Research Paper

NEUTROSOPHIC: A comprehensive review to explore the future

Abstract:

The aim of this paper is twofold: first, it focuses on some important notions such as neutrosophic set, neutrosophic logic, neutrosophic measure, neutrosophic integral, and a single valued neutrosophic set (SVNS). Second, the most important part of the paper is related to the neutrosophic applications. there exists a lot of application in all field such as in information technology , information system and decision support system for example, relational database systems, semantic web services, financial data set detection, new economies growth, decline analysis and etc.. These notion and application may help the researcher in making algorithm to solve problems.

Keywords: neutrosophy, neutrosophic measure, neutrosophic set, neutrosophic integral, a single valued neutrosophic set.

1. Introduction

The important of this paper lies in helping the researchers and anyone interested with neutrosophic and its application in many field, so first, defining some notion and application to use it in solving problems, making algorithms. Because all of above we need first to define what mean by neutrosophic. Neutrosophic Science means development and applications of neutrosophic logic/set/measure/integral probability etc. and their applications in any field.

First neutrosophy applied in many field in order to solve problems related to indeterminacy. Neutrosophy has been introduced some years ago by Florentin Smarandache *[1]* as a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra and The Fundamental thesis of neutrosophy is that every idea has not only a certain degree of truth, as is generally assumed in many-valued logic contexts, but also a falsity degree and an indeterminacy degree that have to be considered independently from each other, so Smarandache define neutrosophy as we see above. Smarandache seems to understand such “indeterminacy” both in a subjective and an objective sense, i.e. as uncertainty as well as imprecision, vagueness, error, doubtfulness, etc.

In the other way neutrosophic is defining as Let < *A* > be an item. < *A* > can be a notion, an attribute, an idea, a proposition, a theorem, a theory, etc. And let < *antiA* > be the opposite of < *A* >; while < *neutA* > be neither < *A* > nor < *antiA* > but the neutral (or Indeterminacy, unknown) related to < *A* >. For example, if < *A* > = victory, then < *antiA* > = defeat, While < *neutA* > = tie game. If < *A* > is the degree of truth value of a proposition, then < *antiA* > is the degree of falsehood of the proposition, while < *neutA* > is the degree of indeterminacy (i.e. neither true nor false) of the proposition. Also, if < *A* > = voting for a candidate, < *antiA* > = voting against that candidate, while < *neutA* > = not voting at all, or casting a blank vote, or casting a black vote. In the case when < *antiA* > does not exist, we consider its measure be null {M (antiA) =0}. And similarly when < neutA > does not exist, its measure is null {m (neutA) = 0}.

An aim of this paper is twofold: to present some notions and applications of neutrosophic that has been successfully applied, and thus to broaden the range of its potential users. The structure of this paper is as follows: Section 2, discusses the biological foundations of the neutrosophic. Section 3, define the notions first Neutrosophic logic. Section 4, defines Neutrosophic set. Section 5, defines Neutrosophic measure. Section 6, defines neutrosophic integral. Section 7, defines single valued neutrosophic set. Section 8, we introduce the most important application of neutrosophic in many fields to make use of it. Section 9, the most important part of this paper to define the future work and how to use this applications.

Literature review

2.1 **From fuzzy to neutrosophic values**:

to define the neutrosophic first, define the source of this science. The neutrosophy has an extension of classical set and fuzzy set. First we define The Classical sets are called crisp sets either an element *belongs* to a set or not, i.e..,$X \in A \quad OR \quad X \notin A$. Now turn to the Definition of fuzzy logic *[2]* .the Fuzzy "not clear, distinct, or precise; blurred which depend upon their contexts. Fuzzy logic is a convenient way to map an input space to an output space. Mapping input to output is the starting point for everything. Consider the following examples:

1) Information about how good your service was at restaurant, a fuzzy logic system can tell you what the tip should be.
2) With your specification of how hot you want the water, a fuzzy logic system can adjust the faucet valve to the right setting.
3) With information about how far away the subject of your photograph is, a fuzzy logic system can focus the lens for you.
4) With information about how fast the car is going and how hard the motor is working, a fuzzy logic system can shift gears for you.

The Atanassov define fuzzy logic *[3]* as replace The two-point set of classical truth values {0, 1} is replaced by the real unit interval [0, 1]: each real value in [0, 1] is intended to represent a different degree of truth, ranging from 0, corresponding to false in classical logic, to 1, corresponding to true. The standard logical connectives are defined as functions on [0, 1], such as $x \wedge y = min(x, y), x \vee y = max(x, y)$ and so given a sentence p whose truth degree is $v(p) = t \in [0, 1],$ in fuzzy logic it is implicitly assumed that it also has a falsity degree given by $(1-t)$. This need not hold in general in the so-called intuitionistic fuzzy logic. IFS have become a popular topic of investigation in the fuzzy set community. The first public statement of this notion was made in 1983 , and the first widely accessible reference was published in 1986 . An intuitionistic fuzzy set in the sense of Atanassov *[4]* is defined by a pair of membership functions (F^+, F^-) denoted by IF, where $F^+ (u)$ is the degree of membership of (u) in IF and $F^- (u)$ is its degree of non-membership.

