Pentapartitioned Fermatean Neutrosophic Alpha Phi Normal Topological Spaces

Gürel Bozma
Bülent Ecevit Üniversitesi

Nazmiye Gonul Bilgin
Bülent Ecevit Üniversitesi

BEŞ BİLEŞENLİ FERMATEAN NÖTROSOFİK ALPHA PHI NORMAL TOPOLOJİK UZAYLAR

Prof. Dr. Yusuf KAYA
Zonguldak Bülent Ecevit Üniversitesi, Fen Fakültesi,
ORCID ID: 0000-0003-2647-1356
Doç. Dr. Nazmiye GÖNÜL BİLGİN
Zonguldak Bülent Ecevit Üniversitesi, Fen Fakültesi
ORCID ID: 0000-0001-6300-6889
Doktora Öğrencisi Gürel BOZMA
Zonguldak Bülent Ecevit Üniversitesi, Fen Fakültesi
ORCID ID: 0000-0002-0916-5894

ÖZET

Matematiğin birçok dalının kümeler teorisi üzerine inşa edilmesi nedeniyle bu teori her an yeni konuların çalışılmaya başlandığı alan olma özelliğini korumaktadır. Klasik küme teorisi mühendislik başta olmak üzere matematik dışında birçok günlük hayat problemine de çözüm ararken başlangıç noktası olmuştur. Hayatın daha karmaşık hale gelmesi ile birlikte bu teori yetersiz hale gelmiş ve yeni küme kavramları ortaya çıkmıştır. Fuzzy küme ve intuitionistic küme teorisinden sonra son günlerin popüler yaklaşımlarından biri de Neutrosophic küme teorisidir. Elemanların bir kümeye ait olmalarını derecelendirme yaklaşımına dayanan fuzzy küme teorisi elemanların dahil olmamasının derecesine ilişkin bilgiler verecek şekilde intuitionistic küme şeklinde genişletilmiş daha sonra çelişki ve belirsizlik durumlarının derecelerini de içerecek şekilde Neutrosophic kümeler haline getirilmiştir. Bu teoriyi matematiğin tüm disiplinlerine uygulamaya yönelik hızla önemli çalışmalar yapılmış olmakla birlikte günlük hayat uygulamaları da en az onlar kadar popülerdir.

Chang (1968) tarafından bulanık kümeler üzerine topolojik uzayların tanımlanmasının ardından Coker (1997) intuitionistic fuzzy kümeler için topoloji kavramını tanımlamıştır. Salama ve Alblowi (2012) den sonra neutrosophic kümeler için de topoloji alanındaki çalışmalar hız kazanmıştır. Bu alanda birçok matematikçi hem teorik hem de diğer alanlarla ilişkili uygulama çalışmaları yapmaktadır.

Bazı özel koşulları sağlayan üyelik derecelerinin temel alındığı çalışmamızda neutrosophic $\alpha\psi$ −topolojik uzaya ilişkin bazı önemli özellikleri inceleyeceğiz. neutrosophic $\alpha\psi$ −kapalı ve neutrosophic $\alpha\psi$ −normal kümeleri kullanarak neutrosophic $\alpha\psi$ −normal uzay, kuvvetli neutrosophic $\alpha\psi$ −normal uzay tanımları verilecektir. Ayrıca verilen kavramlar arasındaki ilişkileri belirleyen önemli teoremler kanıtlanacaktır.

Anahtar Kelimeler: Beş Bileşenli Nötrosofik Küme, Beş Bileşenli Fermatean Nötrosofik Topoloji, Beş Bileşenli Fermatean Nötrosofik $\alpha\psi$ −kapalı Küme.

PENTAPARTITIONED FERMATEAN NEUTROSOPHIC ALPHA PHI NORMAL TOPOLOGICAL SPACES

ABSTRACT

Since many branches of mathematics are built on set theory, this theory maintains its feature of being the field where new subjects are started to be studied at any time. Classical set theory has been the starting point when searching for solutions to many daily life problems besides mathematics, especially engineering. As life became more complex, this theory became insufficient and new set concepts emerged. After fuzzy set and intuitionistic set theory, one of the popular approaches in recent days is Neutrosophic set theory. Fuzzy set theory, which is based on the grading approach of the elements belonging to a set, was expanded as an intuitionistic set to provide information about the degree of inclusion of the elements, and then turned into Neutrosophic sets to include the degrees of contradiction and uncertainty situations. Although important studies have been made to apply this theory to all disciplines of mathematics, daily life applications are at least as popular.

After the definition of topological spaces on fuzzy sets by Chang (1968), Coker (1997) defined the concept of topology for intuitionistic fuzzy sets. After Salama and Alblowi (2012), studies in the field of topology for neutrosophic sets accelerated. Many mathematicians in this field do both theoretical and practical studies related to other fields.

In our study, which is based on membership degrees that meet some special conditions, we will examine some important features of the neutrosophic $\alpha\psi-$topological space. Definitions of neutrosophic $\alpha\psi-$normal space, strong neutrosophic $\alpha\psi-$normal space will be given using neutrosophic $\alpha\psi-$closed and neutrosophic $\alpha\psi-$normal sets. In addition, important theorems that determine the relationships between the given concepts will be proved.

Keywords: Pentapartitioned Neutrosophic Set, Pentapartitioned Fermatean Neutrosophic Topology, Pentapartitioned Fermatean Neutrosophic $\alpha\psi-$closed Set.

1. INTRODUCTION AND PRELIMINARIES

Fuzzy topological spaces was defined and developed in [1]. After that, Atannasov [2] described the notion of intuitionistic fuzzy sets, Çoker [3] constructed the intuitionistic fuzzy topological spaces. The concept of neutrosophic crisp topological spaces were defined in [4,5].

Neutrosophic set theory, which was founded by Samarandache and carried to many different areas, was built to eliminate the deficiencies of fuzzy and intuitionistic fuzzy theories. After the definition of neutrosophic sets, important researches has been carried out to adapt and develop the studies in the field of fuzzy and intuitionistic fuzzy to this new field.

Neutrosophic topology has been studied by many authors especially Samarandache, Salama and Alblowi [6-9]. Neutrosophic α-open and regular open set defined in [10]. Later, Ray and Dey investigated neutrosophic point in 2021,[21].

Now, let us give the basic definitions we will need about neutrosophic sets.

Definition 1.1. [6] A neutrosophic set V on a fixed set X is given below:

$V = \{< r, H_V(r), E_V(r), S_V(r) >: r \in X\}$, where $H_V, E_V, S_V: X \to [0,1]$ are the truth, indeterminacy and falsity membership functions.

Later, the number of membership functions used was increased so that the concepts involved in the uncertainty situation were more clearly stated. Let's give the definition of the set which the concepts of contradiction and ignorance are included.

Definition 1.2. [15] A pentapartitioned neutrosophic set Z (in short P-NS) on X is given below:

$Z = \{< r, H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r) >: r \in X\}$

where $H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r) \in [0,1]$ are the truth, contradiction, ignorance, unknown, falsity membership values of every $r \in X$. Also, for all $r \in X$, $0 \le H_Z(r) + A_Z(r) + O_Z(r) + M_Z(r) + S_Z(r) \le 5$ if truth, falsity, contradiction, ignorance, unknown are independent components.

But for all $r \in X$, truth, falsity are dependent components and contradiction, ignorance, unknown are independent components and so

$0 \le H_Z(r) + A_Z(r) + O_Z(r) + M_Z(r) + S_Z(r) \le 4$.

Now, we give (1_{PN}) and (0_{PN}) We noted that there are different definitions for this sets but we use as in Definition 1.3

Definition 1.3. [15] Let X be a fixed set. Then, (1_{PN}) and (0_{PN}) on X are defined as follows:

(i) $1_{PN} = \{\langle r, 1,1,0,0,0\rangle: r \in X\}$,

(ii) $0_{PN} = \{\langle r, 0,0,1,1,1\rangle: r \in X\}$.

For pentapartitioned neutrosophic set general set operations is given in [15].

Remark 1.1. [15] Let M be a P-NS. Clearly, $0_{PN} \subseteq M \subseteq 1_{PN}$.

Let's give the definition of a fermatean neutrophic set with the help of membership functions with certain conditions.

