NEUTROSOPHIC g^*-CLOSED SETS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract

A neutrosophic set is a mathematical approach that helps with challenges involving data that is indeterminate, imprecise, or inconsistent. The goal of this manuscript is to present the notion of neutrosophic g^*-closed sets and neutrosophic g^*-open sets. In this situation, we prove various neutrosophic generalized theorems. The findings support previous methodologies in the literature and are backed up by various examples and an application.

Keywords: Neutrosophic $g*$ continuity, Neutrosophic Set, Neutrosophic $g*$ closed sets ,Neutrosophic Topology, Neutrosophic $g*$ continuity mapping, Generalized Neutrosophic Set, Neutrosophic continuity mapping.

1 Introduction and Preliminaries

Generalization topological spaces is a classical subject which is a type of classical topological spaces. The concept of fuzzy sets was introduced by Zadeh in his classical paper.[3] In 1968, Chang[19] used fuzzy sets to introduce the notion of fuzzy topological spaces. Çoker[2,18] defined and introduced the intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets Atanassov's[22] (Ifs for short) on a universe X. Neutrosophic system was defined by Smarandache In various recent papers, has laid the foundation for a whole family of new mathematical theories generalizing both their fuzzy and classical counterparts, and have wide range of real applications for the fields of Medicine, Information Systems,decision making, Applied Mathematic, Computer Science. F. Smarandache and A. Al Shumrani obtained the concept of neutrosophic topology on the non-general and standard interval.[20,21] Several authors was extended this principle with many applications (see[10–14,24–26]). In this paper we study and introduce about generalized neutrosophic sets. g^*-closed sets in neutrosophic topological space. We also discussed with details about properties and relationships with other classes of early defined forms. We also introduced application of neutrosophic g^*-closed sets namely neutrosophic $T^*_{\frac{1}{2}}$ space and ${}^*T_{\frac{1}{2}}$ space. Several interesting characterizations and properties are also discussed.

For non-empty fixed set $\mathcal{R}$ and each element $r \in \mathcal{R}$ to the set $\tilde{R}$. A neutrosophic set[16] (NS for short) $\tilde{R}$ is an object having the form $\tilde{R} = \{\langle r, \mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\rangle : r \in \mathcal{R}\}$, where
$\mu_{\tilde{R}}(k)$ represent the degree of member ship function,
$\sigma_{\tilde{R}}(k)$ represent the degree of indeterminacy,
and $\gamma_{\tilde{R}}(k)$ represent the degree of non-membership.
Neutrosophic sets in $\mathcal{S}$ will be denoted by $\tilde{R}, \mathcal{W}, B, G$, etc., and although subsets of $\mathcal{R}$ will be denoted by $\tilde{R}, \tilde{B}, T$, etc.

A neutrosophic set $\tilde{R} = \{\langle k, \mu_{\tilde{R}}(k), \sigma_{\tilde{R}}(k), \gamma_{\tilde{R}}(k)\rangle : k \in \mathcal{R}\}$ can be identified to an ordered triple $\langle \mu_{\tilde{R}}(k), \sigma_{\tilde{R}}(k), \gamma_{\tilde{R}}(k)\rangle$ in $]0^-, 1^+[$ on $\mathcal{RR}$.

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2 Historical Background

Definition 2.1. [16] Let $\mathcal{Z}$ be a non-empty set. A neutrosophic set (NS for short) $\tilde{R}$ is an object having the form $\tilde{R} = \{\langle r, \mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$, where $\gamma_{\tilde{R}}(r)$, $\sigma_{\tilde{R}}(r), \mu_{\tilde{R}}(r)$, and the degree of non-membership (namely $\gamma_{\tilde{R}}(r)$), the degree of indeterminacy (namely $\sigma_{\tilde{R}}(r)$), and the degree of membership function (namely $\mu_{\tilde{R}}(r)$), of each element $r \in \mathcal{Z}$ to the set $\tilde{R}$.

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the Neutrosophic sets (NSs) 0_N and 1_N[16] in $\mathcal{Z}$ are introduced as follows:

$1 - 0_N$ can be defined as four types:

1. $0_N = \{\langle r, 0, 0, 1\rangle : r \in \mathcal{Z}\}$,
2. $0_N = \{\langle r, 0, 1, 1\rangle : r \in \mathcal{Z}\}$,
3. $0_N = \{\langle r, 0, 1, 0\rangle : r \in \mathcal{Z}\}$,
4. $0_N = \{\langle r, 0, 0, 0\rangle : r \in \mathcal{Z}\}$.

2- 1_N can be defined as four types:

1. $1_N = \{\langle r, 1, 0, 0\rangle : r \in \mathcal{Z}\}$,
2. $1_N = \{\langle r, 1, 0, 1\rangle : r \in \mathcal{Z}\}$,
3. $1_N = \{\langle r, 1, 1, 0\rangle : r \in \mathcal{Z}\}$,
4. $1_N = \{\langle r, 1, 1, 1\rangle : r \in \mathcal{Z}\}$.

Definition 2.2. [16] Let X be a non-empty set, and $GNSS\ H$ and K in the form $H = \{r, \mu_H(r), \sigma_H(r), \gamma_H(r)\}$, $K = \{r, \mu_K(r), \sigma_K(r), \gamma_K(r)\}$, then we may consider two possible definitions for subsets $(H \subseteq K)$
$(H \subseteq K)$ may be defined as

1. Type 1: $H \subseteq K \Leftrightarrow \mu_H(r) \leqslant \mu_K(r), \sigma_H(r) \leqslant \sigma_K(r),$ and $\gamma_H(r) \geqslant \gamma_K(r)$ or
2. Type 1: $H \subseteq K \Leftrightarrow \mu_H(r) \leqslant \mu_K(r), \sigma_H(r) \geqslant \sigma_K(r),$ and $\gamma_H(r) \geqslant \gamma_K(r)$.

Definition 2.3. [16] Let $\{A_j : j \in J\}$ be a arbitrary family of NSS in $\mathcal{Z}$, then

1. $\cap A_j$ may be defined as two types:
   -Type 1: $\cap A_j = \langle r, \underset{j\in J}{\wedge}\mu_{Aj}(r), \underset{j\in J}{\wedge}\sigma_{Aj}(r), \underset{j\in J}{\vee}\gamma_{Aj}(r)\rangle$
   -Type 2: $\cap A_j = \langle r, \underset{j\in J}{\wedge}\mu_{Aj}(r), \underset{j\in J}{\vee}\sigma_{Aj}(r), \underset{j\in J}{\vee}\gamma_{Aj}(r)\rangle$.
2. $\cup A_j$ may be defined as two types:
   -Type 1: $\cup A_j = \langle r, \underset{j\in J}{\vee}\mu_{Aj}(r), \underset{j\in J}{\vee}\sigma_{Aj}(r), \underset{j\in J}{\wedge}\gamma_{Aj}(r)\rangle$
   -Type 2: $\cup A_j = \langle r, \underset{j\in J}{\vee}\mu_{Aj}(r), \underset{j\in J}{\wedge}\sigma_{Aj}(r), \underset{j\in J}{\wedge}\gamma_{Aj}(r)\rangle$

Definition 2.4. Let $\tilde{R} = \langle \mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\rangle$ be an NS on $\mathcal{Z}$.[2] The complement of the set $\tilde{R}(C(\tilde{R})$, for short) may be defined as follows:

1. $C(\tilde{R}) = \{\langle r, 1 - \mu_{\tilde{R}}(r), 1 - \gamma_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$,
2. $C(\tilde{R}) = \{\langle r, \gamma_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \mu_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$,
3. $C(\tilde{R}) = \{\langle r, \gamma_{\tilde{R}}(r), 1 - \sigma_{\tilde{R}}(r), \mu_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$.

