NEUTROSOPHIC TRIPLET STRUCTURES

and

NEUTROSOPHIC EXTENDED TRIPLET STRUCTURES

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Neutrosophic Triplets [1, 2, 3, 10] were introduced by F. Smarandache & M. Ali in 2014 – 2016: http://fs.unm.edu/NeutrosophicTriplets.htm

Neutrosophic Extended Triplets were introduced by F. Smarandache [4, 5, 8] in 2016: http://fs.unm.edu/NeutrosophicTriplets.htm

Let *U* be a universe of discourse, and *(N, *)* a set included in it, endowed with a well-defined binary law *.

1. Definition of Neutrosophic Triplet (NT).

A neutrosophic triplet is an object of the form $<x, neut(x), anti(x)>$, for $x \in N$, where $neut(x) \in N$ is the neutral of *x*, different from the classical algebraic unitary element if any, such that:

$$x*neut(x) = neut(x)*x = x$$

and $anti(x) \in N$ is the opposite of *x* such that:

$$x*anti(x) = anti(x)*x = neut(x).$$

In general, an element *x* may have more *neut*'s and more *anti*'s.

2. Definition of Neutrosophic Extended Triplet (NET).

A neutrosophic extended triplet is a neutrosophic triplet, defined as above, but where the *neutral* of *x* {denoted by $_e neut(x)$ and called "extended neutral"} is allowed to also be equal to the classical algebraic unitary element (if any). Therefore, the restriction "different from the classical algebraic unitary element if any" is released.

As a consequence, the "extended opposite" of *x*, denoted by $_e anti(x)$, is also allowed to be equal to the classical inverse element from a classical group.

Thus, a neutrosophic extended triplet is an object of the form $<x, {}_e neut(x), {}_e anti(x)>$, for $x \in N$, where $_e neut(x) \in N$ is the *extended neutral* of *x*, which can be equal or different from the classical algebraic unitary element if any, such that:

$$x * {}_e neut(x) = {}_e neut(x) * x = x$$

and $anti(x) \in N$ is the *extended opposite* of *x* such that:

$$x * {}_e anti(x) = {}_e anti(x) * x = {}_e neut(x).$$

In general, for each $x \in$ N there are many exist $_e neut$'s and $_e anti$'s.

3. Definition of Neutrosophic Triplet (Strong) Set (NTSS).

The set *N* is called a neutrosophic triplet (strong) set if for any $x \in N$ there exist $neut(x) \in N$ and $anti(x) \in N$.

4. Definition of Neutrosophic Extended Triplet (Strong) Set (NETSS).

The set *N* is called a neutrosophic extended triplet (strong) set if for any $x \in N$ there exist $_e neut(x) \in N$ and $_e anti(x) \in N$.

5. Definition of Neutrosophic Triplet Weak Set (NTWS).

The set *N* is called a neutrosophic triplet weak set if for any *x*∈ *N* there exist a neutrosophic triplet <*y*, *neut(y)*, *anti(y)*> included in *N*, such that *x* = *y* or *x* = *neut(y)* or *x* = *anti(y)*.

6. Definition of Neutrosophic Extended Triplet Weak Set (NETWS).

The set *N* is called a neutrosophic extended triplet weak set if for any *x*∈*N* there exist a neutrosophic extended triplet <*y*, ${}_{e}neut(y)$, ${}_{e}anti(y)$> included in *N*, such that *x* = *y* or *x* = ${}_{e}neut(y)$ or *x* = ${}_{e}anti(y)$.

7. Theorem 1.

a) A neutrosophic triplet strong set is also a neutrosophic triplet weak set, but not conversely.

b) A neutrosophic extended triplet strong set is also a neutrosophic extended triplet weak set, but not conversely.

8. Definition of Neutrosophic (Strong) Triplet Group (NTG)

Let *(N, *)* be a neutrosophic (strong) triplet set. Then *(N, *)* is called a neutrosophic (strong) triplet group, if the following classical axioms are satisfied.

1) *(N, *)* is well-defined, i.e. for any *x*, *y* ∈ *N* one has $x*y \in N$.

2) *(N, *)* is associative, i.e. for any *x*, *y*, *z* ∈ *N* one has $x*(y*z) = (x*y)*z$.

NTG, in general, is not a group in the classical way, because it may not have a classical unitary element, nor classical inverse elements.
We consider, that the neutrosophic neutrals replace the classical unitary element, and the neutrosophic opposites replace the classical inverse elements.

9. Definition of Neutrosophic Extended (Strong) Triplet Group (NETG)

Let *(N, *)* be a neutrosophic extended (strong) triplet set. Then *(N, *)* is called a neutrosophic extended (strong) triplet group, if the following classical axioms are satisfied.

