

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/379808642>

Neutrosophic Binary Separation Axioms associated Neutrosophic Binary Kernel Set

Article in International Journal of Neutrosophic Science · April 2024

DOI: 10.54216/IJNS.24011

CITATIONS

0

READS

80

5 authors, including:



[Amer khrija Abed](#)

Al Muthanna University

15 PUBLICATIONS 8 CITATIONS

SEE PROFILE



[Sarab Kazim Hassan](#)

Al Muthanna University

4 PUBLICATIONS 9 CITATIONS

SEE PROFILE



[Qays Hatem Imran](#)

Al-Muthanna University

45 PUBLICATIONS 180 CITATIONS

SEE PROFILE



[Ali Hussein Mahmood Al-Obaidi](#)

University of Babylon

15 PUBLICATIONS 51 CITATIONS

SEE PROFILE



Neutrosophic Binary Separation Axioms associated Neutrosophic Binary Kernel Set

Amer Khrija Abed^{1*}, Sarab Kazim Hassan², Qays Hatem Imran³, Ali H. M. Al-Obaidi⁴, Said Broumi⁵

^{1,2,3}Department of Mathematics, College of Education for Pure Science, Al-Muthanna University, Samawah, Iraq.

⁴Department of Mathematics, College of Education for Pure Science, University of Babylon, Hillah, Iraq.

⁵Laboratory of Information Processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Morocco.

Emails: amer.khrija@mu.edu.iq; sarab.kadhim@mu.edu.iq; qays.imran@mu.edu.iq; aalobaidi@uobabylon.edu.iq; broumisaid78@gmail.com

*Correspondence: amer.khrija@mu.edu.iq

Abstract

The primary purpose of this report is to present an idea that is considered one of the main ideas in neutrosophic binary topological spaces. We plead them the axioms of neutrosophic binary separation axioms associated in the neutrosophic binary kernel. Also we studied its characteristics and the relationships between these new neutrosophic binary separation axioms and their relationships with some other properties.

Keywords: Neutrosophic Set; Neutrosophic Binary Separation; Neutrosophic Binary Kernel Set; binary topological spaces

Phrases: $\mathfrak{NEB}.T.S(X, Y)$, $O(\mathfrak{NEB}.T.S(X, Y))$, $C(\mathfrak{NEB}.T.S(X, Y))$, $\mathfrak{NEBCL}(P, Q)$, $\mathfrak{NEBINT}(P, Q)$, $\mathfrak{NEBKER}(P, Q)$, $S\mathfrak{NEB} - R_0.S$, $\mathfrak{NEB} - R_0.S$, $\mathfrak{NEB} - R_1.S$, $\mathfrak{NEB} - R_2.S$, $\mathfrak{NEB} - T_0.S$, $\mathfrak{NEB} - T_1.S$, $\mathfrak{NEB} - T_2.S$

1. Introduction

The neutrosophic set was introduced by Smarandache in 1998 [1] and explained, neutrosophic set is a generalization of intuitionistic fuzzy set. Salama, Bluey introduced a denotation of neutrosophic topological spaces in 2012 [3], and they introduced neutrosophic topological space as a generalization of the intuitive fuzzy topological space and topological space with the aspect of degree of membership, degree of indeterminacy degree of non-membership of each element. The connotation of separation axioms has always been important and indispensable in the branch of topology, research related to this concept has played fundamental and very important roles in many topological studies. Also, using the conception of neutrosophic topological space, many types of separation axioms are presented and studied [4], [5], [6], [7], [8], [9], [10], [11]. S. N. Jothi introduced the topology between two universal sets whose is defined to be binary topology in 2011 [12]. The binary topology is a binary structure from X to Y which consists of ordered pairs (P, Q) wherever $P \subseteq X$ and $Q \subseteq Y$. So S. N. Jothi introduced and studied of the conception of binary separation axioms in 2012 [13], [14], [15]. A neutrosophic binary structure from X to Y is defined as a neutrosophic set of ordered pairs (P, Q) where P neutrosophic set in X and Q neutrosophic set in Y . In continuation, S. S. Surekha, Sindhu G. and Broumi S. introduced, studied and developed the conception of neutrosophic binary Topological Space which consists of two universal sets and each universal set include its own reality, indeterminacy and false membership values in 2022 [16], [17]. Also A.G.Rose Venish and L. Vidyarani introduced along with some properties of neutrosophic binary crisp points in 2023 [18]. Imran et al. [19], [20], [21] gave fresh insights into the concepts of

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024

topological groups, generalized alpha generalized continuity and generalized semi generalized closed sets that are weakly neutrosophic. In this Article, our primary motivation is to define and recognize the axioms of separation using neutrosophic binary which have implications for the set of neutrosophic binary kernels. But, before we get into that, we've prepared a few definitions and assumptions based on the neutrosophic binary sets and also on the neutrosophic binary kernel sets. In the second part of this article, we defined the weaker dualistic neutrosophic separation axioms and studied their properties and relationships, which we called neutrosophic binary R_L -Spaces, $L = 0, 1, 2$. In the third part, we give the definition of the dualistic neutrosophic separation axioms, we call them neutrosophic binary T_L -Spaces, $L = 0, 1, 2$, and we mention some of their properties. In this part, we also prove their relationship with the previous type. In the latter part we wrote conclusions for this article.

2. Preliminaries

A neutrosophic binary set (P, Q) symbolized $\mathfrak{NEB.S}(P, Q)$, $(X, Y, BT_{\mathfrak{N}})$ is called neutrosophic binary topological spaces on $\mathfrak{NEB.S}(X, Y)$ (represent as $\mathfrak{NEB.T.S}(X, Y)$), the collection of each neutrosophic binary open set in $(X, Y, BT_{\mathfrak{N}})$ symbolized by $O(\mathfrak{NEB.T.S}(X, Y))$, the collection of each neutrosophic binary closed in $(X, Y, BT_{\mathfrak{N}})$ is symbolized by $C(\mathfrak{NEB.T.S}(X, Y))$, the binary neutrosophic closure, neutrosophic binary interior, neutrosophic binary kernel and neutrosophic binary complement of a $\mathfrak{NEB.S}(P, Q)$ in $\mathfrak{NEB.T.S}(X, Y)$ is notation $\mathfrak{NEBCL}(P, Q)$, $\mathfrak{NEBINT}(P, Q)$, $\mathfrak{NEBKER}(P, Q)$ and $(P, Q)^c$, respectively.

Definition 2.1 [14]: A neutrosophic binary topology is a binary structure consistence of two universal sets X and Y is where a family $BT_{\mathfrak{N}} \subseteq P(X) \times P(Y)$, which satisfies the following three conditions:

- (i) $(0_X, 0_Y)$ and $(1_X, 1_Y) \in BT_{\mathfrak{N}}$.
- (ii) $(P_1 \cap Q_1, P_2 \cap Q_2) \in BT_{\mathfrak{N}}$ whereas $(P_1 \cap Q_1) \in BT_{\mathfrak{N}}$ and $(P_2 \cap Q_2) \in BT_{\mathfrak{N}}$.
- (iii) If $\{(P_\ell, Q_\ell) : \ell \in \Delta\}$ is a family of members of $BT_{\mathfrak{N}}$, then $(\bigcup_{\alpha \in \Delta} P_\alpha, \bigcup_{\alpha \in \Delta} Q_\alpha) \in BT_{\mathfrak{N}}$.

The triplet $(X, Y, BT_{\mathfrak{N}})$ is called $\mathfrak{NEB.T.S}(X, Y)$. Every member (P, Q) of $\mathfrak{NEB.T.S}(X, Y)$ belong to $O(\mathfrak{NEB.T.S}(X, Y))$ and the complement of (P, Q) in $\mathfrak{NEB.T.S}(X, Y)$ belong to $(\mathfrak{NEB.T.S}(X, Y))$.

Definition 2.2 [14]:

- a) The order $(0_X, 0_Y)$ can be defined as
 - (i) $0_X = \{\langle x, 0, 0, 0 \rangle : x \in X\}$, $0_Y = \{\langle y, 0, 0, 0 \rangle : y \in Y\}$
 - (ii) $0_X = \{\langle x, 0, 0, 1 \rangle : x \in X\}$, $0_Y = \{\langle y, 0, 0, 1 \rangle : y \in Y\}$
 - (iii) $0_X = \{\langle x, 0, 1, 0 \rangle : x \in X\}$, $0_Y = \{\langle y, 0, 1, 0 \rangle : y \in Y\}$
 - (iv) $0_X = \{\langle x, 0, 1, 1 \rangle : x \in X\}$, $0_Y = \{\langle y, 0, 1, 1 \rangle : y \in Y\}$
- b) The order $(1_X, 1_Y)$ can be defined as
 - (i) $1_X = \{\langle x, 1, 1, 1 \rangle : x \in X\}$, $1_Y = \{\langle y, 1, 1, 1 \rangle : y \in Y\}$
 - (ii) $1_X = \{\langle x, 1, 1, 0 \rangle : x \in X\}$, $1_Y = \{\langle y, 1, 1, 0 \rangle : y \in Y\}$
 - (iii) $1_X = \{\langle x, 1, 0, 1 \rangle : x \in X\}$, $1_Y = \{\langle y, 1, 0, 1 \rangle : y \in Y\}$
 - (iv) $1_X = \{\langle x, 1, 0, 0 \rangle : x \in X\}$, $1_Y = \{\langle y, 1, 0, 0 \rangle : y \in Y\}$

