Neutrosophic Fuzzy Goal Programming Algorithm for Multi-level Multiobjective Linear Programming Problems

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1 Introduction

Most often, the mathematical programming problems consist of only one decision maker who takes the decisions all alone. Apart from that, many decision-making problems involve hierarchical decision structures, each with independent, and most often contradictory in nature. Such decision-making scenarios are termed as decentralized planning problems. Thus, the hierarchical decision-making texture of the problem is formulated as multi-level programming problems (MLPPs). If there are only two decision makers, then it becomes bi-level programming problems, tri-level for three decision makers, and so on. The fundamental concepts behind the MLPPs optimization techniques are that the leader-level decision maker defines his/her goals/target and then seeks the optimal solution from each subordinate level of the organization that has calculated individually. The follower-level decisions are then submitted and satisfied by the leader-level in view of overall benefit of the organizations. There may be more than one linear objective function that are to be optimized by different levels in MLPPs, then such kind of decentralized decision-making problems are termed as multi-level multiobjective linear programming problems (ML-MOLPPs).

There are several research works available in the literature that contribute to the domain of multi-level multiobjective linear programming problems. Based on fuzzy set theory, [1, 6, 19, 20, 22] presented fuzzy programming and fuzzy goal programming approaches to bi-level decision-making problems. Furthermore,

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[8, 10, 12, 14, 15] suggested the fuzzy-based solution procedure for ML-MOLPPs. Later on, intuitionistic fuzzy set theory [7] is also introduced to solve the ML-MOLPPs under intuitionistic fuzzy environment. Recently, [9, 21] discussed the intuitionistic fuzzy techniques to solve the ML-MOLPPs by considering the membership as well as non-membership functions of all objectives at each level. Furthermore, the extension and generalization of fuzzy and intuitionistic fuzzy sets are presented and named as neutrosophic set. First, [18] proposed the neutrosophic set, and later on it was extensively used in the field of mathematical programming problems and their optimization techniques. Only few research work is available that captures the neutrosophic decision set theory in ML-MOLPPs. Only two research articles are cited below that have contributed to neutrosophic ML-MOLPPs domain. Maiti et al. [11] investigated neutrosophic goal programming strategy for ML-MOLPPs with neutrosophic parameters. Pramanik and Dey [13] also suggested a goal programming technique for neutrosophic ML-MOLPPs where the parameters have been taken as triangular or trapezoidal neutrosophic numbers. Thus, this chapter provides more emphasis toward the neutrosophic ML-MOLPPs research area and laid down a concrete base for neutrosophic ML-MOLPPs optimization domain.

In this chapter, the neutrosophic fuzzy goal programming (NFGP) algorithm is introduced to solve the multi-level multiobjective linear programming problems. Two different NFGP procedures based on neutrosophic fuzzy decision set are presented for ML-MOLPPs. To formulate any of these two proposed NFGP models of the ML-MOLPPs, the neutrosophic fuzzy goals of the objectives are determined by finding individual optimal solutions. Marginal evaluations of each objective functions are then depicted by the associated membership functions under neutrosophic environment. These membership functions are converted into neutrosophic fuzzy flexible membership goals by means of incorporating over- and under-deviational variables and assigning highest truth membership value (unity), indeterminacy value (half), and a falsity value (zero) as aspiration levels to each of them. To determine the membership functions of the decision variable vectors monitored by any level decision maker, the optimal solution of the corresponding MOLPP is separately solved. A marginal relaxation of the decisions is prescribed to avoid decision deadlock.

The first proposed NFGP solution algorithm provides an extension of the work presented by [1, 8, 16] under neutrosophic environment, which deals with bi-level linear single-objective programming problems. It also extends the work of [14] by introducing the NFGP algorithm to multi-level programming problems with a multiple linear objective at each level. The final fuzzy model groups the membership functions for the defined neutrosophic fuzzy goals of the objective functions and the decision variable vectors at all levels, which are determined separately for each level except the follower level of the multi-level problem.

The second proposed NFFGP algorithm may be seen as a method for solving multi-level multiobjective programming problems. First, it develops the NFGP model of the leader-level problem to obtain a satisfactory solution to the leader-level decision maker's problem. A marginal relaxation of the leader-level decision maker's decisions is taken into account to avoid a decision deadlock. These decisions of the leader-level decision makers are depicted by membership functions of neutrosophic fuzzy set theory and transferred to the second-level DM (SLDM) as an additional constraint. Then, the SLDM modeled its NFGP model that considers the neutrosophic fuzzy membership goals of the objectives and decision variable vectors of the leader-level decision makers. Afterward, the achieved solution is passed to the third-level DM (TLDM) who seeks the solution in a similar fashion. The same process is carried out until the follower level reaches. Thus, this procedure may be assumed as an extension of the fuzzy mathematical programming algorithm of [16, 17] under the neutrosophic environment.

The remaining part of the chapter is summarized as follows: in Sect. 2, the preliminaries regarding neutrosophic set have been discussed, while Sect. 3 discusses the formulations of multi-level multiobjective programming problems. The proposed neutrosophic fuzzy goal algorithm is developed in Sect. 4, whereas in Sect. 5, a numerical example is presented to show the applicability and validity of the proposed approaches. Finally, conclusions and future scope are discussed based on the present work in Sect. 6.

2 Preliminaries

Some basic preliminaries regarding neutrosophic set are presented in the following section.

Definition 1 ([4]) Let Y be a universe discourse such that $y \in Y$, then a neutrosophic set A in Y is defined by three membership functions namely, truth $\mu_A(y)$, indeterminacy $\lambda_A(y)$, and a falsity $\nu_A(y)$ and is denoted by the following form:

$$A = \{< y,\ \mu_A(y),\ \lambda_A(y),\ \nu_A(y) > \mid y \in Y\},$$

where $\mu_A(y),\ \lambda_A(y)$ and $\nu_A(y)$ are real standard or non-standard subsets belong to $]0^-, 1^+[$, also given as, $\mu_A(y) : Y \rightarrow]0^-, 1^+[$, $\lambda_A(y) : Y \rightarrow]0^-, 1^+[$, and $\nu_A(y) : Y \rightarrow]0^-, 1^+[$. There is no restriction on the sum of $\mu_A(y),\ \lambda_A(y)$ and $\nu_A(y)$, so we have

$$0^- \leq \ sup\ \mu_A(y) + \lambda_A(y) + \ sup\ \nu_A(y) \leq 3^+.$$

Definition 2 ([4]) A single-valued neutrosophic set A over universe of discourse Y is defined as

$$A = \{< y,\ \mu_A(y),\ \lambda_A(y),\ \nu_A(y) > \mid y \in Y\},$$

where $\mu_A(y),\ \lambda_A(y)$, and $\nu_A(y) \in [0, 1]$ and $0 \leq \mu_A(y) + \lambda_A(y) + \nu_A(y) \leq 3$ for each $y \in Y$.

Definition 3 ([4]) The complement of a single valued neutrosophic set A is represented as $c(A)$ and defined by $\mu_{c(A)}(y) = \nu_A(y)$, $\lambda_{c(A)}(y) = 1 - \nu_A(y)$ and $\nu_{c(A)}(y) = \mu_A(y)$, respectively.

Definition 4 ([4]) Let A and B be the two single-valued neutrosophic sets, then the union of A and B is also a single-valued neutrosophic set C, that is, $C = (A \cup B)$, whose truth $\mu_C(y)$, indeterminacy $\lambda_C(y)$, and falsity $\nu_C(y)$ membership functions are given by

$\mu_C(y) = \max\ (\mu_A(y),\ \mu_B(y))$
$\lambda_C(y) = \max\ (\lambda_A(y),\ \lambda_B(y))$
$\nu_C(y) = \min\ (\nu_A(y),\ \nu_B(y))$ for each $y \in Y$.

Definition 5 ([4]) Let A and B be the two single-valued neutrosophic sets, then the intersection of A and B is also a single-valued neutrosophic set C, that is, $C = (A \cap B)$, whose truth $\mu_C(y)$, indeterminacy $\lambda_C(y)$, and falsity $\nu_C(y)$ membership functions are given by

$\mu_C(y) = \min\ (\mu_A(y),\ \mu_B(y))$
$\lambda_C(y) = \min\ (\lambda_A(y),\ \lambda_B(y))$
$\nu_C(y) = \max\ (\nu_A(y),\ \nu_B(y))$ for each $y \in Y$.

Definition 6 A solution set $Y^* \in S$ is said to be an efficient solution to the MLMOPPs if and only if there does not exist any other $Y \in S$ such that $O_{ij} \geq O^*_{ij}$ for all $i = 1, 2, \ldots, t;\quad j = 1, 2, \ldots, m_t$, respectively.

Definition 7 For any ML-MOPPs, an efficient solution selected by the decision makers is the best compromise optimal solution which is chosen on the basis of decision makers' explicit and implicit criteria.

