Neutro-R Modules

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Chapter Six

Neutro-R Modules

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ABSTRACT

In this chapter, neutro - R modules are obtained, basic properties and examples related to these structures are given. Furthermore, classical R modules and neutro-R modules are compared. It is shown that the neutro-R modules have a more general structure. Thus, a new structure is obtained by adding the (T, I, F) components, which form the structure of the neutrosophic theory, to the classical R modules (without using neutrosophic sets). Also, it is shown that a neutro-R module can be obtained from every classical R module.

Keywords: R Modules, Neutrosophic Theory, Neutro – Algebraic Structures, Neutro-R Modules

INTRODUCTION

An R module is one of the basic algebraic structures used in abstract algebra. An R module on a ring is a generalization of the concept of a vector space on a field. Here the corresponding scalars are the (unit) elements of an arbitrary given ring R and a (left and / or right) multiplication is defined on the elements of the ring and the elements of the module. A module that takes its scalars from a ring R is called an R module. Therefore, a module is an additive abelian group, like a vector space. An operation, distributed over every parameter on addition and is compatible with the multiplication of the ring, is defined on the elements of the ring and the elements of the module. R modules are very closely related to the representation theory of groups. It is also one of the basic concepts of commutative algebra and homological algebra and is widely used in algebraic geometry and algebraic topology. For this reason, many researchers worked on the R modules [1 - 5]. Recently, Abobala et al. studied AH-Substructures In Strong Refined Neutrosophic Modules [6]; Veliyeva et al. worked on derivative functor of inverse limit functor in the category of neutrosophic soft modules [7]; Cai et al. studied classification of simple Harish-Chandra modules for map (super) algebras related to the Virasoro algebra [8].

Smarandache defined neutrosophic logic and the concept of a neutrosophic set in 1998 [9]. In the concept of neutrosophic logic and neutrosophic sets, there is a degree of membership T, a degree of uncertainty I and a degree of non-membership F. These are defined independently from each other. A neutrosophic value has the form (T, I, F). In other words, in neutrosophy, a situation is handled according to its trueness, its falsity, and its uncertainty. Therefore, neutrosophic logic and neutrosophic sets help us explain many uncertainties in our lives. Therefore, many researchers have made studies on this subject [10-12, 35-78]. Recently, Şahin et al. obtain some operations for interval valued neutrosophic sets [13]; Uluçay et al. studied neutrosophic multigroups and Applications [14]; Hassan et al. introduced Q-neutrosophic soft expert set and its application [15]; Sahin et al. obtained neutrosophic soft expert sets [16]; Uluçay studied interval-valued refined neutrosophic sets and their applications [17]; Khalifa et al. obtained neutrosophic set significance on deep transfer learning models [18]; Kargın et al. studied generalized Hamming similarity measure aased on neutrosophic quadruple numbers and its applications [19]; Şahin et al. obtain Hausdorff Measures on generalized set valued neutrosophic quadruple numbers and decision making applications for adequacy of online education [20].

In 2019, Florentin Smarandache introduced new research areas in neutrosophy, which he called neutro-structures and anti-structures [21, 22]. When evaluating <A> as an element (concept, attribute, idea, proposition, theory, etc.), during the neutrosification process, he worked on three regions; two opposites corresponding to <A> and <antiA> and also a neutral (indeterminate) <neutA> (also called <neutralA>). A neutro-algebra consists of at least one neutro-operation (indeterminate for other items and false for other items) or it is an algebra well-defined for some items (also called internally defined), indeterminate for others, and externally defined for others. Therefore, the subject attracted the attention of many researchers [23–29]. Recently, Smarandache et al. studied neutro-BCK algebras [30]; İbrahim et al. obtained neutro - vector spaces [31]; Al-Tahan et al. studied NeutroOrderedAlgebra and applications [32]: Jiménez et al. indroduced neutroalgebra for the evaluation of barriers to migrants' access in Primary Health Care in Chile [33]; Smarandache studied generalizations and alternatives of classical algebraic structures to neutroalgebraic structures and antialgebraic structures [34].

