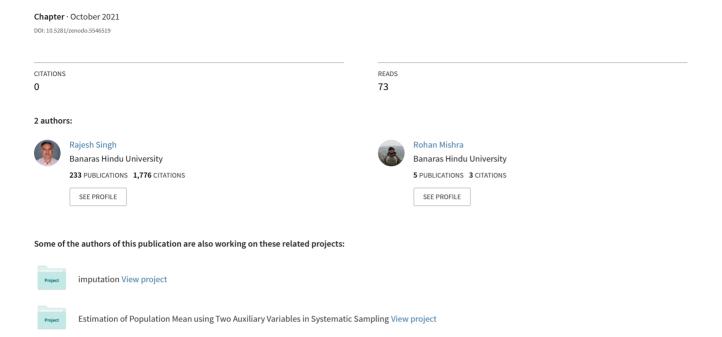
# Neutrosophic transformed ratio estimators for estimating finite neutrosophic population mean



## Neutrosophic transformed ratio estimators for estimating finite neutrosophic population mean

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#### **Abstract**

When the data is vague or indeterminate, estimators proposed under classical statistics fail. Neutrosophic Statistics becomes the only alter- native because it deals with indeterminacy. In this article, we have proposed different transformed neutrosophic ratio estimators to estimate the unknown population mean of neutrosophic data. The Mean squared errors (MSE) up to the first order of approximation, of all the proposed neutrosophic estimators, have been derived. The MSEs of proposed modified neutrosophic ratio estimators are compared with the existing neutrosophic ratio estimators through a theoretical and a simulation study on daily stock prices of Samsung Electronics Co.,Ltd. and Tesla Inc. respectively. The proposed neutrosophic estimators give minimum MSE.

**Keywords:** Neutrosophic Statistics, Classical Statistics, Simulation, Stockprice, Indeterminacy intervals

#### 1 Introduction

Classical statistics and its methods deal with randomness but there are cases where the data at hand is indeterminate or vague or ambiguous or imprecise rather than random. In such situations estimation using classical statistical methods does not yield promising results. Fuzzy logic[1, 2] is one solution to tackle such a problem but still, it ignores indeterminacy.

In such cases, neutrosophic methods are much more reliable. They deal with both randomness and more importantly with indeterminacy. Neutrosophic statistics refers to a set of data such that the data or a part of it is indeterminate and methods to analyze such a data[3].

Neutrosophic statistics is an extension of classical statistics[4] and when the indeterminacy is zero, neutrosophic statistics coincides with classical statistics[3]. Estimation through neutrosophic methods is not a new field yet still unexplored. Some neutrosophic ratio-type estimators has been proposed[4]. In this article first we have provided the terminologies of neutrosophic statics[2] and some related neutrosophic ratio-type estimators[3]. Further, we dealt with the problem of estimating the unknown neutrosophic population mean (daily stock prices of Samsung Electronics Co.,Ltd.[5] and Tesla motors[6]) by proposing three neutrosophic transformed ratio estimators[4]. Their expressions of MSE up to the first order of approximations has been derived and presented. Their MSE has been

compared with similar existing few neutrosophic ratio-type estimators through a theoretical study[5.1] and a simulation study[5.2.1] on daily stock prices of Samsung ElectronicsCo.,Ltd.[5] and Tesla Inc.[6] respectively. A discussion on the results of the study is presented[6]. Finally, concluding remarks and potential research area has been provided[7].

## 2 Terminology

A simple random neutrosophic sample of size n from a classical or neutrosophic population is a sample of n individuals such that at least one of them has some indeterminacy[3]. As presented in [4], a neutrosophic observation is of the form

$$Z_n = Z_L + Z_u I_N$$
, where  $I_N \in [I_L, I_L]$ 

Now consider a simple random neutrosophic sample of size  $n_N \in [n_L, n_U]$  drawn from a finite population of size N and  $y_N(i) \in [y_L, y_U]$  and  $x_N(i)$  are  $i^{th} \in [x_L, x_U]$  neutrosophic sample observation. Here the population mean of neutrosophic survey and auxiliary variable are  $\bar{X}_L, Y_U$  and  $\bar{X}_N \in [X_L, X_U]$  respectively.