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Definition 2.5. [1] A neutrosophic topology (NT for short) and a non empty set $\mathcal{Z}$ is a family $\mathcal{T}$ of neutrosophic subsets of $\mathcal{Z}$ satisfying the following axioms

1. $0_N, 1_N \in \mathcal{T}$
2. $H_1 \cap H_2 \in \mathcal{T}$ for any $H_1, H_2 \in \mathcal{T}$
3. $\cup H_i \in \mathcal{T}, \forall \{H_i | j \in J\} \subseteq \mathcal{T}$.

The pair $(\mathcal{Z}, \mathcal{T})$ is called a neutrosophic topological space (NTS for short).

Definition 2.6. [16] Let $\tilde{R}$ be an NS and $(\mathcal{Z}, \mathcal{T})$ an NT where $\tilde{R} = \{r, \mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\}$. Then,

1. $NCL(\tilde{R}) = \cap\{H : H$ is an NCS in $\mathcal{Z}$ and $\tilde{R} \subseteq H\}$
2. $NInt(\tilde{R}) = \cup\{W : W$ is an NOS in $\mathcal{Z}$ and $W \subseteq \tilde{R}\}$

It can be also shown that $NCl(\tilde{R})$ is an NCS and $NInt(\tilde{R})$ is an NOS in $\mathcal{Z}$. We have

1. $\tilde{R}$ is in $\mathcal{Z}$ iff $NCl(\tilde{R})$.
2. $\tilde{R}$ is an NCS in $\mathcal{Z}$ iff $NInt(\tilde{R}) = \tilde{R}$.

Definition 2.7. [16] Let $\tilde{R} = \{\mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\}$ be a neutrosophic open sets and $B = \{\mu_B(r), \sigma_B(r), \gamma_B(r)\}$ a neutrosophic set on a neutrosophic topological space $(\mathcal{Z}, \mathcal{T})$. Then

1. $\tilde{R}$ is called neutrosophic regular open iff $\tilde{R} = NInt(NCl(\tilde{R}))$.
2. The complement of neutrosophic regular open is neutrosophic regular closed.

Definition 2.8. [9] Let $\tilde{R}$ be an NS and $(\mathcal{Z}, \mathcal{T})$ an NT. Then

1. Neutrosophic regular-open set $(NROS)$ if $\tilde{R} = NInt(NCl(\tilde{R}))$,
2. Neutrosophic regular-closed set $(NROS)$ if $\tilde{R} = NCl(NInt(\tilde{R}))$,
3. Neutrosophic semi-open set $(NSOS)$ if $\tilde{R} \subseteq NCl(NInt(\tilde{R}))$,
4. Neutrosophic semi-closed set $(NSCS)$ if $NInt(NCl(\tilde{R})) \subseteq \tilde{R}$,
5. Neutrosophic pre-open set $(NPOS)$ if $\tilde{R} \subseteq NInt(NCl(\tilde{R}))$,
6. Neutrosophic pre-closed set $(NPCS)$ if $NCl(NInt(\tilde{R})) \subseteq \tilde{R}$,
7. Neutrosophic α-open set $(N\alpha OS)$ if $\tilde{R} \subseteq NInt(NCl(NInt(\tilde{R})))$,
8. Neutrosophic α-closed set $(N\alpha CS)$ if $NCl(NInt(NCl(\tilde{R}))) \subseteq \tilde{R}$.

Definition 2.9. [7]

1. Neutrosophic β-closed set $(N\beta CS)$ if $NInt(NCl(NInt(\tilde{R}))) \subseteq \tilde{R}$,
2. Neutrosophic β-open set $(N\beta OS)$ if $\tilde{R} \subseteq NCl(NInt(NCl(\tilde{R})))$,

Definition 2.10. [17] A (NS) $\tilde{R}$ of an NTs $(\mathcal{Z}, \Gamma)$ is called generalized neutrosophic closed sets ($GNCs$ in short) if $NCl(\tilde{R}) \subseteq \tilde{B}$ wherever $\tilde{R} \subset \tilde{B}$ and $\tilde{B}$ is neutrosophic closed sets in $\mathcal{Z}$.

Definition 2.11. [5] A neutrosophic set $\tilde{R}$ of an NTS $(\mathcal{Z}, \Gamma)$ is neutrosophic generalized pre-closed set ($NGPCS$ in short) if $NPCl(P) \subseteq G$ whenever $P \subseteq G$ and G is an NOS in $\mathcal{Z}$.

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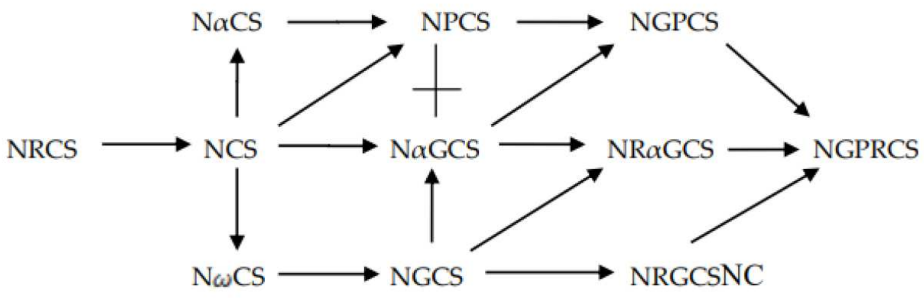
Definition 2.12. [8] Consider a neutrosophic set $\tilde{R}$ in NTS $(\mathcal{Z}, \Gamma)$. Then it is neutrosophic generalized β-closed set ($NG\beta CS$ in short) if $N\beta Cl(P) \subseteq G$ whenever $P \subseteq G$ and G is an NOS.

Definition 2.13. [6] A neutrosophic set $\tilde{R}$ of an NTS $(\mathcal{Z}, \Gamma)$ is neutrosophic generalized semi-closed set ($NGSCS$ in short) if $NScl(P) \subseteq G$ whenever $P \subseteq G$ and G is an NOS in $\mathcal{Z}$.

Definition 2.14. [4] Consider a neutrosophic set $\tilde{R}$ in NTS $(\mathcal{Z}, \Gamma)$. Then it is neutrosophic α-generalized closed set ($N\alpha GCS$ in short) if $N\alpha Cl(P) \subseteq G$ whenever $P \subseteq G$ and G is an NOS.

An NS $\tilde{R}$ is called a neutrosophic generalized open set,neutrosophicgeneralized pre-open set, neutrosophic α-generalized open set, neutrosophic generalized semi-open set, ineutrosophic β-generalized open set ($NGOS$, $NGPOS$, $N\alpha GOS$, $NGSOS$, $NG\beta OS$ respectively), if the complement of $\tilde{R}^c$ is a $NGCS$,$NGPCS$, $N\alpha GCS$, $NGSCS$, $NG\beta CS$ respectively. The following implications[23] are true:



In this diagram by A ⟶ B means A implies B but not conversely and A —|— B means A & B are independent.