Definition 1.4. [16],[17] Let $H_{฿}(r), E_{฿}(r), S_{฿}(r): X \longrightarrow [0,1]$ show the degree of membership indeterminacy and non-membership of every $r \in W$ to ฿. A Fermatean neutrosophic set ฿ has the form $฿ = \{\langle r, H_{฿}(r), E_{฿}(r), S_{฿}(r)\rangle : r \in X\}$ where indeterminacy is an independent component and membership and non-membership are dependent components: for each $r \in X$, $0 \leq H_{฿}(r) + E_{฿}(r) + S_{฿}(r) \leq 2, 0 \leq H_{฿}(r)^3(r) + {S_{฿}}^3(r) \leq 1$ and $H_{฿}(r)^3 + {E_{฿}}^3(r) + {S_{฿}}^3(r) \leq 2$.

Definition 1.5. [18] A pentapartitioned fermatean neutrosophic set (in short P-FNS) Z on X is given below:

$Z = \{\langle r, H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r)\rangle : r \in X\}$ where $H_Z(r), A_Z(r), O_Z(r), M_Z(r), S_Z(r) \in [0,1]$ are the truth, contradiction, ignorance, unknown, falsity membership values of each $r \in X$. Here, $0 \leq H_Z(r) + A_Z(r) + O_Z(r) + M_Z(r) + S_Z(r) \leq 4$, for all $r \in X$, $H_Z(r), S_Z(r)$ are dependent components and $A_Z(r), O_Z(r), M_Z(r)$ is an independent component for each $r \in X$. Also, for each $r \in W$, $0 \leq {H_Z}^3(r) + {S_Z}^3(r) \leq 1$ where $0 \leq {A_Z}^3(r) + {O_Z}^3(r) + {M_Z}^3(r) \leq 3$.

Definition 1.6. [16] Let

$Ţ_1 = \{\langle r, H_{Ţ_1}(r), A_{Ţ_1}(r), O_{Ţ_1}(r), M_{Ţ_1}(r), S_{Ţ_1}(r)\rangle : r \in X\}$,

$Ţ_2 = \{\langle r, H_{Ţ_2}(r), A_{Ţ_2}(r), O_{Ţ_2}(r), M_{Ţ_2}(r), S_{Ţ_2}(r)\rangle : r \in X\}$

be two P-FNSs on X and let $A_{Ţ_i}(r), O_{Ţ_i}(r), M_{Ţ_i}(r)$ is an independent component and $H_{Ţ_i}(r), S_{Ţ_i}(r)$ are dependent components for each $r \in W$, $i \in \{1,2\}$.

i) $Ţ_1 \subseteq Ţ_2$ iff $H_{Ţ_1}(r) \leq H_{Ţ_2}(r), A_{Ţ_1}(r) \leq A_{Ţ_2}(r), O_{Ţ_1}(r) \geq O_{Ţ_2}(r), M_{Ţ_1}(r) \geq M_{Ţ_2}(r), S_{Ţ_1}(r) \geq S_{Ţ_2}(r)$, for every $r \in X$.

ii) The intersection of $Ţ_1$ and $Ţ_2$ is

$Ţ_1 \cap Ţ_2 = \{\langle r, \min\{H_{Ţ_1}(r), H_{Ţ_2}(r)\}, \min\{A_{Ţ_1}(r), A_{Ţ_2}(r)\}, \max\{O_{Ţ_1}(r), O_{Ţ_2}(r)\},$

$\max\{M_{Ţ_1}(r), M_{Ţ_2}(r)\}, \max\{S_{Ţ_1}(r), S_{Ţ_2}(r)\}\rangle : r \in X\}.$

iii) The union of $Ţ_1$ and $Ţ_2$ is

$Ţ_1 \cup Ţ_2 = \{\langle r, \max\{H_{Ţ_1}(r), H_{Ţ_2}(r)\}, \max\{A_{Ţ_1}(r), A_{Ţ_2}(r)\}, \min\{O_{Ţ_1}(r), O_{Ţ_2}(r)\},$

$\min\{M_{Ţ_1}(r), M_{Ţ_2}(r)\}, \min\{S_{Ţ_1}(r), S_{Ţ_2}(r)\}\rangle : r \in X\}.$

iv) ${Ţ_1}^c = \{\langle r, S_{Ţ_1}(r), M_{Ţ_1}(r), 1 - O_{Ţ_1}(r), A_{Ţ_1}(r), H_{Ţ_1}(r)\rangle : r \in X\}$.

2. Pentapartitioned Fermatean Neutrosophic Topologicial Spaces

Before giving the concept of topological space that we will work with, let's give the definition of pentapartitioned fermatean neutrosophic point.

Definition 2.1. Let $\mathcal{N}(X)$ be set all pentapartitioned fermatean neutrosophic sets over X. A neutrosophic set $x_{(a,b,c,d,e)} = \{\langle x, H(x), A(x), O(x), M(x), S(x) : x \in X \rangle\}$ is named pentapartitioned fermatean neutrosophic point iff for any element $y \in X$, $H(y) = a$, $A(y) = b$, $O(y) = c$, $M(y) = d, S(y) = e$ for $y = x$ and $H(y) = 0$, $A(y) = 0$, $O(y) = 1$, $M(y) = 1, S(y) = 1$ for $y \neq x$, where $0 < a \leq 1, 0 < b \leq 1, 0 \leq c < 1, 0 \leq d < 1, 0 \leq e < 1$.

Let f be a mapping from X into Y. If $V = \{\langle y, H_V(y), A_V(y), O_V(y), M_V(y), S_V(y)\rangle : y \in Y\}$ is an P-FNS in Y, then the inverse image of V under f is an P-FNS given by

$$f^{-1}(V) = \{\langle x, f^{-1}(H_V)(x), f^{-1}(A_V)(x), f^{-1}(O_V)(x), f^{-1}(M_V)(x), f^{-1}(S_V)(x)\rangle : x \in X\}.$$

The image of P-FNS $U = \{\langle y, H_U(y), A_U(y), O_U(y), M_U(y), S_U(y)\rangle : y \in Y\}$ under f is an P-FNS described with $f(U) = \{\langle y, f(H_U)(y), f(A_U)(y), f(O_U)(y), f(M_U)(y), f(S_U)(y)\rangle : y \in Y\}$ where

$$f(H_U)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} (H_U)(x), if\ f^{-1}(y) \neq 0 \\ 0 \qquad\qquad otherwise \end{cases},$$

$$f(A_U)(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} (A_U)(x), if\ f^{-1}(y) \neq 0 \\ 0 \qquad\qquad otherwise \end{cases},$$

$$f(O_U)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} (O_U)(x), if\ f^{-1}(y) \neq 0 \\ 1 \qquad\qquad otherwise \end{cases}$$

$$f(M_U)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} (M_U)(x), if\ f^{-1}(y) \neq 0 \\ 1 \qquad\qquad otherwise \end{cases},$$

$$f(S_U)(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} (S_U)(x), if\ f^{-1}(y) \neq 0 \\ 1 \qquad\qquad otherwise \end{cases}$$

for each $y \in Y$.

Definition 2.2. A pentapartitioned fermatean neutrosophic topology on X is a set T_N of pentapartitioned fermatean neutrosophic subsets of X providing

(i) $0_{PN}, 1_{PN} \in T_N$.

(ii) $G \cap H \in T_N$ for every $G, H \in T_N$.

(iii) $\bigcup_{j \in J} G_j \in T_N$ for every $\{G_j : j \in J\} \subseteq T_N$.

Here, the pair (X, T_N) is said a pentapartitioned fermatean neutrosophic topological space (P-FNTS). The elements of T_N are said pentapartitioned fermatean neutrosophic open sets (P-FNOS) in X. P-FNS A in X is said a pentapartitioned fermatean neutrosophic closed set (P-FNCS) $\Leftrightarrow$ its complement A^c is a P-FNOS.