In this chapter; in the second section, basic definitions of a classical R module [1], and the definitions of neutro-group and neutro-ring are given [31]. In the third section, neutro-R module is defined and its basic properties are given. The similarities and differences between the classical R module and the neutro - R module are given. It is shown that a neutro-R module can be obtained from every classical R module. In the last part, results and suggestions were given.

BACKGROUND

Definition1. [1] Let (G, #) be an abelian group, (R, +, .) be a commutative ring and let *: RxG→R be a binary operation. If the following conditions are satisfied, then G is called an R module. For p, r ϵ R and s, t ϵ G,

i) p* (s # t) = (p*s) # (p*t)

ii) (p + r) *s = (p*s) + (r*s)

iii) s*(p.r) = (s*p).r

iv) s*1 = s (1 being the identity element in G)

Definition2. [1] Let G be an R module. A subring N of R is an R submodule if N is also an R module under the same action of G.

Definition3. [1] Let (R, +, .) be a commutative ring; $*_1$: $RxG_1 \rightarrow R$, $*_2$: $RxG_2 \rightarrow R$ be two binary operations, and let (G_1, $\#_1$) and (G_2, $\#_2$) be two R modules. If the following conditions are satisfied, then $\varphi: G_1 \rightarrow G_2$ is called an R module homomorphism.

For p, r ϵ G_1 and t ϵ R,

i) $\varphi(p\#_1 r) = \varphi(p)\#_2\varphi(r)$

ii) $\varphi(t*_1 p) = t*_2\varphi(p)$.

Definition 4. [31]

i) [Law of neutro-closedeness]

There exists a double (b, n) ∈ G such that b # n ∈ G (degree of truth T) and there exist two doubles (u, v) and (p, q) ∈ G such that [u # v = indeterminate (I) or p # q ∉ G (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1). Because (1, 0, 0) represents the classical well-defined law (100% well-defined law; T = 1, I = 0, F = 0), while (0, 0, 1) represents the outer-defined law (i.e. 100% outer-defined law, or T = 0, I = 0, F = 1).

ii) [Axiom of neutro-associativity]

There exists a triplet (b, n, m) ∈ G such that b # (n # m) = (b # n) # m (degree of truth T) and there exist two triplets (p, q, r) and (u, v, w) ∈ G such that [[p # (q # r)] or [(p # q) # r] = indeterminate (degree of indeterminacy I) or u # (v # w) ≠ (u # v) # w (degree of falsehood F)]; where (T, I, F) is different from (1, 0,

0) and (0, 0, 1). Because (1, 0, 0) represents the classical well-defined law (100% well-defined law; T = 1, I = 0, F = 0), while (0, 0, 1) represents the outer-defined law (i.e. 100% outer-defined law, or T = 0, I = 0, F = 1).

iii) [Axiom of existence of the neutro-identity element]

For at least one b ∈ G, there exists e ∈ G such that b # e = e # b = b (degree of truth T) and there exists e ∈ G such that [b # e or e # b = indeterminate (degree of indeterminacy I) or b # e ≠ b ≠ e # b (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

iv) [Axiom of existence of the neutro-inverse element]

For at least one b ∈ G, there exists u ∈ G such that b # u = u # b = b (degree of truth T) and there exists u ∈ G such that [b # u or u # b = indeterminate (degree of indeterminacy I) or b # u ≠ b ≠ u # b (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

v) [Axiom of neutro-commutativity]

There exists a double (b, n) ∈ G such that b # n = n # b (degree of truth T) and there exists two doubles (u, v) and (p, q) ∈ G such that [u # v or v # u = indeterminate (degree of indeterminacy I) or p # q ≠ q # p (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

Definition 5. [31] A neutro-group is a neutro-algebraic structure which possesses at least one of the axioms {i) – iv)} of Definition 4 and is an alternative to classical group.

Definition 6. [31] A neutro-commutative group is a neutro-algebraic structure which possesses at least one of the axioms {i) – iv)} of Definition 4 and is an alternative to classical commutative group.