1) *(N, *)* is well-defined, i.e. for any $x, y \in N$ one has $x*y \in N$.

2) *(N, *)* is associative, i.e. for any $x, y, z \in N$ one has $x*(y*z) = (x*y)*z$.

NETG, in general, is not a group in the classical way, because it may not have a classical unitary element, nor classical inverse elements.
We consider, that the neutrosophic extended neutrals replace the classical unitary element, and the neutrosophic extended opposites replace the classical inverse elements.

In the case when NETG includes a classical group, then NETG enriches the structure of a classical group, since there may be elements with more extended neutrals and more extended opposites.

10. Definition of Neutrosophic Triplet Ring (NTR)

1) Neutrosophic Triplet Ring is a set endowed with two binary laws *(N, *, #)*,

such that:

a) *(N, *)* is a commutative neutrosophic triplet (strong) group;

which means that:
- N is a set of neutrosophic (strong) triplets with respect to the law * (i.e. if x belongs to N, then $neut^*(x)$ and $anti^*(x)$, defined with respect to the law *,

also belong to N); we use the notations $neut^*(.)$ and *respectively* $anti^*(.)$ to mean

with respect to the law *;

- the law * is well-defined, associative, and commutative on N

(as in the classical sense);

b) *(N, #)* is a set such that the law # on N is well-defined and associative

(as in the classical sense);

c) the law # is distributive with respect to the law *

(as in the classical sense).

11. Definition of Neutrosophic Extended Triplet Ring (NETR)

1) Neutrosophic Extended Triplet Ring is a set endowed with two binary laws *(N, *, #)*, such that:

a) *(N, *)* is a commutative neutrosophic extended triplet group;

which means that:
- *N* is a set of neutrosophic extended triplets with respect to the law *

(i.e. if *x* belongs to *N*, then $_{e}neut^{*}(x)$ and $_{e}anti^{*}(x)$, defined with respect to the law *, also belong to *N*);

- the law * is well-defined, associative, and commutative on *N*

(as in the classical sense);

b) *(N, #)* is a set such that the law # on *N* is well-defined and associative

(as in the classical sense);

c) the law # is distributive with respect to the law *

(as in the classical sense).

12. Remarks on Neutrosophic Triplet Ring:

1) The Neutrosophic Triplet Ring is defined on the steps of the classical ring,

the only two distinctions are that:

- the classical unit element with respect to the law * is replaced

by $neut^{*}(x)$ with respect to the law * for each *x* in *N* into the NTR;

- in the same way, the classical inverse element of an element *x* in *N*,

with respect to the law *, is replaced by $anti^{*}(x)$ with respect to the law * in *N*.

2) A Neutrosophic Triplet Ring, in general, is different from a classical ring.

13. Remarks on Neutrosophic Extended Triplet Ring:

1) Similarly, The Neutrosophic Exteded Triplet Ring is defined on the steps of the classical ring, the only two distinctions are that:

- the classical unit element with respect to the law * is extended to $_{e}neut^{*}(x)$ with respect to the law * for each x in N into the NETR;

- in the same way, the classical inverse element of an element x in N, with respect to the law *, is extended to $_{e}anti^{*}(x)$ with respect to the law * in N.

2) A Neutrosophic Extended Triplet Ring, in general, is different from a classical ring.

14. Definition of Hybrid Neutrosophic Triplet Ring (HNTR)

The Hybrid Neutrosophic Triplet Ring is a set N endowed with two binary laws $(N, *, \#)$, such that:

a) $(N, *)$ is a commutative neutrosophic triplet (strong) group; which means that:

- N is a neutrosophic triplets strong set with respect to the law * (i.e. if x belongs to N, then $neut^{*}(x)$ and $anti^{*}(x)$, defined with respect to the law *, also belong to N);

- the law * is well-defined, associative, and commutative on N (as in the classical sense);

b) $(N, \#)$ is a neutrosophic triplet strong set with respect to the law # (i.e. if x belongs to N, then $neut^{\#}(x)$ and $anti^{\#}(x)$, defined with respect to the law #, also belong to N);

- the law # is well-defined and non-associative on N (as in the classical sense);

c) the law # is distributive with respect to the law * (as in the classical sense).