Definition 2.3 [14] : The complement $(P, Q)^c$ of the $\mathfrak{NEB.S}(P, Q) = \{\langle x, \mu_P(x), \sigma_P(x), \gamma_P(x) \rangle : x \in X, \langle y, \mu_Q(y), \sigma_Q(y), \gamma_Q(y) \rangle : y \in Y\}$ on a $\mathfrak{NEB.T.S}(X, Y)$ can be defined as

- (i) $(P, Q)^c = \{\langle x, 1 - \mu_P(x), \sigma_P(x), 1 - \gamma_P(x) \rangle : x \in X, \langle y, 1 - \mu_Q(y), \sigma_Q(y), 1 - \gamma_Q(y) \rangle : y \in Y\}$
- (ii) $(P, Q)^c = \{\langle x, \gamma_P(x), \sigma_P(x), \mu_P(x) \rangle : x \in X, \langle y, \gamma_Q(y), \sigma_Q(y), \mu_Q(y) \rangle : y \in Y\}$
- (iii) $(P, Q)^c = \{\langle x, \gamma_P(x), 1 - \sigma_P(x), \mu_P(x) \rangle : x \in X, \langle y, \gamma_Q(y), 1 - \sigma_Q(y), \mu_Q(y) \rangle : y \in Y\}$

Definition 2.4 [14]: Let $(P_1, Q_1) = \{\langle \mu_{P_1}(x), \sigma_{P_1}(x), \gamma_{P_1}(x) \rangle, \langle \mu_{Q_1}(y), \sigma_{Q_1}(y), \gamma_{Q_1}(y) \rangle\}$ and $(P_2, Q_2) = \{\langle \mu_{P_2}(x), \sigma_{P_2}(x), \gamma_{P_2}(x) \rangle, \langle \mu_{Q_2}(y), \sigma_{Q_2}(y), \gamma_{Q_2}(y) \rangle\}$ be two neutrosophic binary sets. Then

- (i) $(P_1, Q_1) \subseteq (P_2, Q_2)$ if and only if $\mu_{P_1}(x) \leq \mu_{P_2}(x)$, $\sigma_{P_1}(x) \leq \sigma_{P_2}(x)$, $\gamma_{P_1}(x) \geq \gamma_{P_2}(x)$ for each $x \in X$, $\mu_{Q_1}(y) \leq \mu_{Q_2}(y)$, $\sigma_{Q_1}(y) \leq \sigma_{Q_2}(y)$, $\gamma_{Q_1}(y) \geq \gamma_{Q_2}(y)$ for each $y \in Y$

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024

- (ii) $(P_1, O_1) \subseteq (P_2, O_2)$ if and only if $\sigma_{O_1}(y) \geq \sigma_{O_2}(y), \mu_{P_1}(x) \leq \mu_{P_2}(x), \sigma_{P_1}(x) \geq \sigma_{P_2}(x), \gamma_{P_1}(x) \geq \gamma_{P_2}(x)$ for each $x \in X, \mu_{O_1}(y) \leq \mu_{O_2}(y), \gamma_{O_1}(y) \geq \gamma_{O_2}(y)$ for $\{ \langle x, \mu_{P_1}(x) \wedge \mu_{P_2}(x), \sigma_{P_1}(x) \wedge \sigma_{P_2}(x), \gamma_{P_1}(y) \vee \gamma_{P_2}(y) \rangle, \langle y, \mu_{O_1}(y) \wedge \mu_{O_2}(y), \sigma_{O_1}(y) \wedge \sigma_{O_2}(y), \gamma_{O_1}(y) \vee \gamma_{O_2}(y) \rangle \}$ each $y \in Y$
- (iii) $(P_1, O_1) \cap (P_2, O_2) =$
- (iv) $(P_1, O_1) \cap (P_2, O_2) = \{ \langle x, \mu_{P_1}(x) \wedge \mu_{P_2}(x), \sigma_{P_1}(x) \vee \sigma_{P_2}(x), \gamma_{P_1}(y) \vee \gamma_{P_2}(y) \rangle, \langle y, \mu_{O_1}(y) \wedge \mu_{O_2}(y), \sigma_{O_1}(y) \vee \sigma_{O_2}(y), \gamma_{O_1}(y) \vee \gamma_{O_2}(y) \rangle \}$
- (v) $(P_1, O_1) \cup (P_2, O_2) =$
- $\{ \langle x, \mu_{P_1}(x) \vee \mu_{P_2}(x), \sigma_{P_1}(x) \vee \sigma_{P_2}(x), \gamma_{P_1}(y) \wedge \gamma_{P_2}(y) \rangle, \langle y, \mu_{O_1}(y) \vee \mu_{O_2}(y), \sigma_{O_1}(y) \vee \sigma_{O_2}(y), \gamma_{O_1}(y) \wedge \gamma_{O_2}(y) \rangle \}$
- (vi) $(P_1, O_1) \cup (P_2, O_2) = \{ \langle x, \mu_{P_1}(x) \vee \mu_{P_2}(x), \sigma_{P_1}(x) \vee \sigma_{P_2}(x), \gamma_{P_1}(y) \wedge \gamma_{P_2}(y) \rangle, \langle y, \mu_{O_1}(y) \vee \mu_{O_2}(y), \sigma_{O_1}(y) \vee \sigma_{O_2}(y), \gamma_{O_1}(y) \wedge \gamma_{O_2}(y) \rangle \}$.

Definition 2.5 [14]: Let (X, Y, BT_N) be a $\mathcal{NEB}$.T.S, a $\mathcal{NEB}$.S $(P, O) = \{ \langle x, \mu_P(x), \sigma_P(x), \gamma_P(x) \rangle : x \in X, \langle y, \mu_O(y), \sigma_O(y), \gamma_O(y) \rangle : y \in Y \}$ is namely the neutrosophic binary point [shortly, $\mathcal{NEB}$.P] if and only if for each element $(x^*, y^*) \in (X, Y)$

$$\begin{cases} \mu_P(x^*) = \alpha_1, \mu_P(y^*) = \alpha_2 \\ \sigma_P(x^*) = \beta_1, \sigma_P(y^*) = \beta_2 \text{ for } (x^*, y^*) = (x, y) \\ \gamma_P(x^*) = \delta_1, \gamma_P(y^*) = \delta_2 \end{cases}$$

$$\begin{cases} \mu_P(x^*) = 0, \mu_P(y^*) = 0 \\ \sigma_P(x^*) = 1, \sigma_P(y^*) = 1 \text{ for } (x^*, y^*) \neq (x, y). \\ \gamma_P(x^*) = 1, \gamma_P(y^*) = 1 \end{cases}$$

$(x_{\alpha_1, \beta_1, \delta_1}, y_{\alpha_2, \beta_2, \delta_2})$, for the neutrosophic binary point $(x_{\alpha_1, \beta_1, \delta_1}, y_{\alpha_2, \beta_2, \delta_2})$, (x, y) will be called its support, The complement of the $(x_{\alpha_1, \beta_1, \delta_1}, y_{\alpha_2, \beta_2, \delta_2})$ will be denoted by $(x_{\alpha_1, \beta_1, \delta_1}, y_{\alpha_2, \beta_2, \delta_2})^c$.

Definition 2.6 [14]: Let (X, Y, BT_N) be a $\mathcal{NEB}$.T.S and a $\mathcal{NEB}$.S $(P, O) \subseteq (X, Y)$. The subset (P, O) is namely neutrosophic binary neighborhood of a $\mathcal{NEB}$.P (x, y) if and only if there exists $(P_0, O_0) \in O(\mathcal{NEB}$.T.S $(X, Y))$ so that $(x, y) \in (P_0, O_0) \subseteq (P, O)$.

Definition 2.7 [14]: Let (X, Y, BT_N) be a $\mathcal{NEB}$.T.S and $(P, O) \subseteq (X, Y)$,

- (i) The ordered pair $((P, O)_N^{1*}, (P, O)_N^{2*})$ is called $\mathcal{NEBCL}(P, O)$ where $(P, O)_N^{1*} = \cap \{ P_\ell : (P_\ell, O_\ell) \in C(\mathcal{NEB}$.T.S $(X, Y)) \& (P, O) \subseteq (P_\ell, O_\ell) \}$ and $(P, O)_N^{2*} = \cap \{ O_\ell : (P_\ell, O_\ell) \in C(\mathcal{NEB}$.T.S $(X, Y)) \& (P, O) \subseteq (P_\ell, O_\ell) \}$.

- (ii) The ordered pair $((P, O)^{1^\circ}, (P, O)^{2^\circ})$ is called $\mathcal{NEBINT}(P, O)$ where $(P, O)_N^{1^\circ} = \cup \{ P_\ell : (P_\ell, O_\ell) \in O(\mathcal{NEB}$.T.S $(X, Y)) \& (P, O) \supseteq (P_\ell, O_\ell) \}$ $(P, O)_N^{2^\circ} = \cup \{ O_\ell : (P_\ell, O_\ell) \in O(\mathcal{NEB}$.T.S $(X, Y)) \& (P, O) \supseteq (P_\ell, O_\ell) \}$

Theorem 2.8 [14]: Let $(P_1, O_1), (P_2, O_2) \subseteq (X, Y)$ and (X, Y, BT_N) be $\mathcal{NEB}$.T.S. Then the following statements are fulfilled:

- (i) $\mathcal{NEBINT}(P_1, O_1) \subseteq (P_1, O_1)$.
- (ii) If $\mathcal{NEBINT}(P_1, O_1)$ is neutrosophic binary open if and only if $\mathcal{NEBINT}(P_1, O_1) = (P_1, O_1)$.
- (iii) If $(P_1, O_1) \subseteq (P_2, O_2)$, then $\mathcal{NEBINT}(P_1, O_1) \subseteq \mathcal{NEBINT}(P_2, O_2)$.
- (iv) $\mathcal{NEBINT}(\mathcal{NEBINT}(P_1, O_1)) = \mathcal{NEBINT}(P_1, O_1)$.

