Lagrange Multipliers and Neutrosophic Nonlinear Programming Problems Constrained by Equality Constraints

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Abstract:

Operations research science is defined as the science that is concerned with applying scientific methods to complex problems in managing and directing large systems of people, including resources and tools in various fields, private and governmental work, in peace and war, in politics, administration, economics, planning and implementation in various domains. It uses scientific methods that take the language of mathematics as a basis for it and uses computer, without which it would not have been possible to achieve numerical solutions to the raised problems, those that need correct solutions, when the solutions abound and the options are multiple, so we need a decision based on correct scientific foundations and takes into account all the circumstances and changes that you can encounter the decision-maker during the course of work, and nothing is left to chance or luck, but rather everything that enters into the account and plays its role in decision-making, and we get that when we use the concepts of neutrosophic science to reformulate what the science of operations research presented in terms of methods and methods to solve many practical problems, so we will present in this research a study aimed at shedding light on the most important methods used to solve nonlinear models, which is the Lagrangian multiplier method for nonlinear models constrained by equality and then re-formulated using the concepts of neutrosophic science.

key words:

Operations research - nonlinear programming - Lagrange multiplier - neutrosophic science–Neutrosophic nonlinear programming–Lagrange neutrosophic multiplier

Introduction:

 Science is the basis for managing life affairs and human activities. Operations research science is the applied aspect of mathematics. It is one of the most important modern sciences that are concerned with practical issues and meet the desire and demand of decision-makers to obtain ideal decisions through the methods it presented that are appropriate for all issues, meaning that it depends on the attached data. In every issue, these data are values that have been developed through observation, and their accuracy depends on who collects them. They are also specific for a certain period of time that may not be appropriate for subsequent circumstances. Therefore, they are inaccurate data that are not fully confirmed. Before the advent of neutrosophic science, we used to rely on this data. To study factual issues and accept the results as they are, but after the neutrosophic science that made a great revolution in all fields. Its ability to produce optimal results upon which ideal decisions can be built that take into account all circumstances. Many researchers interested in scientific development tended to reformulate many scientific concepts using this science, so we have neutrosophic numbers - neutrosophic groups - neutrosophic probabilities - neutrosophic statistics - neutrosophic differentiation - neutrosophic integration - neutrosophic linear programming - neutrosophic dynamic programming - neutrosophic simulation, etc. and many more [1-28].

The method of nonlinear programming is one of the most important methods presented by the science of operations research, because most of the practical issues devolve into nonlinear models. Therefore, in a previous research, we formulated some basic concepts of nonlinear programming using the concepts of neutrosophy. [26] ,In addition to what we have done earlier, in this research we will paraphrase Lagrangian multiplication method for nonlinear models constrained by equal constraints using the concepts of

neutrosophic science, by taking the data of the issue under study, neutrosophic numbers It has the following standard form $N = a + bI$ where real or composite coefficients, represents a and b specified part fbI The indefinite part of the number N indeterminacy and could be $[\lambda_1, \lambda_2]$ or $\{\lambda_1, \lambda_1\}$ or something else is any set close to the true value a.

Discussion:

We know that if the nonlinear programming issue consists of only a target function and the target function is convex or (concave), then there is a single optimal solution at a point where all the derivatives are non-existent. The only optimal, but in most realistic issues the goal is to find the maximum or minimum value of a target function that is subject to several restrictions. To calculate one of the variables in terms of the rest of the variables, then replace it with the objective function statement, thus obtaining a new objective function and a new unrestricted example problem. We will use this technique as mentioned in the references [29,30].

To convert nonlinear restricted neutrosophic problems into nonlinear non-restricted neutrosophic problems, and for that, the following information taken from the references must be recalled [27, 28]:

1- The mathematical model is a neutrosophic model if it has something of indeterminacy (non-determination), when collecting data describing a realistic issue and this data is incomplete or has something vague or may contain some contradictions or may be unclear, then we will be unable to build an accurate traditional model This will lead us To build an approximate model (with thickness)

2- The neutrosophic number is a number written by the following standard formula $N = a + bI$ where real or composite coefficients, represents a and b specified part fbI The indefinite part of the number N indeterminacy and could be $[\lambda_1, \lambda_2]$ or $\{\lambda_1, \lambda_1\}$ or something else is any set close to the true value a.

3- The derivative of the neutrophilic function can be written in the following simplified form: $f_N(x)$

$$f_N'(x) = \lim_{h \to 0} \frac{[\inf f(x+h) - \inf f(x), \sup f(x+h) - \sup f(x)]}{[\inf H, \sup H]}$$

This definition is a generalization of the traditional derivative definition.