It is worth pointing out that this may be seen as a fuzzification of the idea of sub-definite set, introduced some years before by Narin'yani who separately handles the (ordinary) set F^+ of elements known as belonging to the sub-definite set and the (ordinary) set F^- of elements known as not belonging to it, with the condition $F^+ \cap F^- = \emptyset$ (together with some bounds on the cardinalities of F^+ and F^-). Such a condition is extended to the two membership functions F^+ and F^-, which for IFS are supposed to verify the constraint $F^+(u) + F^-(u) \leq 1$. And also A so-called "Intuitionistic fuzzy set theory" was independently introduced by Takeuti and Titani *[5]* as a set theory developed in (a kind of) intuitionistic logic. Takeuti-Titani's intuitionitic fuzzy logic is simply an extension of intuitionistic logic, i.e. All formulas provable in the intuitionistic logic are provable in their logic.

They give a sequent calculus which extends Heyting intuitionistic logic LJ, an extension that does not collapse to classical logic and keeps the flavour of intuitionism. The name "intuitionistic" in Atanassov's theory of IFSs was most probably motivated by the inequality Which is supposed to express the rejection of the excluded middle law, like in intuitionistic logic Such mathematical objects look reasonable and interesting from the point of view of the theory of fuzzy sets as well as from application viewpoints, but it can be argued that the name "intuitionistic fuzzy sets" (IFSs) for Atanassov theory is unsuitable and misleading, at least for the following three reasons *:*

1) Intuitionistic fuzzy set theory by Takeuti and Titani is an absolutely legitimate approach, in the scope of intuitionistic logic, but it has nothing to do with Atanassov's intuitionistic fuzzy sets.

2) As well known, the law of excluded middle is not valid in fuzzy logic in general; so, the fact that IFSs do not satisfy

It is no sufficient reason to use the name "intuitionistic". Much worse, as analyzed in the paper by Cattaneo and Ciucci, the connectives of IFS theory violate properties of intuitionistic logic by validating the double negation (involution) axiom "I $\overline{\overline{IF}} =$ "IF" which is not valid in intuitionistic logic. (Recall that axioms of intuitionistic logic extended by the axiom of double negation imply classical logic, and thus imply excluded middle; see e.g. Kleene *[6]*). On the other hand, the axiom of non-contradiction $A \cap \bar{A} = 0$ is valid in intuitionistic logic but it is not satisfied in IFS theory by the commonly used involutive negation.

3) Finally, the philosophical ideas behind intuitionism in general, and intuitionistic mathematics and intuitionistic logic in particular have a strong tendency toward constructivist points of view. There are no relationship between these ideas and the basic intuitive ideas of IFS theory.

The main objection to the terminology used by Atanassov is the fact that he calls "intuitionistic fuzzy set theory" something which accepts rules and principles (as double negation) that, added to the intuitionistic logic, make it classical, i.e. Nothing from intuitionism remains. Calling the Atanassov theory intuitionistic leads to a misunderstanding.

This is the time to give Smarandache's original definition of the neutrosophic set of truth values. The neutrosophy is advanced of fuzzy logic, so we are now define the Neutrosophy has been introduced some years ago by Florentin Smarandache as Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra . The Fundamental thesis of neutrosophy is that every idea has not only a certain degree of truth, as is generally assumed in many-valued logic contexts, but also a falsity degree and an indeterminacy degree that have to be considered independently from each other. Smarandache seems to understand such "indeterminacy" both in a subjective and an objective sense, i.e. as uncertainty as well as imprecision, vagueness, error, doubtfulness, etc.

In other way neutrosophic is defining as Let *A:* be an item. This item can be a notion, an attribute, an idea, a proposition, a theorem, a theory, etc. And let *antiA* be the opposite of ***A***; while *neutA* be neither *A* nor antiA but the neutral (or Indeterminacy, unknown) related to *A*. For example, if *A* = victory, then *antiA* = defeat, While *neutA* = tie game. If *A*: is the degree of truth value of a proposition, then *antiA*: is the degree of falsehood of the proposition, while *neutA*: is the degree of indeterminacy (i.e. neither true nor false) of the proposition. Also, if *A* = voting for a candidate, *antiA* = voting against that candidate, while *neutA* = not voting at all, or casting a blank vote, or casting a black vote. In the case when *antiA* does not exist, we consider its measure be null {m (antiA) =0}. And similarly when *neutA* does not exist, its measure is null {m (neutA) = 0}.