Diagram 1

Definition 2.15. [9] Let $(\mathcal{Z}, \Gamma)$ be a neutrosophic topological space and $x_{r,t,s}$ be a neutrosophic point in $\mathcal{Z}$. A neutrosophic set S of $\mathcal{Z}$ is called a neutrosophic neighbourhood if there exists a neutrosophic open set $x_{r,t,s}$ in $\mathcal{Z}$ such that $p_\epsilon \in x_{r,t,s} \leqslant S$.

Definition 2.16. [27] A neutrosophic point $x_{r,t,s}$ is said to be q-coincident with a neutrosophic set G, denoted by $x_{r,t,s}qG$, if and only if there exists $x_{r,t,s} \nsubseteq G^c$. A neutrosophic set H is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with G, denoted by HqG if and only if $H \nsubseteq G^c$.

Definition 2.17. [27] A neutrosophic point (NP in short), $x_{r,t,s}$ is said to be a neurosophic cluster point of a neutrosophic set (Ns in short) G if HqG for each neutrosophic open q-neighborhood H of $x_{r,t,s}$. The union of all neutrosophic cluster points of G is called the neutrosophic closure of G and denoted by $NCl(G)$.

Definition 2.18. [15] A neutrosophic set (Ns in short), $(\mathcal{Z}, \Gamma)$ is said to be neutrosophic-$T_{\frac{1}{2}}$ ($NT_{\frac{1}{2}}$ in short) space if every $GNCs$ in $\mathcal{Z}$ is an NCs in $\mathcal{Z}$.

3 Neutrosophic g^* Closed Sets

In this section we introduce neutrosophic g^*-closed sets in neutrosophic topological spaces and discuss some of their properties.

Definition 3.1. A neutrosophic point (NP in short) $x_{r,t,s}$ is said to be an neutrosophic θ-cluster point of a neutrosophic set (Ns in short) $\tilde{R}$ if and only if $Cl(H)q\tilde{R}$ for each q-neighborhood H.The set of all neutrosophic θ-cluster points of $\tilde{R}$ is said to be neutrosophic θ-closure of $\tilde{R}$ and is denoted by $Cl_\theta(\tilde{R})$. An Ns $\tilde{R}$ will be called an neutrosophic θ-closed ($N\theta CS$ for short) if and only if $\tilde{R} = Cl_\theta(\tilde{R})$. The complement of θCS is said to be neutrosophic θ-open set ($N\theta OS$ for short).

Definition 3.2. A neutrosophic set H in $(\mathcal{Z}, \Gamma)$ is said to be an neutrosophic g^*-closed set (NG^*CS in short) if $Cl(H) \subseteq W$ whenever $H \subseteq W$ and W is an $NGOS$ in $(\mathcal{Z}, \Gamma)$. The family of all G^*CSs of an NTS $(\mathcal{Z}, \Gamma)$ is denoted by $NG^*C(\mathcal{Z})$.

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Example 3.3. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $F = \langle x, (\frac{3}{10}, \frac{4}{10}, \frac{4}{10}), (\frac{4}{10}, \frac{3}{10}, \frac{1}{10})\rangle$. Then the Neutrosophic set $H = \langle x, , (\frac{7}{10}, \frac{6}{10}, \frac{6}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10})\rangle$ is an NG^*CS in $(\mathcal{Z}, \Gamma)$.

Example 3.4. Let $X = \{u, v, w\}$ and $A = \langle(\frac{4}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{4}{10})\rangle$, $B = \langle(\frac{7}{10}, \frac{6}{10}, \frac{5}{10}), (\frac{3}{10}, \frac{4}{10}, \frac{5}{10}), (\frac{3}{10}, \frac{4}{10}, \frac{4}{10})\rangle$, and $C = \langle(\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10})\rangle$. Then $\tau = \{0_N, 1_N, A, B\}$ is a neutrosophic topology on X. Then the Neutrosophic set $C = \langle(\frac{6}{10}, \frac{4}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{6}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{6}{10}, \frac{4}{10})\rangle$ is an NG^*CS in $(\mathcal{Z}, \Gamma)$.

Example 3.5. $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $F = \langle x, (\frac{6}{10}, \frac{5}{10}, \frac{3}{10}), (\frac{8}{10}, \frac{5}{10}, \frac{2}{10})\rangle$. Then the Neutrosophic set $H = \langle x, , (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{7}{10})\rangle$ is not an NG^*CS in $(\mathcal{Z}, \Gamma)$.

Theorem 3.6. *Every NCS is a NG^*CS but not conversely.*

Proof. Let H be a NCS in $(\mathcal{Z}, \Gamma)$. Then $Cl(H) = H$. Let $H \subseteq W$ and W is an $NGOS$ in $(\mathcal{Z}, \Gamma)$. Therefore $Cl(H) = H \subseteq W$. Thus H is an NG^*CS in $(\mathcal{Z}, \Gamma)$. □

Example 3.7. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$, where $F = \langle x, (\frac{4}{10}, \frac{2}{10}, \frac{4}{10}), (\frac{6}{10}, \frac{7}{10}, \frac{5}{10})\rangle$. Then the Neutrosophic set $H = \langle x, , (\frac{6}{10}, \frac{7}{10}, \frac{6}{10}), (\frac{1}{10}, \frac{1}{10}, \frac{1}{10})\rangle$ is an NG^*CS in $(\mathcal{Z}, \Gamma)$ but not an NCS in Γ.

Theorem 3.8. *Every NG^*CS in neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is an $NGCS$ but not conversely.*

Proof. Let H be an NG^*CSCS in $(\mathcal{Z}, \Gamma)$. Let $H \subseteq W$ and W is a NOS in $(\mathcal{Z}, \Gamma)$. Since every NOS is $NGOS$ and since H is an NG^*CS in $\mathcal{Z}$. Therefore $Cl(H) \subseteq W$ whenever $H \subseteq W$, W is an NOS in $\mathcal{Z}$. Thus H is an $NGCS$ in $\mathcal{Z}$. □

Example 3.9. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $F = \langle x, (\frac{3}{10}, \frac{7}{10}, \frac{4}{10}), (\frac{6}{10}, \frac{3}{10}, \frac{5}{10})\rangle$. Then the NS $H = \langle x, , (\frac{3}{10}, \frac{8}{10}, \frac{6}{10}), (\frac{7}{10}, \frac{2}{10}, \frac{1}{10})\rangle$ is an $NGCS$ but not an NG^*CS in Γ.

Theorem 3.10. *Every $NG\theta CS$ in neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is an $N\alpha GCS$ but not conversely.*

Proof. It is obvious because every $N\theta CS$ is NCS and by Theorem 3.6 the proof is clear. □

Example 3.11. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F, H, F \cup H\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (0, 0, \frac{5}{10}), (1, 1, 0)\rangle$ and $H = \langle x, , (1, \frac{5}{10}, 0), (0, \frac{5}{10}, 1)\rangle$. Then the NS $F = \langle x, (1, 1, 0), (0, 0, \frac{5}{10})\rangle$ is an $NG\theta CS$ but not an $N\alpha GCS$ in Γ.