3 Description of ML-MOLPPs

Assume that a t- level multiobjective programming problem with minimization-type objective functions at different level. Consider that DM_i represents the i-th level decision maker and controls over the decision variable $\mathbf{y}_i = (y_{i1},\ y_{i2},\ \ldots,\ y_{in_i}) \in R^{n_i}$ for all $i = 1,\ 2,\ \ldots,\ t$, where $\mathbf{y} = (\mathbf{y}_1,\ \mathbf{y}_2,\ \ldots,\ \mathbf{y}_t) \in R^n$ such that $n = n_1 +_2 + \ldots + n_t$. Furthermore, we assume that

$$O_i(\mathbf{y}) = O_i(\mathbf{y}_1,\ \mathbf{y}_2,\ \ldots,\ \mathbf{y}_t) : R^{n_1} \times R^{n_2} \times \cdots \times R^{n_t} \ \rightarrow\ R^{m_i},\ \ \forall\ i = 1,\ 2,\ \ldots,\ t \quad (1)$$

represents the vector-set of a well-defined linear objective function to the i-th decision makers, $i = 1,\ 2,\ \ldots,\ t$. The equivalent mathematical expressions for the ML-MOLPPS with minimization-type objectives can be stated as follows:

[1st level]

$$\underset{y_1}{\text{Min}}\; O_1(\mathbf{y}) = \underset{y_1}{\text{Min}}\; \left(o_{11}(\mathbf{y}),\; o_{12}(\mathbf{y}),\; \ldots,\; o_{1m_1}(\mathbf{y})\right)$$

where $\mathbf{y}_2,\ \mathbf{y}_3,\ \ldots,\ \mathbf{y}_t$ solves

[2nd level]

$$\underset{y_2}{\text{Min}}\; O_2(\mathbf{y}) = \underset{y_2}{\text{Min}}\; \left(o_{21}(\mathbf{y}),\; o_{22}(\mathbf{y}),\; \ldots,\; o_{2m_2}(\mathbf{y})\right) \tag{2}$$

...

where $\mathbf{y}_t$ solves

[t-th level]

$$\underset{y_t}{\text{Min}}\; O_t(\mathbf{y}) = \underset{y_t}{\text{Min}}\; \left(o_{t1}(\mathbf{y}),\; o_{t2}(\mathbf{y}),\; \ldots,\; o_{tm_t}(\mathbf{y})\right)$$

subject to

$$\begin{aligned}\mathbf{y} \in \mathbf{S} = \big\{&\mathbf{y} \in R^n | G_1 y_1 + G_1 y_1 + \cdots + G_t y_t (\leq\ or\ =\ or\ \geq)\ \mathbf{q},\ \mathbf{y} \geq 0,\\ &\mathbf{q} \in R^m\big\} \neq \phi,\end{aligned} \tag{3}$$

where

$$\begin{aligned}o_{ij}(\mathbf{y}) =& c_1^{ij}\mathbf{y}_1 + c_2^{ij}\mathbf{y}_2 + \cdots + c_t^{ij}\mathbf{y}_t,\ \ i = 1,\ 2,\ \ldots,\ t,\ \ j = 1,\ 2,\ \ldots,\ m_i\\ =& c_{11}^{ij}\mathbf{y}_{11} + c_{12}^{ij}\mathbf{y}_{12} + \cdots + c_{1n_1}^{ij}\mathbf{y}_{1n_1} + c_{21}^{ij}\mathbf{y}_{21} + c_{22}^{ij}\mathbf{y}_{22} + \cdots\\ &+ c_{2n_2}^{ij}\mathbf{y}_{2n_2} + \cdots c_{t1}^{ij}\mathbf{y}_{t1} + c_{t2}^{ij}\mathbf{y}_{t2} + \cdots + c_{tn_t}^{ij}\mathbf{y}_{tn_t}\end{aligned} \tag{4}$$

such that $\mathbf{S}$ is the multi-level convex constraints in feasible decision set under multi-level multiobjective programming problems. The notation $m_i,\ \ i = 1,\ 2,\ \ldots,\ t$ denotes the number of objective function under i-th decision maker, m is the number of constraints, $c_k^{ij} = \left(c_{k1}^{ij},\ c_{k2}^{ij},\ \cdots,c_{kn_k}^{ij}\right),\ \ k = 1,\ 2,\ \ldots,\ t,\ \ c_{kn_k}^{ij}$ are constants, and the coefficient matrices of size $m \times n_i$ are depicted as $G_i,\ \ \forall\ i = 1,\ 2,\ \ldots,\ t$.

4 Proposed Neutrosophic Fuzzy Goal Programming Techniques

In the past few decades, it has been observed that the situation may arise in real-life decision-making problems where the indeterminacy or neutral thoughts about element into the feasible set exist. Indeterminacy/neutral thoughts are the region of the negligence of a proposition's value and lie between truth and a falsity degree. Therefore, the further generalization of fuzzy set (FS) [20] and intuitionistic fuzzy set (IFS) [7] is presented by introducing a new member into the feasible decision set. First, [18] investigated the neutrosophic set (NS) which comprises three membership functions, namely truth (degree of belongingness), indeterminacy (degree of belongingness up to some extent), and a falsity (degree of non-belongingness) functions of the element into the neutrosophic decision set (see [2, 3, 5]).

In ML-MOLPPs, if an imprecise aspiration level under neutrosophic environment is assigned to each of the objectives at each level of the ML-MOLPPs, then such neutrosophic objectives are termed as neutrosophic goals and dealt with neutrosophic decision-making techniques. Hence, the marginal evaluation of each neutrosophic goals is characterized through three different membership functions, namely truth, indeterminacy, and a falsity membership functions by defining the tolerance limits for attainment of their respective aspiration levels.

4.1 Characterization of Different Membership Functions Under Neutrosophic Environment

In multi-level decision-making problems, each DM intends to minimize its own objectives in each level over the same feasible region depicted by the system of constraints; hence, the individual optimal solutions are obtained by them and can be regarded as the aspiration levels of their associated neutrosophic goals.

Assume that $\mathbf{y}^{ij} = (\mathbf{y}_1^{ij}, \mathbf{y}_2^{ij}, \ldots, \mathbf{y}_t^{ij})$ and $o_{ij}^{\min}, \quad i = 1, 2, \ldots, t, \quad j = 1, 2, \ldots, m_i$ be the best individual optimal solutions of each DMs at each level, respectively. Furthermore, consider that $l_{ij} \geq o_{ij}^{\min}$ denotes the aspiration level assigned to the ij-th objective $o_{ij}(\mathbf{y})$ (where ij means that when $i = t$ for t-th level decision makers then $j = 1, 2, \ldots, m_i$). Moreover, also consider that $\mathbf{y}^{i*} = (\mathbf{y}_1^{i*}, \mathbf{y}_2^{i*}, \ldots, \mathbf{y}_t^{i*}), \quad i = 1, 2, \ldots, t-1$, be the optimal solutions for the t-th level decision makers of ML-MOLPPs. Consequently, the neutrosophic goals of each objective function at each level and the vector-set of neutrosophic goals for the decision variables monitored by t-th level decision makers can be stated as follows:

$$o_{ij}(\mathbf{y}) \tilde{<} l_{ij}, \quad i = 1, 2, \ldots, t, \quad j = 1, 2, \ldots, m_i \quad \text{and}$$
$$\mathbf{y}_i \tilde{=} \mathbf{y}_i^{i*}, \quad i = 1, 2, \ldots, t-1,$$

where $\tilde{<}$ and $\tilde{=}$ represent the degree of neutrosophy of the aspiration levels.

One can note that the solutions $\mathbf{y}^{ij} = (\mathbf{y}_1^{ij}, \mathbf{y}_2^{ij}, \ldots, \mathbf{y}_t^{ij}); \quad i = 1, 2, \ldots, t, \quad j = 1, 2, \ldots, m_i$ are probably different due to the conflicting nature of the objective functions at each level for all the decision makers. Therefore, it can be obvious to consider that the values of $o_{gm}(\mathbf{y}_1^{gm}, \mathbf{y}_2^{gm}, \ldots, \mathbf{y}_t^{gm}) \geq$

$o_{ij}^{\min}$, $g = 1, 2, \ldots, t$, $m = 1, 2, \ldots, m_i$, and $\forall\ ij \neq gm$ with all values greater than $o_{gm}^{u} = \max\,[\,o_{ij}\,(\mathbf{y}_1^{gm}, \mathbf{y}_2^{gm}, \ldots, \mathbf{y}_t^{gm}),\ i = 1, 2, \ldots, t,\ j = 1, 2, \ldots, m_i$ and $ij \neq gm]$ are absolutely unacceptable to the objective function $o_{gm}(\mathbf{y}) = o_{gm}(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_t)$. As a result, $o_{gm}(\mathbf{y})$ can be taken as the upper tolerance limit $u_{gm}(\mathbf{y})$ of the neutrosophic goal to the objective functions. The upper and lower bounds for ij-th objective function under the neutrosophic environment can be obtained as follows:

$$U_{ij}^{\mu} = u_{ij}, \quad L_{ij}^{\mu} = l_{ij} \qquad \text{for truth membership}$$

$$U_{ij}^{\lambda} = L_{ij}^{\mu} + a_{ij}, \quad L_{ij}^{\lambda} = L_{ij}^{\mu} \qquad \text{for indeterminacy membership}$$

$$U_{ij}^{\nu} = U_{ij}^{\mu}, \quad L_{ij}^{\nu} = L_{ij}^{\mu} + b_{ij} \qquad \text{for falsity membership,}$$

where a_{ij} and $b_{ij} \in (0, 1)$ are predetermined real numbers.