3. Neutrosophic Logic

Neutrosophic logic was created by Florentin Smarandache (1995) *[7,8]* and is an extension/combination of the fuzzy logic, intuitionistic logic, paraconsistent logic, and the three-valued logics that use an indeterminate value.

In neutrosophic logic, there is an easy way; every logical variable *(x)* is described by an ordered triple. $x= (t, i, f)$ where ***(t)*** is the degree of truth, ***(f)*** is the degree of false and ***(i)*** is the level of indeterminacy. As a particular case, one can split the Indeterminate ***(I)*** into Contradiction (true and false), and Uncertainty (true or false), and we get an extension of Belnap's four-valued logic.

Even more, one can split ***(I)*** into Contradiction, Uncertainty, and Unknown, and we get a five-valued logic .In a general Refined Neutrosophic Logic, (**T**) can be split into subcomponents $(T_1, T_2, ..., T_p)$ and ***(I)*** into$(I_1, I_2, ..., I_r)$ and***(F)*** into $(F_1, F_2, ...,F_s)$ where $[p + r + s = n \geq 1]$. Even more: ***T, I,*** *and/or* ***F*** (or any of their subcomponents $T_j, I_k,$ *and/or* F_l) can be countable or uncountable infinite sets. For software engineering proposals the classical unit interval [0, 1] may be used.

T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1). As example: a statement can be between *[0.4, 0.6]* true, *{0.1}* or between *(0.15, 0.25)* indeterminate, and either *0.4 or 0.6* false.

Another example: Jack wants to invite Kate to the homecoming banquet. Kate may or may not accept the invitation. In neutrosophic Terms, the statement "Kate will accept the invitation" can be described in the following Way: it is 60% true, 40% indeterminate and 30% false.

Note: An alternate definition used by Smarandache is to have the three values of the ordered triple integers greater than or equal to zero and $t + i + f$ < or = or > 100. This definition is of course consistent with percentages. For our purposes, the use of real numbers that sum to 1.00 is preferable, as it is more consistent with other logics.

A more general definition later developed by Smarandache (1999-2002) "Let T, I, F be standard or non-standard real subsets of the non-standard unit interval] $^{-}0$, 1^{+} [with

$$\text{Sup T} = \text{t\_sup}, \ \inf \text{T} = \text{t\_inf},$$
$$\sup \text{I} = \text{i\_sup}, \ \inf \text{I} = \text{i\_inf},$$
$$\text{Sup F} = \text{f\_sup}, \ \inf \text{F} = \text{f\_inf},$$
$$\text{and} \quad \text{n\_sup} = \text{t\_sup+i\_sup+f\_sup},$$
$$\text{n\_inf} = \text{t\_inf+i\_inf+f\_inf}.$$

The sets *T, I, F* are not necessarily intervals, but may be any real sub-unitary subsets: discrete or continuous; single-element, finite, or (countable or uncountable) infinite; union or intersection of various subsets; etc. They may also overlap. The real subsets could represent the relative errors in determining *(t, i, f)* (in the case when the subsets *T, I, F* are reduced to points). Statically *T, I, F* are subsets.

4. Neutrosophic Set.

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (Smarandache1999), and is a powerful general formal framework, which generalizes the above mentioned sets from philosophical point of view. Smarandache (1999) gave the following definition of a neutrosophic set *[9].*

4.1Definition: Let (*X*) be a space of points (objects), with a generic element in (*X*) denoted by (*x*) (Smarandache, 1999) . A neutrosophic set (*A*) in (*X*) is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of$]\, 0^-,\ 1^+\, [$. That is $T_A(x)$: $X \rightarrow]\, 0^-,\ 1^+\, [$, $I_A(x)$: $X \rightarrow]0^-,\ 1^+\, [$, and $F_A(x)$: $X \rightarrow\ ]0^-,\ 1^+\, [$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

4.2Definition: The complement of a neutrosophic set (*A*) is denoted by *c (A)* and is defined as $T_{c\,(A)}\,(x) = \{1^+\} - T_A(x)$, $I_{c\,(A)}\,(x) = \{1^+\} - I_A(x)$, and $F_{c\,(A)}\,(x) = \{1^+\} - F_A(x)$. For every *(x)* in *(X)* (Smarandache, 1999) .

4.3 Definition: A neutrosophic set (*A*) is contained in the other neutrosophic set *(B)*, $A \subseteq B$ if and only if $(\inf T_A(x) \le \inf T_B(x),\ \sup T_A(x) \le \sup T_B(x),\ \inf I_A(x) \ge \inf I_B(x),\ \sup I_A(x) \ge \sup I_B(x),\ \inf F_A(x) \ge \inf F_A(x)$, and $\sup F_A(x) \ge \sup F_B(x)$ for every *(x)* in *(X)* (Smarandache, 1999).

