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More on Fuzzy Neutrosophic Sets and Fuzzy Neutrosophic Topological Spaces

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Abstract: *This paper signifies the basic properties of fuzzy neutrosophic sets.It aims to introduce topological spaces into fuzzy neutrosophic set which gives birth to fuzzy neutrosophic topological spaces and characterized some of its properties.*

Keywords*: Neutrosophic set,Neutrosophic topological spaces, Fuzzy neutrosophic set ,Fuzzy neutrosophic topological spaces.*

1. Introduction:

Smarandache [12] initiated the concept of neutrosophic set which overcomes the inherent difficulties that existed in fuzzy sets[14] and intuitionistic fuzzy sets [3].Following this, the neutrosophic sets are explored to different heights in all fields of science and engineering.I.Arockiarani et al. defined the notion of fuzzy neutrosophic sets [1].In this paper the topology for fuzzy neutrosophic set is introduced and also some basic properties of fuzzy neutrosophic sets are derived.

2. Preliminaries:

2.1. Definition [7]:

A Neutrosophic set A on the universe of discourse X is defined as A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to\ ]^-0, 1^+[$ and $^-0 \le T_A(x)+I_A(x)+F_A(x) \le 3^+$.

2.2. Definition [1]:

A Fuzzy neutrosophic set A on the universe of discourse X is defined as A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

2.3. Definition [8]:

An intuitionistic neutrosophic set is defined by $A^* = \left\langle x, T_{A^*}(x), I_{A^*}(x), F_{A^*}(x)\right\rangle$ where $\min\left\{T_{A^*}(x), F_{A^*}(x)\right\} \le 0.5$, $\min\left\{T_{A^*}(x), I_{A^*}(x)\right\} \le 0.5$ *and* $\min\left\{F_{A^*}(x), I_{A^*}(x)\right\} \le 0.5$ *for all* $x \in X$

With the condition $0 \le T_{A^*}(x) + I_{A^*}(x) + F_{A^*}(x) \le 2$

2.4. Definition [1]:

A Fuzzy neutrosophic set A is a subset of a Fuzzy neutrosophic set B (i.e.,) A $\subseteq$ B for all x if $T_A(x) \le T_B(x)$, $I_A(x) \le I_B(x)$, $F_A(x) \ge F_B(x)$

2.5. Definition [1]:

Let X be a non empty set, and $A=\langle x,T_A(x),I_A(x),F_A(x)\rangle, B=\langle x,T_B(x),I_B(x),F_B(x)\rangle$ be two Fuzzy neutrosophic sets. Then $A \cup B=\langle x, \max(T_A(x),T_B(x)), \max(I_A(x),I_B(x)), \min(F_A(x),F_B(x))\rangle$ $A \cap B=\langle x, \min(T_A(x),T_B(x)), \min(I_A(x),I_B(x)), \max(F_A(x),F_B(x))\rangle$

2.6. Definition [1]:

The difference between two Fuzzy neutrosophic sets A and B is defined as A\B (x)= $\langle x, \min(T_A(x),F_B(x)), \min(I_A(x),1-I_B(x)), \max(F_A(x),T_B(x))\rangle$

2.7. Definition [1]:

A Fuzzy neutrosophic set A over the universe X is said to be null or empty Fuzzy neutrosophic set if $T_A(x) = 0$,$I_A(x) = 0$, $F_A(x) = 1$ for all $x \in X$. It is denoted by 0_N

2.8. Definition [1]:

A Fuzzy neutrosophic set A over the universe X is said to be absolute (universe) Fuzzy neutrosophic set if $T_A(x) = 1$, $I_A(x) = 1$, $F_A(x) = 0$ for all $x \in X$. It is denoted by 1_N

2.9. Definition [1]:

The complement of a Fuzzy neutrosophic set A is denoted by A^c and is defined as $A^c=\langle x,T_{A^c}(x),I_{A^c}(x),F_{A^c}(x)\rangle$ where $T_{A^c}(x)=F_A(x)$, $I_{A^c}(x)=1-I_A(x)$, $F_{A^c}(x)=T_A(x)$ The complement of a Fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.