Text of the constrained neutrosophic nonlinear programming problem:

Based on what is stated in the reference [29], we offer the following:

If it is required to find the (maximum or minimum), value of a function Neutrosophic is continuous, derivable $y_0 = f_N(x_1, x_2, ---, x_n)$

and subject to neutrosophic constraint $g_N(x_1, x_2, ---, x_n) = \alpha$ Also this function is continuous and derivable.

1- We choose the variable x_n in the constraint and express it with the remaining $n - 1$ variables as follows:

$$x_n = H_N(x_1, x_2, ---, x_{n-1})$$

Then we substitute the target function we get

$$y_0 = \overline{f_N}(x_1, x_2, ---, x_{n-1}, H_N(x_1, x_2, ---, x_{n-1}))$$

Thus, the problem turned into an unrestricted one. Traditional methods can be used to obtain a maximum or minimum limit, because the necessary and sufficient condition for the boundary points is that the first derivatives do not exist:

$$\frac{\partial y_0}{\partial x_j} = 0 \quad ; j = 1, 2, ---(n-1)$$

Using the chain rule, we get the following:

$$\frac{\partial y_0}{\partial x_j} = \frac{\partial \overline{f_N}}{\partial x_j} + \frac{\partial \overline{f_N}}{\partial x_n} \cdot \frac{\partial H_N}{\partial x_j} \quad ; j = 1, 2, ---(n-1)$$

Since the $g_N(x_1, x_2, ---, x_n) = \alpha$

We get

$$\frac{\partial g_N}{\partial x_j} + \frac{\partial g_N}{\partial x_n} \cdot \frac{\partial H_N}{\partial x_j} = 0 \quad ; j = 1,2,---(n-1)$$

$$\Rightarrow \frac{\partial H_N}{\partial x_j} = - \left. \frac{\partial g_N}{\partial x_j} \middle/ \frac{\partial g_N}{\partial x_n} \right. \quad ; j = 1,2,---(n-1)$$

On condition: $\frac{\partial g_N}{\partial x_n} \neq 0$

Then we find

$$\frac{\partial y_0}{\partial x_j} = \frac{\partial \overline{f_N}}{\partial x_j} - \left[\frac{\partial \overline{f_N}}{\partial x_n} \cdot \left. \frac{\partial g_N}{\partial x_j} \middle/ \frac{\partial g_N}{\partial x_n} \right. \right] = 0 \quad ; j = 1,2,---(n-1)$$

If the resulting solution vector is the vector that achieves a (maximum or minimum) value, then$(x_{N1}^*, x_{N2}^*, ---, x_{Nn}^*)$, They are the values that make the function maximum or minimum

We symbolize λ_N the following:

$$\lambda_N = \left. \frac{\partial f_N}{\partial x_n} \middle/ \frac{\partial g_N}{\partial x_n} \right.$$

So it is:

$$\frac{\partial \overline{f_N}}{\partial x_j} - \lambda_N \frac{\partial g_N}{\partial x_j} = 0 \quad ; j = 1,2,--,n$$

The supporting condition is:

$$g_N(x_1, x_2, ---, x_n) = \alpha$$

And so we have obtained $n+1$ Equation and $n+1$ unknown, these conditions are necessary for an optimal solution, provided that all derivatives do not exist $\frac{\partial g_N}{\partial x_j} \neq 0$ when$(x_{N1}^*, x_{N2}^*, ---, x_{Nn}^*)$

From the above, we can write the following:

$$y_0 = \overline{f_N}(x_1, x_2, ---, x_n) - \lambda_N[g_N(x_1, x_2, ---, x_n) - \alpha]$$

Then we calculate the derivatives:

$$\frac{\partial y_0}{\partial x_j} = \frac{\partial f_N}{\partial x_j} - \lambda_N \frac{\partial g_N}{\partial x_j} = 0$$

$$\frac{\partial y_0}{\partial \lambda} = -[g_N(x_1, x_2, ---, x_n) - \alpha]$$

Example:

This example is shown in reference [29] (values are dependent and constraints are classic values)

Find the minimum value of the function:

$$f(x) = 3x_1^2 + x_2^2 + 2x_1 x_2 + 6x_1 + 2x_2$$

Subject to constraint:

$$2x_1 - x_2 = 4$$

solution:

We form the Lagrangian function:

$$L(x, \lambda) = 3x_1^2 + x_2^2 + 2x_1 x_2 + 6x_1 + 2x_2 - \lambda(2x_1 - x_2 - 4)$$