Example 2.1 Let $X = \{r_1, r_2\}$ for all $k \in \{1,2,3,4\}$ $ꓔ_{n_k}$ be P-FNS:

$ꓔ_{n_1} = \{\langle r_1, 0.2,0.3,0.5,0.2,0.6\rangle, \langle r_2, 0.3,0.6,0.4,0.2,0.7\rangle\}$

$ꓔ_{n_2} = \{\langle r_1, 0,0,1,1,1\rangle, \langle r_2, 0,0,1,1,1\rangle\}$

$ꓔ_{n_3} = \{\langle r_1, 0.2,0.2,0.9,0.2,0.7\rangle, \langle r_2, 0.1,0.3,0.5,0.7,0.8\rangle\}$

$ꓔ_{n_4} = \{\langle r_1, 1,1,0,0,0\rangle, \langle r_2, 1,1,0,0,0\rangle\}$

where for all $j \in \{1,2\}$, $0 \le {H_{ꓔ_{n_k}}}^3(r_j) + {S_{ꓔ_{n_k}}}^3(r_j) \le 1$ and $0 \le {A_{ꓔ_{n_k}}}^3(r_j) + {O_{ꓔ_{n_k}}}^3(r_j) + {M_{ꓔ_{n_k}}}^3(r_j) \le 3$. Then, $T_N = \{0_N, 1_N, ꓔ_{n_1}, ꓔ_{n_2}, ꓔ_{n_3}, ꓔ_{n_4}\}$ is a pentapartitioned fermatean neutrosophic topology. Also, $ꓔ_{n_1} \cup ꓔ_{n_2} = ꓔ_{n_1} \cup ꓔ_{n_3} = ꓔ_{n_1}$, $ꓔ_{n_1} \cup ꓔ_{n_4} = ꓔ_{n_2} \cup ꓔ_{n_4} = ꓔ_{n_3} \cup ꓔ_{n_4} = ꓔ_{n_4}$, $ꓔ_{n_2} \cup ꓔ_{n_3} = ꓔ_{n_3}$ and $ꓔ_{n_1} \cup ꓔ_{n_2} \cup ꓔ_{n_3} = ꓔ_{n_1}$, $ꓔ_{n_1} \cup ꓔ_{n_2} \cup ꓔ_{n_4} = ꓔ_{n_2} \cup ꓔ_{n_3} \cup ꓔ_{n_4} = ꓔ_{n_1} \cup ꓔ_{n_2} \cup ꓔ_{n_3} \cup ꓔ_{n_4} = ꓔ_{n_4}$.

Further, $ꓔ_{n_1} \cap ꓔ_{n_2} = ꓔ_{n_2} \cap ꓔ_{n_3} = ꓔ_{n_2} \cap ꓔ_{n_4} = ꓔ_{n_2}$, $ꓔ_{n_1} \cap ꓔ_{n_3} = ꓔ_{n_3} \cap ꓔ_{n_4} = ꓔ_{n_3}$, $ꓔ_{n_1} \cap ꓔ_{n_4} = ꓔ_{n_1}$, $ꓔ_{n_1} \cap ꓔ_{n_2} \cap ꓔ_{n_3} = ꓔ_{n_1} \cap ꓔ_{n_2} \cap ꓔ_{n_4} = ꓔ_{n_1} \cap ꓔ_{n_2} \cap ꓔ_{n_3} \cap ꓔ_{n_4} = ꓔ_{n_2}$ and $0_{PN} = ꓔ_{n_2}$, $1_{PN} = ꓔ_{n_4}$.

Now, we examine some properties of these spaces, similar to the works done on neutrosophic topological spaces.

Definition 2.3. On (X, τ_N), let $ꓔ = \{\langle r, H_ꓔ(r), A_ꓔ(r), O_ꓔ(r), M_ꓔ(r), S_ꓔ(r)\rangle : r \in X\}$ be a P-FNS. Then

i) The pentapartitioned fermatean neutrosophic interior of $ꓔ$, denoted by $PFN\,Int(ꓔ)$ is the union of all pentapartitioned fermatean neutrosophic open subsets of $ꓔ$. Clearly $PFN\,Int(ꓔ)$ is the largest neutrosophic open subset of X included in $ꓔ$.

ii) The pentapartitioned fermatean neutrosophic closure of $ꓔ$ denoted with $PFN\,Cl(ꓔ)$ is the intersection of all P-FNCSs containing $ꓔ$. Here, $PFN\,Cl(ꓔ)$ is the smallest P-FNCS which contains $ꓔ$.

Example 2.2. Let $X = \{r_1, r_2\}$ for all $k \in \{1,2,3\}$ $ꓔ_{n_k}$ be P-FNS:

$ꓔ_{n_1} = \{\langle r_1, 0.5,0.6,0.4,0.5,0.1\rangle, \langle r_2, 0.7,0.4,0.6,0.7,0.5\rangle\}$,

$ꓔ_{n_2} = \{\langle r_1, 0.5,0.4,0.6,0.8,0.3\rangle, \langle r_2, 0.6,0.4,0.7,0.8,0.9\rangle\}$,

$ꓔ_{n_3} = \{\langle r_1, 0.4,0.3,0.8,0.9,0.4\rangle, \langle r_2, 0,0.3,0.8,0.8,1\rangle\}$.

Then, $T_N = \{0_{PN}, 1_{PN}, ꓔ_{n_1}, ꓔ_{n_2}, ꓔ_{n_3}\}$ is a pentapartitioned fermatean neutrosophic topology.

Also $0_{PN}, 1_{PN}, Ƭ_{n_1}, Ƭ_{n_2}, Ƭ_{n_3}$ P-FNOS in (X, τ_N) and their complements $1_{PN}, 0_{PN}$,

$Ƭ_{n_1}{}^{c} = \{\langle r_1, 0.1,0.5,0.6,0.6,0.5\rangle, \langle r_2, 0.5,0.7,0.4,0.4,0.7\rangle\}$

$Ƭ_{n_2}{}^{c} = \{\langle r_1, 0.3,0.4,0.4,0.4,0.5\rangle, \langle r_2, 0.9,0.8,0.3,0.4,0.6\rangle\}$,

$Ƭ_{n_3}{}^{c} = \{\langle r_1, 0.4,0.9,0.2,0.3,0.4\rangle, \langle r_2, 1,0.8,0.2,0.3,0\rangle\}$

and $1_{PN} \cap Ƭ_{n_1}{}^{c} = \{\langle r_1, \min\{1,0.1\}, \min\{1,0.5\}, \max\{0,0.6\}, \max\{0,0.6\}, \max\{0,0.5\}\rangle,$

$\langle r_2, \min\{1,0.5\}, \min\{1,0.7\}, \max\{0,0.4\}, \max\{0,0.4\}, \max\{0,0.7\}\rangle, r_1, r_1 \in X\}$

$= \{\langle r_1, 0.1,0.5,0.6,0.6,0.5\rangle, \langle r_2, 0.5,0.7,0.4,0.4,0.7\rangle\} = Ƭ_{n_1}{}^{c}$, $1_{PN} \cap Ƭ_{n_2}{}^{c} = Ƭ_{n_2}{}^{c}$, $1_{PN} \cap Ƭ_{n_3}{}^{c} = Ƭ_{n_3}{}^{c}$

$\{0_{PN}, 1_{PN}, Ƭ_{n_1}{}^{c}, Ƭ_{n_2}{}^{c}, Ƭ_{n_3}{}^{c}\}$ are P-FNCS in (X, τ_N).

Suppose that $Ƭ = \{\langle r_1, 0.4,0.3,0.4,0.5,0.1\rangle, \langle r_2, 0.6,0.3,0.6,0.7,0.6\rangle\}$ be a P-FNTS on X. Then, $PFN\ Int(Ƭ) = \{\langle r_1, 0.4,0.3,0.8,0.9,0.4\rangle, \langle r_2, 0,0.3,0.8,0.8,1\rangle\}$ and $PFN\ Cl(Ƭ) = 1_{PN}$.

b) Let $Ƭ_{n_1} = \{\langle r_1, 0.2,0.3,0.5,0.2,0.6\rangle, \langle r_2, 0.3,0.6,0.4,0.2,0.7\rangle\}$,

$Ƭ_{n_2} = \{\langle r_1, 0.2,0.2,0.9,0.2,0.7\rangle, \langle r_2, 0.1,0.3,0.5,0.7,0.8\rangle\}$.

In this case $T_N = \{0_{PN}, 1_{PN}, Ƭ_{n_1}, Ƭ_{n_2}\}$ is a pentapartitioned fermatean neutrosophic topology. Here $0_{PN}, 1_{PN}, Ƭ_{n_1}, Ƭ_{n_2}$ P-FNOS in (X, τ_N) and their complements $1_{PN}, 0_{PN}$,

$Ƭ_{n_1}{}^{c} = \{\langle r_1, 0.6,0.2,0.5,0.3,0.2\rangle, \langle r_2, 0.7,0.2,0.6,0.6,0.3\rangle\}$,

$Ƭ_{n_2}{}^{c} = \{\langle r_1, 0.7,0.2,0.1,0.2,0.2\rangle, \langle r_2, 0.8,0.7,0.5,0.3,0.1\rangle\}$

and $1_{PN} \cap Ƭ_{n_1}{}^{c} = \{\langle r_1, \min\{1,0.6\}, \min\{1,0.2\}, \max\{0,0.5\}, \max\{0,0.3\}, \max\{0,0.2\}\rangle,$

$\langle r_2, \min\{1,0.7\}, \min\{1,0.2\}, \max\{0,0.6\}, \max\{0,0.6\}, \max\{0,0.3\}\rangle, r_1, r_1 \in X\}$

$= \{\langle r_1, 0.6,0.2,0.5,0.3,0.2\rangle, \langle r_2, 0.7,0.2,0.6,0.6,0.3\rangle\} = Ƭ_{n_1}{}^{c}$, $1_{PN} \cap Ƭ_{n_2}{}^{c} = Ƭ_{n_2}{}^{c}$

are P-FNCS in (X, τ_N). Suppose that $Ƭ = \{\langle r_1, 0.7,0.7,0.8,0.8,0.6\rangle, \langle r_2, 0.5,0.5,0.5,0.6,0.6\rangle\}$ be a P-FNS on X. Then, $PFN\ Int(Ƭ) = 0_{PN}$ and $PFN\ Cl(Ƭ) = 1_{PN}$.