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024

(v) $(P_1, Q_1) \subseteq \text{NEBCL}(P_1, Q_1)$.

(vi) If $(P_1, Q_1) \in C(\text{NEB.T.S}(X, Y))$ if and only if $\text{NEBCL}(P_1, Q_1) = (P_1, Q_1)$.

(vii) If $(P_1, Q_1) \subseteq (P_1, Q_1)$, then $\text{NEBCL}(P_1, Q_1) \subseteq \text{NEBCL}(P_2, Q_2)$.

Definition 2.9: Let (X, X, BT_N) be a NEB.T.S and $(P, Q) \subseteq (X, Y)$. The ordered pair $((P, Q)^{1**}, (P, Q)^{2**})$ is called $\text{NEBKER}(P, Q)$ where

$$(P, Q)^{1**} = \cap \{P_\ell: (P_\ell, Q_\ell) \in O(\text{NEB.T.S}(X, Y)) \& (P, Q) \subseteq (P_\ell, Q_\ell)\} \text{ and}$$

$$(P, Q)^{2**} = \cap \{Q_\ell: (P_\ell, Q_\ell) \in O(\text{NEB.T.S}(X, Y)) \& (P, Q) \subseteq (P_\ell, Q_\ell)\}.$$

For example: Let $X = \{p_1, p_2\}$ and $Y = \{q_1, q_2\}$ be the universe. Let $BT_N = (0_X, 0_Y), (1_X, 1_Y), (P, Q)$ be the NEB.T.S . So that $(P, Q) = \{ \langle X, (0.7, 0.5, 0.3) \rangle, \langle 0.6, 0.5, 0.4 \rangle \rangle, \langle Y, (0.6, 0.5, 0.4) \rangle, \langle 0.7, 0.5, 0.3 \rangle \}$. Let $(P_0, Q_0) = \{ \langle X, (0.4, 0.5, 0.8) \rangle, \langle 0.3, 0.4, 0.7 \rangle \rangle, \langle Y, (0.5, 0.5, 0.6) \rangle, \langle 0.4, 0.3, 0.5 \rangle \}$. Then $\text{NEBKER}(P_0, Q_0) = (P, Q)$

Theorem 2.10: Let (X, Y, BT_N) be a NEB.T.S and $(x, y) \in X \times Y$, then $\text{NEBKER}(P, Q) = \{(x, y) \in X \times Y : \text{NEBCL}(\{x\}, \{y\}) \cap (P, Q) \neq (0_X, 0_Y)\}$.

Proof: Let $(x, y) \in \text{NEBKER}(P, Q)$ & suppose that $\text{NEBCL}(\{x\}, \{y\}) \cap (P, Q) = (0_X, 0_Y)$. Hence $(x, y) \notin [(X, Y) - (\text{NEBCL}(\{x\}, \{y\}))]$ which is belong to $O(\text{NEB.T.S}(X, Y))$ & containing (P, Q) , since $(x, y) \in \text{NEBKER}(P, Q)$. This is a ambivalence.

Conversely, let $(x, y) \in X \times Y$ be so that $\text{NEBCL}(\{x\}, \{y\}) \cap (P, Q) \neq (0_X, 0_Y)$. If possible, let $\text{NEBCL}(\{x\}, \{y\}) \cap (P, Q) \neq (0_X, 0_Y)$ & suppose that $(x, y) \notin \text{NEBKER}(P, Q)$, then there exists $(P_1, Q_1) \in O(\text{NEB.T.S}(X, Y))$ & containing (P, Q) , $(x, y) \notin (P_1, Q_1)$. Let $(x^*, y^*) \in \text{NEBCL}(\{x\}, \{y\}) \cap (P, Q)$. Hence, (P_1, Q_1) is a neutrosophic binary neighborhood of (x^*, y^*) which $(x, y) \notin (P_1, Q_1)$. By this ambivalence $(x, y) \in \text{NEBKER}(P, Q)$.

Theorem 2.11: Let (X, Y, BT_N) be a NEB.T.S and a NEB.P $(x, y), (x^*, y^*) \in (X, Y)$, then $(x, y) \in \text{NEBKER}(\{x^*\}, \{y^*\})$ if and only if $(x^*, y^*) \in \text{NEBCL}(\{x\}, \{y\})$

Proof: Suppose that $(x, y) \notin \text{NEBKER}(\{x^*\}, \{y^*\})$, then there exists $(P, Q) \in O(\text{NEB.T.S}(X, Y))$ so that $(x^*, y^*) \in (P, Q)$. But $(x, y) \notin (P, Q)$; therefore, we have $(x^*, y^*) \notin \text{NEBCL}(\{x\}, \{y\})$. By comparable procedure we can prove the opposite.

Theorem 2.12: If (x, y) and (x^*, y^*) each NEB.P in the $\text{NEB.T.S}(X, Y)$, then $\text{NEBKER}(\{x\}, \{y\}) \neq \text{NEBKER}(\{x^*\}, \{y^*\})$ if and only if $\text{NEBCL}(\{x\}, \{y\}) \neq \text{NEBCL}(\{x^*\}, \{y^*\})$.

Proof: Suppose that $\text{NEBKER}(\{x\}, \{y\}) \neq \text{NEBKER}(\{x^*\}, \{y^*\})$, consequently there is $(x_1, y_1) \in X \times Y$ so that $(x_1, y_1) \in \text{NEBKER}(\{x\}, \{y\})$ & $(x_1, y_1) \notin \text{NEBKER}(\{x^*\}, \{y^*\})$, it follows that $(\{x\}, \{y\}) \cap \text{NEBCL}(\{x_1\}, \{y_1\}) \neq (0_X, 0_Y)$, from this we get $(\{x\}, \{y\}) \in \text{NEBCL}(\{x_1\}, \{y_1\})$. Now, $(x_1, y_1) \notin \text{NEBKER}(\{x^*\}, \{y^*\})$, we have $(\{x^*\}, \{y^*\}) \cap \text{NEBCL}(\{x_1\}, \{y_1\}) = (0_X, 0_Y)$, since $(\{x\}, \{y\}) \in \text{NEBCL}(\{x_1\}, \{y_1\})$, $\text{NEBCL}(\{x\}, \{y\}) \subset \text{NEBCL}(\{x_1\}, \{y_1\})$ & $(\{x^*\}, \{y^*\}) \cap \text{NEBCL}(\{x_1\}, \{y_1\}) = (0_X, 0_Y)$, therefore $\text{NEBCL}(\{x\}, \{y\}) \neq \text{NEBCL}(\{x^*\}, \{y^*\})$.

Conversely, suppose that $\text{NEBCL}(\{x\}, \{y\}) \neq \text{NEBCL}(\{x^*\}, \{y^*\})$, consequently there exist $(x_2, y_2) \in X \times Y$ such that $(x_2, y_2) \in \text{NEBCL}(\{x\}, \{y\})$ & $(x_2, y_2) \notin \text{NEBCL}(\{x^*\}, \{y^*\})$, there exist $(P, Q) \in O(\text{NEB.T.S}(X, Y))$ including (x_2, y_2) & therefore (x, y) but not (x^*, y^*) , namely $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$, hence $\text{NEBKER}(\{x\}, \{y\}) \neq \text{NEBKER}(\{x^*\}, \{y^*\})$.

Theorem 2.13: Let (X, Y, BT_N) be a NEB.T.S and $(P_1, Q_1), (P_2, Q_2) \subseteq (Y, Y)$. Then :

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024

- i) $(x, y) \in \text{NEBKER}(P_1, Q_1)$ if and only if $(P_1, Q_1) \cap (G, H) \neq (0_X, 0_Y)$ for each $(G, H) \in O(\text{NEB.T.S}(X, Y))$ containing (x, y) .
- ii) $(P_1, Q_1) \subseteq \text{NEBKER}(P_1, Q_1)$ and $\text{NEBKER}(P_1, Q_1) \subseteq (P_1, Q_1)$ if $(P, Q) \in O(\text{NEB.T.S}(X, Y))$
- iii) If $(P_1, Q_1) \subseteq (P_2, Q_2)$, then $\text{NEBKER}(P_1, Q_1) \subseteq \text{NEBKER}(P_2, Q_2)$.

Proof: Obvious.

Definition 2.14: Let $(P, Q), (P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$, $(P, Q), (P^*, Q^*)$ are called disjoint if $(P \cap P^*, Q \cap Q^*) = (0_X, 0_Y)$.

