Neutro Algebra and Neutro Group

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Chapter 9
Neutro Algebra and Neutro Group

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ABSTRACT

This chapter improves and extends several definitions, properties, and applications of neutro-algebras. In classical algebraic structures, the laws of compositions on a given set are well-defined. In fuzzy algebraic structures, the laws of compositions on a given set are not well-defined exactly but with a membership value. This chapter examines those structures which have huge applications or strong potential for applications outside of mathematics, namely computer science or engineering. The study emphasizes that the nature of neutro-algebra and anti-neutro algebra differs from that of the classical algebra and fuzzy algebra. This chapter presents Venn diagram for neutro-function and establishes some relations between the partial algebra and neutro-algebra. The main contribution of the chapter is the introduction of the notion of neutro-group.

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INTRODUCTION

The early years of the 20th century brought forth uncertainty in scientific study. In 1965, Zadeh (1965) defined the Fuzzy Set (FS) to deal with uncertainty in non-stochastic sense. Then, FS has been extended and generalized to incorporate indeterminacy and inconsistency in many directions (Atanassov, 1986; Smarandache, 1998). Single-Valued Neutrosophic Set (Wang, Smarandache, Zhang, & Sunderraman, 2010) was proposed as a subclass of Neutrosophic Set (NS) (Smarandache, 1998). Smarandache showed extreme interest in neutrosophy (Smarandache, 1998) and published a plethora of new theories dealing with neutrosophics (Smarandache 2013; 2017a; 2017b; 2019; 2020a; 2020b; 2021). The theory of NSs got enormous popularity after the publication of international journal, namely, "Neutrosophic Sets and Systems". The main thrusts of NSs in research areas include decision making (Peng & Dai, 2020), conflict resolution (Pramanik & Roy, 2014), image processing (Koundal, Gupta & Singh, 2016), video tracking (Hu, Ye, Fan, Shen, Huang & Pi, 2017), fault diagnosis (Ye, 2017), air surveillance (Fan, Hu & Li, 2019), cluster analysis (Ye, 2014), and so on.

Theories and applications of NSs can be found in the studies (Smarandache & Pramanik, 2016; 2018; El-Hefenawy et al., 2016; Broumi et al., 2018; Khan et al., 2018; Pramanik, Mallick & Dasgupta, 2018; Nguyen, Son, Ashour & Dey, 2019; Peng & Dai, 2020; Pramanik, 2020; Muzaffar, Nafis, & Sohail, 2020).

Smarandache (2019) introduced the neutro-defined and anti-defined laws, as well as the neutro-axiom and anti-axiom, inspired by neutrosophy (Smarandache, 1998), that helped in creating the new research areas called neutro-structures and anti-structures. Classical algebraic structure is an algebraic structure dealing only with classical axioms which are totally true.

Then, an neutro-algebraic structure (Smarandache, 2020a) is an algebraic structure that has at least one neutro-axiom and one anti-axioms. Again, in all classical algebraic structures, the axioms (associativity, commutativity, etc.) defined on a set are totally true, but it is again a restrictive case, because there are numerous situations in science and in any domain of knowledge when an axiom defined on a set may be only partially-true (and partially-false), that are called neutro-axiom, or totally false that are called anti- axiom. Therefore, a neutro-algebra (Smarandache, 2020b) is an algebra that has at least one neutro-operation or one neutro-axiom (axiom that is true for some elements, Indeterminate for other elements, and false for the other elements).

In 2013, Agboola and Davvaz (2013) deduced the connections between NSs and algebraic hyper-structures. Several researchers presented the neutrosophic hyper-group (Agboola & Davvaz, 2013), neutrosophic hyper vector spaces (Agboola & Akinleye, 2015), neutro hyper-group (Ibrahim & Agboola, 2020a), neutrosophic canonical hyper-group (Agboola & Davvaz, 2014) and neutrosophic hyper-rings (Ibrahim & Agboola, 2020b).

Agboola (2020a, 2020b, 2020c) introduced formally the notions of neutro-groups, neutro-sub-groups, neutro-rings, neutro-subrings, neutro-ideal, neutro-quotient-rings, anti-groups and proved a few properties of these structures and their sub-structures. Rezaei and Smarandache (2020) presented the notion of neutro-BE-algebras and anti-BE-algebras. Agboola and Ibrahim (2020) introduced the concept of anti-rings and established some of their properties. Ibrahim and Agboola (2020c) introduced the ideas of neutro-vector spaces.

A partial algebra (Burmeister, 1982; 1993) is an algebra that has at least one partial operation, and all its axioms are classical axioms, which are true for all elements. Smarandache (2020b) established that neutro-algebra is a generalization of partial algebra, and provided a few examples of neutro-algebras that are not partial algebras. In the same study, Smarandache (2020b) introduced the neutro-function and neutro-operation.