Theorem 3.12. *Every NG^*CS in neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is an $N\alpha GCS$ but not conversely.*

Proof. Let H be an NG^*CS in $(\mathcal{Z}, \Gamma)$. By Theorem 3.8 H is an $NGOS$ in $(\mathcal{Z}, \Gamma)$. Since $\alpha Cl(H) \subseteq Cl(H)$ and H is a $NGCS$ in $\mathcal{Z}$. Therefore $\alpha Cl(H) \subseteq Cl(H) \subseteq W$ whenever $H \subseteq W$, W is an NOS in $\mathcal{Z}$. Thus H is an $NG * CS$ in $\mathcal{Z}$. □

Example 3.13. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $F = \langle x, (\frac{3}{10}, \frac{4}{10}, \frac{4}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{5}{10})\rangle$. Then the NS $H = \langle x, , (\frac{1}{10}, \frac{4}{10}, \frac{6}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{2}{10})\rangle$ is an $NG * CS$ but not an $N\alpha GCS$ in Γ.

Theorem 3.14. *Every $NGRCS$ in neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is an NG^*CS but not conversely.*

Proof. Let H be an $NRCS$ in $(\mathcal{Z}, \Gamma)$. Then $H = Cl(Int(H))$. Let $H \subseteq W$, W is an $NGOS$ in $\mathcal{Z}$. Thus $Cl(H) \subseteq Cl(Int(A))$. This implies $Cl(H) \subseteq H \subset W$. Thus H is an NG^*CS in Γ. H is an $NG * CS$ in $\mathcal{Z}$. □

Example 3.15. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{3}{10}, \frac{7}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{3}{10}, \frac{4}{10})\rangle$. Then the NS $H = \langle x, (\frac{7}{10}, \frac{6}{10}, \frac{3}{10}), (\frac{3}{10}, \frac{4}{10}, \frac{3}{10})\rangle$ is an $NG * CS$ but not an $NGRCS$ in Γ.

Theorem 3.16. *Every NG^*CS in neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is an $NG\beta CS$ but not conversely.*

Proof. Let H be an NG^*CS in $(\mathcal{Z}, \Gamma)$. Let $H \subseteq W$, W is an NOS in $\mathcal{Z}$. Since every NOS is $NGOS$ and since H is an NG^*CS in $(\mathcal{Z}, \Gamma)$. By hypothesis $Cl(H) \subseteq W$. Which implies $Cl(Int(H)) \subseteq W$. Thus $Int(Cl(Int(H))) \subseteq W$. Therefore $\beta Cl(H) \subseteq W$. Thus H is an $NG\beta CS$ in $(\mathcal{Z}, \Gamma)$. □

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Example 3.17. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{5}{10}, \frac{6}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{4}{10}, \frac{3}{10})\rangle$. Then the NS $H = \langle x, (\frac{5}{10}, \frac{7}{10}, \frac{3}{10}), (\frac{5}{10}, \frac{3}{10}, \frac{2}{10})\rangle$ is an $NG * CS$ but not an $NG\beta CS$ in Γ.

Remark 3.18. An $NG * CS$ is independent from $N\alpha CS$, $NSGCS$, $NPCS$, and $NSCS$ as seen from the following example.

Example 3.19. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{3}{10}, \frac{4}{10}, \frac{4}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{3}{10})\rangle$. Then the NS $H = \langle x, (\frac{1}{10}, \frac{4}{10}, \frac{5}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{4}{10})\rangle$ is an $N\alpha CS$, $NPCS$, $NSCS$ and $NSCS$ but not an NG^*CS in $(\mathcal{Z}, \Gamma)$.

Example 3.20. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{4}{10}, \frac{3}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{7}{10}, \frac{3}{10})\rangle$. Then the NS $H = \langle x, (\frac{6}{10}, \frac{7}{10}, \frac{4}{10}), (\frac{1}{10}, \frac{1}{10}, \frac{4}{10})\rangle$ is anNG^*CS but neither $NSCS$ or $N\alpha CS$ in $(\mathcal{Z}, \Gamma)$.

Example 3.21. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (0, \frac{9}{10}, \frac{7}{10}), (\frac{5}{10}, \frac{1}{10}, \frac{3}{10})\rangle$. Then the NS $H = \langle x, (0, \frac{3}{10}, \frac{3}{10}), (\frac{7}{10}, \frac{7}{10}, \frac{4}{10})\rangle$ is an $NPCS$ but not NG^*CS in $(\mathcal{Z}, \Gamma)$.

Example 3.22. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, H_1, H_2\}$ is a N_Γ on $\mathcal{Z}$. where $H_1 = \langle x, (\frac{5}{10}, \frac{2}{10}, \frac{2}{10}), (\frac{5}{10}, \frac{7}{10}, \frac{5}{10})\rangle$ and $H_2 = \langle x, (\frac{8}{10}, \frac{8}{10}, \frac{3}{10}), (\frac{2}{10}, \frac{2}{10}, \frac{4}{10})\rangle$. Then the NS $H_2 = \langle x, (\frac{5}{10}, \frac{7}{10}, \frac{5}{10}), (\frac{5}{10}, \frac{3}{10}, \frac{4}{10})\rangle$ is an NG^*CS but not an $NPGCS$ in $(\mathcal{Z}, \Gamma)$.

Example 3.23. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, H_1, H_2\}$ is a N_Γ on $\mathcal{Z}$. where $H_1 = \langle x, (\frac{5}{10}, \frac{2}{10}, \frac{2}{10}), (\frac{5}{10}, \frac{7}{10}, \frac{5}{10})\rangle$ and $H_2 = \langle x, (\frac{8}{10}, \frac{8}{10}, \frac{8}{10}), (\frac{2}{10}, \frac{2}{10}, \frac{2}{10})\rangle$. Then the NS $H_2 = \langle x, (\frac{7}{10}, \frac{7}{10}, \frac{7}{10}), (\frac{3}{10}, \frac{3}{10}, \frac{3}{10})\rangle$ is an NG^*CS but not an $NSGCS$ in $(\mathcal{Z}, \Gamma)$.

Remark 3.24. Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then intersection of any two NG^*CSs is not an NG^*CS in general as seen in the following example.

Example 3.25. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $W = \langle x, (\frac{5}{10}, 0, \frac{1}{10}), (\frac{1}{10}, 1, \frac{9}{10})\rangle$. Then the NS $H_1 = \langle x, (\frac{2}{10}, 1, \frac{7}{10}), (\frac{7}{10}, 0, \frac{1}{10})\rangle$, $H_2 = \langle x, (\frac{6}{10}, 0, \frac{1}{10}), (\frac{3}{10}, 1, 1)\rangle$ are NG^*CS's in $(\mathcal{Z}, \Gamma)$, but $H_1 \cap H_2$ is not an NG^*CS in $\mathcal{Z}$.