3. Neutrosophic Binary R_L -Spaces, $L = 0, 1, 2$

Definition 3.1: A $\text{NEB.T.S}(X, Y)$ is said to be:

- (i) Neutrosophic Binary Sober $-R_0$ space $[\text{SNEB} - R_0.S]$ if $\bigcap_{(x,y) \in (X,Y)} \text{NEBCL}(\{x\}, \{y\}) = (0_X, 0_Y)$.
- (ii) Neutrosophic Binary $-R_0$ space $[\text{NEB} - R_0.S]$ if $(P, Q) \in O(\text{NEB.T.S}(X, Y))$ and $\text{NEB.P}(x, y) \in (P, Q)$, then $\text{NEBCL}(\{x\}, \{y\}) \subseteq (P, Q)$.
- (iii) Neutrosophic Binary $-R_1$ space $[\text{NEB} - R_1.S]$ if for each $\text{NEB.P}(x, y), (x^*, y^*)$ in (X, Y) with $\text{NEBCL}(\{x\}, \{y\}) \neq \text{NEBCL}(\{x^*\}, \{y^*\})$, there are disjoint $(P, Q), (P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$, so that $\text{NEBCL}(\{x\}, \{y\}) \subseteq (P, Q)$ and $\text{NEBCL}(\{x^*\}, \{y^*\}) \subseteq (P^*, Q^*)$.
- (iv) Neutrosophic Binary $-R_2$ space $[\text{NEB} - R_2.S]$ if for each $\text{NEB.P}(x, y) \notin (G, H)$ and $(G, H) \in C(\text{NEB.T.S}(X, Y))$ with $\text{NEBCL}(\text{BNEBCL}(\{x\}, \{y\})) \neq \text{NEBCL}(\text{BNEBCL}(G, H))$, there are disjoint $(P, Q), (P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$, so that $\text{NEBCL}(\text{BNEBCL}(G, H)) \subseteq (P, Q)$ and $\text{NEBCL}(\text{BNEBCL}(\{x\}, \{y\})) \subseteq (P^*, Q^*)$.

For example: Let $X = \{p_1, p_2\}$ and $Y = \{q_1, q_2\}$ be the universe. Let $BT_N = (0_x, 0_y), (1_x, 1_y), (P, Q), (P_1, Q_1), (P_2, Q_2)$ be the NEB.T.S. So that $(P_0, Q_0) = \{ \langle X, (0.7, 0.5, 0.3) \rangle, \langle 0.6, 0.5, 0.1 \rangle \}, \langle Y, (0.6, 0.5, 0.4) \rangle, \langle 0.7, 0.5, 0.1 \rangle \}$. Let $(P_1, Q_1) = \{ \langle X, (0.7, 0.5, 0.3) \rangle, 0_X \rangle, \langle Y, (0.6, 0.5, 0.4) \rangle, 0_Y \rangle \}$, $(P_2, Q_2) = \{ \langle X, 0_X \rangle, \langle 0.6, 0.5, 0.1 \rangle \rangle, \langle Y, 0_Y \rangle, \langle 0.7, 0.5, 0.1 \rangle \}$. Then (X, Y) is $\text{NEB} - R_2.S$.

Remark 3.2: Every $\text{NEB} - R_L.S$ is an $\text{NEB} - R_{L-1}.S$, $L = 2, 1$.

Theorem 3.3: A $\text{NEB.T.S}(X, Y)$ is $\text{SNEB} - R_0.S$ if and only if $\text{NEBCL}(\{x\}, \{y\}) \neq (X, Y)$ for each $(x, y) \in (X, Y)$.

Proof: presume that $(X, Y) = \text{SNEB} - R_0.S$ & that there is a $\text{NEB.P}(x^*, y^*)$ in (X, Y) so that $\text{NEBCL}(\{x^*\}, \{y^*\}) = (X, Y)$, then $(x^*, y^*) \notin (P, Q)$ which $(P, Q) \subseteq O(\text{NEB.T.S}(X, Y))$, this implicate that to $(x^*, y^*) \in \bigcap_{(x,y) \in (X,Y)} \text{NEBCL}(\{x\}, \{y\})$, but and so on a ambivalence.

Consequently, $\text{NEBCL}(\{x\}, \{y\}) = (X, Y)$ for any $(x, y) \in (X, Y)$. If there exists a $\text{NEB.P}(x^*, y^*)$ in (X, Y) so that $(x^*, y^*) \in \bigcap_{(x,y) \in (X,Y)} \text{NEBCL}(\{x\}, \{y\})$, then any $(P, Q) \in O(\text{NEB.T.S}(X, Y))$ containing (x^*, y^*) must contain all NEB.P of (X, Y) , this implicate that the $\text{NEB.T.S}(X, Y)$ is the unique containing (x, y) & belong to $O(\text{NEB.T.S}(X, Y))$, hence $\text{NEBKER}(\{x^*\}, \{y^*\}) = (X, Y)$ which is a ambivalence, therefore (X, Y) is $\text{SNEB} - R_0.S$.

Theorem 3.4: A $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - R_0.S$ if and only if for each $(P, Q) \in C(\text{NEB.T.S}(X, Y))$ and $(x, y) \in (P, Q)$, then $\text{NEBKER}(\{x\}, \{y\}) \subseteq (P, Q)$.

Proof: Let a $\text{NEB.T.S}(X, Y)$ be a $\text{NEB} - R_0.S$ & $(P, Q) \in C(\text{NEB.T.S}(X, Y))$ & $(x, y) \in (P, Q)$. Then for each $(x^*, y^*) \notin (P, Q)$ implicate $(x^*, y^*) \in (P, Q)^c \in O(\text{NEB.T.S}(X, Y))$, then $\text{NEBCL}(\{x^*\}, \{y^*\}) \subseteq (P, Q)^c$, also $(x, y) \notin \text{NEBCL}(\{x^*\}, \{y^*\})$. Hence $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$. Thus $\text{NEBKER}(\{x\}, \{y\}) \subseteq (P, Q)$.

Conversely: suppose for all $(P, Q) \in C(\text{NEB.T.S}(X, Y))$ & $(x, y) \in (P, Q)$, then $\text{NEBKER}(\{x\}, \{y\}) \subseteq (P, Q)$ & let $(U, V) \in O(\text{NEB.T.S}(X, Y))$, $(x, y) \in (P_1, Q_1)$ then for each $(x^*, y^*) \notin (P_1, Q_1)$ implicate $(x^*, y^*) \in ((P_1, Q_1))^c \in C(\text{NEB.T.S}(X, Y))$ implicate that $\text{NEBKER}(\{x^*\}, \{y^*\}) \subseteq (P_1, Q_1)^c$.

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024

Therefore $(x, y) \notin \text{NEBKER}(\{x^*, y^*\})$ & $(x^*, y^*) \notin \text{NEBCL}(\{x, y\})$. Hence $\text{NEBCL}(\{x, y\}) \subseteq (P_1, Q_1)$. That's mean (X, Y) is an $\text{NEB} - R_0$. S.

Corollary 3.5: A NEB.T.S (X, Y) is an $\text{NEB} - R_0$. S if and only if for each $(P, Q) \in O(\text{NEB.T.S}(X, Y))$ and $(x, y) \in (P, Q)$, then $\text{NEBCL}(\text{NEBKER}(\{x, y\})) \subseteq (P, Q)$.

Theorem 3.6: A NEB.T.S (X, Y) is a $\text{NEB} - R_0$. S if and only if for any NEB.P (x, y) in (X, Y) , $\text{NEBCL}(\{x, y\}) = \text{NEBKER}(\{x, y\})$.

Proof: NEB.T.S (X, Y) is a $\text{NEB} - R_0$. S. Now If $\text{NEBCL}(\{x, y\}) \neq \text{NEBKER}(\{x, y\})$ for each NEB.P $(x, y) \in (X, Y)$, then there are NEB.P $(x^*, y^*) \in (X, Y)$ so that $(x, y) \neq (x^*, y^*)$ & $(x^*, y^*) \in \text{NEBCL}(\{x, y\})$, $(x^*, y^*) \notin \text{NEBKER}(\{x, y\})$ this implicate, there exist $(P, Q) \in O(\text{NEB.T.S}(X, Y))$, $(x^*, y^*) \notin (P, Q)$, from this we get $\text{NEBCL}(\{x, y\}) \not\subseteq (P, Q)$ this is ambivalence, therefore $\text{NEBCL}(\{x, y\}) = \text{NEBKER}(\{x, y\})$.

Conversely, suppose that $\text{NEBCL}(\{x, y\}) = \text{NEBKER}(\{x, y\})$, then for all $(P, Q) \in O(\text{NEB.T.S}(X, Y))$, $(x, y) \in (P, Q)$, let $(x^*, y^*) \in \text{NEBKER}(\{x, y\})$, then $(x, y) \in \text{NEBCL}(\{x^*, y^*\})$ & hence $(x^*, y^*) \in (P, Q)$. This implicate that $\text{NEKER}(\{x, y\}) \subseteq (P, Q)$, thus $\text{NEBCL}(\{x, y\}) \subseteq (P, Q)$, therefore (X, Y) is an $\text{NEB} - R_0$. S.

Theorem 3.7: A NEB.T.S (X, Y) is an $\text{NEB} - R_0$. S if and only if for any two NEB.P (x, y) , and (x^*, y^*) in (X, Y) with $\text{NEBKER}(\{x, y\}) \neq \text{NEBKER}(\{x^*, y^*\})$, then $\text{NEBKER}(\{x, y\}) \cap \text{NEBKER}(\{x^*, y^*\}) = (0_X, 0_Y)$.

Proof: let A NEB.T.S (X, Y) is an is $\text{NEB} - R_0$. S, then for any two NEB.P (x, y) & (x^*, y^*) in (X, Y) if $\text{NEKER}(\{x, y\}) \neq \text{NEBKER}(\{x^*, y^*\})$, hence $\text{NEBCL}(\{x, y\}) \neq \text{NEBCL}(\{x^*, y^*\})$. In fact, $(x_0, y_0) \in \text{NEBKER}(\{x, y\}) \cap \text{NEBKER}(\{x^*, y^*\})$, then we have $(x, y), (x^*, y^*) \in \text{NEBCL}(\{x_0, y_0\})$, we obtain that $\text{NEBCL}(\{x, y\}) = \text{NEBCL}(\{x^*, y^*\}) = \text{NEBCL}(\{x_0, y_0\})$, which is impossible. Then $\text{NEBKER}(\{x, y\}) \cap \text{NEBKER}(\{x^*, y^*\}) = (0_X, 0_Y)$.