Now, the regular open set and related definitions given for neutrosophic spaces in [10] will be transferred to the topological spaces we have defined.

Definition 2.4. A pentapartitioned fermatean neutrosophic subset $Ƭ$ of a P-FNTS (X, T_N) is said to be a pentapartitioned fermatean neutrosophic regular open set (P-FNROS) if $Ƭ \subseteq PFN\ Int[PFN\ Cl(Ƭ)]$. The complement of a pentapartitioned fermatean neutrosophic regular open set is said a pentapartitioned fermatean neutrosophic regular closed (pre-closed) set (P-FNRCS) in X.

Example 2.3. a) Let $X = \{r_1, r_2\}$ non-empty set. Clearly (X, τ_N) be an P-FNTS, where

$Ť_{n_1} = \{\langle r_1, 0.3,0.3,0.3,0.3,0.4\rangle, \langle r_2, 0.4,0.4,0.4,0.4,0.3\rangle: r_1, r_2 \in X\}$,

$Ť_{n_2} = \{\langle r_1, 0.4,0.4,0.1,0.1,0.4\rangle, \langle r_2, 0.5,0.5,0.3,0.3,0.1\rangle: r_1, r_2 \in X\}$

$\tau_N = \{0_{PN}, 1_{PN}, Ť_{n_1}, Ť_{n_2}\}$ Here complements of $0_N, 1_N, Ť_{n_1}, Ť_{n_2}$ are $1_{PN}, 0_{PN}$,

${Ť_{n_1}}^c = \{\langle r_1, 0.4,0.3,0.7,0.3,0.3\rangle, \langle r_2, 0.3,0.4,0.6,0.4,0.4\rangle\}$,

${Ť_{n_2}}^c = \{\langle r_1, 0.4,0.1,0.9,0.4,0.4\rangle, \langle r_2, 0.1,0.3,0.7,0.5,0.5\rangle\}$ and they are P-FNCS in (X, τ_N).

Let $Ť = \{\langle r_1, 0.3,0.3,0.2,0.2,0.9\rangle, \langle r_2, 0.3,0.3,0.3,0.3,0.4\rangle: r_1, r_2 \in X\}$ be a P-FNS on X. Then, $PFN\ Int(Ť) = 0_{PN}$ and $PFN\ Cl(Ť) = 1_{PN}$. $PFN\ Int[PFN\ Cl(Ť)] = 1_{PN}$

So we get $Ť \subseteq PFN\ Int[PFN\ Cl(Ť)]$, then $Ť$ is (P-FNROS).

b) Let $X = \{r_1, r_2\}$ for all $k \in \{1,2\}$ $Ť_{n_k}$ be P-FNS:

$Ť_{n_1} = \{\langle r_1, 0.2,0.3,0.5,0.2,0.6\rangle, \langle r_2, 0.3,0.6,0.4,0.2,0.7\rangle\}$,

$Ť_{n_2} = \{\langle r_1, 0.2,0.2,0.9,0.2,0.7\rangle, \langle r_2, 0.1,0.3,0.5,0.7,0.8\rangle\}$.

In this case $T_N = \{0_{PN}, 1_{PN}, Ť_{n_1}, Ť_{n_2}\}$ is a pentapartitioned fermatean neutrosophic topology. Here $0_N, 1_N, Ť_{n_1}, Ť_{n_2}$ P-FNOS in (X, τ_N) and their complements $1_{PN}, 0_{PN}$,

${Ť_{n_1}}^c = \{\langle r_1, 0.6,0.2,0.5,0.3,0.2\rangle, \langle r_2, 0.7,0.2,0.6,0.6,0.3\rangle\}$,

${Ť_{n_2}}^c = \{\langle r_1, 0.7,0.2,0.1,0.2,0.2\rangle, \langle r_2, 0.8,0.7,0.5,0.3,0.1\rangle\}$ are P-FNCS in (X, τ_N). Suppose that $Ť = \{\langle r_1, 0.1,0.1,0.5,0.2,0.6\rangle, \langle r_2, 0.1,0.1,0.4,0.2,0.7\rangle\}$ be a P-FNTS on X. Then, $PFN\ Int(Ť) = 0_N$ and $PFN\ Cl(Ť) = 1_{PN}$, $PFN\ Int[PFN\ Cl(Ť)] = 1_{PN}$.

Thus $Ť \subseteq PFN\ Int[PFN\ Cl(Ť)]$, $Ť$ is (P-FNROS) and pre-open set.

Definition 2.5 Let (X, T_N) be a P-FNTS and $Ť$ be a P-FNS. Then pentapartitioned fermatean neutrosophic α-interior of $Ť$, (briefly $PFN\alpha\ Int(Ť)$) is given as the union of each pentapartitioned fermatean neutrosophic regular open subsets of $Ť$. Equivalently, it could be as given below:

$PFN\alpha\ Int(Ť) = \cup\{B: B \subseteq Ť\ and\ B\ is\ a\ \text{P} - \text{FNROS}\ in\ X\}$.

Example 2.4 Let $X = \{r_1, r_2\}$ non-empty set. Clearly (X, τ_N) be an P-FNTS, $\tau_N = \{0_{PN}, 1_{PN}, Ť_{n_1}, Ť_{n_2}\}$ where,

$Ť_{n_1} = \{\langle r_1, 0.5,0.5,0.7,0.7,0.2\rangle, \langle r_2, 0.6,0.6,0.7,0.7,0.3\rangle: r_1, r_2 \in X\}$,

$Ť_{n_2} = \{\langle r_1, 0.5,0.5,0.6,0.6,0.2\rangle, \langle r_2, 0.8,0.8,0.6,0.6,0.3\rangle: r_1, r_2 \in X\}$,

$Ƭ_{n_1}{}^{c} = \{\langle r_1, 0.2,0.7,0.3,0.5,0.5\rangle, \langle r_2, 0.3,0.7,0.3,0.6,0.6\rangle : r_1, r_2 \in X\}$,

$Ƭ_{n_2}{}^{c} = \{\langle r_1, 0.2,0.6,0.4,0.5,0.5\rangle, \langle r_2, 0.3,0.6,0.4,0.8,0.8\rangle : r_1, r_2 \in X\}$,

Consider the P-FNS $Ƭ = \{\langle r_1, 0.4,0.4,0.6,0.6,0.2\rangle, \langle r_2, 0.7,0.7,0.6,0.6,0.3\rangle : r_1, r_2 \in X\}$

$PFN\ Cl(Ƭ) = 1_{PN}$, $PFN\ Int[PFN\ Cl(Ƭ)] = 1_{PN}$ is regular open set. So, $PFN\alpha\ Int(Ƭ) = 1_{PN}$.

Definition 2.6 A subset $Ƭ$ of a P-FNTS (X, T_N) is said

i) a pentapartitioned fermatean neutrosophic semi-open set if $Ƭ \subseteq PFN\ Cl[PFN\ Int(Ƭ)]$ and a pentapartitioned fermatean neutrosophic semi-closed set if $PFN\ Int[PFN\ Cl(Ƭ)] \subseteq Ƭ$,

ii) a pentapartitioned fermatean neutrosophic α-open set if $Ƭ \subseteq PFN\ Int\big[PFN\ Cl[PFN\ Int(Ƭ)]\big]$ and a pentapartitioned fermatean neutrosophic α-closed set if $PFN\ Cl\big[PFN\ Int[PFN\ Cl(Ƭ)]\big] \subseteq Ƭ$.