Theorem 3.26. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then the union of any two NG^*CS is an NG^*CS.*

Proof. Let H and K be any two NG^*CSs in $(\mathcal{Z}, \Gamma)$ and let $H \cup K \subseteq W$, where W is a $NGOS$ in $\mathcal{Z}$. Therefore $H \subseteq W$ or $K \subseteq W$ or both subset of W. Since H and K are NG^*CS, $Cl(H) \subseteq W$ and $Cl(K) \subseteq W$. Therefore $Cl(H \cup K) \subseteq W$. Thus $H \cup K$ is an NG^*CS. □

Theorem 3.27. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then H is an NG^*CS in $(\mathcal{Z}, \Gamma)$ iff $\rceil(H_1 q H_2)$ implies $\rceil(Cl(H_1) q H_2)$ for every $NGCS$ H_2 of $\mathcal{Z}$.*

Proof. Necessity: Let H_2 be an $NGCS$ of $(\mathcal{Z}, \Gamma)$ such that $\rceil(H_1 q H_2)$. Then by Definition, $H_1 \subseteq H_2^c$ where H_2^c is an $NGOS$ in $\mathcal{Z}$. Then $Cl(H_1) \subseteq H_2^c$. Hence $\rceil(Cl(H_1) q H_2)$.
Sufficiency: Let H_2 is an $NGCS$ in $\mathcal{Z}$. Therefore $\rceil(H_1 q H_2)$, by hypothesis, $\rceil(Cl(H_1) q H_2)$. Then we have $Cl(H_1) \subseteq H_2^c$ whenever $H_1 \subseteq H_2^c$ and H_2^c is an $NGOS$ in $\mathcal{Z}$. Hence H_1 is an NG^*CS in $\mathcal{Z}$. □

Theorem 3.28. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. If H is an NG^*CS in $(\mathcal{Z}, \Gamma)$, such that $H_1 \subseteq H_2 \subseteq Cl(H_1)$. Then H_2 is also a NG^*CS $(\mathcal{Z}, \Gamma)$.*

Proof. Let W be a $NGOS$ in $(\mathcal{Z}, \Gamma)$ such that $H_2 \subseteq W$, Since $H_1 \subseteq H_2$, $H_1 \subseteq W$ and W be an $NGOS$. Since H_1 is an NG^*CS, $Cl(H_1) \subseteq W$. Therefore $H_2 \subseteq Cl(H_1)$, by hypothesis. $Cl(H_2) \subseteq Cl(Cl(H_1)) \subseteq W$. Then we have $Cl(H_2) \subseteq W$. Hence H_2 is an NG^*CS in $\mathcal{Z}$. □

Theorem 3.29. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then $NGO(\Gamma) = NGC(\Gamma)$ iff every NS in $(\mathcal{Z}, \Gamma)$ is an NG^*CS in $\mathcal{Z}$.*

Proof. Necessity: Suppose that $NGO(\Gamma) = NGC(\Gamma)$. Let $H_1 \subseteq W$ and W is an $NGOS$ in Γ. This implies $Cl(H_1) \subseteq Cl(W)$. Since W is an $NGOS$ in Γ. Since by hypothesis W is an $NGCS$ in Γ, $Cl(W) \subseteq W$. This implies $Cl(H_1) \subseteq W$. Therefore H_1 is an NG^*CS in Γ.
Sufficiency: Suppose that every NS in $(\mathcal{Z}, \Gamma)$ is an NG^*CS in Γ. Let $W \in NO(\Gamma)$, then $W \in NGO(\Gamma)$. Since $W \subseteq W$ and W is NOS in Γ, by hypothesis $Cl(W) \subseteq W$. That is $W \in NGC(\Gamma)$. Hence $NGO(\Gamma) \subseteq NGC(\Gamma)$. Let $H \in NGC(\Gamma)$ then H^c is an $NGOS$ in Γ. But $NGO(\Gamma) \subseteq NGC(\Gamma)$. Therefore $H^c \in NGC(\Gamma)$. That is $H \in NGO(\Gamma)$. Hence $NGC(\Gamma) \subseteq NGO(\Gamma)$. Thus $NGO(\Gamma) \subseteq NGC(\Gamma)$. □

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Theorem 3.30. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. If H is NG^*CS and $NGOS$ in $(\mathcal{Z}, \Gamma)$, then H_1 is an NCS in $\mathcal{Z}$.*

Proof. Let H_1 is an $NGOS$ in $\mathcal{Z}$. Since $H_1 \subseteq H_1$, by hypothesis $Cl(H_1) \subseteq H_1$. But from the Definition, $H_1 \subseteq Cl(H_1)$. Therefore $Cl(H_1) = H_1$. Hence H_1 is an NCS of $\mathcal{Z}$. □

Theorem 3.31. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. If H is an NG^*CS in $(\mathcal{Z}, \Gamma)$ and $w(u, v)$ be a neutrosophic point in $\mathcal{Z}$ such that $w(u, v)qCl(H)$ then $Cl(w(u, v))qH$.*

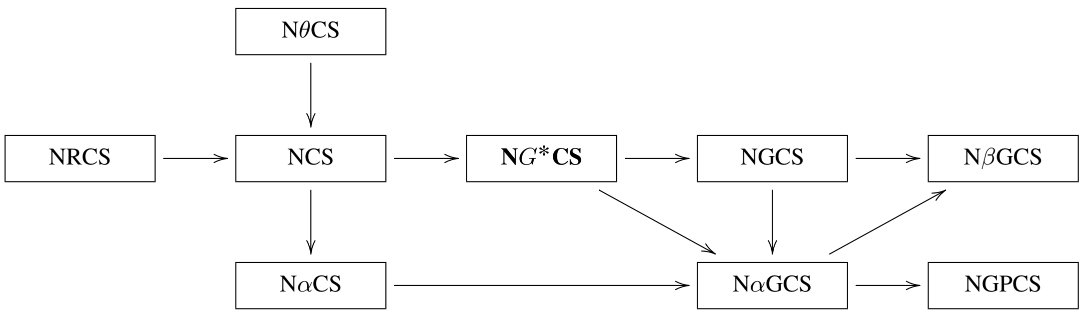
Proof. Let H be an NG^*CS in $\mathcal{Z}$ and let $w(u, v)qCl(H)$. Let $\rceil(Cl(w(u, v))qH)$, then $H \subseteq [Cl(w(u, v))]^c$, where $[Cl(w(u, v))]^c$ is an $NGOS$ in $\mathcal{Z}$. Since $Cl(H) \subseteq [Cl(w(u, v))]^c \subseteq [w(u, v)]^c$, by hypothesis, we have$\rceil(w(u, v)qCl(H))$, which is a contradiction to the hypothesis. Therefore $Cl((w(u, v))qH$. □

Theorem 3.32. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. If H is an NOS and an NG^*CS in $(\mathcal{Z}, \Gamma)$, then*

1. *H is an $NROS$ in $\mathcal{Z}$,*
2. *H is an $NRCS$ in $\mathcal{Z}$.*

Proof. (1) Let H be an NOS and an NG^*CS in $\mathcal{Z}$. Hence $Cl(H) \subseteq H$. That is $Int(Cl(H)) \subseteq H$. Since H is an NOS, H is an $NPOS$ in $\mathcal{Z}$. Then $H \subseteq Int(Cl(H))$. Therefore $H = Int(Cl(H))$. Hence H is an $NROS$ in $\mathcal{Z}$.
(2) Let H be an NOS and an NG^*CS in $\mathcal{Z}$. Then $Cl(H) \subseteq H$. That is $Cl(int(H)) \subseteq H$. Since H is an NOS, H is an $NSOS$ in $\mathcal{Z}$. By hypothesis, we have $H \subseteq Cl(Int(H))$. This implies $H = Int(Cl(H))$. Hence H is an $NRCS$ in $\mathcal{Z}$. □