Conversely : assume that for any two NEB.P (x, y) & (x^*, y^*) in (X, Y) & $\text{NEBKER}(\{x, y\}) \cap \text{NEBKER}(\{x^*, y^*\}) = (0_X, 0_Y)$ then $\text{NEBCL}(\{x, y\}) \cap \text{NEBCL}(\{x^*, y^*\}) = (0_X, 0_Y)$. In fact, $(x_0, y_0) \in \text{NEBCL}(\{x, y\}) \cap \text{NEBCL}(\{x^*, y^*\})$, this implicate that (x, y) & $(x^*, y^*) \in \text{NEBKER}(\{x_0, y_0\})$, this means $\text{NEBCL}(\{x, y\}) \cap \text{NEBCL}(\{x^*, y^*\}) \neq (0_X, 0_Y)$. Hence by hypothesis, we get $\text{NEBKER}(\{x, y\}) = \text{NEBKER}(\{x_0, y_0\})$, by similar way it follows that $\text{NEBKER}(\{x^*, y^*\}) = \text{NEBKER}(\{x_0, y_0\})$ which is a ambivalence. Then $\text{NEBCL}(\{x, y\}) \cap \text{NEBCL}(\{x^*, y^*\}) = (0_X, 0_Y)$, therefore A NEB.T.S (X, Y) is $\text{NEB} - R_0$. S.

Corollary 3.8: A NEB.T.S (X, Y) is an is $\text{NEB} - R_0$. S if and only if for any two NEB.P (x, y) , and (x^*, y^*) in (X, Y) with $\text{NEBCL}(\{x, y\}) \neq \text{NEBCL}(\{x^*, y^*\})$, then $\text{NEBCL}(\{x, y\}) \cap \text{NEBCL}(\{x^*, y^*\}) = (0_X, 0_Y)$.

Theorem 3.9: If a NEB.T.S (X, Y) is $\text{NEB} - R_1$. S, then (X, Y) is $\text{NEB} - R_0$. S.

Proof: Let $(x, y) \in (P, Q) \in O(\text{NEB.T.S}(X, Y))$, if $(x^*, y^*) \notin (P, Q)$, then since $(x, y) \notin \text{NEBCL}(\{x^*, y^*\})$, $\text{NEBCL}(\{x, y\}) \neq \text{NEBCL}(\{x^*, y^*\})$, hence there exist $(P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$ so that $\text{NEBCL}(\{x^*, y^*\}) \subseteq (P^*, Q^*)$ & $(x, y) \notin (P_1, Q_1)$, which implicate $(x^*, y^*) \notin \text{NEBCL}(\{x, y\})$, thus $\text{NEBCL}(\{x, y\}) \subseteq (P, Q)$, therefore, (X, Y) is $\text{NEB} - R_0$. S.

Theorem 3.10: A NEB.T.S (X, Y) is an $\text{NEB} - R_1$. S if and only if for any two NEB.P (x, y) , and (x^*, y^*) in (X, Y) with $\text{NEBKER}(\{x, y\}) \neq \text{NEBKER}(\{x^*, y^*\})$, then there exist $(G, H), (G^*, H^*) \in C(\text{NEB.T.S}(X, Y))$, so that $\text{NEBKER}(\{x, y\}) \subseteq (G, H)$, $\text{NEBKER}(\{x, y\}) \cap (G^*, H^*) = (0_X, 0_Y)$ and $\text{NEBKER}(\{x^*, y^*\}) \subseteq (G^*, H^*)$, $\text{NEBKER}(\{x^*, y^*\}) \cap (G, H) = (0_X, 0_Y)$ and $(G, H) \cup (G^*, H^*) = (1_X, 1_Y)$.

Proof: Let a NEB.T.S (X, Y) be $\text{NEB} - R_1$. S, then two NEB.P (x, y) , and (x^*, y^*) in (X, Y) with $\text{NEBKER}(\{x, y\}) \neq \text{NEBKER}(\{x^*, y^*\})$, this mean $\text{NEBCL}(\{x, y\}) \neq \text{NEBCL}(\{x^*, y^*\})$, since (X, Y) is $\text{NEB} - R_0$. S then there exist disjoint $(P, Q), (P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$, so that $\text{NEBCL}(\{x, y\}) \subseteq (P, Q)$

& $\text{NEBCL}(\{\mathfrak{x}^*, \mathfrak{y}^*\}) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$, also $(\mathcal{P}, \mathcal{Q})^c, (\mathcal{P}^*, \mathcal{Q}^*)^c \in \mathcal{C}(\text{NEB.T.S}(X, Y))$ so that $(\mathcal{P}, \mathcal{Q})^c \cup (1_X, 1_Y)$. Now $(G, H) = (\mathcal{P}, \mathcal{Q})^c$ & $(G^*, H^*) = (\mathcal{P}^*, \mathcal{Q}^*)^c$. Thus $(\mathfrak{x}, \mathfrak{y}) \in (\mathcal{P}, \mathcal{Q}) \subseteq (G^*, H^*)$ & $(\mathfrak{x}^*, \mathfrak{y}^*) \in (\mathcal{P}^*, \mathcal{Q}^*) \subseteq (G, H)$ implicate that $\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (\mathcal{P}, \mathcal{Q}) \subseteq (G^*, H^*)$ & $\text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \subseteq (\mathcal{P}^*, \mathcal{Q}^*) \subseteq (G, H)$.

Conversely : suppose any two $\text{NEB.P}(\mathfrak{x}, \mathfrak{y}), (\mathfrak{x}^*, \mathfrak{y}^*) \in (X, Y)$ with $\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \neq \text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})$, there exist $(G, H), (G^*, H^*) \in \mathcal{C}(\text{NEB.T.S}(X, Y))$, so that $\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (G, H)$, $\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \cap (G^*, H^*) = (0_X, 0_Y)$ & $\text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \subseteq (G^*, H^*)$, $\text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \cap (G, H) = (0_X, 0_Y)$ & $(G, H) \cup (G^*, H^*) = (1_X, 1_Y)$, then $(G, H)^c, (G^*, H^*)^c \in \mathcal{C}(\text{NEB.T.S}(X, Y))$ & $(G, H)^c \cap (G^*, H^*)^c = (0_X, 0_Y)$. Now take $(G, H)^c = (\mathcal{P}^*, \mathcal{Q}^*)$ & $(G^*, H^*)^c = (\mathcal{P}, \mathcal{Q})$. Thus $\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (\mathcal{P}, \mathcal{Q})$ & $\text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$, $(\mathcal{P}, \mathcal{Q}) \cap (\mathcal{P}^*, \mathcal{Q}^*) = (0_X, 0_Y)$, hence $(\mathfrak{x}, \mathfrak{y}) \in (\mathcal{P}, \mathcal{Q})$ & $(\mathfrak{x}^*, \mathfrak{y}^*) \in (\mathcal{P}^*, \mathcal{Q}^*)$ implicate that $(\mathfrak{x}, \mathfrak{y}) \notin \text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})$ & $(\mathfrak{x}^*, \mathfrak{y}^*) \notin \text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\})$, from this we get $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (\mathcal{P}, \mathcal{Q})$ & $\text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$. Therefore a $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - R_1.S$.

Corollary 3.11: A $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - R_1$ if and only if for any two $\text{NEB.P}(\mathfrak{x}, \mathfrak{y})$, and $(\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) with $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \neq \text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})$, then there exist disjoint $(\mathcal{P}, \mathcal{Q}), (\mathcal{P}^*, \mathcal{Q}^*) \in \mathcal{O}(\text{NEB.T.S}(X, Y))$ which $\text{NEBKER}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \subseteq (\mathcal{P}, \mathcal{Q})$ and $\text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$.

Proof: Let a $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - R_1$ & then any two $\text{NEB.P}(\mathfrak{x}, \mathfrak{y})$, and $(\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) with $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \neq \text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})$, then there exist disjoint $(\mathcal{P}, \mathcal{Q}), (\mathcal{P}^*, \mathcal{Q}^*) \in \mathcal{O}(\text{NEB.T.S}(X, Y))$ so that $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (\mathcal{P}, \mathcal{Q})$ & $\text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$. Since for each $(\mathfrak{x}, \mathfrak{y}) \in (X, Y)$ $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) = \text{NEBCL}(\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}))$ & (X, Y) is an $\text{NEB} - R_0$, then $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) = \text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}))$

Consequently $\text{NECL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \subseteq (\mathcal{P}, \mathcal{Q})$ & $\text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$.

Conversely: Assume any two $\text{NEB.P}(\mathfrak{x}, \mathfrak{y}), (\mathfrak{x}^*, \mathfrak{y}^*) \in (X, Y)$ with $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \neq \text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})$, there exist disjoint $(\mathcal{P}, \mathcal{Q}), (\mathcal{P}^*, \mathcal{Q}^*) \in \mathcal{O}(\text{NEB.T.S}(X, Y))$ such that $\text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \subseteq (\mathcal{P}, \mathcal{Q})$ & $\text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$. Also, then $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq \text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}))$ for each $(\mathfrak{x}, \mathfrak{y}) \in (X, Y)$. So we obtain $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (\mathcal{P}, \mathcal{Q})$ & $\text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$. Consequently, (X, Y) is an $\text{NEB} - R_1$.

