Neutrosophic Fuzzy lattice Via Fuzzy Partial Ordering

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4 authors, including:

Mullai Murugappan
Alagappa University

P.K. Santhi
Jayaraj Annapackiam College for Women(Autonomous), Periyakulam

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R. Meenakshi[1], **M. Mullai**[2,*], **P. K. Santhi**[3], **S. Pandiselvi**[4]
[1,4]Department of Mathematics, Seethalakshmi Achi College for Women, Pallathur-630 107, India
[2]Department of Mathematics, Alagappa University, Karaikudi - 630 004, Tamil Nadu, India
[3]Department of Mathematics, Jayaraj Annapackiam College for Women (Autonomous),Periyakulam, Theni(Dist)Tamilnadu, India
Emails: meenakshi.sac@rediffmail.com[1]; mullaim@alagappauniversity.ac.in[2]; pksanthimat@annejac.ac.in[3];selvi.jasmine90@gmail.com[4]

Abstract

In this paper, we introduce the concept of neutrosophic fuzzy lattice via fuzzy partial ordering. The definition of neutrosophic fuzzy lattice and equipotent are developed with suitable examples. Cartesian product and some equivalent properties of neutrosophic fuzzy lattice are discussed . Also some theorems on neutrosophic fuzzy lattice are developed here.

Keywords: Fuzzy lattice; Fuzzy partial ordering; ephimorphism; Neutrosophic Fuzzy lattice.

1 Introduction

The relation between logics of algebra and modern algebra was worked by many mathematicians from Boolean. The notion of a fuzzy set from the crisp set was introduced by L.A. Zadeh[10] and the study of fuzzy algebraic structures was initiated by Rosenfeld, since then various algebraic structures were converted to fuzzy algebra.

Lattice structure has been found to be extremely important in the areas of communication systems and information analysis. Some system models often include exessive complexity of the situation which in turn may lead to consequence where it is difficult to formulate the model or the model is too complicated to be used in practice.

Nanda.S[8] defined the notion of fuzzy lattice latter Kanakana Chakraborty[2] has modified the definition for fuzzy lattice. Vasantha Kandasamy and Florentin Smarandache[9] has introduced Neutrosophic Lattices, which made us to develop the structure Neutrosophic fuzzy lattices via fuzzy partial ordering. The concept of neutrosophic subtraction algebras and neutrosophic subtraction semigroups was developed by M.A. Ibrahim, et.al. in.[4]

NeutroAlgebras and AntiAlgebras are developed and neutroalgebra is a generalization of partial algebra was discussed by Florentin Smarandache in.[3] Also, Image and Inverse Image of Neutrosophic Cubic Sets in UP-Algebras under UP-Homomorphisms and Magnification of MBJ-Neutrosophic Translation on G-Algerbra are discussed in.[5,6] In this article we introduce the concept of neutrosophic fuzzy lattice via fuzzy partial ordering with definitions and example and developed some theorems on neutrosophic fuzzy lattice.

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2 Preliminaries

Let χ be any set and let ν be a fuzzy relation defined over χ. Then ν is said to be
(i) Max-min transitive, if $\nu.\nu \subseteq \nu$ or more explicitly, $\forall (x_1, x_2, x_3) \in \chi^3\ \mu_{\nu(x_1,x_3)} \geq min\{\mu_{\nu(x_1,x_2)}, \mu_{\nu(x_2,x_3)}\}$
(ii) Reflexive, if $\forall x_1 \in \chi, \mu_{\eta(x_1,x_2)} = 1$
(iii) Perfect antisymmetric, if $\forall (x_1, x_2) \in \chi^2, x_1 \neq x_2, \mu_{\nu(x_1,x_2)} > 0 \Rightarrow \mu_{\nu(x_2,x_1)} = 0$, where $\mu_{\nu(x-1,x_2)}$ represent the membership value of the pair $(x_1, x_2) \in \nu$.
The fuzzy relation $\bar{S}$ defined over a set ν is said to be fuzzy partial ordering if and only if it is reflexive, max-min transitive and perfectly antisymmetric. A set χ along with a fuzzy partial ordering $\bar{S}$ defined on it is called a fuzzy partially ordered set.