Definition 7. [31]

Let R be a nonempty set and let + : R × R → R and . : R × R → R be two binary operations on R.

i) [Law of neutro-Closedness with respect to addition]

There exists a double (b, n) ∈ R such that b + n ∈ R (degree of truth T) and there exists two doubles (u, v) and (p, q) ∈ R such that [u + v = indeterminate (degree of indeterminacy I) or p + q ∉ R (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

ii) [Axiom of neutro-associativity with respect to addition]

There exists a triplet (b, n, m) ∈ R such that b + (n + m) = (b + n) + m (degree of truth T) and there exists two triplets (p, q, r), (u, v, w) ∈ R such that [[p + (q + r)] or [(p + q) + r] = indeterminate (degree of indeterminacy I) or u + (v + w) ≠ (u + v) + w (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

iii) [Axiom of existence of the neutro-identity element with respect to addition]

For at least one b ∈ R, there exists an e ∈ R such that b + e = e + b = b (degree of truth T) and for at least one b ∈ R, there exists an e ∈ R such that [b + e or e + b = indeterminate (degree of indeterminacy I) or b + e ≠ b ≠ e + b (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

iv) [Axiom of existence of the neutro-inverse element with respect to addition]

For at least one b ∈ R, there exists a u ∈ R such that b + u = u + b = b (degree of truth T) and for at least one b ∈ R, there exists a u ∈ R such [b + u or u + b indeterminate (degree of indeterminacy I) or b + u ≠ b ≠ u + b (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

v) [Axiom of neutro-commutativity with respect to addition]

There exists a double (b, n) ∈ R such that b+ n = n + b (degree of truth T) and there exists two doubles (u, v) and (p, q) ∈ R such that [u + v or v + u = indeterminate (degree of indeterminacy I) or p + q ≠ q + p (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

vi) [Law of neutro-closedness with respect to multiplication]

There exists a double (b, n) ∈ R such that b . n∈ R (degree of truth T) and there exist two doubles (u, v), (p, q) ∈ R such that [u . v = indeterminate (degree of indeterminacy I) or p . q ∉ R (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

vii) [Axiom of neutro-associativity with respect to multiplication]

There exists a triplet (b, n, m) ∈ R such that b . (n . m) = (b . n) . m (degree of truth T) and there exist two triplets (p, q, r), (u, v, w) ∈ R such that [[p . (q . r)] or [(p . q) . r] = indeterminate (degree of indeterminacy I) or u . (v . w) ≠(u . v) . w (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

viii) [Axiom of neutro-left distribution]

There exists a triplet (b, n, m) ∈ R such that b . (n + m) = b .n + b .m (degree of truth T) and there exist two triplets (p, q, r), (u, v, w) ∈ R such that [p . (q + r) or p .q + p . r = indeterminate (degree of indeterminacy I) or u . (v + w) ≠u . v + u .w (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

ix) [Axiom of neutro-Right distribution]

There exists a triplet (b, n, m) ∈ R such that (n + m) . b = n . b + m .b (degree of truth T) and there exists two triplets (p, q, r), (u, v, w) ∈ R such that [(q + r) . p or q . p + r . p = indeterminate (degree of indeterminacy I) or (v + w) .u≠ v . u + w .u (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

x) [Axiom of neutro-commutativity with respect to multiplication]

There exists a double (b, n) ∈ R such that b . n = n .b (degree of truth T) and there exist two doubles (u, v) and (p, q) ∈ R such that [u . v or v . u = indeterminate (degree of indeterminacy I) or p . q ≠ q . p (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

Definition 8. [31] A neutro-ring is a neutro-algebraic structure which possesses at least one of the axioms {i) – ix)} of Definition 7 and is an alternative to classical ring.

Definition 9. [31] A commutative neutro-ring is a neutro-algebraic structure which possesses at least one of the axioms {i) – x)} of Definition 7 and is an alternative to classical commutative ring.

NEUTRO-R MODULES

Throughout this section, "$=^U$" is used to symbolize the conditions where an equality is indeterminate. For example, "a $=^U$ b" means that it is not certain whether a is equal to b.