Thus, the different membership functions, namely truth $\mu_{o_{ij}}(o_{ij}(\mathbf{y}))$, indeterminacy $\lambda_{o_{ij}}(o_{ij}(\mathbf{y}))$, and a falsity $\nu_{o_{ij}}(o_{ij}(\mathbf{y}))$ membership functions for the ij-th neutrosophic goals can be stated as follows:

$$\mu_{o_{ij}}(o_{ij}(\mathbf{y})) = \begin{cases} 1 & \text{if } o_{ij}(\mathbf{y}) \leq L_{ij}^{\mu} \\ 1 - \frac{o_{ij}(\mathbf{y}) - L_{ij}^{\mu}}{U_{ij}^{\mu} - L_{ij}^{\mu}} & \text{if } L_{ij}^{\mu} \leq o_{ij}(\mathbf{y}) \leq U_{ij}^{\mu} \\ 0 & \text{if } o_{ij}(\mathbf{y}) \geq U_{ij}^{\mu} \end{cases} \tag{5}$$

$$\lambda_{o_{ij}}(o_{ij}(\mathbf{y})) = \begin{cases} 1 & \text{if } o_{ij}(\mathbf{y}) \leq L_{ij}^{\lambda} \\ 1 - \frac{o_{ij}(\mathbf{y}) - L_{ij}^{\lambda}}{U_{ij}^{\lambda} - L_{ij}^{\lambda}} & \text{if } L_{k}^{I} \leq o_{ij}(\mathbf{y}) \leq U_{ij}^{\lambda} \\ 0 & \text{if } o_{ij}(\mathbf{y}) \geq U_{ij}^{\lambda} \end{cases} \tag{6}$$

$$\nu_{o_{ij}}(o_{ij}(\mathbf{y})) = \begin{cases} 1 & \text{if } o_{ij}(\mathbf{y}) \geq U_{ij}^{\nu} \\ 1 - \frac{U_{ij}^{\nu} - o_{ij}(\mathbf{y})}{U_{ij}^{\nu} - L_{ij}^{\nu}} & \text{if } L_{ij}^{\nu} \leq o_{ij}(\mathbf{y}) \leq U_{ij}^{\nu} \\ 0 & \text{if } o_{ij}(\mathbf{y}) \leq L_{ij}^{\nu}. \end{cases} \tag{7}$$

To construct the different membership functions for the decision variables monitored by i-th decision makers, first, the optimal solution for the t-th level MOLPPs, $\mathbf{y}^{i*} = (\mathbf{y}_1^{i*}, \mathbf{y}_2^{i*}, \ldots, \mathbf{y}_t^{i*}),\ i = 1, 2, \ldots, t-1$, should be carried out by using any appropriate method for MOLPPs optimization techniques.

Suppose that $T_k^{i\alpha}$ and $T_k^{i\beta}$, $i = 1, 2, \ldots, t-1$, $k = 1, 2, \ldots, n_i$ be the maximum negative and positive tolerance limits on the decision variables imposed by the i-th level decision makers. Usually, the tolerances T_{ik}^{-} and T_{ik}^{+} may not be equal. The upper and lower bounds for ik-th decision variable vectors under the neutrosophic environment can be stated as follows:

$$\mu^U_{y_{ik}} = y^*_{ik} + T^{i\beta}_k,\ \ \mu^L_{y_{ik}} = y^*_{ik} - T^{i\alpha}_k \qquad \text{for truth membership}$$
$$\lambda^U_{y_{ik}} = \mu^L_{y_{ik}} + a_{ik},\ \ \lambda^L_{y_{ik}} = \mu^L_{y_{ik}} \qquad \text{for indeterminacy membership}$$
$$\nu^U_{y_{ik}} = \mu^U_{y_{ik}},\ \ \nu^L_{y_{ik}} = \mu^L_{y_{ik}} + b_{ik} \qquad \text{for falsity membership,}$$

where a_{ik} and $b_{ik} \in (0, 1)$ are predetermined real numbers.

For each of the n_i components of the decision variable vector $\mathbf{y}^*_{ik} = (\mathbf{y}^*_{i1}, \mathbf{y}^*_{i2}, \ldots, \mathbf{y}^*_{in_i})$ controlled by the leader $(t-1)$-th level decision makers, the different linear membership functions under neutrosophic environment such as truth $\mu_{y_{ik}}(y_{ik})$, indeterminacy $\lambda_{y_{ik}}(y_{ik})$, and a falsity $\nu_{y_{ik}}(y_{ik})$ can be furnished as follows:

$$\mu_{y_{ik}}(y_{ik}) = \begin{cases} \frac{y_{ik}-\mu^L_{y_{ik}}}{y^*_{ik}-\mu^L_{y_{ik}}} & \text{if } \mu^L_{y_{ik}} \le y_{ik} \le y^*_{ik} \\ \frac{\mu^U_{y_{ik}}-y_{ik}}{\mu^U_{y_{ik}}-y^*_{ik}} & \text{if } y^*_{ik} \le y_{ik} \le \mu^U_{y_{ik}} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

$$\lambda_{y_{ik}}(y_{ik}) = \begin{cases} \frac{y_{ik}-\lambda^L_{y_{ik}}}{y^*_{ik}-\lambda^L_{y_{ik}}} & \text{if } \lambda^L_{y_{ik}} \le y_{ik} \le y^*_{ik} \\ \frac{\lambda^U_{y_{ik}}-y_{ik}}{\lambda^U_{y_{ik}}-y^*_{ik}} & \text{if } y^*_{ik} \le y_{ik} \le \lambda^U_{y_{ik}} \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

$$\nu_{y_{ik}}(y_{ik}) = \begin{cases} \frac{y_{ik}-\nu^L_{y_{ik}}}{y^*_{ik}-\nu^L_{y_{ik}}} & \text{if } \nu^L_{y_{ik}} \le y_{ik} \le y^*_{ik} \\ \frac{\nu^U_{y_{ik}}-y_{ik}}{\nu^U_{y_{ik}}-y^*_{ik}} & \text{if } y^*_{ik} \le y_{ik} \le \nu^U_{y_{ik}} \\ 1 & \text{otherwise.} \end{cases} \tag{10}$$

Also, it should be noted that the range of y_{ik} may be shifted according to the decision makers' choices.

In a neutrosophic decision environment, the neutrosophic goals comprising the decision makers' objective functions at different level and the neutrosophic goals of the decision variable vectors are monitored by leader $(t-1)$-th level decision makers. The attainment degrees to their aspiration levels to the extent possible are virtually achieved by the possible achievement of their respective memberships, namely truth, indeterminacy, and a falsity membership functions to their utmost degrees. Obviously, this aspect of neutrosophic fuzzy programming approach enables a neutrosophic fuzzy goal programming technique as a justified approach for solving the leader t-th level MOLPPs and consequently ML-MOLPPs.

4.2 Neutrosophic Fuzzy Goal Programming Approach

In neutrosophic programming approaches, the neutrosophic membership degrees can be transformed into neutrosophic membership goals according to their respective maximum degrees of attainment. The highest degree of truth membership function that can be achieved is unity (1), the indeterminacy membership function is neutral and independent with the highest attainment degree half (0.5), and the falsity membership function can be achieved with the highest attainment degree zero (0). Now, the transformed membership goals under the neutrosophic environment can be expressed as follows:

$$\left.\begin{array}{l} \mu_{o_{ij}}(o_{ij}(\mathbf{y})) + d_{ij\mu}^{-} - d_{ij\mu}^{+} = 1, \\ \lambda_{o_{ij}}(o_{ij}(\mathbf{y})) + d_{ij\lambda}^{-} - d_{ij\lambda}^{+} = 0.5, \\ \nu_{o_{ij}}(o_{ij}(\mathbf{y})) + d_{ij\nu}^{-} - d_{ij\nu}^{+} = 0, \end{array}\right\} \forall\ i = 1,\ 2,\ \ldots,\ t,\quad j = 1,\ 2,\ \ldots,\ m_i \tag{11}$$

$$\left.\begin{array}{l} \mu_{y_{ik}}(y_{ik}) + d_{ik\mu}^{-} - d_{ik\mu}^{+} = 1, \\ \lambda_{y_{ik}}(y_{ik}) + d_{ik\lambda}^{-} - d_{ik\lambda}^{+} = 0.5, \\ \nu_{y_{ik}}(y_{ik}) + d_{ik\nu}^{-} - d_{ik\nu}^{+} = 0, \end{array}\right\} \forall\ i = 1,\ 2,\ \ldots,\ t-1,\quad k = 1,\ 2,\ \ldots,\ n_i \tag{12}$$

or equivalently represented as follows:

$$\left.\begin{array}{l} 1 - \frac{o_{ij}(\mathbf{y}) - L_{ij}^{\mu}}{U_{ij}^{\mu} - L_{ij}^{\mu}} + d_{ij\mu}^{-} - d_{ij\mu}^{+} = 1, \\ 1 - \frac{o_{ij}(\mathbf{y}) - L_{ij}^{\lambda}}{U_{ij}^{\lambda} - L_{ij}^{\lambda}} + d_{ij\lambda}^{-} - d_{ij\lambda}^{+} = 0.5, \\ 1 - \frac{U_{ij}^{\nu} - o_{ij}(\mathbf{y})}{U_{ij}^{\nu} - L_{ij}^{\nu}} + d_{ij\nu}^{-} - d_{ij\nu}^{+} = 0, \end{array}\right\} \forall\ i = 1,\ 2,\ \ldots,\ t,\quad j = 1,\ 2,\ \ldots,\ m_i \tag{13}$$

$$\left.\begin{array}{l} \frac{y_{ik} - \mu_{y_{ik}}^{L}}{y_{ik}^{*} - \mu_{y_{ik}}^{L}} + d_{ik\mu}^{\alpha-} - d_{ik\mu}^{\alpha+} = 1, \\ \frac{\mu_{y_{ik}}^{U} - y_{ik}}{\mu_{y_{ik}}^{U} - y_{ik}^{*}} + d_{ik\mu}^{\beta-} - d_{ik\mu}^{\beta+} = 1, \\ \frac{y_{ik} - \lambda_{y_{ik}}^{L}}{y_{ik}^{*} - \lambda_{y_{ik}}^{L}} + d_{ik\lambda}^{\alpha-} - d_{ik\lambda}^{\alpha+} = 0.5, \\ \frac{\lambda_{y_{ik}}^{U} - y_{ik}}{\lambda_{y_{ik}}^{U} - y_{ik}^{*}} + d_{ik\lambda}^{\beta-} - d_{ik\lambda}^{\beta+} = 0.5, \\ \frac{y_{ik} - \nu_{y_{ik}}^{L}}{y_{ik}^{*} - \nu_{y_{ik}}^{L}} + d_{ik\nu}^{\alpha-} - d_{ik\nu}^{\alpha+} = 0, \\ \frac{\nu_{y_{ik}}^{U} - y_{ik}}{\nu_{y_{ik}}^{U} - y_{ik}^{*}} + d_{ik\nu}^{\beta-} - d_{ik\nu}^{\beta+} = 0, \end{array}\right\} \forall\ i = 1,\ 2,\ \ldots,\ t-1,\quad k = 1,\ 2,\ \ldots,\ n_i, \tag{14}$$

where $d_{ik}^{-} = (d_{ik}^{\alpha-},\ d_{ik}^{\beta-})$, $d_{ik}^{+} = (d_{ik}^{\alpha+},\ d_{ik}^{\beta+})$; $d_{ij.}^{-},\ d_{ij.}^{+},\ d_{ik}^{\alpha-},\ d_{ik}^{\beta-},\ d_{ik}^{\alpha+},\ d_{ik}^{\beta+} \geq 0$; and $d_{ik}^{\beta-} \times d_{ik}^{\beta+} = 0,\ \forall\ i = 1,\ 2,\ \ldots,\ t-1,\ \ k = 1,\ 2,\ \ldots,\ n_i$ are the over and under deviations for truth, indeterminacy, and a falsity membership goals from their respective aspiration levels under neutrosophic environment.

In goal programming strategy, the over- and/or under-deviational variable vectors are considered in the objective function to minimize them and solely depend on the nature of objective function that is being optimized. In the proposed neutrosophic goal programming technique, the over-deviational variables for neutrosophic goals of each objective function, $d_{ij.}^{+}\ \forall\ i = 1, 2, \ldots, t,\ \ j = 1, 2, \ldots, m_i$ and the over and under-deviational variables for the neutrosophic fuzzy goals of the decision variable vectors, $d_{ik}^{\alpha+}$, $d_{ik}^{\alpha-}$, $d_{ik}^{\beta+}$ and $d_{ik}^{\beta-}\ \forall\ i = 1, 2, \ldots, (t-1)$, $k = 1, 2, \ldots, n_i$ are needed to be minimized to attain the neutrosophic fuzzy goals.

4.3 Neutrosophic Fuzzy Goal Programming Approach for ML-MOLPPs

The proposed neutrosophic fuzzy goal programming (NFGP) algorithm for solving the multi-level multiobjective linear programming problems (ML-MOLPPs) is presented, and the two new algorithms are suggested under neutrosophic environment.

4.3.1 The First NFGP Algorithm for ML-MOLPPs

First of all, the first NFGP algorithm proposed in this chapter groups over the different membership functions for the prescribed neutrosophic fuzzy goals of the objective functions at each levels; it also groups the different membership functions of the neutrosophic fuzzy goals of the decision variable vector of the t-th leader-level problems that are optimized individually under neutrosophic environment. Thus, by assuming the goal attainment problems at the same preference level, the equivalent proposed neutrosophic fuzzy multi-level multiobjective linear goal programming model of the ML-MOLPPs under neutrosophic environment can be expressed as follows:

$$\begin{aligned}
\text{Min F} = & \sum_{j=1}^{m_1} w_{1j\mu}^{+} d_{1j\mu}^{+} + \sum_{j=1}^{m_2} w_{2j\mu}^{+} d_{2j\mu}^{+} + \cdots + \sum_{j=1}^{m_t} w_{tj\mu}^{+} d_{tj\mu}^{+} \\
& + \sum_{j=1}^{m_1} w_{1j\lambda}^{+} d_{1j\lambda}^{+} + \sum_{j=1}^{m_2} w_{2j\lambda}^{+} d_{2j\lambda}^{+} + \cdots + \sum_{j=1}^{m_t} w_{tj\lambda}^{+} d_{tj\lambda}^{+} \\
& - \sum_{j=1}^{m_1} w_{1j\nu}^{+} d_{1j\nu}^{-} - \sum_{j=1}^{m_2} w_{2j\nu}^{+} d_{2j\nu}^{-} - \cdots - \sum_{j=1}^{m_t} w_{tj\nu}^{+} d_{tj\nu}^{-} \\
& + \sum_{k=1}^{n_1} \left(w_{1k\cdot}^{\alpha} . (d_{1k\cdot}^{\alpha-} + d_{1k\cdot}^{\alpha+}) + w_{1k\cdot}^{\beta} . (d_{1k\cdot}^{\beta-} + d_{1k\cdot}^{\beta+}) \right)
\end{aligned}$$

$$+\sum_{k=1}^{n_2}\left(w^{\alpha}_{2k\cdot}.(d^{\alpha-}_{2k\cdot}+d^{\alpha+}_{2k\cdot})+w^{\beta}_{2k\cdot}.(d^{\beta-}_{2k\cdot}+d^{\beta+}_{2k\cdot})\right)\quad\cdots\quad\cdots\quad\cdots$$

$$+\sum_{k=1}^{n_{t-1}}\left(w^{\alpha}_{t-1k\cdot}.(d^{\alpha-}_{t-1k\cdot}+d^{\alpha+}_{t-1k\cdot})+w^{\beta}_{t-1k\cdot}.(d^{\beta-}_{t-1k\cdot}+d^{\beta+}_{t-1k\cdot})\right)$$

subject to

$$\mu_{o_{1j}}(o_{1j}(\mathbf{y}))+d^{-}_{1j\mu}-d^{+}_{1j\mu}=1,\quad j=1,\ 2,\ \ldots,\ n_1$$

$$\mu_{o_{2j}}(o_{2j}(\mathbf{y}))+d^{-}_{2j\mu}-d^{+}_{2j\mu}=1,\quad j=1,\ 2,\ \ldots,\ n_2$$

$$\ldots$$

$$\mu_{o_{tj}}(o_{tj}(\mathbf{y}))+d^{-}_{tj\mu}-d^{+}_{tj\mu}=1,\quad j=1,\ 2,\ \ldots,\ n_t$$

$$\lambda_{o_{1j}}(o_{1j}(\mathbf{y}))+d^{-}_{1j\lambda}-d^{+}_{1j\lambda}=0.5,\quad j=1,\ 2,\ \ldots,\ n_1$$

$$\lambda_{o_{2j}}(o_{2j}(\mathbf{y}))+d^{-}_{2j\lambda}-d^{+}_{2j\lambda}=0.5,\quad j=1,\ 2,\ \ldots,\ n_2$$

$$\ldots$$

$$\lambda_{o_{tj}}(o_{tj}(\mathbf{y}))+d^{-}_{tj\lambda}-d^{+}_{tj\lambda}=0.5,\quad j=1,\ 2,\ \ldots,\ n_t$$

$$\nu_{o_{1j}}(o_{1j}(\mathbf{y}))+d^{-}_{1j\nu}-d^{+}_{1j\nu}=0,\quad j=1,\ 2,\ \ldots,\ n_1$$

$$\nu_{o_{2j}}(o_{2j}(\mathbf{y}))+d^{-}_{2j\nu}-d^{+}_{2j\nu}=0,\quad j=1,\ 2,\ \ldots,\ n_2$$