Remark 3.33. From above the following implication between NG^*CS and the other existed NCS's and none of these implications is reversible



Theorem 3.34. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. If H is an NG^*CS in $(\mathcal{Z}, \Gamma)$, then $Cl(H)\backslash H$ contains non-empty $NGCS$.*

Proof. Suppose that H is an NG^*CS of $(\mathcal{Z}, \Gamma)$ and let W be non-empty $NGCS$ of $(\mathcal{Z}, \Gamma)$, such that $W \subseteq Cl(H)\backslash H$. Therefore $H \subseteq \mathcal{Z}\backslash W$. By hypothesis, we have H is NG^*CS and $\mathcal{Z}\backslash W$ is $NGOS$, then $Cl(H) \subseteq \mathcal{Z}\backslash W$. This implies $W \subseteq \mathcal{Z}\backslash Cl(H)$. So $W \subseteq (\mathcal{Z}\backslash cl(H)) \cap (cl(H)\backslash H) \subseteq (\mathcal{Z}\backslash Cl(H)) \cap cl(H) = \phi$. Hence W is empty set. □

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4 Neutrosophic g^*-Open Sets

In this section, we introduce Neutrosophic g^*-open set in Neutrosophic topological space and study some of their properties.

Definition 4.1. A neutrosophic set H (Ns in short) is said to be Neutrosophic g^*-open set (NG^*OS in short) in $(\mathcal{Z},\Gamma)$ if the complement H^c is an NG^*CS in $\mathcal{Z}$. The family of all NG^*OS's of an NTS $(\mathcal{Z},\Gamma)$ is denoted by $NG^*O(\mathcal{Z})$.

Theorem 4.2. *Let $(\mathcal{Z},\Gamma)$ be neutrosophic topological space. Then A subset H_1 of $\mathcal{Z}$ is an NG^*OS if and only if $H_2 \subseteq Int(H_1)$ whenever H_2 is an $NGCS$ in $\mathcal{Z}$ and $H_2 \subseteq H_1$.*

Proof. Necessity: Let H_1 is an NG^*OS in $\mathcal{Z}$. Let H_2 be an $NGCS$ in $\mathcal{Z}$ and $H_2 \subseteq H_1$. Then ${H_2}^c$ is an $NGOS$ in $\mathcal{Z}$ such that ${H_1}^c \subseteq H2^c$. Since ${H_1}^c$ is an NG^*CS, we have $Cl({H_1}^c) \subseteq {H_2}^c$. Hence $(Int(H_1))^c \subseteq {H_2}^c$. Therefore $H_2 \subseteq Int(H_1)$.
Sufficiency: Let $H_2 \subseteq Int(H_1)$ whenever H_2 is an $NGCS$ in and $H_2 \subseteq H_1$. Then ${H_1}^c \subseteq {H_2}^c$ and ${H_2}^c$ is an $NGOS$. By hypothesis, $(Int(H_1))^c \subseteq {H_2}^c$, which implies $Cl({H_1}^c) \subseteq {H_2}^c$. Therefore ${H_1}^c$ is an NG^*CS of $\mathcal{Z}$. Hence H_1 is an NG^*OS in $\mathcal{Z}$. □

Theorem 4.3. *Let $(\mathcal{Z},\Gamma)$ be neutrosophic topological space. Then for every NOS is an NG^*OS but not conversely.*

Proof. Let H be an NOS. Then H^c is an NCS. By Theorem 3.6, every NCS is an NG^*CS. Therefore H^c is an NG^*CS. Hence H is an NG^*OS. □

Example 4.4. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{5}{10}, \frac{3}{10}, \frac{3}{10}), (\frac{7}{10}, \frac{8}{10}, \frac{8}{10})\rangle$. Then the NS $H = \langle x, (\frac{2}{10}, \frac{2}{10}, \frac{2}{10}), (\frac{7}{10}, \frac{8}{10}, \frac{8}{10})\rangle$ is an NG^*OS but not an NCS in $(\mathcal{Z},\Gamma)$.

Theorem 4.5. *Let $(\mathcal{Z},\Gamma)$ be neutrosophic topological space. Then every $NROS$ is an NG^*OS but not conversely.*

Proof. Let H be an $NROS$ in $(\mathcal{Z},\Gamma)$. Then H^c is an $NRCS$. By Theorem 3.14, every $NRCS$ is an NG^*CS. Therefore H^c is an NG^*CS. Hence H is an NG^*OS. □

Example 4.6. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{3}{10}, \frac{7}{10}, \frac{7}{10}), (\frac{6}{10}, \frac{3}{10}, \frac{3}{10})\rangle$. Then the NS $H = \langle x, (\frac{3}{10}, \frac{4}{10}, \frac{4}{10}), (\frac{7}{10}, \frac{6}{10}, \frac{6}{10})\rangle$ is an NG^*OS but not an $NROS$ in $(\mathcal{Z},\Gamma)$.

Theorem 4.7. *Let $(\mathcal{Z},\Gamma)$ be neutrosophic topological space. Then every NG^*OS is an $NGOS$ but not conversely.*

Proof. Let H be an NG^*OS in $(\mathcal{Z},\Gamma)$. Then H^c is an NG^*CS. By Theorem 3.8, every NG^*CS is an $NGCS$. Therefore H^c is an $NGCS$. Hence H is an $NGOS$. □

Example 4.8. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{5}{10}, \frac{2}{10}, \frac{3}{10}), (\frac{4}{10}, \frac{2}{10}, \frac{2}{10})\rangle$. Then the NS $H = \langle x, (\frac{4}{10}, \frac{2}{10}, \frac{4}{10}), (\frac{6}{10}, \frac{6}{10}, \frac{6}{10})\rangle$ is an $NGOS$ but not an NG^*OS in $(\mathcal{Z},\Gamma)$.

Theorem 4.9. *Let $(\mathcal{Z},\Gamma)$ be neutrosophic topological space. Then every NG^*OS is a $N\alpha GOS$ but not conversely.*

Proof. Let H be a NG^*OS in $(\mathcal{Z},\Gamma)$. Then H^c is a NG^*CS. By Theorem 3.12, every NG^*CS is a $N\alpha GCS$. Therefore H^c is a $N\alpha GCS$. Hence H is a $N\alpha GOS$. □

Example 4.10. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $H = \langle x, (\frac{4}{10}, \frac{2}{10}, \frac{3}{10}), (\frac{6}{10}, \frac{7}{10}, \frac{6}{10})\rangle$. Then the NS $H = \langle x, (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{4}{10}, \frac{3}{10}, \frac{6}{10})\rangle$ is an $N\alpha GOS$ but not a NG^*OS in $(\mathcal{Z},\Gamma)$.

Theorem 4.11. *Let $(\mathcal{Z},\Gamma)$ be neutrosophic topological space. Then The intersection of any two NG^*OS's is a NG^*OS.*

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Proof. Let H_1 and H_2 be the any two NG^*OS's in $\mathcal{Z}$, ${H_1}^c$ and ${H_2}^c$ are NG^*CS sets. By Theorem 3.26, ${H_1}^c \cup {H_2}^c$ is an NG^*CS in $\mathcal{Z}$. Therefore $(H_1 \cap H_2)^c$ is an NG^*CS. Thus $H_1 \cap H_2$ is an NG^*OS in $\mathcal{Z}$. □

Remark 4.12. Let $(\mathcal{Z}, \Gamma)$ be $N\Gamma$ Space. The union of any two NG^*OS's is not a NG^*OS in general as seen in the following example.