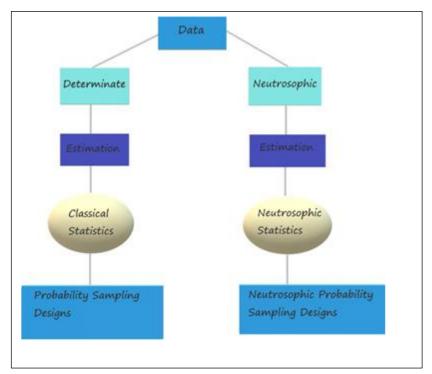


Fig. 1 Flow diagram describing the application of Classical Statistics and Neutrosophic Statistics

 $C_{yN} \in [C_{yN_L}, C_{yN_U}]$  and  $C_{xN} \in [C_{xN_L}, C_{xN_U}]$  are population coefficient of variation of neutrosophic survey and auxiliary variables respectively. In addition,  $\rho_{xyN} \in [\rho_{xyN_L}, \rho_{xyN_U}]$ ,  $\beta_1(x_N) \in [\beta_1(x_{N_L}), \beta_1(x_{N_U})]$  and  $\beta_2(x_N) \in [\beta_2(x_{N_L}), \beta_2(x_{N_U})]$  are the correlation coefficient between the neutrosophic survey and auxiliary variables, coefficient of skewness and coefficient of kurtosis of the neutrosophic auxiliary variable respectively.

The MSE of a neutrosophic estimator is of the form,  $MSE(\bar{y}_N) \in [MSE_L, MSE_U]$ 

The error terms in neutrosophic statistics are:

$$\bar{e}_{vN} = \bar{y}_{-}\{N\} - \bar{Y}_{N}$$

$$\overline{e}_{xN}=\overline{x}_N-\overline{X}_N,$$

$$E(\bar{e}_{yN}) = E(\bar{e}_{xN}) = 0,$$

$$\overline{e}_{yN}^2 = \frac{N-n}{Nn} \overline{Y}_N^2 C_{yN}^2$$

$$\bar{e}_{xN}^2 = \frac{N-n}{Nn} \bar{X}_N^2 C_{xN}^2$$

$$\bar{e}_{xN}\bar{e}_{yN} = \frac{N-n}{Nn}\bar{X}_N\bar{Y}_N\rho_{xyN}C_{xN}C_{yN}$$

Where

$$\bar{e}_{vN} \in [\bar{e}_{vN_I}, \bar{e}_{vN_{II}}]$$

$$\bar{e}_{xN} \in [\bar{e}_{xN_L}, \bar{e}_{xN_U}]$$

$$\bar{e}_{\nu N}^2 \in [\bar{e}_{\nu N_I}^2, \bar{e}_{\nu N_{II}}^2]$$

$$\bar{e}_{xN}^2 \in [\bar{e}_{xN_L}^2, \bar{e}_{xN_U}^2]$$

## 3 Some related neutrosophic estimators

Tahir et al.[4] proposed ratio-type estimators given by

$$\bar{y}_R = \frac{\bar{y}_N}{\bar{x}_N} \bar{X}_N,\tag{1}$$

$$\bar{y}_{USr} = \bar{y}_N \frac{\bar{x}_N \beta_2(xN) + C_{xN}}{\bar{x}_N \beta_2(xN) + C_{xN}} \tag{2}$$

where  $\bar{y}_N \in [\bar{y}_{NL}, \bar{y}_{NU}]$  an  $y_R \in [y_{RL}, y_{RU}]$ ,  $y_{USr} \in [\bar{y}_{USr_L}, y_{USr_U}]$ . Their expressions of MSE are

$$MSE(\bar{y}_R) = \frac{N-n}{Nn} Y_N^2 [C_{yN}^2 + C_{xN}^2 - 2C_{xN}C_{yN}\rho_{xyN}], \tag{3}$$

and

$$MSE(\bar{y}_{USr}) = \frac{N-n}{Nn} Y_N^2 \left[ C_{yN}^2 + \left( \frac{\bar{x}_N \beta_2(xN)}{\bar{x}_N \beta_2(xN) + C_{xN}} \right) C_{xN}^2 - 2 \left( \frac{\bar{x}_N \beta_2(xN)}{\bar{x}_N \beta_2(xN) + C_{xN}} \right) C_{xN} C_{yN} \rho_{xyN} \right], \tag{4}$$

where 
$$C_{yN}^2 \in \left[ C_{yN_L}^2, C_{yN_U}^2 \right]$$
,  $C_{xN}^2 \in \left[ C_{xN_L}^2, C_{xN_U}^2 \right]$ , and  $\rho_{xyN} \in \left[ \rho_{xy} N_L, \rho_{xy} N_U \right]$ .