And the Abstract of Smarandache (1995) defined the notion of neutrosophic sets, which is a generalization of Zadeh's fuzzy set and Atanassov's intuitionistic fuzzy set. In this paper, we first define the neutrosophic set neutrosophic set as: is a generalization of a classical set and a fuzzy set. Generally, a Neutrosophic set is denoted as *<T, I, F>*. An element *x (t, i, f)* belongs to the set in the Following way: ***(t)*** it is t true, ***(i)*** indeterminate, and ***(f)*** false in the set, where *(t, i, and f)* are real Numbers taken from sets *(T, I, and F)* with no restriction on *(T, I,F)*nor on their *sum[m=t+i+f]*.
First we can see in the classical set, *((i = 0), (t and f are either 0 or 1)).* And In a fuzzy set, there is no indeterminacy so $(i= 0,\ 0 \le t, f \le 1$ and $t + f = 1)$.
On the other side, in a neutrosophic set, the indeterminacy is the most things in the neutrosophy so, $0 \le t, f, i \le 1$.
Let *(U)* be a universe of discourse, and *(M)* a set included in *(U).* An element *(x)* from*(U)* is noted with respect to the set *(M)* as *x(T, I, F)* and belongs to *(M)* in the following way: it is ***(t)*** % true in the set, ***(I)*** % indeterminate (unknown if it is) in the set, and ***(f)*** % false, where *(t)* varies in *(T), (i)* varies in *(I), (f)* varies in Statically *(T, I, F)* are subsets, but dynamically *(T, I, F)* are functions/operators depending on many known or unknown parameters.

5. Neutrosophic Measure

First of all Neutrosophic measure *[10]* is generalization of the classical measure for the case when the space contains some indeterminacy. Let *(X)* is a neutrosophic space, and *(Σ) an* σ -neutrosophic algebra over *(X).* A neutrosophic Measure *(v)* is defined by for neutrosophic set $A \in \Sigma$ by $V: X \rightarrow R^3$, $V\,(A) = (m\,(A),\ m\,(neutA),\ m\,(antiA),$ (1)

With *(antiA)* = the opposite of *(A),* and *(neutA)* = the neutral (Indeterminacy) neither *(A)* nor anti *(A)* (as defined above);

For any $A \subseteq X$ and $A \in \Sigma$,

M (A) means measure of the determinate part of *(A); m (neutA)* means measure of indeterminate part of *(A);* and m *(antiA)* means measure of the determinate part of *(antiA);*

Where ν is a function that satisfies the following two properties:

a) Null empty set: $v(\Phi) = (0, 0, 0)$.

b) Countable Additivity (or σ -Additivity): For all Countable collections $\{A_n\}_{n\in L}$ of disjoint neutrosophic sets in (Σ).

And neutrosophic measure space is a triplet (X, v, Σ).

5.1 Normalized Neutrosophic Measure.

A neutrosophic measure is called normalized if

$V(x) = (m(X), m(neutX), m(antiX)) = (x_1, x_2, x_3)$,

With $x_1+x_2+x_3 = 1$, and $x_1 \geq 0$ $x_2 \geq 0$ $x_3 \geq 0$.

Where, of course, X is the whole neutrosophic measure space.

5.2 Finite Neutrosophic Measure Space.

Let $A \subset X$. We say that $v(A) = (a_1, a_2, a_3)$ is finite if all $(a_1, a_2,$ and $a_3)$ are finite real numbers. A neutrosophic measure space (X, v, Σ) is called finite if $V(X) = (a, b, c)$ such that all *a, b,* and *c* are finite (rather than infinite).

5.3 σ -Finite Neutrosophic Measure.

A neutrosophic measure is called σ -finite if *(X)* can be decomposed into a countable union of neutrosophically measurable sets of fine neutrosophic measure. Analogously, a set *(A)* in *(X)* is said to have an σ -finite neutrosophic measure if it is a countable union of sets with finite neutrosophic measure.

5.4 The Properties of Neutrosophic Measure:

a) Monotonicity:

If A_1 and A_2 are neutrosophically measurable, with $A_1 \subseteq A_2$, where

$V(A_1) = (m(A_1), m(neutA_1), m(antiA_1))$,

$V(A_2) = (m(A_2), m(neutA_2), m(antiA_2))$, then

$M(A_1) \leq m(A_2)$, $m(neutA_1) \leq m(neutA_2)$, $m(antiA_1) \geq m(antiA_2)$

$V(x) = (x_1, x_2, x_3)$ and $v(y) = (y_1, y_2, y_3)$ we say that $v(x) \leq v(y)$, if $x_1 \leq y_1, x_2 \leq y_2, and x_3 \geq y$

$b) additivity$: $A_1 \cap A_2 = \emptyset$ then $v(A_1 \cup A_2) = v(A_1) + (A_2)$.