The pre-closure (respectively, α-closure and semi-closure) of a subset $Ƭ$ of a P-FNTS (X, T_N) is the intersection of every pre-closed (respectively, α-closed, semi-closed) sets which contain $Ƭ$ and is demonstrated with $PFNpCl(Ƭ)$ (respectively, $PFN\alpha Cl(Ƭ)$).and $PFNsCl(Ƭ)$

Example 2.5 Let $X = \{r_1, r_2\}$ non-empty set. Clearly (X, τ_N) be an P-FNTS, where

$\tau_N = \{0_{PN}, 1_{PN}, Ƭ_{n_1} = \{\langle r_1, 0.3,0.3,0.3,0.3,0.4\rangle, \langle r_2, 0.4,0.4,0.4,0.4,0.3\rangle : r_1, r_2 \in X\}$,

$Ƭ_{n_2} = \{\langle r_1, 0.4,0.4,0.1,0.1,0.4\rangle, \langle r_2, 0.5,0.5,0.3,0.3,0.1\rangle : r_1, r_2 \in X\}\}$ Here $0_{PN}, 1_{PN}, Ƭ_{n_1}, Ƭ_{n_2}$ P-FNOS in (X, τ_N) and their complements $1_N, 0_N$,

$Ƭ_{n_1}{}^{c} = \{\langle r_1, 0.4,0.3,0.7,0.3,0.3\rangle, \langle r_2, 0.3,0.4,0.6,0.4,0.4\rangle\}$,

$Ƭ_{n_2}{}^{c} = \{\langle r_1, 0.4,0.1,0.9,0.4,0.4\rangle, \langle r_2, 0.1,0.3,0.7,0.5,0.5\rangle\}$ are P-FNCS in (X, τ_N).

$Ƭ_1 = \{\langle r_1, 0.3,0.3,0.1,0.1,0.4\rangle, \langle r_2, 0.4,0.4,0.3,0.3,0.1\rangle : r_1, r_2 \in X\}$ be a P-FNS on X. Then, $PFN\ Int(Ƭ_1) = Ƭ_{\boldsymbol{n_1}} = \{\langle r_1, 0.3,0.3,0.3,0.3,0.4\rangle, \langle r_2, 0.4,0.4,0.4,0.4,0.3\rangle : r_1, r_2 \in X\}$

and $PFN\ Cl[PFN\ Int(Ƭ_1)] = 1_{PN}$. Thus $Ƭ_1 \subseteq PFN\ Cl[PFN\ Int(Ƭ_1)]$, $Ƭ_1$ is pentapartitioned fermatean neutrosophic semi-open set. So;

$PFN\ Int\big[PFN\ Cl[PFN\ Int(Ƭ_1)]\big] = 1_{PN}$.

Thus $Ƭ_1 \subseteq PFN\ Int\big[PFN\ Cl[PFN\ Int(Ƭ_1)]\big]$, $Ƭ_1$ is pentapartitioned fermatean neutrosophic α-open set.

Example 2.6 Let $X = \{r_1, r_2\}$ for all $k \in \{1,2\}$ $Ƭ_{n_k}$ be P-FNS:

$Ƭ_{n_1} = \{\langle r_1, 0.2,0.3,0.5,0.2,0.6\rangle, \langle r_2, 0.3,0.6,0.4,0.2,0.7\rangle\}$,

$\daleth_{n_2} = \{\langle r_1, 0.2,0.2,0.9,0.2,0.7\rangle, \langle r_2, 0.1,0.3,0.5,0.7,0.8\rangle\}$.

In this case $T_N = \{0_{PN}, 1_{PN}, \daleth_{n_1}, \daleth_{n_2}\}$ is a pentapartitioned fermatean neutrosophic topology. Here $0_{PN}, 1_{PN}, \daleth_{n_1}, \daleth_{n_2}$ P-FNOS in (X, τ_N) and their complements $1_{PN}, 0_{PN}$,

$\daleth_{n_1}{}^c = \{\langle r_1, 0.6,0.2,0.5,0.3,0.2\rangle, \langle r_2, 0.7,0.2,0.6,0.6,0.3\rangle\}$,

$\daleth_{n_2}{}^c = \{\langle r_1, 0.7,0.2,0.1,0.2,0.2\rangle, \langle r_2, 0.8,0.7,0.5,0.3,0.1\rangle\}$ are P-FNCS in (X, τ_N).

Suppose that $\daleth = \{\langle r_1, 0.5,0.1,0.8,0.8,0.6\rangle, \langle r_2, 0.6,0.1,0.6,0.6,0.3\rangle\}$ be a P-FNTS on X. Then, $PFN\ Int(\daleth) = 0_{PN}$ and $PFN\ Cl(\daleth) = \{\langle r_1, 0.6,0.2,0.5,0.3,0.2\rangle, \langle r_2, 0.7,0.2,0.6,0.6,0.3\rangle\} = \daleth_{n_1}{}^c$. $PFN\ Int[PFN\ Cl(\daleth)] = 0_{PN}$, $PFN\ Int[PFN\ Cl(\daleth)] \subseteq \daleth$, $\daleth$ semi-closed set.

$PFN\ Cl[PFN\ Int(\daleth)] = \{\langle r_1, 0.6,0.2,0.5,0.3,0.2\rangle, \langle r_2, 0.7,0.2,0.6,0.6,0.3\rangle\} = \daleth_{n_1}{}^c$,

$\daleth \subseteq PFN\ Cl[PFN\ Int(\daleth)]$, $\daleth$ semi-open set. $PFN\ Int\big[PFN\ Cl[PFN\ Int(\daleth)]\big] = 0_{PN}$

$PFN\ Cl\big[PFN\ Int[PFN\ Cl(\daleth)]\big] = 0_{PN}, PFN\ Cl\big[PFN\ Int[PFN\ Cl(\daleth)]\big] \subseteq \daleth$, $\daleth$ is pentapartitioned fermatean neutrosophic α-closed set.

Now, the definition of neutrosophic generalized semi-open set will be given in the next definition.

In the next definitions, the definition of neutrosophic generalized semi-open(closed) set will be given with the method in [11] for the space we are working on.

Definition 2.7 A subset $\daleth$ of P-FNTS (X, T_N) is called pentapartitioned fermatean neutrosophic generalized-semi closed set [$PFNgs$-closed set] in X if $PFNs\ Cl(\daleth) \subseteq B$, whenever $\daleth \subseteq B$ and B is pentapartitioned fermatean neutrosophic open set.$\daleth^c$ is pentapartitioned fermatean neutrosophic generalized-open set in X [$PFNgs -$ open set] .

Now, we give some definition of using ψ-closed set in our spaces same as in [12].

Definition 2.8 A subset $\daleth$ of a P-FNTS (X, T_N) is said

i) a pentapartitioned fermatean neutrosophic semi- generalized closed set [$PFNsg -$ closed set] if $PFNsCl(\daleth) \subseteq B$ whenever $\daleth \subseteq B$ and B is semi-open in (X, T_N).

ii) a pentapartitioned fermatean neutrosophic ψ-closed set [$PFN\psi -$ closed set] if $PFNsCl(\daleth) \subseteq B$ whenever $\daleth \subseteq B$ and B is $PFNsg$-open in (X, T_N).

iii) The ψ-closure of a subset $\daleth$ of a P-FNTS (X, T_N) is the intersection of every $PFN\ \psi$-closed sets that contain $\daleth$ and is denoted by $PFN\psi Cl(\daleth)$.

iv) A pentapartitioned fermatean neutrosophic $\alpha\psi$-closed (PFN $\alpha\psi$-closed) set is given as if $PFN\psi\ cl(\text{Ꞇ}) \subseteq B$ whenever $\text{Ꞇ} \subseteq B$ and B is a PFNα-open set in (X, T_N). Its complement is said a pentapartitioned fermatean neutrosophic $\alpha\psi$-open (PFN $\alpha\psi$-open) set.

Example 2.7 Let $X = \{r_1, r_2\}$ non-empty set. Clearly (X, τ_N) be an P-FNTS, where

$\text{Ꞇ}_{n_1} = \{\langle r_1, 0.3,0.3,0.7,0.3,0.4\rangle, \langle r_2, 0.3,0.4,0.6,0.4,0.4\rangle : r_1, r_2 \in X\}$,

$\text{Ꞇ}_{n_2} = \{\langle r_1, 0.4,0.4,0.1,0.1,0.4\rangle, \langle r_2, 0.5,0.5,0.3,0.3,0.1\rangle : r_1, r_2 \in X\}$

$\tau_N = \{0_{PN}, 1_{PN}, \text{Ꞇ}_{n_1}, \text{Ꞇ}_{n_2}\}$.