$$\ldots$$

$$\nu_{o_{tj}}(o_{tj}(\mathbf{y}))+d^{-}_{tj\nu}-d^{+}_{tj\nu}=0,\quad j=1,\ 2,\ \ldots,\ n_t$$

$$\mu_{y_{1k}}(y_{1k})+d^{-}_{1k\mu}-d^{+}_{1k\mu}=1,\quad k=1,\ 2,\ \ldots,\ n_1$$

$$\mu_{y_{2k}}(y_{2k})+d^{-}_{2k\mu}-d^{+}_{2k\mu}=1,\quad k=1,\ 2,\ \ldots,\ n_2$$

$$\ldots$$

$$\mu_{y_{t-1k}}(y_{t-1k})+d^{-}_{t-1k\mu}-d^{+}_{t-1k\mu}=1,\quad k=1,\ 2,\ \ldots,\ n_{t-1}$$

$$\lambda_{y_{1k}}(y_{1k})+d^{-}_{1k\lambda}-d^{+}_{1k\lambda}=0.5,\quad k=1,\ 2,\ \ldots,\ n_1$$

$$\lambda_{y_{2k}}(y_{2k})+d^{-}_{2k\lambda}-d^{+}_{2k\lambda}=0.5,\quad k=1,\ 2,\ \ldots,\ n_2$$

$$\ldots$$

$$\lambda_{y_{t-1k}}(y_{t-1k})+d^{-}_{t-1k\lambda}-d^{+}_{t-1k\lambda}=0.5,\quad k=1,\ 2,\ \ldots,\ n_{t-1}$$

$$\nu_{y_{1k}}(y_{1k})+d^{-}_{1k\nu}-d^{+}_{1k\nu}=0,\quad k=1,\ 2,\ \ldots,\ n_1$$

$$\nu_{y_{2k}}(y_{2k})+d^{-}_{2k\nu}-d^{+}_{2k\nu}=0,\quad k=1,\ 2,\ \ldots,\ n_2$$

$$\ldots$$

$$\nu_{y_{t-1k}}(y_{t-1k}) + d^-_{t-1k\nu} - d^+_{t-1k\nu} = 0, \quad k = 1,\ 2,\ \ldots,\ n_{t-1}$$

$$G_1 y_1 + G_1 y_1 + \cdots + G_t y_t (\leq \ or \ = \ or \ \geq)\ \mathbf{q},\ \mathbf{y} \geq 0$$

$$d^-_{ij\cdot},\ d^+_{ij\cdot} \geq 0 \text{ and } d^-_{ij\cdot} \times d^+_{ij\cdot} = 0, \quad \forall\ i = 1,\ 2,\ \ldots,\ t, \quad j = 1,\ 2,\ \ldots,\ m_i$$

$$d^-_{ik\cdot},\ d^+_{ik\cdot} \geq 0 \text{ and } d^-_{ik\cdot} \times d^+_{ik\cdot} = 0, \quad \forall\ i = 1,\ 2,\ \ldots,\ t-1, \quad k = 1,\ 2,\ \ldots,\ n_i \tag{15}$$

Now the above model in Eq. (15) can be represented as follows:

$$\begin{aligned}
\text{Min F} = & \sum_{j=1}^{m_1} w^+_{1j\mu} d^+_{1j\mu} + \sum_{j=1}^{m_2} w^+_{2j\mu} d^+_{2j\mu} + \cdots + \sum_{j=1}^{m_t} w^+_{tj\mu} d^+_{tj\mu} \\
& + \sum_{j=1}^{m_1} w^+_{1j\lambda} d^+_{1j\lambda} + \sum_{j=1}^{m_2} w^+_{2j\lambda} d^+_{2j\lambda} + \cdots + \sum_{j=1}^{m_t} w^+_{tj\lambda} d^+_{tj\lambda} \\
& - \sum_{j=1}^{m_1} w^+_{1j\nu} d^-_{1j\nu} - \sum_{j=1}^{m_2} w^+_{2j\nu} d^-_{2j\nu} - \cdots - \sum_{j=1}^{m_t} w^+_{tj\nu} d^-_{tj\nu} \\
& + \sum_{k=1}^{n_1} \left(w^\alpha_{1k\cdot} (d^{\alpha-}_{1k\cdot} + d^{\alpha+}_{1k\cdot}) + w^\beta_{1k\cdot} (d^{\beta-}_{1k\cdot} + d^{\beta+}_{1k\cdot}) \right) \\
& + \sum_{k=1}^{n_2} \left(w^\alpha_{2k\cdot} (d^{\alpha-}_{2k\cdot} + d^{\alpha+}_{2k\cdot}) + w^\beta_{2k\cdot} (d^{\beta-}_{2k\cdot} + d^{\beta+}_{2k\cdot}) \right) \quad \cdots \quad \cdots \quad \cdots \\
& + \sum_{k=1}^{n_{t-1}} \left(w^\alpha_{t-1k\cdot} (d^{\alpha-}_{t-1k\cdot} + d^{\alpha+}_{t-1k\cdot}) + w^\beta_{t-1k\cdot} (d^{\beta-}_{t-1k\cdot} + d^{\beta+}_{t-1k\cdot}) \right)
\end{aligned}$$

subject to

$$1 - \frac{o_{ij}(\mathbf{y}) - L^\mu_{ij}}{U^\mu_{ij} - L^\mu_{ij}} + d^-_{tj\mu} - d^+_{tj\mu} = 1, \quad i = 1,\ 2,\ \ldots,\ t, \quad j = 1,\ 2,\ \ldots,\ m_i$$

$$1 - \frac{o_{ij}(\mathbf{y}) - L^\lambda_{ij}}{U^\lambda_{ij} - L^\lambda_{ij}} + d^-_{tj\lambda} - d^+_{tj\lambda} = 0.5, \quad i = 1,\ 2,\ \ldots,\ t, \quad j = 1,\ 2,\ \ldots,\ m_i$$

$$1 - \frac{U^\nu_{ij} - o_{ij}(\mathbf{y})}{U^\nu_{ij} - L^\nu_{ij}} + d^-_{tj\nu} - d^+_{tj\nu} = 0, \quad i = 1,\ 2,\ \ldots,\ t, \quad j = 1,\ 2,\ \ldots,\ m_i$$

$$\frac{y_{ik} - \mu^L_{y_{ik}}}{y^*_{ik} - \mu^L_{y_{ik}}} + d^-_{1k\mu} - d^+_{1k\mu} = 1, \quad i = 1,\ 2,\ \ldots,\ t-1, \quad k = 1,\ 2,\ \ldots,\ n_i$$

$$\frac{\mu^U_{y_{ik}} - y_{ik}}{\mu^U_{y_{ik}} - y^*_{ik}} + d^-_{1k\mu} - d^+_{1k\mu} = 1, \quad i = 1, 2, \ldots, t-1, \quad k = 1, 2, \ldots, n_i$$

$$\frac{y_{ik} - \lambda^L_{y_{ik}}}{y^*_{ik} - \lambda^L_{y_{ik}}} + d^-_{1k\lambda} - d^+_{1k\lambda} = 0.5, \quad i = 1, 2, \ldots, t-1, \quad k = 1, 2, \ldots, n_i$$

$$\frac{\lambda^U_{y_{ik}} - y_{ik}}{\lambda^U_{y_{ik}} - y^*_{ik}} + d^-_{1k\lambda} - d^+_{1k\lambda} = 0.5, \quad i = 1, 2, \ldots, t-1, \quad k = 1, 2, \ldots, n_i$$

$$\frac{y_{ik} - \nu^L_{y_{ik}}}{y^*_{ik} - \nu^L_{y_{ik}}} + d^-_{1k\nu} - d^+_{1k\nu} = 0, \quad i = 1, 2, \ldots, t-1, \quad k = 1, 2, \ldots, n_i$$

$$\frac{\nu^U_{y_{ik}} - y_{ik}}{\nu^U_{y_{ik}} - y^*_{ik}} + d^-_{1k\nu} - d^+_{1k\nu} = 0, \quad i = 1, 2, \ldots, t-1, \quad k = 1, 2, \ldots, n_i$$

$$G_1 y_1 + G_1 y_1 + \cdots + G_t y_t (\leq \ or \ = \ or \ \geq) \, \mathbf{q}, \ \mathbf{y} \geq 0$$

$$d^-_{ij\cdot}, \ d^+_{ij\cdot} \geq 0 \text{ and } d^-_{ij\cdot} \times d^+_{ij\cdot} = 0, \ \ \forall \ i = 1, 2, \ldots, t, \ \ j = 1, 2, \ldots, m_i$$

$$d^{\alpha-}_{ik\cdot}, \ d^{\alpha+}_{ik\cdot} \geq 0 \text{ and } d^{\alpha-}_{ik\cdot} \times d^{\alpha+}_{ik\cdot} = 0, \ \ \forall \ i = 1, 2, \ldots, t-1, \ \ k = 1, 2, \ldots, n_i$$

$$d^{\beta-}_{ik\cdot}, \ d^{\beta+}_{ik\cdot} \geq 0 \text{ and } d^{\beta-}_{ik\cdot} \times d^{\beta+}_{ik\cdot} = 0, \ \ \forall \ i = 1, 2, \ldots, t-1, \ \ k = 1, 2, \ldots, n_i, \tag{16}$$

where F represents the neutrosophic achievement function comprising the weighted over-deviational variables $d^+_{ij\cdot}$, $\forall \ i = 1, 2, \ldots, t, \ \ j = 1, 2, \ldots, m_i$ of the neutrosophic goals l_{ij} and the under-deviational and over-deviational variables $d^{\alpha-}_{ik\cdot}$, $d^{\alpha+}_{ik\cdot}$, $d^{\beta-}_{ik\cdot}$ and $d^{\beta+}_{ik\cdot}$, $\forall \ i = 1, 2, \ldots, t-1, \ \ k = 1, 2, \ldots, n_i$ for the neutrosophic goals of all the decision variable vectors for the leader $t-1$-th levels. The corresponding weights $w^+_{ij\cdot}$, $w^{\alpha}_{ik\cdot}$ and $w^{\beta}_{ik\cdot}$ depict the relative importance of attaining the aspired levels of the respective neutrosophic goals under the given constraints in the hierarchical decision-making scenarios.