## 4 Proposed neutrosophic transformed ratio estimator

Motivated by [7] and [8] we propose the following neutrosophic transformed ratio estimators

$$\bar{y}_{USn} = \bar{y}_N \frac{\bar{X}_N C_{xN} + \beta_2(x_N)}{\bar{x}_N C_{xN} + \beta_2(x_N)} \tag{5}$$

where  $\bar{y}_N \in \left[\bar{y}_{N_L}, y_{N_U}\right]$ 

$$\bar{y}_{YT_1} = \bar{y}_N \frac{\bar{x}_N \beta_2(x_N) + \beta_1(x_N)}{\bar{x}_N \beta_2(x_N) + \beta_1(x_N)} \tag{6}$$

and where  $\bar{y}_{YT_1} \in \left[\bar{y}_{YT_{1L}}, \bar{y}_{YT_{1U}}\right]$  and

$$\bar{y}_{YT_1} = \bar{y}_N \frac{\bar{x}_N \beta_2(x_N) + \beta_1(x_N)}{\bar{x}_N \beta_2(x_N) + \beta_1(x_N)}.$$
 (7)

where  $\bar{y}_{YT_1} \in [\bar{y}_{YT_{1L}}, y_{YT_{1U}}]$ . In order to derive the MSE of  $\bar{y}_{USn}$ , we rewrite (5) using error terms,

$$\overline{y}_{USn} = (\overline{Y}_N + \overline{e}_N) \left[ \frac{\overline{X}_N C_{xN} + \beta_2(x_N)}{(\overline{X}_N + \overline{e}_{xN}) C_{xN} + \beta_2(x_N)} \right]$$

$$\bar{y}_{USn} = (\bar{Y}_N + \bar{e}_N)(1 + \bar{y}_{USn} = \bar{Y}_N + e_N) \left(1 + \frac{\beta_2(x_N)}{\bar{X}C_{xN}}\right) \left(1 + \frac{\bar{e}_{xN}}{\bar{X}_N} + \frac{\beta_2(x_N)}{\bar{X}C_{xN}}\right)^{-1}$$
(8)

Taking  $1 + \frac{\beta_2(x_N)}{\overline{X}C_{xN}}$  as  $\lambda_1$ , we get

$$\bar{y}_{USn} = (\bar{Y}_N + \bar{e}_{xN})\lambda_1 + \left(\frac{\bar{e}_{xN}}{\bar{X}_N} + \lambda_1\right)^{-1}$$
(9)

Expanding (9), we get

$$\bar{y}_{USn} = (\bar{Y}_N + \bar{e}_{xN}) \lambda_1 \left( \frac{1}{\lambda_1} - \frac{\bar{e}_{xN}}{\bar{X}_N \lambda_1^2} + \frac{\bar{e}_{xN}^2}{X_N^2 \lambda_1^3} \right)$$
(10)

On further simplification we get

$$\overline{y}_{USn} = \overline{Y}_{N} - \frac{\overline{Y}_{N} \overline{e}_{xN}}{\overline{X}_{N} \lambda_{1}} + \frac{\overline{Y}_{N} \overline{e}_{xN}^{2}}{\overline{X}_{N}^{2} \lambda_{1}^{2}} + \overline{e}_{yN} - \frac{\overline{e}_{xN} \overline{e}_{yN}}{\overline{X}_{N} \lambda_{1}}$$

$$(11)$$

Simplifying (11), subtracting  $\overline{Y}_N$  from both sides, squaring and taking expectation, we get

$$MSE(\bar{y}_{USn}) = \frac{N-n}{Nn} \bar{Y}^2 \left( C_{yN}^2 + \left( \frac{\bar{x}_N C_{xN}}{\bar{x}_N C_{xN} + \beta_2(x_N)} \right)^2 C_{xN}^2 - 2 \left( \frac{\bar{x}_N C_{xN}}{\bar{x}_{N} C_{xN} + \beta_2(x_N)} \right) C_{xN} C_{yN} \rho_{xyn} \right)$$
(12)

where  $MSE(\bar{y}_{USn}) \in [MSE(\bar{y}_{USn_L}), MSE(\bar{y}_{USn_U})].$ 

In order to derive the MSE of  $\overline{y}_{YT_1}$ , we rewrite (6) using error terms,

$$\overline{y}_{YT_1} = (\overline{Y}_N + \overline{e}_N) \ [ \frac{\overline{x}\beta_2(x_N) + \beta_1(x_N)}{(\overline{X}_N + \overline{e}_{xN})\beta_2(x_N) + \beta_1(x_N)},$$

$$\overline{y}_{YT_1} = (\overline{Y}_N + \overline{e}_N) \left( 1 + \frac{\beta_1(x_N)}{\overline{X}\beta_2(x_N)} \right) \left( 1 + \frac{\overline{e}_{xN}}{X_N} + \frac{\beta_1(x_N)}{\overline{X}\beta_2(x_N)} \right)^{-1}$$
(13)