Let χ be a fuzzy partially ordered set with a fuzzy partial order $\bar{S}$ defined over it with each $x_1 \in \nu$, we associate two fuzzy sets $x_1 \in \nu$ and $x_2 \in \nu$ such that
(i) The dominating class $\bar{S} \geq (x_1)(x_2) = \bar{S}(x_2, x_1)$
(ii) The dominating class $\bar{S} \leq (x_1)(x_2) = \bar{S}(x_1, x_2)$

Let α be a non fuzzy subset of ν. Then, the fuzzy upper bound of α is denoted by $U_{\varphi(\alpha)} = \bigcap_{x_1 \in \alpha} \bar{P} \geq (x_1)$ and the fuzzy lower bound of α is denoted by $L_{\varphi(\alpha)} = \bigcup_{x_1 \in \alpha} \bar{P} \leq (x_1)$. Let $\bar{\tau}$ be a fuzzy partially ordered set and let $\bar{\sigma}$ be a fuzzy subset of $\bar{\tau}$. Then $\bar{\sigma}$ is said to be a fuzzy lattice in $\bar{\tau}$, if every pair of elements in $\bar{\tau}$ has a fuzzy lower bound L_φ and fuzzy upper bound U_φ, where both L_φ and U_φ are fuzzy subsets of $\bar{\tau}$ satisfying the following two conditions:

$$\begin{aligned} \mu_{max\{U_\varphi\}(x_1)} &\geq \mu_{\bar{\sigma}(x_1)}, \forall x_1 \in \bar{\tau} \\ \mu_{min\{L_\varphi\}(x_1)} &\geq \mu_{\bar{\sigma}(x_1)}, \forall x_1 \in \bar{\tau} \end{aligned}$$

Let N(P) be a partially ordered set with 0, 1, I , $1 + I = 1 \bigcup I \in N(P)$.
Define min and max on N(P) such that $max\{x, y\}$ and $min\{x, y\} \in N(P)$.

3 Neutrosophic fuzzy lattice Via fuzzy lattice Partial Ordering

Let **N(L)** be a neutrosophic fuzzy lattice. Then N(L) is
(i) maximum, if $U_{T(x,y)} = \bigcap N(P) \geq \chi$
(ii) indeterminacy, if $L_{I(x,y)} = \bigcup N(P) \leq \chi$
(iii) minimum, if $L_{F(x,y)} = \bigcup N(P) \leq \chi, \forall \chi \in N(L)$
Let **N(L)** be a neutrosophic fuzzy lattice, if a fuzzy subset μ_σ of **N(L)** is said to be a neutrosophic fuzzy lattice **N(L)**, if it satisfies the following axioms
(i) $\mu_{maxT(x,y)} \geq \mu_\sigma$
(ii) $\mu_{maxI(x,y)} \geq \mu_\sigma$
(iii) $\mu_{minF(x,y)} \geq \mu_\sigma$ Let us consider $(Z_4, *)$ and let $0 \leq \mu_\sigma \leq 3$

$*$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

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Then, $U_{T(x,y)} = \bigcap N(P)$, $L_{I(x,y)} = \bigcup N(P)$, $L_{F(x,y)} = \bigcup N(P)$

$$\begin{aligned}
L_{\varphi(0,1)} &= \{0,1,2,3\} \\
L_{\varphi(0,2)} &= \{0,2,0,2\} \\
L_{\varphi(0,3)} &= \{0,3,2,1\} \\
L_{\varphi(1,2)} &= \{0,2,2,3\} \\
L_{\varphi(1,3)} &= \{0,3,2,3\} \\
L_{\varphi(2,3)} &= \{0,3,2,2\} \\
U_{\varphi(0,1)} &= \{0,0,0,0\} \\
U_{\varphi(0,2)} &= \{0,0,0,0\} \\
U_{\varphi(0,3)} &= \{0,0,0,0\} \\
U_{\varphi(1,2)} &= \{0,1,0,2\} \\
U_{\varphi(1,3)} &= \{0,1,2,1\} \\
U_{\varphi(2,3)} &= \{0,2,0,1\}
\end{aligned}$$

