacy						Right Indeterminacy					
			3	17	99	577	3363	19601	3	17	99	577	3363	19601
			2.9	16.3	94.9	553.1	3223.7	18789	3.7	21.1	122.9	716.3	4174.9	24333.1
			2.6	14.2	82.6	481.4	2805.8	16353	5.8	33.4	194.6	1134.2	6610.6	38529.4
			2.54	13.78	80.14	467.1	2722.2	15866	6.22	35.86	208.9	1217.8	7097.7	41368.7
			1.8	8.6	49.8	290.2	1691.4	9858.2	11.4	66.2	385.8	2248.6	13106	76386.2
			2	10	58	338	1970	11482	10	58	338	1970	11482	66922
			2.2	11.4	66.2	385.8	2248.6	13106	8.6	49.8	290.2	1691.4	9858.2	57457.8
			2.4	12.8	74.4	433.6	2527.2	14730	7.2	41.6	242.4	1412.8	8234.4	47993.6
			Left Falsity						Right Falsity					
			1	3	17	99	577	3363	17	99	577	3363	19601	114243
			1.2	4.4	25.2	147	855.6	4987	15.6	90.8	529	3084	17977	104779
			3	17	99	577	3363	19601	3	17	99	577	3363	19601
			2.86	16	93.26	544	3168	18464	3.98	22.74	132	772	4500	26226
			3	17	99	577	3363	19601	3	17	99	577	3363	19601
			2	10	58	338	1970	11482	10	58	338	1970	11482	66922
			2.92	16.4	95.72	558	3252	18951	3.56	20.28	118	688.4	4013	23387
			1	3	17	99	577	3363	17	99	577	3363	19601	114243

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Table 11: The first 6 elements of Neutrosophic Mersenne Sequence for α, β, and γ cuts.

α	β	γ	M_0	M_1	M_2	M_3	M_4	M_5	M_0	M_1	M_2	M_3	M_4	M_5
			Left Truth						Right Truth					
1	0	1	1	3	7	15	31	63	1	3	7	15	31	63
0.9	0.05	0.9	0.9	2.8	6.6	14.2	29.4	59.8	1.2	3.4	7.8	16.6	34.2	69.4
0.8	0.2	0	0.8	2.6	6.2	13.4	27.8	56.6	1.4	3.8	8.6	18.2	37.4	75.8
0.7	0.23	0.07	0.7	2.4	5.8	12.6	26.2	53.4	1.6	4.2	9.4	19.8	40.6	82.2
0.6	0.6	0	0.6	2.2	5.4	11.8	24.6	50.2	1.8	4.6	10.2	21.4	43.8	88.6
0.5	0.5	0.5	0.5	2	5	11	23	47	2	5	11	23	47	95
0.4	0.4	0.04	0.4	1.8	4.6	10.2	21.4	43.8	2.2	5.4	11.8	24.6	50.2	101.4
0.3	0.3	1	0.3	1.6	4.2	9.4	19.8	40.6	2.4	5.8	12.6	26.2	53.4	107.8
			Left Indeterminacy						Right Indeterminacy					
			1	3	7	15	31	63	1	3	7	15	31	63
			0.95	2.9	6.8	14.6	30.2	61.4	1.1	3.2	7.4	15.8	32.6	66.2
			0.8	2.6	6.2	13.4	27.8	56.6	1.4	3.8	8.6	18.2	37.4	75.8
			0.77	2.54	6.08	13.16	27.32	55.64	1.46	3.92	8.84	18.68	38.36	77.72
			0.4	1.8	4.6	10.2	21.4	43.8	2.2	5.4	11.8	24.6	50.2	101.4
			0.5	2	5	11	23	47	2	5	11	23	47	95
			0.6	2.2	5.4	11.8	24.6	50.2	1.8	4.6	10.2	21.4	43.8	88.6
			0.7	2.4	5.8	12.6	26.2	53.4	1.6	4.2	9.4	19.8	40.6	82.2
			Left Falsity						Right Falsity					
			0	1	3	7	15	31	3	7	15	31	63	127
			0.1	1.2	3.4	7.8	16.6	34.2	2.8	6.6	14.2	29.4	59.8	120.6
			1	3	7	15	31	63	1	3	7	15	31	63
			0.93	2.86	6.72	14.4	29.88	60.76	1.14	3.28	7.56	16.12	33.24	67.48
			1	3	7	15	31	63	1	3	7	15	31	63
			0.5	2	5	11	23	47	2	5	11	23	47	95
			0.96	2.92	6.84	14.7	30.36	61.72	1.08	3.16	7.32	15.64	32.28	65.56
			0	1	3	7	15	31	3	7	15	31	63	127

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Table 12: The first 6 elements of Neutrosophic Mersenne-Lucas Sequence for α, β, and γ cuts.

α	β	γ	ML_0	ML_1	ML_2	ML_3	ML_4	ML_5	ML_0	ML_1	ML_2	ML_3	ML_4	ML_5
			Left Truth						Right Truth					
1	0	1	3	5	9	17	33	65	3	5	9	17	33	65
0.9	0.05	0.9	2.9	4.8	8.6	16.2	31.4	61.8	3.2	5.4	9.8	18.6	36.2	71.4
0.8	0.2	0	2.8	4.6	8.2	15.4	29.8	58.6	3.4	5.8	10.6	20.2	39.4	77.8
0.7	0.23	0.07	2.7	4.4	7.8	14.6	28.2	55.4	3.6	6.2	11.4	21.8	42.6	84.2
0.6	0.6	0	2.6	4.2	7.4	13.8	26.6	52.2	3.8	6.6	12.2	23.4	45.8	90.6
0.5	0.5	0.5	2.5	4	7	13	25	49	4	7	13	25	49	97
0.4	0.4	0.04	2.4	3.8	6.6	12.2	23.4	45.8	4.2	7.4	13.8	26.6	52.2	103.4
0.3	0.3	1	2.3	3.6	6.2	11.4	21.8	42.6	4.4	7.8	14.6	28.2	55.4	109.8
			Left Indeterminacy						Right Indeterminacy					
			3	5	9	17	33	65	3	5	9	17	33	65
			2.95	4.9	8.8	16.6	32.2	63.4	3.1	5.2	9.4	17.8	34.6	68.2
			2.8	4.6	8.2	15.4	29.8	58.6	3.4	5.8	10.6	20.2	39.4	77.8
			2.77	4.54	8.08	15.16	29.32	57.64	3.46	5.92	10.84	20.68	40.36	79.72
			2.4	3.8	6.6	12.2	23.4	45.8	4.2	7.4	13.8	26.6	52.2	103.4
			2.5	4	7	13	25	49	4	7	13	25	49	97
			2.6	4.2	7.4	13.8	26.6	52.2	3.8	6.6	12.2	23.4	45.8	90.6
			2.7	4.4	7.8	14.6	28.2	55.4	3.6	6.2	11.4	21.8	42.6	84.2
			Left Falsity						Right Falsity					
			2	3	5	9	17	33	5	9	17	33	65	129
			2.1	3.2	5.4	9.8	18.6	36.2	4.8	8.6	16.2	31.4	61.8	122.6
			3	5	9	17	33	65	3	5	9	17	33	65
			2.93	4.86	8.72	16.44	31.88	62.76	3.14	5.28	9.56	18.12	35.24	69.48
			3	5	9	17	33	65	3	5	9	17	33	65
			2.5	4	7	13	25	49	4	7	13	25	49	97
			2.96	4.92	8.84	16.68	32.36	63.72	3.08	5.16	9.32	17.64	34.28	67.56
			2	3	5	9	17	33	5	9	17	33	65	129

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