To assign the different relative importance of the neutrosophic goals adequately, we have suggested the weighting scheme with the aid of u_{ij} and l_{ij}. The weighting scheme to each weight $w^+_{ij\cdot}$, $w^{\alpha}_{ik\cdot}$, and $w^{\beta}_{ik\cdot}$ has been stated as follows:

$$w^+_{ij\cdot} = \frac{1}{u_{ij} - l_{ij}}, \ \ \forall \ i = 1, 2, \ldots, t, \ \ j = 1, 2, \ldots, m_i \tag{17}$$

$$w^{\alpha}_{ik\cdot} = \frac{1}{T^{i\alpha}_k} \text{ and } w^{\beta}_{ik\cdot} = \frac{1}{T^{i\beta}_k}, \ \ \forall \ i = 1, 2, \ldots, t-1, \ \ k = 1, 2, \ldots, n_i. \tag{18}$$

The NFGP model (16) gives the most satisfactory solution for the decision makers at all levels by attaining the aspired level of different neutrosophic membership goals at utmost possible in neutrosophic decision environment. The solution method is quite simple and demonstrated with the help of numerical examples in Sect. 5.

The step-wise solution procedure for the first NFGP algorithm for solving ML-MOLPPs can be stated as follows:

1. Solve each objectives individually for all levels under given constraints in order to find the maximum and minimum values of each objectives at all levels.
2. Depict the goals and upper tolerance limits—u_{ij}, l_{ij} ; $\forall\ i = 1, 2, \ldots, t,\ j = 1, 2, \ldots, m_i$—for each objectives at all levels.
3. Calculate the weights, $w^{+}_{ij.} = \dfrac{1}{u_{ij} - l_{ij}},\ \forall\ i = 1, 2, \ldots, t,\ j = 1, 2, \ldots, m_i$ and set $g = 1$.
4. Evaluate the different membership functions $\mu_{o_{gj}}(o_{gj}(\mathbf{y}))$, $\lambda_{o_{gj}}(o_{gj}(\mathbf{y}))$ and $\nu_{o_{gj}}(o_{gj}(\mathbf{y}))$, $j = 1, 2, \ldots, m_g$ for each objective function under neutrosophic environment.
5. Develop the model given in Eq. (22) for the g-th level MOLPPs.
6. Obtain the value of $\mathbf{y}^{g*} = (\mathbf{y}^{g*}_1, \mathbf{y}^{g*}_2, \ldots, \mathbf{y}^{g*}_t)$ by solving model given in Eq. (22).
7. Impose the maximum negative and positive tolerance limits on the decision variable vectors $\mathbf{y}^{g*}_g = (\mathbf{y}^{g*}_{g1}, \mathbf{y}^{g*}_{g2}, \ldots, \mathbf{y}^{g*}_{gn_g})$, $T^{g\alpha}_k$ and $T^{g\beta}_k$; $k = 1, 2, \ldots, n_g$.
8. Calculate the weights $w^{\alpha}_{gk.} = \dfrac{1}{T^{g\alpha}_k}$ and $w^{\beta}_{gk.} = \dfrac{1}{T^{g\beta}_k}$, $k = 1, 2, \ldots, n_g$.
9. Evaluate the different membership functions $\mu_{y_{gk}}(y_{gk})$, $\lambda_{y_{gk}}(y_{gk})$ and $\nu_{y_{gk}}(y_{gk})$ for the decision variable vectors $\mathbf{y}^{g*}_g = (\mathbf{y}^{g*}_{g1}, \mathbf{y}^{g*}_{g2}, \ldots, \mathbf{y}^{g*}_{gn_g})$ given in Eq. (12).
10. If $g > t - 1$, then proceed to step 11, otherwise go to step 4.
11. Depict the different membership functions $\mu_{o_{tj}}(o_{tj}(\mathbf{y}))$, $\lambda_{o_{tj}}(o_{tj}(\mathbf{y}))$ and $\nu_{o_{tj}}(o_{tj}(\mathbf{y}))$, $j = 1, 2, \ldots, m_t$ for each objective function at the p-th level under neutrosophic environment.
12. Calculate the weights, $w^{+}_{tj.} = \dfrac{1}{u_{tj} - l_{tj}},\ \forall\ j = 1, 2, \ldots, m_t$.
13. Formulate the model given in Eq. (16) under neutrosophic environment and solve it to get the satisfactory solution of the ML-MOLPPs.

4.3.2 The Second NFGP Algorithm for ML-MOLPPs

In the first NFGP algorithm, the final model contains the different membership functions for the neutrosophic goals of the decision variable vectors monitored by $t - 1$ levels, which separately solves for the i-th level MOLPPs. The second NFGP algorithm solves t MOLPPs that considers the decisions of the leader levels. After initialization steps 1 to 3 in first algorithm, the solution methods initiate with MOLPP of the first decision maker obtaining the compromise solution. A marginal evaluation of the first decision maker's decisions is taken into account to get rid of the decision deadlock. Hence, decisions of the first decision maker are depicted by the different membership functions under neutrosophic environment and sent to the second decision maker as additional auxiliary constraints. Afterward, the second decision maker considers the neutrosophic membership goals of the objectives as well as decision variable vectors of the first decision maker. After that, the achieved solution is passed to the third decision maker who tries to find out the optimal solution in a similar fashion. The processes of finding the optimal solution are repeated until the follower level is reached and consequently, the process is terminated.

The step-wise solution procedure for the second NFGP algorithm for solving ML-MOLPPs can be stated as follows:

1. Follow the same procedure from steps 1 to 9 as discussed in the first NFGP algorithm.
2. Formulate the model given in Eq. (16) for the ML-MOLPPs with $t = g$ under neutrosophic environment.
3. Solve the model given in Eq. (16) to get $\mathbf{y}^{g*} = (\mathbf{y}_1^{g*}, \ \mathbf{y}_2^{g*}, \ \ldots, \ \mathbf{y}_t^{g*})$.
4. Establish $g = g + 1$.
5. If $g > t$, then terminating with a satisfactory solution results $\mathbf{y}^{g*} = (\mathbf{y}_1^{g*}, \ \mathbf{y}_2^{g*}, \ \ldots, \ \mathbf{y}_t^{g*})$ to the ML-MOLPPs, otherwise proceed to step 7 of the first NFGP algorithm.

According to the solution priority, the second NFGP algorithm can be used to obtain the direct solution of the ML-MOLPPs to decisions of the first-level decision maker. After that, it directs the solutions to the decisions of second-level decision maker by preserving the solution closer to the decisions of first-level decision maker. Thus, the process goes on until the last level of the ML-MOLPPs preserving the solution closer to the decision of the leader levels.

5 Numerical Illustrations

The following numerical example consisting of tri-level multiobjective linear programming problems is presented to show the validity and applicability of the proposed NFGP optimization algorithms.

[1st level]

$$\underset{y_1}{\text{Min}}\ O_1(\mathbf{y}) = \underset{y_1}{\text{Min}}\ \left(o_{11}(\mathbf{y}) = y_1 - y_2 - 4y_3,\ o_{12}(\mathbf{y}) = -y_1 + 3y_2 - 4y_3\right),$$

where $\mathbf{y}_2$ and $\mathbf{y}_3$ solve

[2nd level]

$$\underset{y_2}{\text{Min}}\ O_2(\mathbf{y}) = \underset{y_2}{\text{Min}}\ \left(o_{21}(\mathbf{y}) = 2y_1 - y_2 + 2y_3,\ o_{22}(\mathbf{y}) = 2y_1 + y_2 - 3y_3,\right.$$
$$\left. o_{23}(\mathbf{y}) = 3y_1 - y_2 + y_3\right),$$

where $\mathbf{y}_3$ solves

Table 1 Individual minimum and maximum values for each objectives

	1st level		2nd level			3rd level	
	o_{11}	o_{12}	o_{21}	o_{22}	o_{23}	o_{31}	o_{32}
$\min_S o_{ij}$	−2.5	−3.5	−1	−1	−1	−0.5	0
$\max_S o_{ij}$	1	3	4	2	5	8.5	2

[3rd level]

$$\underset{y_3}{\text{Min}}\ O_3(\mathbf{y}) = \underset{y_3}{\text{Min}}\ \left(o_{31}(\mathbf{y}) = 7y_1 + 3y_2 - 4y_3,\ o_{32}(\mathbf{y}) = y_1 + y_3\right)$$

subject to

$$y_1 + y_2 + y_3 \le 3, \quad y_1 + y_2 - y_3 \le 1,$$

$$y_1 + y_2 + y_3 \ge 1, \quad -y_1 + y_2 + y_3 \le 1,$$

$$y_3 \le 0.5, \quad y_1,\ y_2,\ y_3 \ge 0.$$

The individual minimum and maximum values of each objective function for all the three levels of MOLPP under the given constraints S is furnished in Table 1. To apply the proposed NFGP algorithms, the aspiration levels and leader tolerance limits to the objective functions may be taken as the minimum and maximum individual optimal solutions.