Here $0_{PN}, 1_{PN}, \text{Ꞇ}_{n_1}, \text{Ꞇ}_{n_2}$ P-FNOS in (X, τ_N) and their complements $1_{PN}, 0_{PN}$,

${\text{Ꞇ}_{n_1}}^c = \{\langle r_1, 0.4,0.3,0.3,0.3,0.3\rangle, \langle r_2, 0.4,0.4,0.4,0.4,0.3\rangle\}$,

${\text{Ꞇ}_{n_2}}^c = \{\langle r_1, 0.4,0.1,0.9,0.4,0.4\rangle, \langle r_2, 0.1,0.3,0.7,0.5,0.5\rangle\}$ are P-FNCS in (X, τ_N).

Then the P-FNS $G = \{\langle r_1, 0.6,0.6,0.1,0.1,0.4\rangle, \langle r_2, 0.9,0.9,0.2,0.2,0.1\rangle : r_1, r_2 \in X\}$ is an pentapartitioned fermatean neutrosophic semi-open set in (X, τ_N).

$\text{Ꞇ} = \text{Ꞇ}_{\boldsymbol{n_1}}$, $PFN\ Cl(\text{Ꞇ}) = {\text{Ꞇ}_{n_1}}^c$, $PFN\ Int[PFN\ Cl(\text{Ꞇ})] = \text{Ꞇ}_{n_1}$.

$PFNsCl(\text{Ꞇ}) \subseteq G$ where $\text{Ꞇ} = \text{Ꞇ}_{\boldsymbol{n_1}} \subseteq G$ and $G \subseteq PFN\ Cl[PFN\ Int(G)] = 1_{PN}$.

So Ꞇ is pentapartitioned fermatean neutrosophic generalized-semi closed set.

Example 2.8 Let $X = \{r_1, r_2\}$ non-empty set. Clearly (X, τ_N) be an P-FNTS, where

$\text{Ꞇ}_{n_1} = \{\langle r_1, 0.3,0.3,0.6,0.6,0.5\rangle, \langle r_2, 0.4,0.4,0.5,0.6,0.7\rangle : r_1, r_2 \in X\}$,

$\text{Ꞇ}_{n_2} = \{\langle r_1, 0.4,0.4,0.5,0.5,0.5\rangle, \langle r_2, 0.5,0.5,0.5,0.6,0.7\rangle : r_1, r_2 \in X\}$,

and $\tau_N = \{0_{PN}, 1_{PN}, \text{Ꞇ}_{n_1}, \text{Ꞇ}_{n_2}\}$. Here

$G = \{\langle r_1, 0.5,0.5,0.5,0.4,0.4\rangle, \langle r_2, 0.7,0.6,0.5,0.5,0.5\rangle : r_1, r_2 \in X\}$,

${\text{Ꞇ}_{n_1}}^c = \{\langle r_1, 0.5,0.6,0.4,0.3,0.3\rangle, \langle r_2, 0.7,0.6,0.5,0.4,0.4\rangle : r_1, r_2 \in X\}$,

${\text{Ꞇ}_{n_2}}^c = \{\langle r_1, 0.5,0.5,0.5,0.4,0.4\rangle, \langle r_2, 0.7,0.6,0.5,0.5,0.5\rangle : r_1, r_2 \in X\}$,

$B = \{\langle r_1, 0.4,0.4,0.5,0.5,0.4\rangle, \langle r_2, 0.5,0.5,0.5,0.6,0.7\rangle : r_1, r_2 \in X\}\}$,

$G^c = \{\langle r_1, 0.4,0.4,0.5,0.5,0.5\rangle, \langle r_2, 0.5,0.5,0.5,0.6,0.7\rangle : r_1, r_2 \in X\}$.

$PFN\ Int(B) = \text{Ꞇ}_{n_2}$, $PFN\ Cl(\text{Ꞇ}_{n_2}) = {\text{Ꞇ}_{n_2}}^c$, $B \subseteq {\text{Ꞇ}_{n_2}}^c$ So B a pentapartitioned fermatean neutrosophic semi-open set.

$G^c \subseteq B,\ PFN\ Cl(G^c) = ꓔ_{n_2}{}^c,\ PFN\ Int\left(ꓔ_{n_2}{}^c\right) = ꓔ_{n_2}, ꓔ_{n_2} \subseteq G^c$ so G^c a pentapartitioned fermatean neutrosophic semi-closed set and pentapartitioned fermatean neutrosophic semi-closure.

$ꓔ_{n_2} = PFNs\ Cl(G^c) \subseteq B,\ \ G^c$ is pentapartitioned fermatean neutrosophic generalized-semi closed set and G is pentapartitioned fermatean neutrosophic generalized-semi open set.

Here $ꓔ = G^c$ and $ꓔ \subseteq G, PFNsCl(G^c) = ꓔ_{n_2}, ꓔ_{n_2} \subseteq G$ then G^c is $PFN\psi$-closed set,

$PFN\psi Cl(ꓔ) = \ G^c.\ ꓔ = \{\langle r_1, 0.4{,}0.4{,}0.5{,}0.5{,}0.5\rangle, \langle r_2, 0.5{,}0.5{,}0.5{,}0.6{,}0.7\rangle : r_1, r_2 \in X\} = G^c$

$PFN\ Int(ꓔ) = ꓔ_{n_2}, PFN\ Cl\left(ꓔ_{n_2}\right) = ꓔ_{n_2}{}^c,\ PFN\ Int\left(ꓔ_{n_2}{}^c\right) = ꓔ_{n_2}, ꓔ \subseteq ꓔ_{n_2}$ so G is PFNα-open set. $G^c \subseteq ꓔ$, $ꓔ_{n_2} = PFN\psi Cl(G^c) \subseteq ꓔ$ so $ꓔ$ is PFN $\alpha\psi$-closed in (X, τ_N).

Definition 2.9 Let $ꓔ$ be an PFNS in PFNTS (X, T_N). Then

PFN$\alpha\psi$ $Int(ꓔ) = \cup \{B : B\ is\ a\ PFN\alpha\psi\ OS\ in\ X\ and\ B \subseteq ꓔ\}$ is said to be a pentapartitioned fermatean neutrosophic $\alpha\psi$-interior of $ꓔ$;

PFN$\alpha\psi$ $Cl(ꓔ) = \cap \{B : B\ is\ a\ PFN\alpha\psi\ CS\ in\ X\ and\ B \supseteq ꓔ\}$ is said to be a pentapartitioned fermatean neutrosophic $\alpha\psi$-closure of $ꓔ$.

The family of all pentapartitioned fermatean neutrosophic $\alpha\psi$-open ($\alpha\psi$-closed) in a pentapartitioned fermatean neutrosophic topological space (X, T_N) is denoted by P$FN\alpha\psi - OS$ (P$FN\alpha\psi - CS$).

Example 2.9 $ꓔ$, PFN $\alpha\psi$-closed set in the previous example. The intersection of closures covering this set is $ꓔ_{n_2}{}^c$.

Proposition 2.1 Let (X, τ_N) be a PFNTS. Then the following situations are true:

i) Every pentapartitioned fermatean neutrosophic α-open (α-closed) set is pentapartitioned fermatean neutrosophic $\alpha\psi$ open ($\alpha\psi$ closed) set.

ii) Every pentapartitioned fermatean neutrosophic open (closed) set is pentapartitioned fermatean neutrosophic $\alpha\psi$-open ($\alpha\psi$-closed) set.

Proof. The proof is obtained using the Definition 2.9.

Remark 2.1 As can be seen from the examples 2.8 and 2.9, a pentapartitioned fermatean neutrosophic $\alpha\psi$-closed set does not have to be pentapartitioned fermatean neutrosophic closed set in X.

In next propositions, we give some main properties of union (intersection) of P$FN\alpha\psi - OS$, after that we define regular $\alpha\psi$-open as in [13] in PFNTS.

Proposition 2.2 Let (X, τ_N) be a PFNTS. Then the union (intersection) of any family of P$FN\alpha\psi - OS$ (P$FN\alpha\psi - CS$) is in P$FN\alpha\psi - OS$ (resp. $PFN\alpha\psi - CS$).

Proposition 2.3 Let (X, τ_N) be a PFNTS. Let Ƭ be an P$FN\alpha - OS$ and B be an P$FN\alpha\psi - OS$. Then Ƭ $\cap B$ is an P$FN\alpha\psi - OS$.