Theorem 3.12: A $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - R_1.S$ if and only if for each $\text{NEB.P}(\mathfrak{x}, \mathfrak{y})$, and $(\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) with $\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \neq \text{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})$ there exist $(\mathcal{P}, \mathcal{Q}), (\mathcal{P}^*, \mathcal{Q}^*) \in \mathcal{O}(\text{NEB.T.S}(X, Y))$ and it is disjoint so that $\text{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (\mathcal{P}, \mathcal{Q})$ and $\text{NEBCL}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\}) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$.

Proof: Obviously.

Theorem 3.13: A $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - R_2.S$ if and only if for each $(G, H) \in \mathcal{C}(\text{NEB.T.S}(X, Y))$ and $(\mathfrak{x}, \mathfrak{y}) \notin (K, H)$ with $\text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \neq \text{NEBCL}(\text{NEBKER}(K, H))$, then there exist disjoint $(\mathcal{P}, \mathcal{Q}), (\mathcal{P}^*, \mathcal{Q}^*) \in \mathcal{O}(\text{NEB.T.S}(X, Y))$, such that $(\mathfrak{x}, \mathfrak{y}) \in (\mathcal{P}, \mathcal{Q})$ and $(K, H) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$.

Proof: Let a $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - R_2.S$ if and only if for all $(\mathfrak{x}, \mathfrak{y}) \notin (K, H)$ & $(K, H) \in \mathcal{C}(\text{NEB.T.S}(X, Y))$ with $\text{NECL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \neq \text{NEBCL}(\text{NEBKER}(K, H))$, then there exist disjoint $(\mathcal{P}, \mathcal{Q}), (\mathcal{P}^*, \mathcal{Q}^*) \in \mathcal{O}(\text{NEB.T.S}(X, Y))$ such that $\text{NEBCL}(\text{NEBKER}(K, H)) \subseteq (\mathcal{P}, \mathcal{Q})$ & $\text{NEBCL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \subseteq (\mathcal{P}^*, \mathcal{Q}^*)$ implicate that $(K, H) \subseteq (\mathcal{P}, \mathcal{Q})$ & $(\mathfrak{x}, \mathfrak{y}) \in (G, H)$.

Conversely: Assume that $(\mathfrak{x}, \mathfrak{y}) \notin (K, H)$ & $(K, H) \in \mathcal{C}(\text{NEB.T.S}(X, Y))$ with $\text{NECL}(\text{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \neq \text{NEBCL}(\text{NEBKER}(K, H))$, then there exist disjoint $(\mathcal{P}, \mathcal{Q}), (\mathcal{P}^*, \mathcal{Q}^*) \in \mathcal{O}(\text{NEB.T.S}(X, Y))$ so that $(K, H) \subseteq (\mathcal{P}, \mathcal{Q})$ & $(\mathfrak{x}, \mathfrak{y}) \in (\mathcal{P}^*, \mathcal{Q}^*)$. Since $\text{NEBKER}(K, H) \subseteq (\mathcal{P}, \mathcal{Q})$, this is implicate to $\text{NEBKER}(K, H) \cap (\mathcal{P}^*, \mathcal{Q}^*) = (0_X, 0_Y)$, thus $(\mathfrak{x}, \mathfrak{y}) \notin \text{NEBCL}(\text{NEBKER}(K, H))$, this explain $\text{NEBCL}(\text{NEBKER}(K, H)) \subseteq (K, H) = \text{NEBCL}(K, H) =$

$\mathfrak{NEBKER}(K, H) \ \& \ \mathfrak{NEBCL}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) = \mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})$ for each $(\mathfrak{x}, \mathfrak{y}) \in (X, Y)$. Thus $\mathfrak{NEBCL}(\mathfrak{NEBKER}(K, H)) \subseteq (P, O) \ \& \ \mathfrak{NEBCL}(\mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})) \subseteq (P^*, O^*)$.

Theorem 3.14: A $\mathfrak{NEB.T.S} (X, Y)$ is $\mathfrak{NEB} - R_2.S$ if and only if for each $(\mathfrak{x}, \mathfrak{y}) \notin (K, H) \ \& \ (K, H) \in C(\mathfrak{NEB.T.S} (X, Y))$ with $\mathfrak{NEBKER}(G, H) \neq \mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})$, then there exist $(G_1, H_1), (G_2, H_2) \in C(\mathfrak{NEB.T.S} (X, Y))$, such that $\mathfrak{NEBKER}(G, H) \subseteq (K_1, H_1)$, $\mathfrak{NEBKER}(K, H) \cap (K_2, H_2) = (0_X, 0_Y)$ and $\mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (K_2, H_2)$, $\mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \cap (K_1, H_1) = (0_X, 0_Y)$ $(K_1, H_1) \cup (K_2, H_2) = (1_X, 1_Y)$.

Proof: Let a $\mathfrak{NEB.T.S} (X, Y)$ is $\mathfrak{NEB} - R_2.S$ if and only if for all $(\mathfrak{x}, \mathfrak{y}) \notin (K, H) \ \& \ (K, H) \in C(\mathfrak{NEB.T.S} (X, Y))$, then there exist $(P, O), (P^*, O^*) \in O(\mathfrak{NEB.T.S} (X, Y))$ so that $(K, H) \subseteq (P, O)$, $(\mathfrak{x}, \mathfrak{y}) \in (P^*, O^*) \ \& \ (P, O) \cap (P^*, O^*) = (0_X, 0_Y)$, then $(P, O)^C \ \& \ (P^*, O^*)^C \in C(\mathfrak{NEB.T.S} (X, Y))$ such that $(P, O)^C \cup (P^*, O^*)^C = (1_X, 1_Y)$. Put $(K_2, H_2) = (P, O)^C \ \& \ (K_1, H_1) = (P^*, O^*)^C$, also $\mathfrak{NEBKER}(K, H) \subseteq (P, O) \subseteq (K_1, H_1)$, $\mathfrak{NEKER}(K, H) \cap (K_2, H_2) = (0_X, 0_Y) \ \& \ \mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (P^*, O^*)^C \subseteq (K_2, H_2)$, $\mathfrak{NEKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \cap (K_1, H_1) = (0_X, 0_Y) \ \& \ (K_1, H_1) \cup (K_2, H_2) = (1_X, 1_Y)$.

Conversely : $(\mathfrak{x}, \mathfrak{y}) \notin (K, H) \ \& \ (K, H) \in C(\mathfrak{NEB.T.S} (X, Y))$ with $\mathfrak{NEBKER}(K, H) \neq \mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})$, then there exist $(K_1, H_1), (K_2, H_2) \in C(\mathfrak{NEB.T.S} (X, Y))$, such that $\mathfrak{NEBKER}(G, H) \subseteq (K_1, H_1)$, $\mathfrak{NEBKER}(K, H) \cap (K_2, H_2) = (0_X, 0_Y) \ \& \ \mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \subseteq (K_2, H_2)$, $\mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \cap (K_1, H_1) = (0_X, 0_Y) \ \& \ (K_1, H_1) \cup (K_2, H_2) = (1_X, 1_Y)$. Then $(K_1, H_1)^C \ \& \ (K_2, H_2)^C \in C(\mathfrak{NEB.T.S} (X, Y))$ so that $(K_1, H_1)^C \cap (K_2, H_2)^C = (0_X, 0_Y)$ and $\mathfrak{NEBKER}(K, H) \cap (K_1, H_1)^C = (0_X, 0_Y)$, $\mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\}) \cap (K_2, H_2)^C = (0_X, 0_Y)$, also $(K, H) \subseteq (K_2, H_2)^C \ \& \ (\mathfrak{x}, \mathfrak{y}) \in (K_1, H_1)^C$. Thus (X, Y) is $\mathfrak{NEB} - R_2.S$.

4. Neutrosophic Binary T_L -Spaces, $L = 0, 1, 2$

Definition 4.1: A $\mathfrak{NEB.T.S} (X, Y)$ is

- (i) Neutrosophic Binary- T_0 space $[\mathfrak{NEB} - T_0.S]$ if for each elements $(\mathfrak{x}, \mathfrak{y}) \neq (\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) , there are $(P, O), (P^*, O^*) \in O(\mathfrak{NEB.T.S} (X, Y))$ so that $(\mathfrak{x}, \mathfrak{y}) \in (P, O)$ and $(\mathfrak{x}^*, \mathfrak{y}^*) \notin (P, O)$ or $(\mathfrak{x}, \mathfrak{y}) \notin (P, O)$ and $(\mathfrak{x}^*, \mathfrak{y}^*) \in (P, O)$.
- (ii) Neutrosophic Binary- T_1 space $[\mathfrak{NEB} - T_1.S]$ if for each elements $(\mathfrak{x}, \mathfrak{y}) \neq (\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) , there are $(P, O), (P^*, O^*) \in O(\mathfrak{NEB.T.S} (X, Y))$, so that $(\mathfrak{x}, \mathfrak{y}) \in (P, O)$ and $(\mathfrak{x}^*, \mathfrak{y}^*) \notin (P, O)$ or $(\mathfrak{x}^*, \mathfrak{y}^*) \in (P^*, O^*)$ and $(\mathfrak{x}, \mathfrak{y}) \notin (P^*, O^*)$.
- (iii) Neutrosophic Binary- T_2 space $[\mathfrak{NEB} - T_2.S]$ if for each elements $(\mathfrak{x}, \mathfrak{y}) \neq (\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) , there are disjoint $(P, O), (P^*, O^*) \in O(\mathfrak{NEB.T.S} (X, Y))$, so that $(\mathfrak{x}, \mathfrak{y}) \in (P, O)$ and $(\mathfrak{x}^*, \mathfrak{y}^*) \in (P^*, O^*)$.