This chapter establishes that neutro-algebra is a generalization of partial algebra, and provides a few examples of neutro-algebras that are not partial algebras. This chapter also analyzes the neutro-function and neutro-operation. The main contribution of the chapter is the introduction of the notion of neutro-group.

The rest of the chapter is organized as follows:

Next section presents the preliminaries of neutro-algebra, partial algebra, neutro-functions, etc. The following next section presents the main results of the chapter namely, Venn diagram of neutro-function, relation between neutro-algebra and partial algebra and neutro-group. The following section concludes the chapter by stating the contribution and future scope of the research.

PRELIMINARIES

This section presents some basic definitions, results to develop of the present study.

Definition 1: (Smarandache, 2019) A neutro algebra is an algebra which has at least one neutro- operation or one neutro- axiom (axiom that is true for some elements, indeterminate for other elements, and false for other elements), and no anti-axiom.

Note 1: The neutro-algebra is a generalization of partial algebra, which is an algebra that has at least one partial operation, while all its axioms are totally true (classical axioms).

Note 2: Comparison between the partial algebra and the neutro-algebra (Smarandache, 2020b)

1. When the neutro-algebra has no neutro-axiom and no outer-operation, it coincides with the partial algebra.
2. There are neutro-algebras that have no neutro-operations, but have neutro-axioms. These are different from partial algebras.
3. Neutro-algebras that have both neutro-operations and neutro-axioms.
4. These are different from partial algebras too.

Definition 2: (Reichel, 1984) A function f: X→Y is called a partial function if it is well-defined for some elements in X, and undefined for all the other elements in X. Therefore, there exist some elements $a \in$ X such that $f(a) \in$ Y (well-defined), and for all other elements b ∈ X, one has f(b) is undefined.

Definition 3: (Smarandache, 2019) A function f: X→Y is called a neutro-function if it has elements in X for which the function is well-defined {degree of truth (T)}, elements in X for which the function is indeterminate {degree of indeterminacy (I)}, and elements in X for which the function is outer-defined {degree of falsehood (F)}, where T, I, F ∈ [0, 1], with (T, I, F) ≠ (1, 0, 0) that represents the (Total) Function, and (T, I, F) ≠ (0, 0, 1) that represents the anti-function.

Definition 4: (Smarandache, 2020a) A function $f(x)$ is said to be classical function if $f(x)$ is well-defined for all the elements in its domain of definition.

Definition 5: (Smarandache, 2020a) A function $f(x)$ is said to be neutro-function if the function is partially well-defined (T), partially indeterminate (I), and partially outer-defined (F) on its domain of definition, where (T, I, F) are different from (1, 0, 0), and (0,0, 1).

Definition 6: (Smarandache, 2019) A function $f(x)$ is said to be anti-function if the function is outer-defined for all the elements in its domain of definition.

Definition 7: (Smarandache, 2020a) The (classical) universal algebra (or general algebra) is a branch of mathematics that studies classes of (classical) algebraic structures.

Definition 8: (Smarandache, 2020a) An algebraic structure (or algebra) is a nonempty set A endowed with some (totally well-defined) operations (functions) on A, and satisfying some (classical) axioms (totally true) - according to the universal algebra.

Definition 9: (Smarandache, 2020a) A partial algebra is an algebra defined on a nonempty set P_A that is endowed with some partial operations (or partial functions: partially well-defined, and partially undefined). While the axioms (laws) defined on a partial algebra are all totally (100%) true.

Classification of Algebras

Smarandache (2020b) classified algebras as follows:

1. A classical algebra is a nonempty set C_A that is endowed with total operations (or total functions, i.e., true for all set's elements) and (classical) axioms (also true for all set's elements).
2. A neutro-algebra (or neutro-algebraic structure) is a nonempty set N_A that is endowed with at least one neutro-operation (or neutro-function), or one neutro-axiom that is referred to the set's (partial-, neutro-, or total-) operations, and no anti-operation and no anti-axiom.
3. An anti-algebra (or anti-algebraic structure) is a nonempty set A_A that is endowed with at least one anti-operation (or anti-function) or at least one anti-axiom.

Therefore, the neutrosophic triplet is of the form:

<Algebra, neutro-algebra (which includes the partial algebra), anti-algebra>.

Definition 10: (Smarandache, 2020b) The universal neutro-algebra (or general neutro-algebra) is a branch of neutrosophic mathematics that studies classes of neutro- algebras and anti-algebras.

MAIN RESULTS

This section establishes the main results of this chapter.