Taking  $1+\frac{\beta_1(x_N)}{\overline{X}_N\beta_2(x_N)}$  as  $\lambda_2$ , we get

$$\bar{\mathbf{y}}_{\mathrm{YT}_{1}} = (\bar{\mathbf{Y}}_{\mathrm{N}} + \bar{\mathbf{e}}_{\mathrm{xN}})\lambda_{2} \left(\frac{\bar{\mathbf{e}}_{\mathrm{xN}}}{\bar{\mathbf{x}}_{\mathrm{N}}} + \lambda_{2}\right)^{-1} \tag{14}$$

Expanding (14),

$$\bar{y}_{YT_{1}} = (\bar{Y}_{N} + \bar{e}_{xN}) \lambda_{2} \left( \frac{1}{\lambda_{2}} - \frac{\bar{e}_{xN}}{\bar{X}_{N} \lambda_{2}^{2}} + \frac{\bar{e}_{xN}^{2}}{\bar{X}_{N}^{2} \lambda_{2}^{3}} \right)$$
(15)

On further simplification we get

$$\overline{y}_{YT_1} = \overline{Y}_N - \frac{\overline{Y}_N \overline{e}_{xN}}{\overline{X}_N \lambda_2} + \frac{\overline{Y}_N \overline{e}_{xN}^2}{\overline{X}_N^2 \lambda_2^2} + \overline{e}_{yN} - \frac{\overline{e}_{xN} \overline{e}_{yN}}{\overline{X}_N \lambda_2}$$

$$(16)$$

Subtracting  $\overline{Y}_N$  from both sides, squaring and taking expectation, we get

$$MSE(\bar{y}_{YT_1}) = \frac{N-n}{Nn} \, \bar{Y}_N^2 \, \left( \, C_{yN}^2 \, + \, \left( \frac{\bar{x}_N \beta_2(x_N)}{\bar{x}_N \beta_2(x_N) + \beta_1(x_N)} \right)^2 C_{xN}^2 - 2 \left( \frac{\bar{x}_N \beta_2(x_N)}{\bar{x}_N \beta_2(x_N) + \beta_1(x_N)} \right) C_{xN} C_{yN} \rho_{xyN}$$
(17)

Where  $MSE(\bar{y}_{YT_1}) \in [MSE(\bar{y}_{YT_1L}), MSE(\bar{y}_{YT_1L})].$ 

Similarly, in order to derive the MSE of  $\overline{y}_{YT_2}\text{, we re-write (7) using error terms,}$ 

$$\overline{y}_{YT_2} = (\overline{Y}_N + \overline{e}_N) [\frac{\overline{X\beta_1(x_N) + \beta_2(x_N)}}{(\overline{X}_N + \overline{e}_{xN})\beta_1(x_N) + \beta_2(x_N)},$$

$$\overline{y}_{YT_2} = (\overline{Y}_N + \overline{e}_N) \left( 1 + \frac{\beta_2(x_N)}{\overline{X}\beta_1(x_N)} \right) \left( 1 + \frac{\beta_2(x_N)}{\overline{X}\beta_1(x_N)} + \frac{\beta_2(x_N)}{\overline{X}\beta_1(x_N)} \right)^{-1}$$
(18)

Taking  $1 \, + \, \frac{\beta_2(x_N)}{\overline{X}_N \beta_1(x_N)} \,$  as  $\lambda_3$  , we get\\

$$\bar{y}_{YT_2} = (\bar{Y}_N + \bar{e}_{xN})\lambda_3 \left(\frac{\bar{e}_{xN}}{\bar{X}_N} + \lambda_3\right)^{-1}$$
(19)

Expanding (19), we get

$$\bar{y}_{YT_2} = (\bar{Y}_N + \bar{e}_{xN})\lambda_3(\frac{1}{\lambda_3} - \frac{\bar{e}_{xN}}{\bar{X}_N \lambda_3^2} + \frac{\bar{e}_{xN}^2}{\bar{X}_N^2 \lambda_3^3})$$
(20)

On further simplification we get

$$\overline{y}_{YT_2} = \overline{Y}_N - \frac{\overline{Y}_N \overline{e}_{xN}}{\overline{X}_N \lambda_2} + \frac{\overline{Y}_N \overline{e}_{xN}^2}{\overline{X}_N^2 \lambda_3^2} + \overline{e}_{yN} - \frac{\overline{e}_{xN} \overline{e}_{yN}}{\overline{X}_N \lambda_3}$$
(21)