Therefore $\mu_{minT(x,y)} = \{0,1,0,1\}$
$\mu_{maxI(x,y)} = \{0,1,2,2\}$,
$\mu_{maxF(x,y)} = \{0,1,2,2\}$
where $0 \leq \mu_\sigma \leq 3$
Hence, $(Z_4, *)$ is a neutrosophic fuzzy lattice. Let **N(L)** be a neutrosophic fuzzy lattice and N(L) is said to be equipotent, if it satisfies the following axioms
(i) $\mu_{maxT(x,y)} \geq max\{T_{\mu(x)}, T_{\mu(y)}\}$
(ii) $\mu_{minI(x,y)} \geq min\{I_{\mu(x)}, I_{\mu(y)}\}$
(iii) $\mu_{minF(x,y)} \geq min\{F_{\mu(x)}, F_{\mu(y)}\}$, $\forall x,y \in N(L)$ Let $I : N(X) \longrightarrow N(L)$ is called a neutrosophic fuzzy lattice over L(X). If
(i) $L_{T(x+y)} \geq L\{T(x), T(y)\}$
(iii) $L_{I(x+y)} \leq L\{I(x), I(y)\}$
(ii)$L_{F(x+y)} \geq L\{F(x), F(y)\}, \forall x,y \in L(X)$. A fuzzy set μ in **N(L)** be a neutrosophic fuzzy lattice, if it satifies the following inequalities
(i) $\mu_{\bar{T}(0)} \leq \mu_{\bar{T}(x)}$
(ii) $\mu_{\bar{F}(x)} \leq \mu_{min\{F_\varphi(x)\}}$, $\forall x \in N(L)$ If α and β be two neutrosophic fuzzy lattices of $N(L)$, then the cartesian product $\alpha \times \beta : X \times X \longrightarrow [0,3]$ is defined by $(\alpha \times \beta)(x,y) = min\{L_\alpha(x), L_\beta(y)\}$, where $\{L_\alpha(x), L_\beta(y)\}$ is a greatest least element of $N(L)$. Let $N(L)$ be a neutrosophic fuzzy lattice , then the following hold for any elements l and m in $N(L)$.
(i) $L_{T(l)} \leq L_{T(m)}$
(ii)$L_{I(l)} \leq L_{I(m)}$
(iii) $L_{F(l)} \leq L_{F(m)}$ **Proof**:
For (i) $L_{T(l)} \leq L_{T(m)} \Longrightarrow L_{T(l)} \subseteq L_{T(m)}$
$l \leq m \Longrightarrow max\{l,m\} = m$ and $min\{l,m\} = l$
$\Longrightarrow L_{T(l \vee m)} = L_{T(m)}, L_{T(l \wedge m)} = L_{T(l)}$,
where $max\{l,m\} = l \vee m, min\{l,m\} = l \wedge m$
$L_{T(l)} \vee L_{T(m)} = L_{T(m)}$ and $L_{T(l)} \wedge L_{T(m)} = L_{T(l)}$
For ii: $L_I(l,m) \Longrightarrow L_{I(l)} \leq L_{I(m)}$
$\Longrightarrow L_{I(l)} \vee L_{I(m)} = L_{I(m)}$ and $L_{I(l)} \wedge L_{I(m)} = L_{I(l)}$
For iii: $L_{F(l,m)} \Longrightarrow L_{F(l)} \leq L_{F(m)}$
$L_{F(l)} \vee L_{F(m)} = L_{F(m)}$ and $L_{F(l)} \wedge L_{F(m)} = L_{F(l)}$ If α and β be two neutrosophic fuzzy lattices of $N(L)$. Then $\alpha \times \beta$ is a neutrosophic fuzzy lattice of $N(L) \times N(L)$. Proof:
For any $(l,m) \in N(L) \times N(L)$, we have
$(\alpha \times \beta)(0,0) = min\{L_\alpha(0), L_\beta(0)\} \leq min\{L_\alpha(l), L_\beta(m)\} = (\alpha \times \beta)(l,m)$

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Let (l_1, l_2) and $(m_1, m_2) \in N(L) \times N(L)$, then