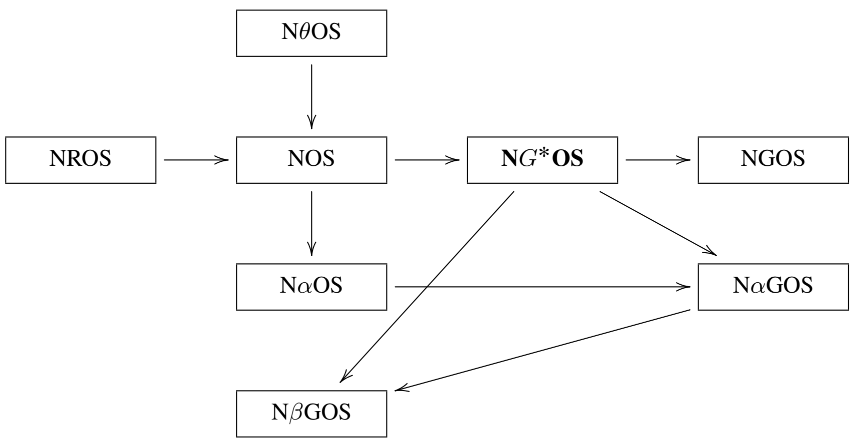
Example 4.13. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $F = \langle x, (\frac{5}{10}, 0, 0), (\frac{1}{10}, 1, 1)\rangle$. Then the NS's $H_1 = \langle x, (\frac{7}{10}, 0, 0), (\frac{2}{10}, 1, 1)\rangle$, $H_1 = \langle x, (\frac{3}{10}, 1, 1), (\frac{6}{10}, 0, 0)\rangle$ is are NG^*OS in $(\mathcal{Z}, \Gamma)$ but $H_1 \cup H_2$ is not an NG^*CS in $(\mathcal{Z}, \Gamma)$.

Theorem 4.14. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then every NG^*OS is an $N\beta GOS$ but not conversely.*

Proof. Let H be an NG^*OS in $(\mathcal{Z}, \Gamma)$. Then H^c is an NG^*CS. By Theorem 4.3, every NG^*CS is an $N\beta GCS$. Therefore H^c is a $N\beta GCS$. Hence H is a $N\beta GOS$. □

Example 4.15. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $F = \langle x, (\frac{5}{10}, \frac{6}{10}, \frac{6}{10}), (\frac{5}{10}, \frac{4}{10}, \frac{4}{10})\rangle$. Then the NS $H = \langle x, (\frac{5}{10}, \frac{3}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{7}{10}, \frac{6}{10})\rangle$ is an $N\beta GOS$ but not an NG^*OS in $(\mathcal{Z}, \Gamma)$.

Remark 4.16. From above the following implication between NG^*OS and the other existed NOS's, $NGOS$'s and none of these implications is reversible



Theorem 4.17. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then if H is a NS of $\mathcal{Z}$ then have the following equivalent properties:*

1. *H in $NG^*O(\Gamma)$,*
2. *$F \subseteq Int(H)$ whenever $F \subseteq H$ and F is a $NGCS$ in $\mathcal{Z}$,*
3. *There exists OS's Q_1 and Q_2 such that $Q_2 \subseteq F \subseteq Q_1$, where $Q_1 = Int(H)$, $F \subseteq H$ and F is a NCS in $\mathcal{Z}$.*

Proof. (1) → (2): Let H in $NG^*O(\mathcal{Z})$. Then H^c is an NG^*CS in $\mathcal{Z}$. Therefore $Cl(H^c) \subseteq W$ whenever $H^c \subseteq W$ and W is a $NGOS$ in $\mathcal{Z}$. By taking the complement on both sides $[Cl(H^c)]^c \supseteq W^c$ whenever $[H^c]^c \supseteq W^c$. Therefore $W^c \subseteq Int(H)$ whenever $W^c \subseteq H$ and W^c is a $NGCS \in \mathcal{Z}$. Replacing W^c by F, $F \subseteq Int(H)$ whenever $F \subseteq H$ and F is a $NGCS \in \mathcal{Z}$.
(2) → (3): $F \subseteq Int(H)$ whenever $F \subseteq H$ and F is a $NGCS \in \mathcal{Z}$. Hence $Int(F) \subseteq F \subseteq Int(H)$ then there exists NOS's Q_1 and Q_2 such that $Q_2 \subseteq F \subseteq Q_1$, where $Q_1 = Int(H)$ and $Q_2 = Int(F)$.
(3) → (1): Suppose that there exists NOS's Q_1 and Q_2 such that $Q_2 \subseteq F \subseteq Q_1$. That is $F \subseteq Int(H)$. Then $Cl(H^c) \subseteq F^c$ whenever $H^c \subseteq F^c$ and F^c is a $NGOS \in \mathcal{Z}$. Hence H^c is a $NG^*CS \in \mathcal{Z}$. Therefore H in $NG^*O(\mathcal{Z})$. □

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Theorem 4.18. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then if H is a NS of $\mathcal{Z}$. Then for every H in $NG^*O(\mathcal{Z})$ and every R in $NS(\mathcal{Z})$, $Int(H) \subseteq R \subseteq H$ implies R in $NG^*O(\mathcal{Z})$.*

Proof. Since $Int(H) \subseteq R \subseteq H$. By taking the complement on both sides, we get $H^c \subseteq R^c \subseteq Cl(H^c)$. Let $R^c \subseteq F$ and F is a $NGOS \in \mathcal{Z}$. Since $H^c \subseteq R^c$, $H^c \subseteq F$. Since H^c is a NG^*CS, $Cl(H^c) \subseteq F$. Therefore $Cl(R^c) \subseteq Cl(H^c) \subseteq F$. Hence R^c is an $NG^*CS \in \mathcal{Z}$. Therefore R is a $G^*OS \in \mathcal{Z}$. That is R in $NG^*O(\mathcal{Z})$. □

5 Separation Axioms of Neutrosophic g^*- Closed Sets

In this section, we study and introduce the concepts of neutrosophic $T^*_{\frac{1}{2}}$ space (N$T^*_{\frac{1}{2}}$ in short), neutrosophic $^*T_{\frac{1}{2}}$ space and some of their basic properties. Also we study some applications of Neutrosophic g^*- Closed Sets.

Definition 5.1. An neutrosophic topological space $(\mathcal{Z}, \Gamma)$ is said to be an neutrosophic $T^*_{\frac{1}{2}}$ space (in short N$T^*_{\frac{1}{2}}$) if every NG^*CS of $(\mathcal{Z}, \Gamma)$ is a NCS of $(\mathcal{Z}, \Gamma)$.