5.5 generalizations

a) Neutrosophic measure *(v)* is continuous from below if, for A_1, A_2…neutrosophically measurable sets with $A_n \subset A_{n+1}$ for all *(n)*, the union of the sets (A_n) is neutrosophically measurable,

$$v(\cup_{n=1}^{\infty} A_n) = lim_{n\to\infty} V(A_n)$$

And neutrosophic measure *(v)* is continuous from above if for A_1, A_2 ...neutrosophically measurable sets,

$A_n \supset A_{n+1}$ For all *(n)*, and least one (A_n) has finite neutrosophic measure, the intersection of sets (A_n) and neutrosophically measurable

$$v(\cap_{n=1}^{\infty} A_n) = lim_{n\leftarrow\infty} v(A_n)$$

On important point we can call that neutrosophic measure is generalization of the fuzzy measure because when $m(neut_A) = 0$ and $m(anti_A)$ is ignored, we get $v(A) = (m(A), 0, 0) \equiv m(A)$and

The two fuzzy measure axioms are verified:

1) If $A = \varphi$ $then v(A) = (0,0,0) \equiv 0$.
2) $A \subseteq B$ $then$ $v(A) \leq v(B)$.

b) The neutrosophic measure is practically a triple classical measure: a classical measure of the determinate part of a neutrosophic object and, another classical measure of the determinate part of the opposite neutrosophic object.

Of course, if the indeterminate part does not exit (its measure is zero) and the measure of the opposite object is ignored, the neutrosophic measure is reduced to classical measure.

5.5.1 Example

Let's see some example of neutrosophic objects and neutrosophic measure

a) If a book of 100 sheets (covers included) has 3 missing sheets, then *v(book)*=(97,3,0) where *(v)* is the neutrosophic measure of the book number of pages.

b) If a surface of (5×5) square meters has cracks of (0.1×0.2) square meters, then *v(surface)=(24.98,0.02,0)* where (v) is the neutrosophic measure of the surface.

6. Neutrosophic integral

Using the neutrosophic measure, the neutrosophic integral *[11]* of a function *(f)* is written as:

$$\int_x f dv$$

Where*(x)* is then neutrosophic measure space and the integral is taken with respect to the neutrosophic measure *(v)*.
Indeterminacy related to integration can occur in multiple ways:

a) With respect to value of the function to be integrated.

b) With respect to the lower or upper limit of integration.

c) With respect to the space and its measure

6.1 First Example of Neutrosophic Integral:

Indeterminacy Related to function's Values Let $f_N: [a, b] \to R$ .Where the neutrosophic function is defined as $f_N(x) = g(x) + i(x)$. With g(x) the determinate part of $f_N(x)$, *and* $i(x)$ the Indeterminate part of $f_N(x)$, where for all *(x)* in *[a, b]* one has*:* $i(x) \in [0, h(x)],\ h(x) \geq 0$.
Therefore the values of the function f_N(x) are Approximate, i.e.

$$f_N(x) \in [g(x), g(x) + h(x)].$$

Similarly, the neutrosophic integral is an Approximation: $\int_a^b fN(X)\, dV = \int_a^b g(x)\, dx + \int_a^b i(x)\, dx$.

7. Single valued neutrosophic set (SVNS)

On the other side there exist an important notion called single valued neutrosophic set, we need to define it. SVNS is an instance of a neutrosophic set, which give us an additional possibility to represent uncertainty, imprecise, incomplete, and inconsistent information which exist in real world. It would be more suitable to apply indeterminate information and inconsistent information measures and can use in real scientific and Engineering applications. In the following, we introduce the definition of an SVNS as (Wanget al., 2010) *[12].*

7.1 Definition ***[12]:*** Let (X) is a space of points (objects) with generic elements in *(X)* denoted by *(x)* (Wang et al., 2010). An SVNS *(A)* in *(X)* is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function. $F_A(x)$ for each point *(x)* in *(X)*, $T_A(x)$, $I_A(x)$, $F_A(x)$ are real standard or nonstandard subsets of
$]\,0^-, 1^+\,[$. That is $T_A(x)\!: x\to]\,0^-, 1^+\,[$, $I_A(x)\!: x\to]\,0^-, 1^+\,[$, $F_A(x)\!: x \to]\,0^-, 1^+\,[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.
We can write the SVN When X is discrete, an SVNS A can be written as:

$$A=\sum_{i=0}^{n} \frac{T_A(x_i), I_A(x_i), F_A(x_i)}{X_i} \ , X_i \in x.$$

When X is continuous, an SVNS A can be written as:

$$A = \int_x \frac{T_A(x), I_A(X), f_A(x)}{x} \quad , x \in x \ .$$

7.2 Definition ***[13]:*** The complement of a neutrosophic set *(A) is denoted by* $\overline{(A)}$ *and is defined as* $T_A^{-}(x)= \{1^+\}\theta T_A(x)$, $I_A^{-}(x) = \{1^+\}\theta I_A(x)$, $F_A^{-}(x) = \{1^+\}\theta F_A(x)$ for every *(x)* in*(X).*

7.3* Definition** ***[14]: A neutrosophic set (A) is contained in the other neutrosophic set *(B)*, $A \subseteq B$ if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$, *and* $\sup F_A(x) \geq \sup F_B(x)$ *for every x in X.*

8. Application of neutrosophic

As neutrosophic have many applications in many fields: information technology, information system, decision support system. In the next sections presented some of theies applications.