The first NFGP algorithm can be elaborated through the solution method of the second NFGP algorithm. Thus, the following is the proposed first NFGP algorithm to tri-level multiobjective linear programming problem with the step-wise solution procedures.

$$First - level\ decision\ maker's\ NFGP\ model:$$

$$\text{Min } F_1 = 0.286d^+_{11\mu} + 0.154d^+_{12\mu} + 0.286d^+_{11\lambda} + 0.154d^+_{12\lambda} - 0.286d^-_{11\nu} - 0.154d^-_{12\nu}$$

subject to

$$-0.286y_1 + 0.286y_2 + 1.143y_3 + d^-_{11\mu} - d^+_{11\mu} = 0.714$$

$$-0.286y_1 + 0.286y_2 + 1.143y_3 + d^-_{11\lambda} - d^+_{11\lambda} = 0.143$$

$$-0.286y_1 + 0.286y_2 + 1.143y_3 + d^-_{11\nu} - d^+_{11\nu} = 0.03$$

$$0.154y_1 - 0.154y_2 + 0.62y_3 + d^-_{12\mu} - d^+_{12\mu} = 0.54$$

$$0.154y_1 - 0.154y_2 + 0.62y_3 + d^-_{12\lambda} - d^+_{12\lambda} = 0.21$$

$$0.154y_1 - 0.154y_2 + 0.62y_3 + d^-_{12\nu} - d^+_{12\nu} = 0.07$$

$$
\begin{aligned}
& y_1+y_2+y_3 \leq 3, \quad y_1+y_2-y_3 \leq 1, \\
& y_1+y_2+y_3 \geq 1, \quad -y_1+y_2+y_3 \leq 1, \\
& y_3 \leq 0.5, \quad y_1,\ y_2,\ y_3 \geq 0, \\
& d_{ij.}^{-},\ d_{ij.}^{+} \geq 0 \text{ and } d_{ij.}^{-} \times d_{ij.}^{+}=0, \quad \forall\ i=1, \quad j=1,\ 2.
\end{aligned} \tag{19}
$$

With the help of optimizing software, the optimal solution of the problem given in Eq. (19) is $\mathbf{y}^{1*} = (0.5, 0, 0.5)$. Assume that the first-level decision maker assigns $y_1^{1*} = 0.5$ along with the negative and positive tolerances $T_1^{1\alpha} = T_1^{1\beta} = 0.5$ and with the weights $w_{11.}^{\alpha} = w_{11.}^{\beta} = \dfrac{1}{0.5} = 2$, respectively.

$$Second-level\ decision\ maker's\ NFGP\ model:$$

$$
\begin{aligned}
& \text{Min } F_1 = 0.286d_{11\mu}^{+} + 0.154d_{12\mu}^{+} + 0.286d_{11\lambda}^{+} + 0.154d_{12\lambda}^{+} - 0.286d_{11\nu}^{-} \\
& -0.154d_{12\nu}^{-} + 0.2d_{21\mu}^{+} + 0.33d_{22\mu}^{+} + 0.167d_{23\mu}^{+} + 0.2d_{21\lambda}^{+} + 0.33d_{22\lambda}^{+} \\
& +0.167d_{23\lambda}^{+} - 0.2d_{21\nu}^{-} - 0.33d_{22\nu}^{-} - 0.167d_{23\nu}^{-} + 2\left[d_{11.}^{-\alpha} + d_{11.}^{+\alpha} + d_{11.}^{-\beta} + d_{11.}^{+\beta}\right] \\
& \text{subject to} \\
& -0.4y_1 + 0.2y_2 - 0.4y_3 + d_{21\mu}^{-} - d_{21\mu}^{+} = 0.2 \\
& -0.4y_1 + 0.2y_2 - 0.4y_3 + d_{21\lambda}^{-} - d_{21\lambda}^{+} = 0.13 \\
& -0.4y_1 + 0.2y_2 - 0.4y_3 + d_{21\nu}^{-} - d_{21\nu}^{+} = 0.04 \\
& -0.667y_1 - 0.33y_2 + y_3 + d_{22\mu}^{-} - d_{22\mu}^{+} = 0.33 \\
& -0.667y_1 - 0.33y_2 + y_3 + d_{22\lambda}^{-} - d_{22\lambda}^{+} = 0.18 \\
& -0.667y_1 - 0.33y_2 + y_3 + d_{22\nu}^{-} - d_{22\nu}^{+} = 0.02 \\
& -0.5y_1 + 0.167y_2 - 0.167y_3 + d_{23\mu}^{-} - d_{23\mu}^{+} = 0.17 \\
& -0.5y_1 + 0.167y_2 - 0.167y_3 + d_{23\lambda}^{-} - d_{23\lambda}^{+} = 0.09 \\
& -0.5y_1 + 0.167y_2 - 0.167y_3 + d_{23\nu}^{-} - d_{23\nu}^{+} = 0.01 \\
& 2y_1 + d_{11\mu}^{-\alpha} - d_{11\mu}^{+\alpha} = 1, \quad 2y_1 + d_{11\mu}^{-\beta} - d_{11\mu}^{+\beta} = 1, \\
& 2y_1 + d_{11\lambda}^{-\alpha} - d_{11\lambda}^{+\alpha} = 0.5, \quad 2y_1 + d_{11\lambda}^{-\beta} - d_{11\lambda}^{+\beta} = 0.5, \\
& 2y_1 + d_{11\nu}^{-\alpha} - d_{11\nu}^{+\alpha} = 0, \quad 2y_1 + d_{11\nu}^{-\beta} - d_{11\nu}^{+\beta} = 0,
\end{aligned}
$$

constraints (19)

$$d_{ik\cdot}^{\alpha-},\ d_{ik\cdot}^{\alpha+} \geq 0 \text{ and } d_{ik\cdot}^{\alpha-} \times d_{ik\cdot}^{\alpha+} = 0,\ \ \forall\ i = 1,\ 2\ \ k = 1. \tag{20}$$
$$d_{ik\cdot}^{\beta-},\ d_{ik\cdot}^{\beta+} \geq 0 \text{ and } d_{ik\cdot}^{\beta-} \times d_{ik\cdot}^{\beta+} = 0,\ \ \forall\ i = 1,\ 2\ \ k = 1.$$

The optimal solution for the second-level NFGP model in Eq. (20) is obtained as $\mathbf{y}^{2*} = (0.5, 0, 0.5),\ (0.5, 0.998, 0.5),\ (0.5, 0.5, 0)$. Suppose that second-level decision maker finalizes $y_1^{2*} = 0.998$ along with the negative and positive tolerances $T_1^{2\alpha} = 0.75$, and $T_1^{2\beta} = 0.25$ and with weights $w_{21\cdot}^{\alpha} = \dfrac{1}{0.75} = 1.333$, and $w_{21\cdot}^{\beta} = \dfrac{1}{0.25} = 4$, respectively.