Definition 2.10 Let (X, τ_N) be a PFNTS and Ƭ be a PFNS of X. Then Ƭ is called to be pentapartitioned fermatean neutrosophic regular $\alpha\psi$-open set if Ƭ $= PFN\alpha\psi\, Int[PFN\alpha\psi\, Cl($Ƭ$)]$. The complement of pentapartitioned fermatean neutrosophic $\alpha\psi$-regular open set (PFN $\alpha\psi$-ROS) is called pentapartitioned fermatean neutrosophic $\alpha\psi$-regular closed set (PFN $\alpha\psi$-RCS) in X.

Lemma 2.1 Assume that Ƭ is a pentapartitioned fermatean neutrosophic subset of a PFNTS (X, τ_N). Then the following relations hold.

i) $X/PFN\alpha\psi\, Int($Ƭ$) = PFN\alpha\psi\, Cl(X/$Ƭ$)$. ii) $X/PFN\alpha\psi\, Cl($Ƭ$) = PFN\alpha\psi\, Int(X/$Ƭ$)$.

Now, some main definitions in [12], [19] will be transferred to the space we are working on.

Definition 2.11 A function $f: (X, \tau_N) \to (Y, \sigma_N)$ is called to be pentapartitioned fermatean neutrosophic $\alpha\psi$-continuous function if the inverse $f^{-1}(B)$ of each PFN open set B in Y is $\alpha\psi$-open set in X.

Definition 2.12 A function $f: (X, \tau_N) \to (Y, \sigma_N)$ is said to be a PFN $\alpha\psi$-irresolute function if $f^{-1}(B)$ is a PFN $\alpha\psi$-open set in X, for every PFN $\alpha\psi$-open set B in Y.

Lemma 2.2 A function $f: (X, \tau_N) \to (Y, \sigma_N)$ is a PFN $\alpha\psi$-irresolute function if and only if $f^{-1}(B)$ is a PFN $\alpha\psi$-closed set in X, for every PFN $\alpha\psi$-closed set B in Y.

Definition 2.13 A function $f: (X, \tau_N) \to (Y, \sigma_N)$ is called to be a PFN $\alpha\psi$-closed function if image set $f($Ƭ$)$ is a PFN $\alpha\psi$-closed set in Y, for every PFN closed set Ƭ in X.

3. PFN $\alpha\psi$ −Normal Spaces

In this section, inspired by the Neutrosophic delta beta normal space given in [19] we constructed pentapartitioned fermatean neutrosophic $\alpha\psi$−normal space and study its characterizations.

Definition 3.1 A PFNTS (X, τ_N) is called to be PFN $\alpha\psi$−normal if for any two disjoint PFN $\alpha\psi$-closed sets Ƭ and Ύ, there exist disjoint PFN $\alpha\psi$-open sets $\mathcal{P}$ and Ԛ where Ƭ $\subseteq \mathcal{P}$ and Ύ $\subseteq$ Ԛ.

Theorem 3.1 Let (X, τ_N) be a PFNTS. Then the following situations are equivalent:

a) (X, τ_N) is PFN $\alpha\psi$-normal.

b) For all PFN $\alpha\psi$-closed set Ͳ and all PFN $\alpha\psi$-open set Ƥ containing Ͳ, there exists a PFN $\alpha\psi$-open set Ɋ containing Ͳ such that Ƥ $\supseteq PFN\ \alpha\psi\ Cl($Ɋ$)$.

c) For all pair of disjoint PFN $\alpha\psi$-closed set Ͳ and ʹϒ, there exists a PFN $\alpha\psi$-open set Ƥ containing Ͳ such that ʹϒ $\cap\ PFN\ \alpha\psi\ Cl($Ƥ$) = 0_{PN}$.

d) For all pair of disjoint PFN $\alpha\psi$-closed set Ͳ and ʹϒ, there exists PFN $\alpha\psi$-open set Ƥ and Ɋ containing Ͳ and ʹϒ respectively where $PFN\ \alpha\psi\ Cl($Ƥ$) \cap PFN\ \alpha\psi\ Cl($Ɋ$) = 0_PN$.

Theorem 3. 2 A PFNTS (X, τ_N) is PFN $\alpha\psi-$normal $\Leftrightarrow$ for all PFN $\alpha\psi-$closed set K and a PFN $\alpha\psi-$open set Z containing K, there exists a PFN $\alpha\psi-$open set Ƥ where $K \subseteq$ Ƥ $\subseteq PFN\alpha\psi\ Cl($Ƥ$) \subseteq Z$.

Theorem 3.3 Let (X, τ_N) be a PFNTS. Then the following statements are equivalent:

a) X is PFN $\alpha\psi-$normal.

b) For any two PFN $\alpha\psi-$open sets Ƥ and Ɋ whose union is 1_{PN}, there exist PFN $\alpha\psi-$closed subsets Ͳ of Ƥ and ʹϒ of Ɋ such that Ͳ $\cup$ ʹϒ $= 1_{PN}$.

Theorem 3.4 Let $f: (X, \tau_N) \to (Y, \sigma_N)$ be a function.

a) If X is PFN $\alpha\psi-$normal and f is injective, PFN $\alpha\psi-$irresolute, PFN $\alpha\psi-$open, then Y is PFN $\alpha\psi-$normal.

b) If Y is PFN $\alpha\psi-$normal and f is PFN $\alpha\psi-$irresolute, PFN $\alpha\psi-$closed, then X is PFN $\alpha\psi-$normal.

Proof a) Suppose X is PFN $\alpha\psi-$normal. Let Ͳ and ʹϒ be disjoint PFN $\alpha\psi-$closed sets in . Using f is PFN $\alpha\psi-$irresolute, $f^{-1}($Ͳ$)$ and $f^{-1}($ʹϒ$)$ are disjoint PFN $\alpha\psi-$closed sets in . Then using X is PFN $\alpha\psi-$normal, there exist disjoint PFN $\alpha\psi-$open sets Ƥ and Ɋ in X where $f^{-1}($Ͳ$) \subseteq$ Ƥ and $f^{-1}($ʹϒ$) \subseteq$ Ɋ. Now $f^{-1}($Ͳ$) \subseteq$ Ƥ implies that Ͳ $\subseteq f($Ƥ$)$ and $f^{-1}($ʹϒ$) \subseteq$ Ɋ implies that ʹϒ $\subseteq f($Ɋ$)$. Since f is a PFN $\alpha\psi-$open map, $f($Ƥ$)$ and $f($Ɋ$)$ are PFN $\alpha\psi-$open in Y. Also Ƥ $\cap$ Ɋ $= 0_N$ implies that $f($Ƥ $\cap$ Ɋ$) = 0_{PN}$ and f is injective, then $f($Ƥ$) \cap f($Ɋ$) = 0_{PN}$. So $f($Ƥ$)$ and $f($Ɋ$)$ are disjoint PFN $\alpha\psi-$open sets in Y containing Ͳ and ʹϒ respectively. Hence, Y is PFN $\alpha\psi-$normal.

Theorem 3.5 Let $f: (X, \tau_N) \to (Y, \sigma_N)$ be a PFN continuous, PFN $\alpha\psi-$open bijection of a PFN normal space X onto a PFN space Y and if all PFN $\alpha\psi-$closed set in Y is PFN closed, so Y is PFN $\alpha\psi-$normal.

Now, we give strongly PFN $\alpha\psi-$normal space then we introduce its properties in PFNTS.

Definition 3.2 A PFNTS (X, τ_N) is said to be strongly PFN $\alpha\psi$−normal if for all pair of disjoint PFN closed sets Ƭ and Ƴ, there are disjoint PFN $\alpha\psi$−open sets Ƥ and Ɋ containing Ƭ and Ƴ respectively.

Theorem 3.6 Every PFN $\alpha\psi$−normal space is strongly PFN $\alpha\psi$−normal.

Proof Let X is PFN $\alpha\psi$−normal, also let Ƭ and Ƴ be disjoint PFN closed sets. So Ƭ and Ƴ are disjoint PFN $\alpha\psi$−closed sets. Since X is PFN $\alpha\psi$−normal, there exist disjoint PFN $\alpha\psi$−open sets Ƥ and Ɋ containing Ƭ and Ƴ respectively. This implies that X is strongly PFN $\alpha\psi$−normal.

Theorem 3.7 Let (X, τ_N) be a PFNTS. Then the following are equivalent:

a) X is strongly PFN $\alpha\psi$−normal.

b) For all PFN closed set E and each PFN open set Ƥ containing E, there exists a PFN $\alpha\psi$−open set Ɋ containing E such that $PFN\alpha\psi\ Cl($Ɋ$) \subseteq$ Ƥ.

c) For each pair of disjoint PFN closed sets Ƭ and Ƴ, there exists a PFN $\alpha\psi$−open set Ƥ containing Ƭ such that $PFN\alpha\psi\ Cl($Ƥ$) \cap$ Ƴ $= 0_{PN}$.