8.1 information system application

When it comes to everyday problems, reference the application of neutrosophic theory in solving common problems in human development and for which there is no science or research field right that identify.
Some of these problems are:

8.1.1 Neutrosophic Database

Neutrosophic database [13] relation R is a subset of cross product $2^{D_1} \times 2^{D_2} \times \ldots \times 2^{D_m}$ where $2^{D_j} = 2^{D_1} - \varphi$. The police could use the neutrosophic database to investigate the murderer through eye-witness instead of dealing with huge list of criminals. We consider a criminal data file. Suppose that one murder has taken place at some area in deem light. The police suspects that the murderer is also from the same area and so police refer to a data file of all thesuspected criminals of that area. Listening to the eye-witness, the police have discovered that the criminal for that murder case has more or less or non-more and less curly hair textureand he is moderately large built. Form the criminal data file, the information table with attributes
"Hair Coverage", "Hair Texture", and "Build" is given by data collected. Now, consider the neutrosophic tolerance relation T_{D_1} where D_1 ="Hair Coverage", where "Hair Coverage"= {FB, FS, Rec., Bald}. The neutrosophic tolerance relationT_{D_2} where D_2 ="Hair Texture" where "Hair Texture"= {Str., Stc., Wavy, Curly}.
Also, neutrosophic tolerance relation T_{D_3} where D_3="Build" where "Build"= {Vl, L, A, S, Vs}.
Now, the job is to find out a list of those criminals who resemble with more or less or non-big hair coverage with more or less or non-curly hair texture and moderately large built. This list will be useful to the police for further investigation. Therefore, according to the information obtained from the eye-witness, police concludes that two of them are the likely murderers, and further investigation now is to be done on them only, instead of dealing with huge list of criminals.

8.1.2 Social Network Analysis e-Learning Systems via Neutrosophic Set

The purpose of this section is to put on view a Social Learning Management System that integrates social activities in e-Learning, employing neutrosophic set to analyze social networks data conducted through learning activities. Results show that recommendations can be enhanced through utilizing proposed system. We will now extend the concepts of Social Learning Management System that integrates social activities in e-Learning presented in *[14-26]* to the case of neutrosophic sets.
E-learning can be thought of as structured learning conducted over an electronic platform. One of the recommendations of Clayton Christensen's Disrupting Class is to take a "student-centric" approach to education, one that responds to students' unique learning styles and preferences.
E-Learning is moving rapidly towards integrating social network activities in presented enhanced learning experience to students. Social Networks are dominating nowadays, and students spend long times there. We can present an effective e-Learning model that integrates social networks activities in e-Learning [27]. This model utilizes the newly presented Neutrosophic Setting analysis of social network data integrated in e-Learning.
Identifying relationships between students is important for e-learning.

8.1.3 Applications of Neutrosophic in the reconciliation of financial market information

Bhattacharya postulated that when the long-term price of a market-traded derivative security (e.g. an exchange-traded option) is observed to deviate from the theoretical price *[28]*; three possibilities should be considered:
(1) The theoretical pricing model is inadequate or inaccurate, which implies that the observed market price may very well be the true price of the derivative security, or
(2) A temporary upheaval has occurred in the market possibly triggered by psychological forces like mass cognitive dissonance that has pushed the market price "out of sync "with the theoretical price as the latter is based on the assumptions of rational economic behavior, or
(3) The nature of the deviation is indeterminate and could be either due to (1) or (2) or a mix of both (1) or (2) or is merely a random fluctuation with no apparent causal connection.

8.2 information technology application

8.2.1 Neutrosophic Security

The new security mechanism which combines the advantages of both neutrosophic classification and the public key infrastructure had been demonstrated. The advantages of the proposed mechanism comparing to other existing mechanisms had been shown firstly by comparing the neutrosophic to the non-classification, showing that neutrosophic is more adaptable and provides a better response in MANET. Also, the PKI is compared to the non-PKI and indeterminacy showing that it provides a far better security with a neglect table amount of delay.to the security level provided by the PKI under the same conditions.