$$\begin{aligned}
(\alpha \times \beta)(l_1, l_2) &= min\{L_{\alpha(l_1)}, L_{\beta(l_2)}\} \\
&\geq min\{min\{L_{\alpha(l_1 \wedge m_1)}, L_{\alpha(m_1)}\}, min\{L_{\beta(l_2 \wedge m_2)}, L_{\beta m_2)}\}\} \\
&= min\{L_{(\alpha\times\beta)(l_1 \wedge m_1, l_2 \wedge m_2)}, L_{(\alpha\times\beta)(m_1,m_2)}\} \\
&= min\{L_{(\alpha\times\beta)(l_1,l_2)\wedge(m_1,m_2)}, L_{(\alpha\times\beta)(m_1,m_2)}\} \\
L_{(\alpha\times\beta)(l_1,l_2)\wedge(m_1,m_2)} &= L_{(\alpha\times\beta)(l_1\wedge m_1),(l_2\wedge m_2)} \\
&= min\{L_{\alpha(l_1\wedge m_1)}, L_{\beta(l_2\wedge m_2)}\} \\
&\geq min\{min\{L_{(\alpha(l_1),\alpha(m_1))}\}, min\{L_{(\beta(l_2),\beta(m_2))}\}\} \\
&= min\{min\{L_{(\alpha(l_1),\beta(l_2))}\}, min\{L_{(\alpha(m_1),\beta(m_2))}\}\} \\
&= min\{L_{(\alpha\times\beta)(l_1,l_2)}, L_{(\alpha\times\beta)(m_1,m_2)}\}
\end{aligned}$$

Hence $\alpha \times \beta$ is a neutrosophic fuzzy lattice of $N(L) \times N(L)$. Let $\chi : \eta \longrightarrow \nu$ be a mapping of neutrosophic fuzzy lattice $N(L)$ and η be a fuzzy lattice of ν. Then the mapping ν_χ is the preimage of ν if $\eta_{\chi(x)} = \nu_{\chi(x)}$, for all $x \in \eta$. Let $\chi : \eta \longrightarrow \nu$ be a homomorphism. If ν is a neutrosophic fuzzy lattice of $N(L)$, then ν_χ is a neutrosophic fuzzy lattice of $N(L)$. **Proof:** For any $l \in N(L)$, we have

$$\nu_{\chi(l)} = \nu\chi(l) = \nu(0)^\chi = \nu\chi(0) = \nu_\chi(0)$$

Let $(l, m) \in N(L)$, then

$$\begin{aligned}
min\{\nu_{\chi(l\wedge m)}, \nu_{\chi(m)}\} &= min\{\nu\chi(l \wedge m), \nu\chi(m)\} \\
&= min\{\nu\chi(l) \wedge \chi(m), \nu\chi(m)\} \\
&\leq \nu\chi(l) = \nu_{\chi(l)} \\
\text{and } min\{\nu_{\chi(l)}, \nu_{\chi(m)}\} &= min\{\nu\chi(l), \nu\chi(m)\} = \nu\chi(l) \wedge \nu\chi(m) \\
&= \nu(\chi(l \wedge m)) = \nu_{\chi(l\wedge m)}
\end{aligned}$$

Let $\chi : \eta \longrightarrow \nu$ be an epimorphism of neutrosophic fuzzy lattice $N(L)$. If ν_η is a neutrosophic fuzzy lattice of η, then ν is a neutrosophic fuzzy lattice χ. **Proof:** Let $m \in \nu$, there exists $l \in \eta$, such that $\chi(l) = m.$
Then $\nu(m) = \nu(\chi(l)) = \nu_{\chi(0)} = \nu(\chi(0)) = \chi(0')$.
Let $(l, m) \in \nu$, then there exists $(\eta_1, \eta_2) \in \eta$, such that $\chi(\eta_1) = l, \chi(\eta_2) = m.$
Hence, we have

$$\begin{aligned}
\nu(l) = \nu(\chi(\eta_1)) = \nu^{\chi(\eta_1)} &\geq min\{\nu_{\chi(\eta_1,\eta_2)}, \nu_{\chi(\eta_2)}\} \\
&= min\{\chi(\eta_1, \eta_2), \chi(\eta_2)\} \\
&= min\{\chi(\eta_1) \wedge \chi(\eta_2), \chi(\eta_2)\} \\
&= min\{\chi(l \wedge m), m)\} \\
\text{and } \nu(l \wedge m) &\geq min\{\chi(\eta_1) \wedge \chi(\eta_2)\} \\
&= min\{\nu(l \wedge m) \geq min\{\chi(l), \chi(m)\}
\end{aligned}$$

Hence, ν is a neutrosophic fuzzy lattice of χ.

4 Conclusion

So far in research algebraic structures were converted to lattice algebraic structure and fuzzy lattice. In this paper, we have converted fuzzy lattice to neutrosophic fuzzy lattice via fuzzy partial ordering. In the same way it will be interesting to convert other fuzzy lattice structure via fuzzy partial ordering to neutrosophic fuzzy lattice.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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