Subtracting  $\bar{Y}_N$  from both sides, squaring and taking expection, we get

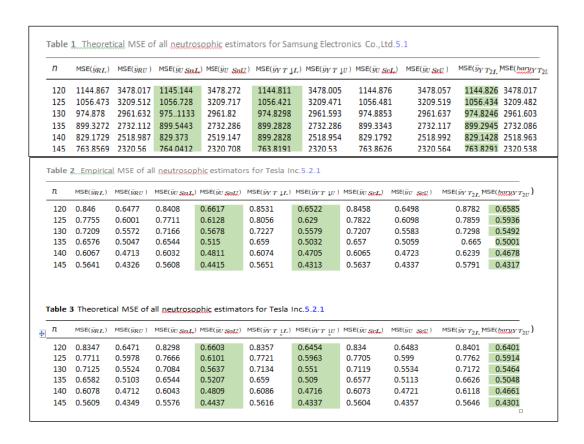
$$MSE(\bar{y}_{YT_2}) = \frac{N-n}{Nn} \, \bar{Y}_N^2 \, \left( \, C_{yN}^2 \, + \left( \frac{\bar{x}_N \beta_1(x_N)}{\bar{x}_N \beta_1(x_N) + \beta_2(x_N)} \right)^2 \, C_{xN}^2 - 2 \, \left( \frac{\bar{x}_N \beta_1(x_N)}{\bar{x}_N \beta_1(x_N) + \beta_2(x_N)} \right) C_{xN} C_{yN} \rho_{xyN}$$
(22)

 $\text{Where MSE}\big(\bar{y}_{YT_2}\big) \in \ \big[\text{MSE}\big(\bar{y}_{YT_{2L}}\big), \ \text{MSE}\big(\bar{y}_{YT_{2U}}\big)\big].$ 

#### 5 Numerical Illustration

### 5.1 Theoretical study on Samsung

The rationale behind taking daily stock prices as a neutrosophic variable isthat the price of a stock every day starts from an opening price (price at which trading starts) and at the end of the trading day reaches a closing price (price at which trading stops for the day). However, it always lies between a high price (the highest price of the day) and a low price (lowest price of the day) which may or may not be the same as the opening price or closing price. We are estimating this high price and low price interval within which the price of the stock lies using daily opening price as an auxiliary variable which is not a neutrosophic variable since its value for each day is fixed and known in advance. In this section, we are conducting a theoretical study using daily stock prices of Samsung Electronics Co.,Ltd.[5] from 1st September 2020 to 1st September 2021 by drawing neutrosophic simple random sample without replacement of sample sizes 120, 125, 130, 135, 140 and 145.



### 5.2 Empirical study

#### 5.2.1 Simulation study on Tesla Inc.

In this section, a simulation study is conducted with 10,000 replication foreach sample, on daily stock prices of Tesla Inc.[6] from 1<sup>st</sup> September 2020 to 1<sup>st</sup> September 2021. We have

calculated the theoretical as well as empirical indeterminacy intervals of MSE for each neutrosophic ratio type estimator for sample sizes same as in the previous section.

#### 6 Discussion

Table-1 shows the result of theoretical study5.1 on Samsung Electronics Co.,Ltd. daily stock price[5]. From the indeterminacy intervals, it is clear that neutrosophic estimator  $\bar{y}_{Y} \tau_{2}$  performs better than the rest of the neutrosophic estimators discussed in this article as its indeterminacy interval of MSE contains the lowest value of MSE.

Table-2 shows the results of the simulation study5.2.1 on Tesla Inc.'s daily stock price[6]. From these indeterminacy intervals, again it is clear that neutrosophic estimator  $\bar{y}_Y$  performs better than the rest of the neutrosophic estimators discussed in this article since its indeterminacy interval of MSEcontains the lowest value of MSE.

In case of Samsung Electronics Co.,Ltd.5.1, the lower values of MSE are being obtained at the lower ends of the indeterminacy intervals interpretating that sample values were closer to the average lower price of the stock for each day, resulting in lower values of MSE while for Tesla Inc.5.2.1, all the 10,000 samples for 6 different sizes had sample values closer to average higher prices for each day resulting in lower MSE at upper end of the indeterminacy interval but in both cases neutrosophic transformed ratio estimator  $\bar{y}_{YT2}$  resulted in lowest MSE[9].

#### 7 Conclusion

In this research article, we aim at solving the problem of estimating the unknown neutrosophic population mean. From the study conducted on the two real neutrosophic populations, we propose that neutrosophic transformed ratio estimator  $\bar{y}_{YT_2}$  should be used for estimating the unknown neutrosophic population mean.

Estimation of neutrosophic population parameters is still unexplored, estimation of the mean using different functions, using multi-auxiliary variables are some future areas of research.

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