$Third - level\ decision\ maker's\ NFGP\ model:$

$$\begin{aligned}
&\text{Min } F_1 = 0.286d_{11\mu}^{+} + 0.154d_{12\mu}^{+} + 0.286d_{11\lambda}^{+} + 0.154d_{12\lambda}^{+} - 0.286d_{11\nu}^{-} \\
&- 0.154d_{12\nu}^{-} + 0.2d_{21\mu}^{+} + 0.33d_{22\mu}^{+} + 0.167d_{23\mu}^{+} + 0.2d_{21\lambda}^{+} + 0.33d_{22\lambda}^{+} \\
&+ 0.167d_{23\lambda}^{+} - 0.2d_{21\nu}^{-} - 0.33d_{22\nu}^{-} - 0.167d_{23\nu}^{-} + 2\left[d_{11\cdot}^{-\alpha} + d_{11\cdot}^{+\alpha} + d_{11\cdot}^{-\beta} + d_{11\cdot}^{+\beta}\right] \\
&+ 1.33(d_{21\cdot}^{-\alpha} + d_{21\cdot}^{+\alpha}) + 4(d_{21\cdot}^{-\beta} + d_{21\cdot}^{+\beta}) \\
&\text{subject to} \\
&- 0.78y_1 + 0.33y_2 + 0.44y_3 + d_{31\mu}^{-} - d_{31\mu}^{+} = 0.06 \\
&- 0.78y_1 + 0.33y_2 + 0.44y_3 + d_{31\lambda}^{-} - d_{31\lambda}^{+} = \\
&- 0.78y_1 + 0.33y_2 + 0.44y_3 + d_{31\nu}^{-} - d_{31\nu}^{+} = \\
&- 0.5y_1 - 0.5y_3 + d_{32\mu}^{-} - d_{32\mu}^{+} = 0 \\
&- 0.5y_1 - 0.5y_3 + d_{32\lambda}^{-} - d_{32\lambda}^{+} = 0 \\
&- 0.5y_1 - 0.5y_3 + d_{32\nu}^{-} - d_{32\nu}^{+} = 0 \\
&1.33y_2 + d_{21\mu}^{-\alpha} - d_{21mu}^{+\alpha} = 1.33,\ \ 4y_1 + d_{21\mu}^{-\beta} - d_{21\mu}^{+\beta} = 3.99, \\
&1.33y_2 + d_{21\lambda}^{-\alpha} - d_{21\lambda}^{+\alpha} = 0.94,\ \ 4y_1 + d_{21\lambda}^{-\beta} - d_{21\lambda}^{+\beta} = 2.35, \\
&1.33y_2 + d_{21\nu}^{-\alpha} - d_{21\nu}^{+\alpha} = 0.35,\ \ 4y_1 + d_{21\nu}^{-\beta} - d_{21\nu}^{+\beta} = 1.86, \\
&\text{constraints (20)} \\
&d_{ik\cdot}^{\alpha-},\ d_{ik\cdot}^{\alpha+} \geq 0 \text{ and } d_{ik\cdot}^{\alpha-} \times d_{ik\cdot}^{\alpha+} = 0,\ \ \forall\ i = 1,\ 2,\ 3\ \ k = 1,\ 2. \\
&d_{ik\cdot}^{\beta-},\ d_{ik\cdot}^{\beta+} \geq 0 \text{ and } d_{ik\cdot}^{\beta-} \times d_{ik\cdot}^{\beta+} = 0,\ \ \forall\ i = 1,\ 2,\ 3\ \ k = 1,\ 2.
\end{aligned} \tag{21}$$

Table 2 Comparison of optimal solutions and satisfactory degrees of the given example

Proposed NFGP algorithm	Baky approach	Abo-Sinna approach	Shih approach
$(o_{11},\ \mu_{11}) = (-2.499, 0.999)$	(−2.498, 0.99)	(−2.21, 0.92)	(−2.21, 0.92)
$(o_{12},\ \mu_{12}) = (0.4941, 0.399)$	(0.494, 0.39)	(−0.569, 0.55)	(−0.569, 0.55)
$(o_{21},\ \mu_{21}) = (1.002, 0.59)$	(1.002, 0.6)	(1.88, 0.56)	(1.88, 0.56)
$(o_{22},\ \mu_{22}) = (0.498, 0.50)$	(0.498, 0.5)	(−0.09, 0.7)	(−0.09, 0.7)
$(o_{23},\ \mu_{23}) = (1.002, 0.67)$	(1.002, 0.67)	(1.09, 0.65)	(1.09, 0.65)
$(o_{31},\ \mu_{31}) = (4.491, 0.47)$	(4.493, 0.45)	(2.62, 0.65)	(2.62, 0.65)
$(o_{32},\ \mu_{32}) = (1, 0.50)$	(1, 0.50)	(0.899, 0.55)	(0.899, 0.55)
$\mathbf{y}* = (0.5, 0.9975, 0.5)$	(0.5, 0.998, 0.5)	(0.339, 0.61, 0.5)	(0.339, 0.61, 0.5)

Table 3 Theoretical comparison of proposed NFGP algorithms with others

Proposed NFGP approach	Other approaches
Proposed approach considers the indeterminacy degree in decision-making process.	Abo-Sinna [1], Baky [8], and Shih et al. [16] cannot deal with indeterminacy in decision-making processes.
The overall satisfactory degree is achieved by attaining the neutrosophic fuzzy goals.	In [1, 8, 16] approaches, satisfactory degree is achieved by attaining the fuzzy goals.
It characterizes neutrosophic membership functions for both objectives and decision variables under neutrosophic environment.	Abo-Sinna [1], Baky [8], and Shih et al. [16] do not cover this aspects.
Additional predetermined parameters in indeterminacy and falsity degrees make the decisions more flexible according to decision makers' choices.	This facility is not provided in Abo-Sinna [1], Baky [8], and Shih et al. [16]

The final optimal solution for the ML-MOLPPs given in Eq. (21) is obtained as $\mathbf{y}^{3*} = (0.5, 0.9975, 0.5)$ with the different objectives values $o_{11} = -2.499$, $o_{12} = 0.4941$, $o_{21} = 1.002$, $o_{22} = 0.498$, $o_{23} = 1.002$, $o_{31} = 4.491$, and $o_{31} = 1$, along with membership functions $\mu_{11} = 0.999$, $\mu_{12} = 0.399$, $\mu_{21} = 0.590$, $\mu_{22} = 0.50$, $\mu_{23} = 0.67$, $\mu_{31} = 0.47$, and $\mu_{31} = 0.50$, respectively. A comparative study is performed among the proposed NFGP algorithm and presented in the Table 2. Other approaches reveal that the solution results are very close to [8], whereas [1, 16] give the same solution results for the presented numerical examples. Furthermore, the theoretical contributions in the domain of ML-MOLPPs are also summarized in Table 3.

6 Conclusions

This chapter proposes two different neutrosophic fuzzy goal programming algorithms for the solutions of ML-MOLPPs. The neutrosophic goal programming model is constructed to minimize the group tolerance of satisfactory degree of all the decision makers and to attain the highest degree for truth (unity), indeterminacy (half), and a falsity (zero) of each kind of the defined membership functions' goals to the utmost possible by minimizing their respective deviational variables and so that obtain the optimal solution for all decision makers. The primary advantages of the proposed two different neutrosophic fuzzy goal programming algorithms that the chances of refusing the solution repeatedly by the leader-level decision maker and reevaluation of the problem again and again by restating the defined membership functions required to reach the optimal solution would not arise.

The first NFGP algorithm considers the different membership functions for the defined neutrosophic goals of the objective functions at all levels as well as the different membership functions for the neutrosophic goals for the decision variable vectors at each level except the follower level of the ML-MOLPPs. The second NFGP algorithm solves the MOLPPs of the ML-MOLPPs by taking into account the decisions of the MOLPPs for the leader level only. A numerical example is presented to show the validity and applicability of the proposed NFGP algorithms with the fact that the degree of indeterminacy may arise in the hierarchical decision-making processes and can be overcome by utilizing the proposed algorithms. In future, it can be applied to real-life applications such as transportation, assignment, vendor selection, inventory control, supply chain, etc. and problems in multi-level decision-making scenarios.

Appendix

The NFGP approach to solve the single-level MOLPPs is presented to facilitate the achievement function $\mathbf{y}^{g*} = (\mathbf{y}_1^{g*},\ \mathbf{y}_2^{g*},\ \ldots,\ \mathbf{y}_t^{g*}),\ \ g = 1,\ 2,\ \cdots,\ t-1$. By using the same notations and symbols of this chapter, the NFGP model can be formulated for any g-th level MOLPPs and can be stated as follows:

$$
\begin{aligned}
&\text{Min } F = \sum_{j=i}^{m_g} w_{gj\mu}^{+} d_{gj\mu}^{+} + w_{gj\lambda}^{+} d_{gj\lambda}^{+} - w_{gj\nu}^{+} d_{gj\nu}^{-} \\
&\text{subject to} \\
&c_1^{gj}\mathbf{y}_1 + c_2^{gj}\mathbf{y}_2 + \cdots + c_t^{gj}\mathbf{y}_t + d_{gj\mu}^{-} - d_{gj\mu}^{+} = 1,\quad j = 1,\ 2,\ \ldots,\ m_g \\
&c_1^{gj}\mathbf{y}_1 + c_2^{gj}\mathbf{y}_2 + \cdots + c_t^{gj}\mathbf{y}_t + d_{gj\lambda}^{-} - d_{gj\lambda}^{+} = 0.5,\quad j = 1,\ 2,\ \ldots,\ m_g \\
&c_1^{gj}\mathbf{y}_1 + c_2^{gj}\mathbf{y}_2 + \cdots + c_t^{gj}\mathbf{y}_t + d_{gj\nu}^{-} - d_{gj\nu}^{+} = 0,\quad j = 1,\ 2,\ \ldots,\ m_g \\
&G_1 y_1 + G_1 y_1 + \cdots + G_t y_t (\leq\ or\ =\ or\ \geq)\ \mathbf{q},\ \mathbf{y} \geq 0 \\
&d_{gj\cdot}^{-},\ d_{gj\cdot}^{+} \geq 0 \text{ and } d_{gj\cdot}^{-} \times d_{gj\cdot}^{+} = 0,\quad j = 1,\ 2,\ \ldots,\ m_g
\end{aligned}
\tag{22}
$$

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