8.2.2 Application of neutrosophic cognitive maps in the analysis of the problems faced by girl students who get married during the period of study

Cognitive Maps based on the data collected from 100 such Students pursuing Under Graduate and Post Graduate courses in Arts and Science colleges located in Coimbatore city, Based on the opinion given by the majority of respondents ,depended in collection of factors $(M_1,M_2,M_3,..)$, and Based on the opinion about the existence of causal relationship Between Two nodes, weightage was assigned. When majority of the respondents opined the existence Of Causal relationship, weightage between two nodes was assigned as *(1)*, otherwise the weightage was assigned as *(0)*. In case, if the majority of the respondents were uncertain about the existence (or) Nonexistence of casual relationship between two nodes, then weightage was assigned as *(I),* which Denotes "Indeterminate" *[29]*.

8.2.3 Application to Robotics

- For the fusion of information received from various sensors, information that can be conflicting in a certain degree the robot uses the fuzzy and neutrosophic logic or set.
- In a real time it is used a neutrosophic dynamic fusion so an autonomous robot can take a decision at any moment

8.3 decision support system application

Once could use the neutrosophic in statistical physics, financial markets, risk management, mathematical biology, neutrosophic linguistic variables, neutrosophic decision making and preference structures, neutrosophic expert systems, Neutrosophic reliability theory, neutrosophic logic to robotics etc.

And in almost any humanistic or scientific field where indeterminacy, unknown, and in general where <*neutA*> *(*neutrality with respect to an item <*A*>*)* occur.

8.3.1 Quantum theory

The Schrödinger's Cat Theory says that the quantum state of a photon can Basically be in more than one place in the same time which, translated to the neutrosophic set, Means that an element (quantum state) belongs and does not belong to a set (a place) in the same time; or an element (quantum state) belongs to two different sets (two different places) in the same time. It is a question of "alternative worlds" theory very well represented by the neutrosophic set theory *[30]*.

8.3.2 Applications of Neutrosophic Logic

Voting (pro, contra, neuter) *[31]*:

The candidate *(C),* who runs for election in a metropolis *(M)* of people with right to vote, will win.
This proposition is say, 20-25% true (percentage of people voting for him).35-45% false (percentage of people voting against him).and 40% or 50% indeterminate (percentage of people not coming to the ballot box, or giving Blank vote-not selecting anyone or giving a negative vote –cutting all candidates on the list).

- Epistemic/subjective uncertainty (which as hidden/unknown parameter).
  Tomorrow it will rain. This proposition is, say, 50% true according to meteorologists who have investigate the past year' weather,20-30% false according to today's very sunny and droughty summer , and 40% undecided.

8.3.3 Applications of Neutrosophic Sets

Philosophical application*:*

- Or, how to calculate the truth-value of Zen doctrine philosophical propositions the present is eternal and comprises in itself the past and the future?

- In eastern philosophy the contradictory utterance from the core of the Taoism and Zen/Chan (which emerged from Buddhism and Taoism) doctrines.
- How to judge the truth-value of a metaphor, or of an ambiguous statement or of a social phenomenon which is positive from a standpoint and negative from another standpoint?

Physics application:

- How to describe a particle $\int$ in the infinite micro – universal of quantum physics that belong to two distinct places $P_1$and P_2 in the same time? $\int \in P_1 and \int \notin P_2$ As a true contradiction, or $\int \in P_1 and \int \in \neg P_1$.
- Don't we better describe, using the attribute "neutrosophic" than "fuzzy" and other, a quantum particle that neither exists nor non- exists? [High degree of indeterminacy].
- A cloud is neutrosophic set because its borders are ambiguous, and each element belongs with a neutrosophic probability to the set.

8.3.4 Applications of Neutrosophic in Production Facility Layout Planning and Design.

The original CRAFT (Computerized Relative Allocation of Facilities Technique) model for cost-optimal relative allocation of production facilities as well as many of its later extensions tends to be quite “heavy” in terms of CPU engagement time due to their heuristic nature. A Modified Assignment (MASS) model (first proposed by Bhattacharya and Khoshnevisan in 2003) increases the computational efficiency by developing the facility layout problem as primarily a Hungarian assignment problem but becomes indistinguishable from the earlier CRAFT - type models beyond the initial configuration *[31]*.

8.3.5 Application in Neutrosophic set

Could provide a new tool to describe the image with uncertain information, which had been applied to image processing techniques such as image segmentation, thresholding and denoise *[32]*.

As we can see a novel image segmentation approach which is proposed based on neutrosophic filtering and level set theory. The image is transformed into neutrosophic set domain, and described using three Membership sets *(T, I and F)*. The directional alpha-mean filter (DAMF) is employed to reduce the Image’s indeterminacy, and the image is segmented on the *(T)* subset after DAMF processing using Level set algorithm. The experimental results show that the proposed method can perform better on clear Images and noisy images, due to the fact that the proposed approach can handle the indeterminacy of The images well. The proposed method can be used widely in many images processing applications.