3. Basic Properties Of Fuzzy Neutrosophic Sets:

3.1. Proposition:

Let A_i 's and B be Fuzzy neutrosophic sets in X ($i \in J$) then $A_i \subseteq B$ for each $i \in J \Rightarrow \cup A_i \subseteq B$

Proof:

Let $A_i \subseteq B$ (i.e.,) $A_1 \subseteq B, A_2 \subseteq B, \ldots\ldots\ldots, A_n \subseteq B$

$$\Rightarrow T_{A_1}(x) \le T_B(x),\ I_{A_2}(x) \le I_B(x),\ F_{A_1}(x) \ge F_B(x)$$

$$T_{A_2}(x) \le T_B(x), I_{A_2}(x) \le I_B(x), F_{A_2}(x) \ge F_B(x) \qquad \ldots\ldots\ldots(1)$$

$$\ldots\ldots\ldots$$

$$T_{A_n}(x) \le T_B(x),\ I_{A_n}(x) \le I_B(x),\ F_{A_n}(x) \ge F_B(x)$$

$$\max(T_{A_1}(x), T_{A_2}(x), \ldots. T_{A_n}(x)) \le T_B(x)$$

$$\max(I_{A_1}(x), I_{A_2}(x), \ldots. I_{A_n}(x)) \le I_B(x)$$

$$\min(F_{A_1}(x), F_{A_2}(x), \ldots\ldots F_{A_n}(x)) \ge F_B(x)$$

$$\cup A_i = \left\langle x, \max(T_{A_1}, T_{A_2}, \ldots T_{A_n}), \max(I_{A_1}, I_{A_2}, \ldots I_{A_n}), \min(F_{A_1}, F_{A_2}, \ldots F_{A_n}) \right\rangle \quad \Rightarrow \cup A_i \subseteq B$$

by (1)

3.2. Proposition:

Let A_i 's and B be Fuzzy neutrosophic sets in X ($i \in J$) then $B \subseteq A_i$ for each $i \in J \Rightarrow B \subseteq \cap A_i$

Proof: Let $B \subseteq A_i$ (i.e.,) $B \subseteq A_1, B \subseteq A_2, \ldots\ldots B \subseteq A_n$

$$\Rightarrow T_B(x) \le T_{A_1}(x),\ I_B(x) \le I_{A_1}(x), F_B(x) \ge F_{A_1}(x)$$

$$T_B(x) \le T_{A_2}(x),\ I_B(x) \le I_{A_2}(x), F_B(x) \ge F_{A_2}(x)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$T_B(x) \le T_{A_n}(x),\ I_B(x) \le I_{A_{n1}}(x), F_B(x) \ge F_{A_n}(x)$$

$$\Rightarrow T_B(x) \le \min(T_{A_1}(x), T_{A_2}(x), \ldots. T_{A_n}(x))$$

$$I_B(x) \le \min(I_{A_1}(x), I_{A_2}(x), \ldots\ldots I_{A_n}(x))$$

$$F_B(x) \le \max(F_{A_1}(x), F_{A_2}(x) \ldots\ldots F_{A_n}(x))$$

$\Rightarrow B \subseteq \cap A_i \ \forall i \in J$ where $\cap A_i = \left\langle x, \min(T_{A_1}, T_{A_2}, \ldots. T_{A_n}), \min(I_{A_1}, I_{A_2}, \ldots\ldots I_{A_n}), \max(F_{A_1}, F_{A_2}, \ldots\ldots F_{A_n}) \right\rangle$

3.3. Proposition:

Let A_i's be Fuzzy neutrosophic sets in X , $i \in J$ then $(i)\ \left(\cup A_i\right)^c = \cap A_i^c \ (ii) \left(\cap A_i\right)^c = \cup A_i^c$

Proof:

$$\cup A_i = \left\langle x, \max(T_{A_1}, T_{A_2}, \ldots T_{A_n}), \max(I_{A_1}, I_{A_2}, \ldots I_{A_n}), \min(F_{A_1}, F_{A_2}, \ldots F_{A_n}) \right\rangle$$

$$\left(\cup A_i\right)^c = \left\langle x, \min(F_{A_1}, F_{A_2}, \ldots F_{A_n}), 1 - \max(I_{A_1}, I_{A_2}, \ldots I_{A_n}), \max(T_{A_1}, T_{A_2}, \ldots T_{A_n}) \right\rangle$$

$$=\left\langle x,\min(F_{A_1},F_{A_2},\ldots\ldots F_{A_n}),\min(1-I_{A_1},1-I_{A_2},\ldots 1-I_{A_n}),\max(T_{A_1},T_{A_2},\ldots T_{A_n})\right\rangle \ldots\ldots(1)$$