15. Definition of Hybrid Neutrosophic Extended Triplet Ring (HNETR)

The Hybrid Neutrosophic Extended Triplet Ring is a set *N* endowed with two binary laws *(N, *, #)*, such that:

a) *(N, *)* is a commutative neutrosophic extended triplet (strong) group; which means that:

- *N* is a neutrosophic extended triplet strong set with respect to the law * (i.e. if *x* belongs to *N*, then $_{e}neut^{*}(x)$ and $_{e}anti^{*}(x)$, defined with respect to the law *, also belong to *N*);

- the law * is well-defined, associative, and commutative on *N* (as in the classical sense);

b) *(N, #)* is a neutrosophic extended triplet strong set with respect to the law # (i.e. if *x* belongs to *N*, then $_{e}neut^{\#}(x)$ and $_{e}anti^{\#}(x)$, defined with respect to the law #, also belong to *N*);

- the law # is well-defined and non-associative on *N* (as in the classical sense);

c) the law # is distributive with respect to the law * (as in the classical sense).

16. Remarks on Hybrid Neutrosophic Triplet Ring

a) A Hybrid Neutrosophic Triplet Ring is a field *(N, *, #)* from which there has been removed the associativity of the second law #.

b) Or, Hybrid Neutrosophic Triplet Ring is a set *(N, *, #)*, such that *(N, *)* is a commutative neutrosophic triplet group, and *(N, #)* is a neutrosophic triplet loop, and the law # is distributive with respect to the law * (as in the classical sense).

17. Remarks on Hybrid Neutrosophic ExtendedTriplet Ring

a) A Hybrid Neutrosophic Extended Triplet Ring is a field *(N, *, #)* from which there has been removed the associativity of the second law #.

b) Or, Hybrid Neutrosophic Extended Triplet Ring is a set *(N, *, #)*, such that *(N, *)* is a commutative neutrosophic extended triplet group, and *(N, #)* is a

neutrosophic extended triplet loop, and the law # is distributive with respect to the law * (as in the classical sense).

18. Definition of Neutrosophic Triplet Field (NTF)

2) Neutrosophic Triplet Field is a set endowed with two binary laws *(N, *, #)*,

such that:

a) *(N, *)* is a commutative neutrosophic triplet group;

which means that:
- *N* is a set of neutrosophic triplets with respect to the law *

(i.e. if *x* belongs to *N*, then $neut^*(x)$ and $anti^*(x)$, defined with respect to the law *, also both belong to *N*);

- the law * is well-defined, associative, and commutative on *N*

(as in the classical sense);

b) *(N, #)* is a neutrosophic triplet group;

which means that:

- *M* is a set of neutrosophic triplets with respect to the law #

(i.e. if *x* belongs to *N*, then $neut^\#(x)$ and $anti^\#(x)$, defined with respect to the law #, also both belong to *N*);

- the law # is well-defined and associative on *N*

(as in the classical sense);

c) the law # is distributive with respect to the law *

(as in the classical sense).

19. Definition of Neutrosophic Extended Triplet Field (NETF)

2) Neutrosophic Extended Triplet Field is a set endowed with two binary laws *(N, *, #)*, such that:

a) *(N, *)* is a commutative neutrosophic extended triplet group;

which means that:
- *N* is a neutrosophic extended triplet set with respect to the law *

(i.e. if *x* belongs to *N*, then $_e neut^*(x)$ and $_e anti^*(x)$, defined with respect to the law *,

also both belong to *N*);

- the law * is well-defined, associative, and commutative on *N*

(as in the classical sense);

b) *(N, #)* is a neutrosophic extended triplet group;

which means that:

- *N* is a set of neutrosophic triplets with respect to the law #

(i.e. if x belongs to *N*, then $_e neut^\#(x)$ and $_e anti^\#(x)$, defined with respect to the law #,

also both belong to *N*);

- the law # is well-defined and associative on *N*

(as in the classical sense);

c) the law # is distributive with respect to the law *

(as in the classical sense).

20. Remarks on Neutrosophic Triplet Field:

1) The Neutrosophic Triplet Field is defined on the steps of the classical field,

the only four distinctions are that:

- the classical unit element with respect to the first law * is extended to

$_e neut^*(x)$ with respect to the first law * for each *x* in *N* into the NTF;

- in the same way, the classical inverse element of an element *x* in *N*, with respect to the first law *, is extended to $_e$*anti***(x)* with respect to the first law * in *N*;

- and the classical unit element with respect to the second law # is extended to $_e$*neut*$^\#$*(x)* with respect to the second law # for each *x* in *N* into the NTF;

- in the same way, the classical inverse element of an element *x* in *N*, with respect to the second law #, is extended to $_e$*anti*$^\#$*(x)* with respect to the second law # in *N*;

2) A Neutrosophic Triplet Field, in general, is different from a classical field.

21. Hybrid Neutrosophic Triplet Field of Type 1 (HNTF1).

It is a set *N* endowed with two laws * and # such that:

1: *(N, *)* is a commutative neutrosophic triplet group;

2: *(N, #)* is a classical group;

3: The law # is distributive over the law *.