Venn-Diagram of Neutro-Function

Let

$$\mathcal{U} = \{ a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15} \}$$

be a universe of discourse, and two of its nonempty subsets

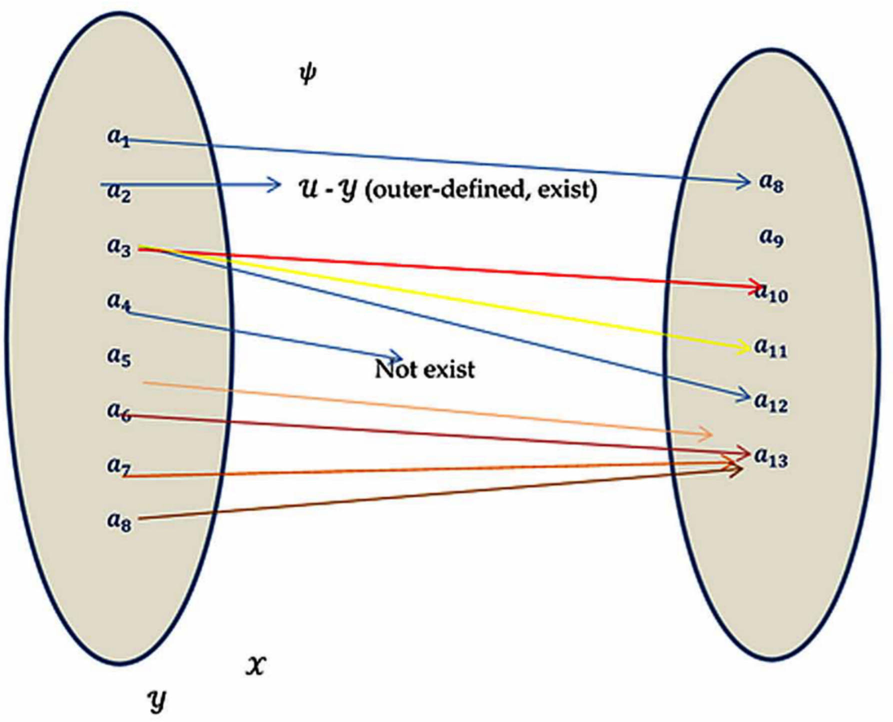
$$\mathcal{X} = \{ a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8 \},\ \mathcal{Y} = \{ a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13} \},$$

and the function ψ constructed as follows:

$\psi : \mathcal{X} \to \mathcal{Y}$ such that

$\psi(a_1) = a_8 \in \mathcal{Y}$ (well-defined);

Figure 1. Venn diagram for neutro function



$\psi(a_2) = a_7 \in \mathcal{U} - \mathcal{Y}$ (outer-defined);

$\psi(a_3)$ = undefined (doesn't exist);

$\psi(a_4) = a_{10}$ or a_{11} or a_{12}

(It does exist, but it is not known exactly), therefore $\psi(a_4)$ = indeterminate;

ψ (Some element among a_5, a_6, a_7, a_8) = a_{13}, {i.e., it can be $\psi(a_5) = a_{13}$ or $\psi(a_6) = a_{13}$, it is not sure about}, therefore ψ (indeterminate) = a_{13}.The Venn-diagram of the above neutro-algebra function is

Shown in Figure 1.

Theorem 1: The neutro -algebra is a generalization of partial algebra.

Proof. Partial algebra is equipped with partially defined operations, in partial algebra that is endowed with some partial operations or partial functions: partially well-defined, and partially undefined. While the axioms (laws) defined on a partial algebra are all totally (almost 100%) true. Whereas, a neutro-algebra is a nonempty set that is endowed with at least one neutro-operation or neutro-function, or one neutro-axiom that is referred to the set's partial-, neutro-, or total-operations, in that

Table 1.

*	*x*	y
x	y	*x*
y	x	*x* or *y*

case undefined and neutro membership will be greater than 0%. Hence, they are neutro-algebras according with the above definition of neutro-algebras.

Theorem 2: The partial algebra is a neutro-algebra but the converse is not true in general.

Proof: The partial algebra is a neutro-algebra is proved in Theorem 3.1.

Remark 1: The converse of the above theorem may not be true in general, which follows from the following counter-example.

Example 1: Let $\mathcal{U} = \{x, y, z\}$ be a universe of discourse, and $\mathcal{X} = \{x, y\}$ be one of its nonempty subsets structure $\mathcal{X} = (\mathcal{X}, *)$, constructed as shown in Table 1 using Cayley Table.

Here, * is an indeterminate-operation. Since, y*y = x or y (indeterminate), but for any element other than y*y, x*y is well-defined. The same, because * is not partially defined (since y*y ≠ undefined), $\mathcal{X}$ cannot be a partial algebra.