8.3.6 Neutrosophy can be widely applied in physics and the like

- Let <A> be a physical entity (i.e. concept, notion, object, space, field, idea, law, property, state, attribute, theorem, theory, etc.), <antiA> be the opposite of <A>, and <neutA> be their neutral (i.e. neither <A> nor <antiA>, but in between).
- Neutrosophic Physics is a mixture of two or three of these entities <A>, <antiA>, and <neutA> that hold together.
- Therefore, we can have neutrosophic fields, and neutrosophic objects, neutrosophic states, etc.
- For example, one of the reasons for 2011 Nobel Prize for physics is "for the discovery of the accelerating expansion of the universe through observations of distant supernovae", but according to Neutrosophy,

There exist seven or nine states of accelerating expansion and contraction and the neutrosophic state in The universe. Another two examples are "a revision to Gödel's incompleteness theorem by neutrosophy" and" Six neutral (neutrosophic) fundamental interactions". In addition, the "partial and temporary unified theory so Far "is discussed (including "partial and temporary unified electromagnetic theory so far", "partial and temporary Unified gravitational theory so far", "partial and temporary unified theory of four fundamental interactions so far", and "partial and temporary unified theory of natural science so far") *[33]*.

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8.3.7 Medical diagnosis under the interval neutrosophic environment

However, by only taking one time inspection, we wonder whether one can obtain a conclusion from a particular person with a particular decease or not. Hence, we have to examine the patient at different time intervals (e.g., two or three times a day) and can obtain that data collected from multiple time inspections for the patient are interval values rather than single values. In this case, the improved cosine measures of INSs (interval neutrosophic sets) are a better tool to find a proper disease diagnosis *[34]*.

9. More Applications

Neutrosophy and Neutrosophic Logic/Set/Probability/Statistics are used in:

- Extenics (to resolve contradictory problems);
- Description Logic, Relational Data Model, Semantic Web Service Agent;
- Image Segmentation; Image Segmentation;
- Remedy for Effective Cure of Diseases using Combined
- Neutrosophic Relational Maps;
- Neutrosophic Research Method;
- Transdisciplinarity, Multispace & Multistructure;
- Qualitative Causal Reasoning on Complex Systems;
- Study on suicide problem using combined overlap block Neutrosophic Cognitive Maps;
- Neutrosophic Topologies;
- Discrimination of outer membrane proteins using reformulated support vector machine based on neutrosophic set;
- support vector machine based on neutrosophic set;
- Decision support tool for knowledge based institution using neutrosophic neutrosophic cognitive maps;
- Imprecise query solving;
- Answering queries in Relational Database using Neutrosophic Logia;
- Ensemble Neural Networks Using Interval Neutrosophic Sets and Bagging;
- Lithofacies Classification from Well Log Data using Neural Networks, Interval Neutrosophic Sets and Quantification of Uncertainty;
- Redesigning Decision Matrix Method with an indeterminacy-based inference process ;

10. Conclusion and future work

The neutrosophic is an important topic to study it as it open the closed door to many application. Since the world is full of indeterminacy, the neutrosophic found their place into research. This paper presented some important definition (notion) of neutrosophic set, neutrosophic logic, neutrosophic measure, neutrosophic integral, and a single valued neutrosophic set.

And introduce some application of neutrosophic in many field (information system ,information technology ,decision support system): neutrosophic relational databases, neutrosophic image (thresholding, denoising, segmentation) processing, neutrosophic linguistic variables, neutrosophic decision making and Preference structures, neutrosophic expert systems, neutrosophic reliability theory, neutrosophic soft computing techniques in e-commerce and e-learning, image segmentation, Neutrosophic Logic to Robotics etc.

In the future, these notions can help researchers and make great use of it in the future in making algorithms to solving problems and manage between these notions to produce a new application or new algorithm of solving decision support problems. So, in the future, these three algorithms can help the researchers and use any one of it to make application. These algorithms in multicriteria decision making to select best alternative (ideal alternative) from group of alternative and group of criteria and now introduce it to see how to manage between them to select best alternative. First, Algorithm called multicriteria decision making based on neutrosophic sets *[35]* .This algorithm presents a new method for handling neutrosophic multicriteria decision-making problems, where the characteristics of the alternatives are represented by neutrosophic sets. Suppose there exists a set of alternatives from which the most preferred Alternative is to be selected. Each alternative is assessed on different criteria.

Second, called Single-valued neutrosophic multicriteria decision-making method *[36]* can use it to select the most preferred alternative based on set of alternative, criteria and the weight of the criterion entered by the decision maker.

Third, a multi-criteria neutrosophic group decision making method based TOPSIS for supplier selection *[37, 38]*. The main purposes of this algorithm is to determining the alternative which is closest to the ideal solution throw one equation to calculate performance weights of decision makers expressed by single valued neutrosophic numbers and linguistic variables.

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