The following optimal solution was obtained:

$$x_1^* = \frac{7}{11} \ , x_2^* = \frac{-30}{11} \ , \qquad \lambda^* = \frac{24}{11}$$

And by testing the function using the Hessian matrix, it was found that this function is a convex function

and the constraint is convex

Then the radius of the solution that we obtained is a minimum limit and the value of the function is:

$$f_N^*(x_1^*, x_2^*) = 3.55$$

As we mentioned earlier, if some of the values in the target function or constraints are undefined, ambiguous, or uncertain values, then the issue becomes a neutrosophic issue clearly)

In the previous example, we will take coefficients x_1^2 uncertain (All values can be undefined; we will suffice with one value in order to clearly convey the idea of using neutrosophic numbers clearly

Find the minimum value of the neutrosophic function:

$$f(x) = \{2,3,4\}x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2$$

Subject to constraint:

$$2x_1 - x_2 = 4$$

The solution:

We form the Lagrangian function:

$$L(x,\lambda) = \{2,3,4\}x_1^2 + x_2^2 + 2x_1x_2 + 6x_1 + 2x_2 - \lambda(2x_1 - x_2 - 4)$$

$$\frac{\partial L}{\partial x_1} = \{4,6,8\}x_1 + 2x_2 + 6 - 2\lambda = 0$$

$$\frac{\partial L}{\partial x_2} = 2x_1 + 2x_2 + 2 + \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = 2x_1 - x_2 + 4 = 0$$

We solve the following set of equations:

$$\{4,6,8\}x_1 + 2x_2 + 6 - 2\lambda = 0$$
$$2x_1 + 2x_2 + 2 + \lambda = 0$$
$$2x_1 - x_2 + 4 = 0$$

We get

We obtained the system of three equations with three unknowns. By solving this system, we get:

$$x_1^* = \left\{\frac{7}{10}, \frac{7}{11}, \frac{7}{12}\right\} , x_2^* = \left\{\frac{-13}{5}, \frac{-30}{11}, \frac{-17}{6}\right\} , \qquad \lambda^* = \left\{\frac{9}{5}, \frac{24}{11}, \frac{15}{6}\right\}$$

They are neutrosophic values, and the value of the target function is as follows:

$$f_N^*(x_1^*, x_2^*) = \{3.1, 3.55, 3.24\}$$

That is, we got the optimal solution vector, which is a neutrosophic value. An extreme of a subordinate, it can be a minor end or a major end. to determine its type we will use the test given in reference [26]

1 –The constraint is a linear function, as it is convex and concave at the same time.

2 –To specify the type of follower $f(x)$(convex or concave) we resort to the Hessian matrix for this function:

$$H_N(x) = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 f}{\partial x_2 \partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} \end{bmatrix}$$

$$H_N(x) = \begin{bmatrix} \{4,6,8\} & 2 \\ 2 & 2 \end{bmatrix}$$

The matrix is symmetric and the main diagonal elements are also positive.

Major basic minor determinants are positive because:

$$|\{4,6,8\}| > 0$$

$$\begin{vmatrix} \{4,6,8\} & 2 \\ 2 & 2 \end{vmatrix} = \{8,12,16\} - 4 = \{4,8,12\} > 0$$

Hence the Hessian matrix of the function $f_N(x)$ Positive knowledge, that is, the function is convex.

From the above, the Vector of the solution that we obtained is a minor limit of the value neutrosophic for to follow him:

$$f_N^*(x_1^*, x_2^*) = \{3.1, 3.55, 3.24\}$$

Conclusion and Results:

In the previous study, we presented an important technique for solving neutrosophic nonlinear models constrained by equal constraints. Through the study, we found that there is a difference in the values of the optimal solution when using the technique and the data are traditional values and using them and the data are neutrosophic values, since the goal of solving examples problems is to find the maximum value that expresses profit or profitability and the smallest value that expresses the amount of loss or cost for a follower of a goal within certain constraints, and since mathematical models are built using data collected on the case under study, and these data are values that represent the current reality of the work environment, and any change in the surrounding conditions leads to a change in the results of the solution, which may cause unexpected losses whose nature is determined by the type of issue under study, it may be human, material, or otherwise, so we focus on the need to use neutrosophic values when collecting data for any realistic issue, values that take into account the worst conditions to the best, as we focus on necessity reformulation of many other techniques for solving nonlinear models using neutrosophic concepts such as projection gradients and vibration–Newton's method–Fibonacci Search ----

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