$$A_i^c=\left\langle x,(F_{A_1},F_{A_2},\ldots F_{A_n}),(1-I_{A_1},1-I_{A_2},\ldots 1-I_{A_n}),(T_{A_1},T_{A_2},\ldots T_{A_n})\right\rangle$$

$$\cap A_i^c=\left\langle x,\min(F_{A_1},F_{A_2},\ldots F_{A_n}),\min(1-I_{A_1},1-I_{A_2},\ldots 1-I_{A_n}),\max(T_{A_1},T_{A_2},\ldots T_{A_n})\right\rangle \ldots\ldots(2)$$

From (1) and (2) $\left(\cup A_i\right)^c=\cap A_i^c$

Proof of (ii) is similar.

3.4. Proposition:

Let A and B be Fuzzy neutrosophic sets then $A\subseteq B \Leftrightarrow B^c\subseteq A^c$

Proof:

Let A and B be Fuzzy neutrosophic sets then

$$A\subseteq B \Leftrightarrow T_A(x)\le T_B(x), I_A(x)\le I_B(x), F_A(x)\ge F_B(x)$$
$$\Leftrightarrow F_B(x)\le F_A(x), 1-I_B(x)\le 1-I_A(x), T_B(x)\ge T_A(x)$$
$$\Leftrightarrow B^c\subseteq A^c$$

3.5. Proposition:

Let A be a Fuzzy neutrosophic set in X then $\left(A^c\right)^c=A$

Proof:

Let A= $\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle$ be a Fuzzy neutrosophic set in X then $A^c=\left\langle x,F_A(x),1-I_A(x),T_A(x)\right\rangle$

Hence $\left(A^c\right)^c=\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle=A$

Note:

$\left(0_N\right)^c=1_N$, $(1_N)^c=0_N$

3.6. Proposition:

Let A be a Fuzzy neutrosophic set in X then the following properties hold: $(i)\ A\cup 0_N=A\ (ii)\ A\cup 1_N=1_N\ (iii)\ A\cap 0_N=0_N\ (iv)\ A\cap 1_N=A$

Proof:

Let A= $\left\langle x,T_A(x),I_A(x),F_A(x)\right\rangle$, $0_N=\langle x,0,0,1\rangle$, $1_N=\langle x,1,1,0\rangle$

$(i)\ A \cup 0_N = \langle x, \max(T_A(x),0), \max(I_A(x),0), \min(F_A(x),1) \rangle$

$= \langle x, T_A(x), I_A(x), F_A(x) \rangle$

$= A$

$(ii)\ A \cup 1_N = \langle x, \max(T_A(x),1), \max(I_A(x),1), \min(F_A(x),0) \rangle$

$= \langle x,1,1,0 \rangle = 1_N$

$(iii)\ A \cap 0_N = \langle x, \min(T_A(x),0), \min(I_A(x),0), \max(F_A(x),1) \rangle$

$= \langle x,0,0,1 \rangle = 0_N$

$(iv)\ A \cap 1_N = \langle x, \min(T_A(x),1), \min(I_A(x),1), \max(F_A(x),0) \rangle$

$= \langle x, T_A(x), I_A(x), F_A(x) \rangle = A$

3.7. Proposition:

Let A and B be two Fuzzy neutrosophic sets in X then $A \cup B = A$ if and only if $B \subseteq A$

Proof:

Let A and B be two Fuzzy neutrosophic sets in X such that $A \cup B = A$. (i.e.,)

$\langle x, \max(T_A(x),T_B(x)), \max(I_A(x),I_B(x)), \min(F_A(x),F_B(x)) \rangle = \langle x, T_A(x), I_A(x), F_A(x) \rangle$

$\Leftrightarrow \max(T_A(x), T_B(x)) = T_A(x)$, $\max(I_A(x), I_B(x)) = I_A(x)$, $\min(F_A(x), F_B(x)) = F_A(x)$

$\Leftrightarrow T_B(x) \le T_A(x)$, $I_B(x) \le I_A(x)$, $F_B(x) \ge F_A(x)$

$\Leftrightarrow B \subseteq A$

3.8. Proposition:

Let A and B be two Fuzzy neutrosophic sets in X then A \ B = B^c \ A^c

Proof:

Let A = $\langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$

$(A \setminus B)(x) = \langle x, \min(T_A(x), F_B(x)), \min(I_A(x), 1-I_B(x)), \max(F_A(x), T_B(x)) \rangle$ $A^c = \langle x, F_A(x), 1-I_A(x), T_A(x) \rangle$, $B^c = \langle x, F_B(x), 1-I_B(x), T_B(x) \rangle$

$(B^c \setminus A^c)(x) = \langle x, \min(F_B(x), T_A(x)), \min(1-I_B(x), I_A(x)), \max(T_B(x), F_A(x)) \rangle$

Hence $(A \setminus B)(x) = (B^c \setminus A^c)(x)$

3.9. Proposition:

Let A , B and C be Fuzzy neutrosophic sets in X then

$(i)\ A \cup B = B \cup A$

$(ii)\ A \cap B = B \cap A$

$(iii)\ A \cup (B \cup C) = (A \cup B) \cup C$

$(iv)\ A \cap (B \cap C) = (A \cap B) \cap C$

$(v)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$(vi)\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$(vii)\ A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

$(viii)\ A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

4. Fuzzy Neutrosophic Topologicalspaces:

4.1. Definition:

A Fuzzy neutrosophic topology on a nonempty set X is a τ of Fuzzy neutrosophic sets in X satisfying the following axioms

$(i)\ 0_N, 1_N \in \tau$

$(ii)\ A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$

$(iii)\ \cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \in \tau$

In this case the pair (X, τ) is called Fuzzy neutrosophic topological space and any Fuzzy neutrosophic set in τ is known as Fuzzy neutrosophic open set in X .

4.2. Example:

Let X= {a,b,c}and consider the family $\tau = \{0_N, 1_N, A_1, A_2, A_3, A_4\}$ where

$A_1 = \{\langle a,0.8,0.7,0.6\rangle, \langle b,0.6,0.5,0.4\rangle, \langle c,0.4,0.7,0.5\rangle\}$

$A_2 = \{\langle a,0.7,0.6,0.4\rangle, \langle b,0.8,0.2,0.3\rangle, \langle c,0.9,0.3,0.2\rangle\}$

$A_3 = \{\langle a,0.8,0.7,0.4\rangle, \langle b,0.8,0.5,0.3\rangle, \langle c,0.9,0.7,0.2\rangle\}$

$A_4 = \{\langle a,0.7,0.6,0.6\rangle, \langle b,0.6,0.2,0.4\rangle, \langle c,0.4,0.3,0.5\rangle\}$

Then (X, τ) is called Fuzzy neutrosophic topological space on X

4.3. Definition:

The complement A^c of a Fuzzy neutrosophic set A in a Fuzzy neutrosophic topological space (X, τ) is called a Fuzzy neutrosophic closed set in X.

4.4. Definition:

Let (X, τ) be a Fuzzy neutrosophic topological space and A= $\langle x, T_A(x), I_A(x), F_A(x) \rangle$ be a Fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$\operatorname{int}(A) = \cup \{ G : G \text{ is a fuzzy neutrosophic open set in } X \text{ and } G \subseteq A \}$$
$$cl(A) = \cap \{ G : G \text{ is a fuzzy neutrosophic closed set in } X \text{ and } A \subseteq G \}$$

4.5. Proposition:

Let (X,τ) be a Fuzzy neutrosophic topological space over X .Then the following properties hold. $(i)\ cl(A^C) = (\operatorname{int} A)^C \quad (ii)\ \operatorname{int} A^C = (cl\, A)^C$

Proof:

Let A= $\langle x, T_A(x), I_A(x), F_A(x) \rangle$

Suppose that the family of Fuzzy neutrosophic open sets G_i contained in A are indexed by the family $\{ \langle x, T_{G_i}(x), I_{G_i}(x), F_{G_i}(x) \rangle : i \in J \}$

Then we see that

$$\operatorname{int} A = \langle x, \max(T_{G_i}(x)), \max(I_{G_i}(x)), \min(F_{G_i}(x)) \rangle$$
$$\Rightarrow (\operatorname{int} A)^c = \langle x, \min(F_{G_i}(x)), 1 - \max(I_{G_i}(x)), \max(T_{G_i}(x)) \rangle \quad \ldots\ldots(1) \quad A^c = \langle x, F_A(x), 1 - I_A(x), T_A(x) \rangle$$

$$\because G_i \subseteq A \quad \forall i \in J$$

$$T_{G_i}(x) \le T_A(x),\ I_{G_i}(x) \le I_A(x),\ F_{G_i}(x) \ge F_A(x) \quad \forall i \in J$$