The symbol " $\in^U$ ", similarly, is used to indicate that the corresponding membership is indeterminate; i.e. if it is not known whether the element a is in the set B, we write a $\in^U$ B.

Definition 10. Let (G, #) be an abelian neutro-group, (R, $+_1$, $._1$) a commutative neutro-ring and let *: RxG → R be a binary operation. If at least one of the following conditions {i, ii, iii, iv, v} is satisfied, then (G, #) is called a neutro-R module.

i) There exists a double (b, n) ∈ (R, G) such that b * n ∈ G (degree of truth T) and there exist two doubles (u, v) and (p, q) ∈ (R, G) such that [p * q ∉ R (degree of falsehood F) or u * v $\in^U$V (indeterminacy (I))]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

ii) There exists a triplet (b, n, m) ∈(R, G, G) such that b * (n # m) = (b *n)#(b * m) (degree of truth T) and there exist two triplets (p, q, r) and (u, v, w) ∈ (R, G, G) such that [p * (q # r)$=^U$(p * q)#(p * r) (degree of indeterminacy I) or [u*(v # w)≠(u *v) # (u* w) (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

iii) There exists a triplet (b, n, m) ∈(R, G, G) such that (b $+_1$ n) * m = (b *m) $+_1$ (n * m) (degree of truth T) and there exist two triplets (p, q, r) and (u, v, w) ∈ (R, R, G) such that [(p $+_1$ q) * r $=^U$ (p * r) $+_1$ (q* r) (degree of indeterminacy I) or [(u $+_1$ v) * w ≠(u * w) $+_1$ (v * w) (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

iv) There exists a triplet (b, n, m) ∈(R, G, G) such that *(n $._1$ m) = (b * n) $._1$ m (degree of truth T) and there exist two triplets (p, q, r) and (u, v, w) ∈ (R, R, G) such that [p * (q $._1$ r) $=^U$(p * q) $._1$ r (degree of indeterminacy I) or u * (v $._1$ w) ≠ (u * v) $._1$ w (degree of falsehood F)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

v) For a double (a, e) ∈ (R, G), there exists an e ∈ G such that a * e = a (degree of truth T) and (for two doubles (b, e), (c, e) ∈ (R, G), there exists e ∈ G such that b * e ≠ b (degree of falsehood F) or c * e $=^U$ c (degree of indeterminacy I)); where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

Note 11. Definition 10 is different from the definition of a classical R module and neutro-R modules are given as an alternative for classical R module. But, for a neutro-R Module, classical R module conditions are valid if neutro-R module is not satisfied as in Definition 10.

Definition 12. Let (G, #) be an abelian group, (R, $+_1$, $._1$) a commutative ring and let *: RxG → R be a binary operation. If at least one of the conditions {i, ii, iii, iv, v} in Definition 10 is satisfied, then (G, #) is called a weak neutro-R module.

Example 13. (ℕ, .) is a neutro-group since the condition iv) of Definition 4 is satisfied. (Classical group conditions are valid for the remaining conditions in Definition 1), where "." is the known multiplication.

As

For all a∈ ℕ, a.1 =1.a = a ,

1 is the unit element for (ℕ, .) . Since 1.1 = 1, 1∈ ℕ has an inverse. But, for a ∈ ℕ \ {1}, there is no b ∈ ℕ \ {1} such that a.b = b.a = 1. So, every element does not have an inverse.

As it satisfies condition iv) of Definition 7, (ℕ, +, .) is a neutro-ring, where "." and "+" are the known multiplication and addition, respectively. (Again, classical ring conditions are valid for the remaining ones.) For all a∈ ℕ, 0 is the unit element for addition as a + 0 = 0 + a = a. For 0∈ ℕ, 0 + 0 = 0 and hence 0 has an inverse. On the other hand, for all a ∈ ℕ \ {0}, there does not exist any b ∈ ℕ \ {0} such that a + b = b + a = 0. So, every element does not have an inverse.