Theorem 3.8 Let (X, τ_N) be a PFNTS. Then the following are equivalent:

a) X is strongly PFN $\alpha\psi$-normal.

b) For any two PFN open sets Ƥ and Ɋ whose union is 1_N, there exist PFN $\alpha\psi$-closed subsets Ƭ of Ƥ and Ƴ of Ɋ where Ƭ $\cup$ Ƴ $= 1_{PN}$.

Proof a)⇒b) : Let Ƥ and Ɋ be two PFN open sets in a strongly PFN $\alpha\psi$-normal space X where Ƥ $\cup$ Ɋ $= 1_{PN}$. So Ƥ^c and Ɋ^c are disjoint PFN closed sets. Using X is strongly PFN $\alpha\psi$-normal, then there exist disjoint PFN $\alpha\psi$-open sets ß and Ê where Ƥ$^c \subseteq$ ß and Ɋ$^c \subseteq$ Ê. Let Ƭ $=$ ß^c and Ƴ $=$ Ê^c. Hence Ƭ and Ƴ are PFN $\alpha\psi$-closed subsets of Ƥ and Ɋ respectively where Ƭ $\cup$ Ƴ $= 1_{PN}$.

b)⇒a): Let Ƭ and Ƴ be disjoint PFN closed sets in X. Then Ƭ^c and Ƴ^c are PFN open sets such that Ƭ$^c \cup$ Ƴ$^c = 1_{PN}$. By (b), there exists PFN $\alpha\psi$-closed sets M and N such that $M \subseteq$ Ƭ^c, $N \subseteq$ Ƴ^c and $\cup N = 1_N$. Then M^c and N^c are disjoint PFN $\alpha\psi$-open sets containing Ƭ and Ƴ respectively.

Theorem 3.9 Let $f: (X, \tau_N) \to (Y, \sigma_N)$ be a function.

a) If X is strongly PFN $\alpha\psi$-normal and f is injective, PFN continuous, PFN $\alpha\psi$-open, then Y is strongly PFN $\alpha\psi$-normal.

b) If Y is strongly PFN $\alpha\psi$-normal and f is PFN $\alpha\psi$-irresolute, PFN $\alpha\psi$-closed map, then X is strongly PFN $\alpha\psi$-normal.

Proof a) Suppose X is strongly PFN $\alpha\psi$-normal. Let Ͳ and Ύ be disjoint PFN closed sets in. Since f is PFN continuous, $f^{-1}(Ͳ)$ and $f^{-1}(Ύ)$ are PFN closed in. Since X is strongly PFN $\alpha\psi$-normal, there exist disjoint PFN $\alpha\psi$-open sets $\mathcal{P}$ and Ɋ in X such that $f^{-1}(Ͳ) \subseteq \mathcal{P}$ and $f^{-1}(Ύ) \subseteq Ɋ$. Now $f^{-1}(Ͳ) \subseteq \mathcal{P}$ implies that $Ͳ \subseteq f(\mathcal{P})$ and $f^{-1}(Ύ) \subseteq Ɋ$ implies that $Ύ \subseteq f(Ɋ)$. Since f is a PFN $\alpha\psi$-open map, $f(\mathcal{P})$ and $f(Ɋ)$ are PFN $\alpha\psi$-open sets in Y. Also $\mathcal{P} \cap Ɋ = 0_N$ implies that $f(\mathcal{P} \cap Ɋ) = 0_{PN}$ and f is injective, then $f(\mathcal{P}) \cap f(Ɋ) = 0_{PN}$. Thus $f(\mathcal{P})$ and $f(Ɋ)$ are disjoint PFN $\alpha\psi$-open sets in Y containing Ͳ and Ύ respectively. Thus, Y is strongly PFN $\alpha\psi$-normal

4. RESULTS

In this article, some important properties of neutrosophic $\alpha\psi$-topological space are examined. Neutrosophic $\alpha\psi$-normal space using strong neutrosophic $\alpha\psi$-normal space definitions, neutrosophic $\alpha\psi$-closed and neutrosophic $\alpha\psi$-normal sets is given. In addition, important theorems that determine the relationships between the given concepts have been proven.

REFERENCES

[1] Chang, C. L. (1968). Fuzzy topological spaces. Journal of mathematical Analysis and Applications, 24(1), 182-190.
[2] Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87-96.
Çoker, D. (1997). An introduction to intuitionistic fuzzy topological spaces. Fuzzy sets and systems, 88(1), 81-89.
[3] Lupiáñez, F. G. (2008). On neutrosophic topology. Kybernetes. 37(6), 797-800.
Salama, A. A., & Alblowi, S. A. (2012). Neutrosophic set and neutrosophic topological spaces. IOSR Journal of Mathematics, (IOSR-JM), 3(4).
[4] Smarandache, F. (2002). Neutrosophy and neutrosophic logic, first international conference on neutrosophy, neutrosophic logic, set, probability, and statistics, University of New Mexico, Gallup, NM 87301, USA.
[5] Salama, A., & AL-Blowi, S. (2012). Generalized neutrosophic set and generalized neutrosophic topological spaces. Computer Science and Engineering, 2(7), 129–132.
[6] Lupiáñez, F. G. (2008). On neutrosophic topology. The International Journal of Systems and Cybernetics, 37(6), 797–800.
[7] Karatas, S., & Kuru, C. (2016). Neutrosophic topology. Neutrosophic sets and systems, 13(1), 90-95.
[8] Arokiarani, I., Dhavaseelan, R., Jafari, S., & Parimala, M. (2017). On Some New Notions and Functions in Neutrosophic Topological Spaces. Neutrosophic Sets and Systems, 16(1).
[9] Shanthi, V. K., Chandrasekar, S., & Begam, K. S. (2018). Neutrosophic generalized semi closed sets in neutrosophic topological spaces. Infinite Study.
[10] Parimala, M., Smarandache, F., Jafari, S., & Udhayakumar, R. (2018). On Neutrosophic $\alpha\psi-$ Closed Sets. Information. 9(5), 103. https://doi.org/10.3390/info9050103
[11] Vadivel, A., Seenivasan, M., & Sundar, C. J. (2021). An introduction to $\delta-$open sets in a neutrosophic topological spaces. In Journal of Physics: Conference Series 1724(1), 012011. IOP Publishing.
[12] Smarandache, F. (1998). Neutrosophy: neutrosophic probability, set, and logic: analytic synthesis & synthetic analysis.
[13] Mallick, R., & Pramanik, S. (2020). Pentapartitioned neutrosophic set and its properties (Vol. 36). Infinite Study.
[14] Sweety, C.A.C, & Jansi, R. (2021). Fermatean Neutrosophic Sets. International Journal of Advanced Research in Computer and Communication Engineering, 10(6), 24-27.
[15] Gonul Bilgin, N., Pamučar, D., & Riaz, M. (2022). Fermatean Neutrosophic Topological Spaces and an Application of Neutrosophic Kano Method. Symmetry, 14(11), 2442.

[16] Gonul Bilgin N., & Bozma, G. (2022). Pentapartitioned Rough Fermatean Neutrosophic Normed Spaces, 3. International Hasankeyf Scientific Research and Innovation Congress 17-18 December 2022 Batman, 196-210.
[17] Latif, R. M. (2022). Neutrosophic Delta Beta Normal Topological Space. In International Conference on Mathematical Sciences and Statistics 2022 (ICMSS 2022), 47-56. Atlantis Press.
[18] Ray, G. C., & Dey, S. (2021). Neutrosophic point and its neighbourhood structure. Neutrosophic Sets and Systems, 43, 156-168.
[19] Gonul Bilgin N., & Bozma, G. (2022), Dörtlü Nötrosofik Topolojik Uzaylarda Genelleştirilmiş Regüler* Nötrosofik Kapalı Kümeler. Proceeding 2. International Anatolian Scientific Research Congress, 259-268.
[20] Gonul Bilgin N. (2022). Rough Statistical Convergence in Neutrosophic Normed Spaces. Euroasia Journal of Mathematics, Engineering, Natural & Medical Sciences, 9(21), 47-55.
[21] Riaz, M., Smarandache, F., Karaaslan, F., Hashmi, M. R., & Nawaz, I. (2020). Neutrosophic soft rough topology and its applications to multi-criteria decision-making. Infinite Study.