For example: Let $X = \{p_1, p_2\}$ and $Y = \{q_1, q_2\}$ be the universe. Let $BT_{\mathfrak{K}} = \{(0_X, 0_Y), (1_X, 1_Y), (P, O), (P, O)^C\}$ Be the $\mathfrak{NEB.T.S}$. So that $(P, O) = \{X, \langle (1, 0, 0), (0, 1, 1) \rangle, \langle Y, (0, 1, 1), (1, 0, 0) \rangle\}$. Then A $\mathfrak{NEB.T.S} (X, Y)$ is $\mathfrak{NEB} - T_2.S$.

Remark 4.2: Clearly, If a $\mathfrak{NEB.T.S} (X, Y)$ is an $\mathfrak{NEB} - T_L.S$, then is an $\mathfrak{NEB} - T_{L-1}.S$, $L = 1, 2$.

Theorem 4.3: A $\mathfrak{NEB.T.S} (X, Y)$ is an $\mathfrak{NEB} - T_0.S$ if and only if for each $(\mathfrak{x}, \mathfrak{y}) \neq (\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) , either $(\mathfrak{x}^*, \mathfrak{y}^*) \notin \mathfrak{NEBKER}(\{\mathfrak{x}\}, \{\mathfrak{y}\})$ or $(\mathfrak{x}, \mathfrak{y}) \notin \mathfrak{NEBKER}(\{\mathfrak{x}^*\}, \{\mathfrak{y}^*\})$.

Proof: Let (X, Y) be an $\mathfrak{NEB} - T_0.S$ & for each elements $(\mathfrak{x}, \mathfrak{y}) \neq (\mathfrak{x}^*, \mathfrak{y}^*)$ in (X, Y) , there are $(P, O), (P^*, O^*) \in O(\mathfrak{NEB.T.S} (X, Y))$ so that $(\mathfrak{x}, \mathfrak{y}) \in (P, O)$ and $(\mathfrak{x}^*, \mathfrak{y}^*) \notin (P, O)$.

or $(x, y) \notin (P, Q)$ and $(x^*, y^*) \in (P, Q)$. Thus either $(x, y) \in (P, Q)$ & $(x^*, y^*) \notin (P, Q)$ implicate $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$ or $(x, y) \notin (P, Q)$ and $(x^*, y^*) \in (P, Q)$ implicate $(x, y) \notin \text{NEBKER}(\{x^*\}, \{y^*\})$.

Conversely: Suppose that $(x, y) \neq (x^*, y^*)$ in (X, Y) & $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$ or $(x, y) \notin \text{NEBK}$. Then there exists $(P, Q), \in O(\text{NEB.T.S}(X, Y))$ so that $(x, y) \in (P, Q)$, $(x^*, y^*) \notin (P, Q)$ or $(x, y) \notin (P, Q)$, $(x^*, y^*) \in (P, Q)$ & $x \notin G$. Thus $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - T_0.S$.

Theorem 4.4: A $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - T_1.S$ if and only if for each $(x, y) \neq (x^*, y^*)$ in (X, Y) , then $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$ and $(x, y) \notin \text{NEBKER}(\{x^*\}, \{y^*\})$.

Proof: Let (X, Y) Be an $\text{NEB} - T_1.S$ & for each elements $(x, y) \neq (x^*, y^*)$ in (X, Y) , there exist $(P, Q), (P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$, so that $(x, y) \in (P, Q)$, $(x^*, y^*) \notin (P, Q)$ or $(x^*, y^*) \in (P^*, Q^*)$, $(x, y) \notin (P^*, Q^*)$

, implicate $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$ & $(x, y) \notin \text{NEBKER}(\{x^*\}, \{y^*\})$

Conversely: Suppose that $(x, y) \neq (x^*, y^*)$ in (X, Y) & $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$ or $(x, y) \notin \text{NEBKER}(\{x^*\}, \{y^*\})$. Then there exists $(P, Q), (P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$ such that $(x, y) \in (P, Q)$, $(x^*, y^*) \notin (P, Q)$ & $(x^*, y^*) \in (P^*, Q^*)$, $(x, y) \notin (P^*, Q^*)$. Thus $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - T_1.S$.

Theorem 4.5: A $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - T_1.S$ if and only if for each element (x, y) in (X, Y) , then $\text{NEBKER}(\{x\}, \{y\}) = (\{x\}, \{y\})$.

Proof: Let (X, Y) be an $\text{NEB} - T_1.S$ & $\text{NEBKER}(\{x\}, \{y\}) \neq (\{x\}, \{y\})$. Then there exists $(x^*, y^*) \in \text{NEBKER}(\{x\}, \{y\})$ so that $(x^*, y^*) \neq (x, y)$. Hence a $\text{NEB.T.S}(X, Y)$ is not $\text{NEB} - T_1.S$, this is ambivalence. Thus $\text{NEBKER}(\{x\}, \{y\}) = (\{x\}, \{y\})$.

Conversely: Suppose that a $\text{NEB.T.S}(X, Y)$ is not $\text{NEB} - T_1.S$ & $\text{NEBKER}(\{x\}, \{y\}) = (\{x\}, \{y\})$ for each $(x, y) \in (X, Y)$, then $(x^*, y^*) \in \text{NEBKER}(\{x\}, \{y\})$ implicate $\text{NEBKER}(\{x\}, \{y\}) \neq (\{x\}, \{y\})$, this is ambivalence. Thus a $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - T_1.S$.

Theorem 4.6: A $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - T_1.S$ if and only if for each $(x, y) \neq (x^*, y^*)$ in (X, Y) implicate $\text{NEKER}(\{x\}, \{y\}) \cap \text{NEBKER}(\{x^*\}, \{y^*\}) = (0_X, 0_Y)$.

Proof: Let a $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - T_1.S$. Then $\text{NEBKER}(\{x\}, \{y\}) = (\{x\}, \{y\})$ & $\text{NEBKER}(\{x^*\}, \{y^*\}) = (x^*, y^*)$. Thus $\text{NEBKER}(\{x\}, \{y\}) \cap \text{NEBKER}(\{x^*\}, \{y^*\}) = (0_X, 0_Y)$.

Conversely: Suppose that $\text{NEBKER}(\{x\}, \{y\}) \cap \text{NEBKER}(\{x^*\}, \{y^*\}) = (0_X, 0_Y)$ for each $(x, y) \neq (x^*, y^*)$ in (X, Y) & let a $\text{NEB.T.S}(X, Y)$ is not $\text{NEB} - T_1.S$. Then for each for each $(x, y) \neq (x^*, y^*)$ in (X, Y) implicate $(x^*, y^*) \in \text{NEBKER}(\{x\}, \{y\})$ or $(x, y) \in \text{NEKER}(\{x^*\}, \{y^*\})$. Thus $\text{NEBKER}(\{x\}, \{y\}) \cap \text{NEBKER}(\{x^*\}, \{y^*\}) = (0_X, 0_Y)$, therefore ambivalence. Thus a $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - T_1.S$.

Theorem 4.7: A $\text{NEB.T.S}(X, Y)$ is an

i) $\text{NEB} - T_1.S$ if and only if is an $\text{NEB} - T_0.S$ and $\text{NEB} - R_0.S$

ii) $\text{NEB} - T_2.S$ if and only if is an $\text{NEB} - T_1.S$ and $\text{NEB} - R_1.S$

Proof:

i) Let a $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - T_1.S$ & $(x, y) \in (P, Q) \in O(\text{NEB.T.S}(X, Y))$, then for each $(x, y) \neq (x^*, y^*)$ in (X, Y) , $\text{NEBKER}(\{x\}, \{y\}) \cap \text{NEBKER}(\{x^*\}, \{y^*\}) = (0_X, 0_Y)$ implicate that $(x, y) \notin \text{NEKER}(\{x^*\}, \{y^*\})$ & $(x^*, y^*) \notin \text{NEBKER}(\{x\}, \{y\})$. Thus $\text{NEBCL}(\{x\}, \{y\}) = (\{x\}, \{y\})$, therefore a $\text{NEB.T.S}(X, Y)$ is an $\text{NEB} - R_0.S$.

Conversely, suppose that a $\text{NEB.T.S}(X, Y)$ is $\text{NEB} - T_0.S$ & $\text{NEB} - R_0.S$, then for each $(x, y) \neq (x^*, y^*)$ in (X, Y) there exists $(P, Q) \in O(\text{NEB.T.S}(X, Y))$ so that $(x, y) \in (P, Q)$ & $(x^*, y^*) \notin (P, Q)$. Since (X, Y) is an $\text{NEB} - R_0.S$, then $\text{NEBCL}(\{x\}, \{y\}) \subseteq (P, Q)$, there exists $(P^*, Q^*) \in O(\text{NEB.T.S}(X, Y))$ so that $(x^*, y^*) \in (P^*, Q^*)$ & $(x, y) \notin (P^*, Q^*)$, therefore, (X, Y) is an $\text{NEB} - T_1.S$.