Now, define the operation *: ℕ x ℕ → ℕ such that a*b = a/b. Therefore, (ℕ, .) is a neutro-R module as it satisfies conditions i), ii) and iv) of Definition 10. For instance,

i) 1∈ ℕ and 1*1 = 1/1 = 1∈ ℕ. However, 1∈ ℕ, 2∈ ℕ and 1*2 = 1/2 ∉ ℕ.

ii) 1∈ ℕ and 1*(1.1) = 1 = (1*1).1 . Yet, for 1∈ ℕ and 2∈ ℕ, 1*(2.2) = 1/4 ≠ (1*2).2 = 1 .

iv) 1∈ ℕ and 1*(1.1) = 1 = (1*1).(1*1) . But, for 1∈ ℕ, 2∈ ℕ and 3∈ ℕ, 2*(2.3) = 1/3≠ (2*2).(2*3) = $^2/_3$.

Corollary 14. In Example 13,

(ℕ, .) is a neutro-group but not a classical group.

(ℕ, +, .) is a neutro-ring but not a classical ring.

(ℕ, .) is a neutro-R module but not a classical R module.

Thus, a neutro-R module is a more general structure compared to a classical R module.

Theorem 15. A neutro R-module can be obtained from every classical R module.

Proof: Let (G, #) be a classical abelian group, (R, $+_1$, $._1$) be a classical commutative ring and *: RxG → R be a binary operation. We assume that (G, #) be a classical R module.

i) First, we show that a commutative neutro-group can be obtained from a classical commutative group (G, #).

Let a∉G, b ∈ G and let a # b ∉ G. Here, (G ∪ {a}, #) satisfies the condition i) of Definition 4 and therefore it is a neutro-group.

Let c∉ G, b∈ G, d∈ G and let c # (b # d) ≠ (c # b) # d. Now, (G ∪ {c}, #) satisfies condition ii) of Definition 5 and hence it is a neutro-group.

Let f∉ G, b ∈ G and let e # f≠e ≠f # e. Clearly, (G ∪ {f}, #) satisfies condition iii) of Definition 5. So, (G ∪ {f}, #) is a neutro-group. (e is the classical unit element.)

Let g∉ G, b ∈ G and let c # b ≠ e ≠b # c. As (G ∪ {g}, #) satisfies condition iv) of Definition 5, it is a neutro-group.

Let h∉ G, b ∈ G and let h # b≠b # h. Hence, (G ∪ {h}, #) is a commutative neutro-group as it satisfies condition iv) of Definition 5.

Thus, take A = {a, c, f, g, h} and let P(A) be the power set of A. For B ∈P(A) \ ∅, (G ∪ B, #) is a commutative neutro-group. Hence, a commutative neutro-group is obtained from the commutative classical group (G, #).

ii) Similar to i), take the classical commutative ring (R, $+_1$, $._1$). Now, by the 10 conditions of the commutative neutro-ring and letting C = $\{f_1, f_2, \ldots, f_{10}\}$, for D ∈P(C) \ ∅, (R ∪D, $+_1$, $._1$) is a commutative neutro-ring. Here, $f_1, f_2, \ldots, f_{10}$ are the elements which do not satisfy the conditions of the classical commutative ring. As a result, a commutative neutro-ring can be obtained from the classical commutative ring (R, $+_1$, $._1$) .

iii) Consider the commutative neutro-group (G ∪ A, #) or the commutative neutro-ring (R∪D ,$+_1$, $._1$) obtained above. By adding new elements which do not satisfy classical R – module conditions to the above, one can obtain a neutro-R module from the commutative neutro-group (G ∪ A, #).

Theorem 16. A neutro-R module can be obtained from every weak neutro-R module.

Proof: Let (G, #) be a classical abelian group, (R, $+_1$, $._1$) a classical commutative ring and let *: RxG → R be a binary operation. Assume that (G, #) is a neutro-R module. If the conditions i) and ii) of Theorem 15 are satisfied, then the theorem is proved. Condition iii) need not be satisfied, since (R, $+_1$, $._1$) is already a neutro-G module.

Definition 17. Let (G, #) be a classical abelian group, (R, $+_1$, $._1$) be a classical commutative ring and let *: R x G → R be a binary operation. Assume that (G, #) is a neutro-R module. A subgroup M of G is a neutro-R submodule if M is a neutro-R module which satisfies at least one of the conditions of neutro-R module in Definition 10.