Similarly, the axiom of commutatively is totally true, since x*y and y*x are defined, and they are equal: $x*y = x = y*x$.

Therefore, $\mathcal{X}$ equipped with the axiom of commutativity is a neutro-algebra).

Neutro-Group

Definition 11: Let $\mathcal{U}$ be a universe of discourse, and * be the neutro-operation on $\mathcal{U}$. Then the neutro algebra ($\mathcal{U}$, *) is called a neutro-group if the following conditions are satisfied:

1. A classical **axiom** defined on a nonempty set is an axiom that is totally true (i.e., true for all set's elements).
2. A **neutro-axiom** (or neutrosophic axiom) defined on a nonempty set is an axiom that is true for some set's elements [degree of truth (T)], indeterminate for other set's elements [degree of indeterminacy (I)], and false for the other set's elements [degree of falsehood (F)], where $T, I, F \in [0,1]$, with $(T, I, F) \neq (1,0,0)$ that represents the classical axiom, and $(T, I, F) \neq (0,0,1)$ that represents the anti-axiom.

3. An **anti-axiom** defined on a nonempty set is an axiom that is false for all set's elements.

Therefore, we form the neutrosophic triplet as follows:

neutrosophic triplet = **<axiom, neutro-axiom, anti-axiom>**

= <axiom(1, 0, 0), neutro-axiom(t, i, f), anti-axiom(0, 0, 1)>,

where (T, I, F) $\notin$ {(1, 0, 0), (0, 0, 1)}.

Definition 12: An neutro-algebra structure is called an neutro-group if the following properties hold:

Neutro-Closure Property

For any two elements *x*, *y* under the neutro-algebraic operation * on the neutro-algebra set $\mathcal{U}$ is neutro closed, i.e., *x* * *y* has the set's (partial-, neutro-, or total) membership value with classical partial and neutro-algebraic membership values.

Neutro-Associative Property

1. there are some elements a, b, c such that a*(b*c) = (a*b)*c (degree of truth T),
2. there are some elements d, e, f such that d*(e*f) or (d*e)*f are indeterminate (degree of indeterminacy I),
3. there are some elements g, h, j such that g*(h*j) is different from (g*h)*j (degree of falsehood F), where (T, I, F) is different from (1,0,0) that represents the classical axiom, and (T, I, F) is different from (0, 0, 1) that represents the anti-axiom.

Existence of Neutro Unit Element

There exists an element in which the total membership value of x*y= y*x= x is more than 0.5.

Existence of Neutro Inverse Element

If there exists the same unit-element x for x and y, or neut(x)=neut(y)=x, since x*x=x and y*x=x*y=y.

Definition 13: If an neutro-group is cumulative under the neutro operation *, then it is called neutro-commutative group.

Example 2: Let $\mathcal{U} = \{x, y, z, t\}$ be a universe of discourse. The subset $\mathcal{X} = \{x, y, z\}$ is constructed using Cayley Table as shown in Table 2.

Table 2.

*	x	y	z
x	x	z	z
y	x	x	x
z	z	x	t

Neutro-Defined Law of Composition

For example: $x*y = z \in \mathcal{X}$, but $z*z = t \notin \mathcal{X}$.

Neutro-Associativity

For example: $x*(x*z) = x*z = z$ and $(x*x)*z = x*z = z$;

While for example: $x*(y*z) = x*x = x$ and $(x*y)*z = z*z = t \neq x$.

Neutro-Commutativity

Because, for example: $x*z = z*x = z$, but $x*y = z$ while $y*x = x \neq z$.

Neutro-Unit Element

There exists the same unit-element x for x and z, or neut(x) = neut(z) = x, since $x*x = x$ and $z*x = x*z = z$. But there is no unit element for b, because $y*x = x$, not y, for any $x \in \mathcal{X}$ (see the above Cayley Table).

Neutro-Inverse Element

From the composition table, with respect to the same unit element x, there exists an inverse element for x, which is x, or inv(x) = x, because $x*x = x$, and an inverse element for z, which is y, or inv(z) = y, because $z*y = y*z = x$. But there is no inverse element for y, since y has no unit element. Therefore, $(\mathcal{X}, *)$ is a neutro defined neutro commutative neutro- group.

CONCLUSION

This chapter presents Venn diagram for neutro-function and establishes some relations between partial algebra and neutro-algebra. The main contribution of the chapter is the introduction of the notion of neutro-group. In future, the developed concept of neutro-group can further be investigated. Its core properties can deeply be explored further. The main impact of the notion of neutro-group will be in the applications in neutro-algebra and real-life problems.

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CONFLICTS OF INTEREST

The authors declare that the research has been conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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