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024

ii) Let a $\mathfrak{NEB.T.S}(X, Y)$ is an $\mathfrak{NEB} - T_2.S$, this results for each elements $(x, y) \neq (x^*, y^*)$ in (X, Y) there exists disjoint $(P, Q), (P^*, Q^*) \in C(\mathfrak{NEB.T.S}(X, Y))$ so that $(x, y) \in (P, Q)$ & $(x^*, y^*) \in (P^*, Q^*)$ $(x, y) \notin \mathfrak{NEBKER}(\{x\}, \{y\})$ & $(x^*, y^*) \notin \mathfrak{NEBKER}(\{x\}, \{y\})$, then $\mathfrak{NEBCL}(\{x\}, \{y\}) = (\{x\}, \{y\}) \subseteq (P, Q)$ & $\mathfrak{NEBCL}(\{x^*\}, \{y^*\}) = (\{x^*\}, \{y^*\}) \subseteq (P^*, Q^*)$.

Conversely: suppose that a $\mathfrak{NEB.T.S}(X, Y)$ is $\mathfrak{NEB} - T_1.S$ & $\mathfrak{NEB} - R_1.S$, then for each $(x, y) \neq (x^*, y^*)$ in (X, Y) there exists $(P, Q), (P^*, Q^*) \in O(\mathfrak{NEB.T.S}(X, Y))$, so that $(x, y) \in (P, Q)$ & $(x^*, y^*) \notin (P, Q)$, $(x, y) \in (P^*, Q^*)$, $(x, y) \notin (P^*, Q^*)$ implicate that to $\mathfrak{NEBCL}(\{x\}, \{y\}) \neq \mathfrak{NEBCL}(\{x^*\}, \{y^*\})$, since (X, Y) is $\mathfrak{NEB} - R_1.S$, there exist disjoint $(P, Q), (P^*, Q^*) \in O(\mathfrak{NEB.T.S}(X, Y))$, so that $(x, y) \in (P, Q)$ & $(x^*, y^*) \in (P^*, Q^*)$, a $\mathfrak{NEB.T.S}(X, Y)$ is an $\mathfrak{NEB} - T_2.S$.

Corollary 4.8: A $\mathfrak{NEB} - T_0.S(X, Y)$ is $\mathfrak{NEB} - T_2.S$ if and only if for each $(x, y) \neq (x^*, y^*)$ in (X, Y) with $\mathfrak{NEBKER}(\{x\}, \{y\}) \neq \mathfrak{NEBKER}(\{x^*\}, \{y^*\})$, then there exist $(K, H), (K^*, H^*) \in C(\mathfrak{NEB.T.S}(X, Y))$, so that $\mathfrak{NEBKER}(\{x\}, \{y\}) \subseteq (K, H)$, $\mathfrak{NEBKER}(\{x\}, \{y\}) \cap (K^*, H^*) = (0_x, 0_y)$ and $\mathfrak{NEBKER}(\{x^*\}, \{y^*\}) \subseteq (K^*, H^*)$, $\mathfrak{NEBKER}(\{x^*\}, \{y^*\}) \cap (K, H) = (0_x, 0_y)$ & $(K, H) \cup (K^*, H^*) = (1_x, 1_y)$.

Proof: Obviously.

Theorem 4.9: A $\mathfrak{NEB} - R_1.S(X, Y)$ is $\mathfrak{NEB} - T_2.S$ if and only if one of the following satisfies:

- for each $\mathfrak{NEB.P}(x, y)$ in (X, Y) , then $\mathfrak{NEBKER}(\{x\}, \{y\}) = (\{x\}, \{y\})$
- for each two $\mathfrak{NEB.P}(x, y)$ and (x^*, y^*) in (X, Y) with $\mathfrak{NEBKER}(\{x\}, \{y\}) \neq \mathfrak{NEBKER}(\{x^*\}, \{y^*\})$, then $\mathfrak{NEBKER}(\{x\}, \{y\}) \cap \mathfrak{NEBKER}(\{x^*\}, \{y^*\}) = (0_x, 0_y)$.
- for each two $\mathfrak{NEB.P}(x, y)$ and (x^*, y^*) in (X, Y) with $(x, y) \neq (x^*, y^*)$, either $(x, y) \notin \mathfrak{NEBKER}(\{x^*\}, \{y^*\})$ or $(x^*, y^*) \notin \mathfrak{NEBKER}(\{x\}, \{y\})$.
- for each two $\mathfrak{NEB.P}(x, y)$ and (x^*, y^*) in (X, Y) with $(x, y) \neq (x^*, y^*)$, then $(x, y) \notin \mathfrak{NEBKER}(\{x^*\}, \{y^*\})$ and $(x^*, y^*) \notin \mathfrak{NEBKER}(\{x\}, \{y\})$.

Proof: It comes directly from previous results

5. Conclusions

In this present Article, we have defined neutrosophic Binary separation axioms related with the neutrosophic binary kernel, after we presented some of the basic characteristics in neutrosophic binary topological spaces and characteristics of the neutrosophic Binary nucleus. The relationships Between them were also investigated. At the inception of the thierd section, we aforementioned the definitions of the neutrosophic binary topological spaces, neutrosophic binary R_i -Spaces, $i = 0, 1$, we also studied some of the relationships associated with of these separation axioms. Likewise, In the fourth section we defined the neutrosophic binary topological T_L -Space $L = 0, 1, 2$ and calculated the various characteristic and relationships connected with these separation axioms. We prospect that our feedbacks in this manuscript will be advantageous to the research combination and participate to the improvement of some distinct appearance of neutrosophic binary topology.

References

- [1] Florentin Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, RehoBoth, NM. 1999.
- [2] Florentin Smaradache, Neutrosophic Set :- A Generalization of Intuitionistic Fuzzy set. Journal of Defense Resources Management. 1 (2010), 107-116.
- [3] A. A. Salama and S. A. Alblowi, Neutrosophic set and neutrosophic topological space. ISOR J. Mathematics, Vol (3), Issue (4), (2012). pp-31-35.
- [4] Serkan K., and Cemil K., Neutrosophic Topology. Neutrosophic Sets and Systems, Vol. 13, 2016 , 90-95.

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024

- [5] R Suresh, R. & Palaniammal S., Separation axioms in neutrosophic topological spaces. Journal of Physics: Conference Series, (2020) .
- [6] Acikgoz, A. & Esenbel, F. A look on separation axioms in neutrosophic topological. AIP Conference Proceedings, (2021).
- [7] Suman Das and Rakhal Das, Neutrosophic Separation Axioms. Neutrosophic Sets and Systems, Vol. 49, 2022.
- [8] M. M. Abdulkadhi , Q. H. Imran and A. Kh. Abed, On Neutrosophic Generalized Alpha Generalized Separation Axioms. International Journal of Neutrosophic Science (IJNS), Vol. 19, No. 01, PP. 99-106, 2022.
- [9] Sudeep D. Gautam C., Separation Axioms in Neutrosophic Topological Spaces. Neutrosophic Systems with Applications, Vol. 2, 2023, 38.
- [10] Sudeep D. Gautam C., Pre-separation Axioms in Neutrosophic Topological Spaces. International Journal of Neutrosophic Science (IJNS), Vol. 22, No. 01, PP. 15-28, 2023.
- [11] A. K. Abed, E. Abd Ali, A. S. Razzaq, Q. H. Imran, S. Broumi, On Neutrosophic D–Topological Spaces. International Journal of Neutrosophic Science, Vol.22, No.3, 2023, 8-14.
- [12] S. N. Jothi, P. Thangavelu, Topology Between Two Sets. J. Math. Sci. Computer Appl. 1(3) (2011), 95-107.
- [13] S. N. Jothi, Contribution to Binary Topological Spaces, Ph.D. Thesis, Manonmaniam Sundaranar University, Tirunelveli, 2012.
- [14] S. N. Jothi, P. Thangavelu, On Binary Continuity and Separation Axioms. Ultra Sci. 24 (2012), 121-126.
- [15] A. Khrija Abed, Binary D – Separation Axioms in Binary Topological Spaces. Galoitica: Journal of Mathematical Structures and Applications (GJMSA) Vol. 09, No. 02, 2023, PP. 49-55.
- [16] S. S. Surekha, J. Elekhia and G.Sindhu, A study on Neutrosophic Binary Topological space. Stochastic. Modelling and applications, 26(3), 2022, 479-486.
- [17] Surekha S. S., Broumi Said, A Novel Approach on Neutrosophic Binary α gs Neighborhood Points and its Operators. International Journal of Neutrosophic Science (IJNS), Vol. 19, No. 01, PP. 306-313, 2022
- [18] A.G.Rose Venish, L. Vidharani, Neutrosophic Binary Crisp Points And Neutrosophic Binary Neighborhoods. Journal of Neutrosophic and Fuzzy Systems, Vol. 05, No. 01, 2023, PP. 15-22.
- [19] Q. H. Imran, A. H. M. Al-Obaidi, & Smarandache, F., On Some Types of Neutrosophic Topological Groups with Respect to Neutrosophic Alpha Open Sets. Neutrosophic Sets and Systems, 32, (2020), 425-434.
- [20] Q. H. Imran, R. Dhavaseelan, A. H. M. Al-Obaidi and Md. Hanif PAGE, On neutrosophic generalized alpha generalized continuity. Neutrosophic Sets and Systems, 35(2020), 511-521.
- [21] Q. H. Imran, A. H. M. Al-Obaidi, F. Smarandache and S. Broumi, On Neutrosophic Generalized Semi Generalized Closed Sets. International Journal of Neutrosophic Science, vol.18, No.3, 2022, pp.10-20.

Doi: <https://doi.org/10.54216/IJNS.240119>

Received: August 14, 2023 Revised: December 23, 2023 Accepted: April 04, 2024