22. Hybrid Neutrosophic Extended Triplet Field of Type 1 (HNETF1).

It is a set *N* endowed with two laws * and # such that:

1: *(N, *)* is a commutative neutrosophic extended triplet group;

2: *(N, #)* is a classical group;

3: The law # is distributive over the law *.

23. Hybrid Neutrosophic Triplet Field of Type 2 (HNTF2).

It is a set *N* endowed with two laws * and # such that:

1: *(N, *)* is a classical commutative group;

2: *(N, #)* is a neutrosophic triplet group;

3: The law # is distributive over the law *.

24. Hybrid Neutrosophic Triplet Field of Type 2 (HNETF2).

It is a set *N* endowed with two laws * and # such that:

1: *(N, *)* is a classical commutative group;

2: *(N, #)* is a neutrosophic extended triplet group;

3: The law # is distributive over the law *.

25. Applications of Neutrosophic Triplet Structures (NTS) and Neutrosophic Extended Triplet Structures (NETS)

This new fields of Neutrosophic Triplet Structures and Neutrosophic Extended Triplet Structures are very important, because they reflect our everyday life [they are not simple imagination!].

The neutrosophic triplets and neutrosophic extended triplets are based on real triads:

(friend, neutral, enemy), (positive particle, neutral particle, negative particle), (yes, unclear, no), (pro, neutral, against), (victory, tie game, defeat), (taking a decision, undecided, not taking a decision), (accept, pending, reject), and in general *(<A>, <neutA>, <antiA>)* as in *neutrosophy*, which is a new branch of philosophy generalizing the dialectics.

References:

[1] Florentin Smarandache and Mumtaz Ali, *Neutrosophic Triplet Group*, Neural

Computing and Applications, Springer, 1-7, 2016,

https://link.springer.com/article/10.1007/s00521-016-2535-x;

DOI: 10.1007/s00521-016-2535-x.

[2] F. Smarandache, M. Ali, Neutrosophic triplet as extension of matter plasma, unmatter plasma, and antimatter plasma, 69th annual gaseous electronics conference, Bochum, Germany, Veranstaltungszentrum & Audimax, Ruhr-Universitat, 10–14 Oct. 2016,

http://meetings.aps.org/Meeting/GEC16/Session/HT6.111

[3] Florentin Smarandache, Mumtaz Ali, *The Neutrosophic Triplet Group and its Application to Physics*, presented by F. S. to Universidad Nacional de Quilmes, Department of Science and Technology, Bernal, Buenos Aires, Argentina, 02 June 2014.

[4] F. Smarandache, *Neutrosophic Theory and Applications*, Le Quy Don Technical University, Faculty of Information technology, Hanoi, Vietnam, 17th May 2016.

[5] F. Smarandache, Neutrosophic Extended Triplets, Arizona State University, Tempe, AZ, Special Collections, 2016.

[6] F. Smarandache, M. Ali, *Neutrosophic Triplet Field Used in Physical Applications*, (Log Number: NWS17-2017-000061), 18th Annual Meeting of the APS Northwest Section, Pacific University, Forest Grove, OR, USA, June 1-3, 2017;

http://meetings.aps.org/Meeting/NWS17/Session/D1.1

[7] F. Smarandache, M. Ali, *Neutrosophic Triplet Ring and its Applications*, (Log Number: NWS17-2017-000062), 18th Annual Meeting of the APS Northwest Section, Pacific University, Forest Grove, OR, USA, June 1-3, 2017.

http://meetings.aps.org/Meeting/NWS17/Session/D1.2

[8] Florentin Smarandache, *Seminar on Physics (unmatter, absolute theory of relativity, general theory – distinction between clock and time, superluminal and instantaneous physics, neutrosophic and paradoxist physics), Neutrosophic Theory of Evolution, Breaking Neutrosophic Dynamic Systems, and Neutrosophic Extended Triplet Algebraic Structures*, Federal University of Agriculture, Communication Technology Resource Centre, Abeokuta, Ogun State, Nigeria, 19th May 2017.

[9] F. Smarandache, Hybrid Neutrosophic Triplet Ring in Physical Structures, Annual Meeting of the APS Four Corners Section, Fort Collins, CO, USA, & Bulletin of the American Physical Society, October 20-21, 2017;

http://meetings.aps.org/Meeting/4CF17/Session/G1.33

[10] F. Smarandache, Neutrosophic Perspectives: Triplets, Duplets, Multisets,

Hybrid Operators, Modal Logic, Hedge Algebras. And Applications. Pons

Editions, Bruxelles, second edition, 323 p., 2017;

http://fs.unm.edu/NeutrosophicPerspectives-ed2.pdf

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