We obtain that $\{ \langle x, F_{G_i}(x), 1 - I_{G_i}(x), T_{G_i}(x) \rangle : i \in J \}$ is the family of Fuzzy neutrosophic closed sets containing A^c .(i.e.,)

$$cl(A^c) = \{ <x, \min F_{G_i}(x), \min(1 - I_{G_i}(x)), \max(T_{G_i}(x)) > : i \in J \}$$
$$= \{ <x, \min F_{G_i}(x), 1 - \max I_{G_i}(x), \max(T_{G_i}(x)) > : i \in J \} \quad \ldots\ldots.(2)$$

$$\textit{Hence } cl(A^c) = (\operatorname{int} A)^c$$

Similarly we can prove (ii).

4.6. Proposition:

Let $(X,\tau_1)\, and\, (X,\tau_2)$ be two Fuzzy neutrosophic topological spaces. Denote $\tau_1 \cap \tau_2 = \{A : A \in \tau_1 \ and\ A \in \tau_2\}$ then $\tau_1 \cap \tau_2$ is a Fuzzy neutrosophic topological space.

Proof:

Obviously $0_N, 1_N \in \tau_1 \cap \tau_2$

Let $A_1, A_2 \in \tau_1 \cap \tau_2 \quad \Rightarrow A_1, A_2 \in \tau_1\ ,\ A_1, A_2 \in \tau_2$

$\tau_1\ and\ \tau_2$ are Fuzzy neutrosophic topological spaces on X .Then

$A_1 \cap A_2 \in \tau_1\ and\ A_1 \cap A_2 \in \tau_2 \Rightarrow A_1 \cap A_2 \in \tau_1 \cap \tau_2$ Let $\{A_i : i \in J\} \subseteq \tau_1 \cap \tau_2 \ \Rightarrow A_i \in \tau_1$ and $A_i \in \tau_2 \quad \forall\, i \in J$

Since $\tau_1\ and\ \tau_2$ are Fuzzy neutrosophic topological spaces on X

$\bigcup\{A_i : i \in J\} \in \tau_1\ and\ \bigcup\{A_i : i \in J\} \in \tau_2 \Rightarrow \bigcup\{A_i : i \in J\} \in \tau_1 \cap \tau_2$ Therefore $\tau_1 \cap \tau_2$ is a Fuzzy neutrosophic topological space.

Remark:

$\tau_1 \cup \tau_2$ is not a Fuzzy neutrosophic topological space can be seen by the following example.

4.7. Example:

Let X={a,b} , $\tau_1 = \{0_N, 1_N, A\}\, and\, \tau_1 = \{0_N, 1_N, B\}$ where

$A = \{\langle a, 0.8, 0.7, 0.5\rangle, \langle b, 0.9, 0.3, 0.4\rangle\}$

$B = \{\langle a, 0.5, 0.6, 0.3\rangle, \langle b, 0.3, 0.5, 0.7\rangle\}$

Here $\tau_1 \cup \tau_2 = \{0_N, 1_N, A, B\}$, Since $A \cup B, A \cap B \notin \tau_1 \cup \tau_2$, $\tau_1 \cup \tau_2$ is not a Fuzzy neutrosophic topological space

4.8. Definition:

Let (X,τ) be a Fuzzy neutrosophic topological space on X.

i. A family $\beta \subseteq \tau$ is called a base for (X,τ) if and only if each member of τ can be written as the elements of β.

ii. A family $\gamma \subseteq \tau$ is called a sub base for (X,τ) if and only if the family of finite intersections of elements in γ forms a base for (X,τ).In this case the Finite topology τ is said to be generated by γ.

4.9. Proposition:

Let (X,τ) be a Fuzzy neutrosophic topological space over X .Then the following properties hold.

1. 0_N , 1_N are Fuzzy neutrosophic closed sets over X
2. The intersection of any number of Fuzzy neutrosophic closed sets is a Fuzzy neutrosophic closed set over X
3. The union of any two Fuzzy neutrosophic closed sets is a Fuzzy neutrosophic closed set over X

Proof: It is obvious.

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