Example 18. In Example 13, (ℕ, +, .) is a commutative neutro-ring, a*b = a/b is a binary operation and (ℕ, .) is a neutro-R module. Taking A = {1, 2, 3, 4, 5}⊂ ℕ, the neutro-submodule (A, .) is a neutro-R submodule since (A, .) satisfies the condition i) of Definition 10.

Definition 19. Let (R, $+_1$, $._1$) be a commutative neutro – ring; $*_1$: R$\times G_1$→ R and $*_2$: R$\times G_2$→ R be two binary operations; (G_1, $\#_1$) and (G_2, $\#_2$) be two neutro – R modules and let φ be a mapping such that φ: $G_1 \rightarrow G_2$. If at least one of the following conditions {i, ii} is satisfied then φ is called a neutro-R module homomorphism:

i) There exists a double (p, q)∈ (G, G) such that φ(p$\#_1$ q) = φ(p)$\#_2$φ(q) (degree of truth T) and there exist two doubles (s, t), (k, m) ∈ (F, V) such that [φ(s $\#_1$ t) ≠φ(s) $\#_1$φ(t) (degree of falsehood F) or φ(k $\#_1$ m) $=^U$φ(k) $\#_1$ φ(m) (degree of indeterminacy I)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

ii) There exists a double (p, q) ∈ (R, G_1) such that $\varphi(p*_1q) = p*_2\varphi(q)$ (degree of truth T) and there exist two doubles (s, t), (k, m) ∈ (R, G_1) such that [$\varphi(s*_1t) = s*_2\varphi(t)$ (degree of falsehood F) or $\varphi(k*_1m) =^U k*_2\varphi(m)$ (degree of indeterminacy I)]; where (T, I, F) is different from (1, 0, 0) and (0, 0, 1).

Example 20. In Example 13, (ℕ, +, .) is a commutative neutro-ring, a*b = a/b is a binary operation and (ℕ, .) is a neutro-R module. Define the mapping φ :ℕ → ℕ such that φ(x) = 1/|x| . So, φ is a neutro-R module homeomorphism as it satisfies the conditions i) and ii) of Definition 19. Namely, for a ∈ℕ - {0} and b∈ℕ - {0}, if a = b, then φ(a.b) = φ(a).φ(b) . Also, for a ∈ℕ - {0} and b∈ℕ - {0}, if a ≠ b, then φ(a.b)≠φ(a).φ(b) . If a = 0 or b = 0, then it is indeterminate. In addition, for a ∈ℕ - {0} and b∈ℕ - {0}, if a = b, then φ(a.b) = a.φ(b) . But, for a ∈ℕ - {0} and b∈ℕ - {0}, if a ≠ b, then φ(a.b) ≠a.φ(b). If a = 0 or b = 0, then it is indeterminate.

Corollary 21. The mapping φ in Example 20 is a neutro-R module homomorphism but not a classical R module homeomorphism. Thus, a neutro-R module homeomorphism is a more general structure compared to a classical R module homeomorphism.

CONCLUSIONS

In this chapter, the neutro-R module is defined and its basic properties are given. The similarities and differences between the classical R module and the neutro-R module are given. It is shown that a neutro-R module can be obtained from every classical R module. Thus, a new structure is obtained by adding the (T, I, F) components, which form the structure of the neutrosophic theory, to the classical R modules (without using neutrosophic sets). Also, researchers can use this section to obtain new structures of neutro-algebra by redefining the classical R module structures. For instance, neutro-group representations, neutro-commutative algebras, neutro-homological algebras, neutro-algebraic geometry and neutro-algebraic topology can be defined.

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<Algebra, NeutroAlgebra, AntiAlgebra>

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The NeutroAlgebras & AntiAlgebras are a *new field of research*, which is inspired from our real world.

In classical algebraic structures, all axioms are 100%, and all operations are 100% well-defined, but in real life, in many cases these restrictions are too harsh, since in our world we have things that only partially verify some laws or some operations.

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