Theorem 5.2. *A neutrosophic space topological $(\mathcal{Z}, \Gamma)$ is neutrosophic $T^*_{\frac{1}{2}}$ space iff $NG^*OS(\mathcal{Z}) = NOS(\mathcal{Z})$*

Proof. Necessity: Let H be an NG^*CS in $\mathcal{Z}$ then H^c is a NG^*CS in $\mathcal{Z}$. By our assumption H^c is an NCS in $\mathcal{Z}$, this implies H is a NOS of $\mathcal{Z}$. Then NG^*OS of $\mathcal{Z} = NOS(\mathcal{Z})$.
Sufficiency: Let H be a NG^*CS in $\mathcal{Z}$, the H^c is a NG^*OS of $\mathcal{Z}$. By hypothesis H^c is $NOS \in \mathcal{Z}$, therefore H is a $NCS \in \mathcal{Z}$. Then $(\mathcal{Z}, \Gamma)$ is neutrosophic $T^*_{\frac{1}{2}}$ space. □

Theorem 5.3. *Let $(\mathcal{Z}, \Gamma)$ be neutrosophic topological space. Then every neutrosophic $T_{\frac{1}{2}}$ space is neutrosophic $T^*_{\frac{1}{2}}$ space.*

Proof. Assume $(\mathcal{Z}, \Gamma)$ be an N$T_{\frac{1}{2}}$ space and let H be an NG^*CS in $(\mathcal{Z}, \Gamma)$. Since every NG^*CS is a $NGCS$. However, H is an NCS. Hence $\mathcal{Z}$ is an N$T^*_{\frac{1}{2}}$ space. □

Remark 5.4. Every neutrosophic $T^*_{\frac{1}{2}}$ space need not be neutrosophic $T_{\frac{1}{2}}$ space in $(\mathcal{Z}, \Gamma)$ in general as seen from the following example.

Example 5.5. Let $X = \{u, v\}$ $\Gamma_N = \{0_N, 1_N, F\}$ is a N_Γ on $\mathcal{Z}$. where $F = \langle x, (\frac{9}{10}, \frac{9}{10}, \frac{9}{10}), (\frac{1}{10}, \frac{1}{10}, \frac{1}{10}) \rangle$. Clearly $(\mathcal{Z}, \Gamma)$ is an neutrosophic $T^*_{\frac{1}{2}}$ space, but not neutrosophic $T_{\frac{1}{2}}$ space.

Theorem 5.6. *For any neutrosophic space topological $(\mathcal{Z}, \Gamma)$ the following are equivalent:*

1. *$(\mathcal{Z}, \Gamma)$ is neutrosophic $T_{\frac{1}{2}}$ space,*
2. *Every singleton element of $\mathcal{Z}$ is either $NGCS$ or NOS.*

Proof. $(1) \rightarrow (2)$: Let $r \in \mathcal{Z}$ and Suppose r is not a $NGCS$ in $(\mathcal{Z}, \Gamma)$. Then $\mathcal{Z} \backslash r$ is not $NGOS$, This implies $\mathcal{Z}$ is the only $NGOS$ containing $\mathcal{Z} \backslash r$. Therefore $\mathcal{Z} \backslash r$ is a $NGOS$ of $(\mathcal{Z}, \Gamma)$. Since $(\mathcal{Z}, \Gamma)$ is a neutrosophic $T^*_{\frac{1}{2}}$ space, then $\mathcal{Z} \backslash r$ is NCS or equivalently r is an NOS in $(\mathcal{Z}, \Gamma)$.
$(2) \rightarrow (1)$" Let H be an NG^*CS in $(\mathcal{Z}, \Gamma)$. Its Obvious that $H \subseteq Cl(H)$. Let $r \in Cl(H)$. By (2) r is either NOS or $NGCS$. Case (1) let r is NOS. Since $r \in Cl(H)$, then $r \cap H \neq \phi$. So $r \in H$. Then in any case $r \in H$. Thus $H = Cl(H)$ or equivalently H is a NCS. Thus every NG^*CS is NCS. Hence $(\mathcal{Z}, \Gamma)$ is neutrosophic $T^*_{\frac{1}{2}}$ space. Case (2) let r is $NGCS$. If $r \notin H$ then $Cl(H) \backslash H$ contains non-hempty $NGCS$ r. But this is not possible according to the theorem 3.34 as H is NG^*CS. Hence $r \in H$. □

Theorem 5.7. *Let $(\mathcal{Z}, \Gamma)$ be a NTS and $\mathcal{Z}$ is a neutrosophic $T^*_{\frac{1}{2}}$ space. Then the following properties hold:*

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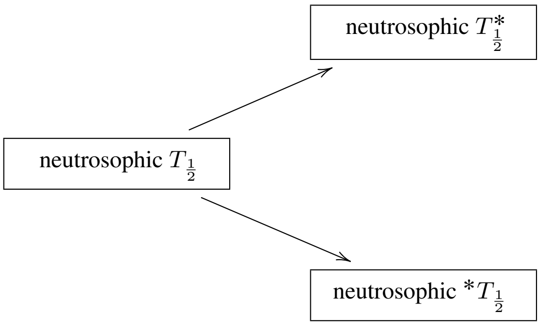
1. *Any intersection of NG^*OS is NG^*OS in $\mathcal{Z}$,*
2. *Any union of NG^*CS is NG^*CS in $\mathcal{Z}$.*

Proof. (1) It is obvious from (1) by taking complement.
(2) Let $H_{j\, j\in I}$ be a collection of NG^*CS's in a neutrosophic $T^*_{\frac{1}{2}}$ space. Thus, every NG^*CS is an NCS of $\mathcal{Z}$. However, the union of NCS is a NCS in $\mathcal{Z}$. Hence $\cup H_{j\, j\in I}$ is an $NCS \in \mathcal{Z}$. Since every NCS is an NG^*CS, $\cup H_{j\, j\in I}$ is an NG^*CS in $\mathcal{Z}$. Therefore any union of NG^*CS's is NG^*CS in $\mathcal{Z}$. □

Definition 5.8. A neutrosophic topological space $(\mathcal{Z},\Gamma)$ is said to be a neutrosophic $^*T_{\frac{1}{2}}$ space (in short N$^*T_{\frac{1}{2}}$ if every $NGCS$ of $(\mathcal{Z},\Gamma)$ is a NG^*CS of $(\mathcal{Z},\Gamma)$.

Remark 5.9. The following diagram, we have provided the relation between N$T^*_{\frac{1}{2}}$, N$^*T_{\frac{1}{2}}$ and N$T_{\frac{1}{2}}$.



Theorem 5.10. *If $(\mathcal{Z},\Gamma)$ is an N$^*T_{\frac{1}{2}}$ space then each $r \in \mathcal{Z}$, r is either NCS or NG^*OS.*

Proof. Suppose $(\mathcal{Z},\Gamma)$ is a N$^*T_{\frac{1}{2}}$ space Let $r \in \mathcal{Z}$ and let that r is $NGCS$ since $\mathcal{Z}$ is the only open set which contains $\mathcal{Z}\backslash r$. Since $(\mathcal{Z},\Gamma)$ is a N$^*T_{\frac{1}{2}}$ then $\mathcal{Z}\backslash r$ is an NG^*CS or equivalently r is NG^*OS. □

Theorem 5.11. *For any neutrosophic topological space $(\mathcal{Z},\Gamma)$, neutrosophic $T_{\frac{1}{2}}$ space is neutrosophic $^*T_{\frac{1}{2}}$ space.*

Proof. Let $\mathcal{Z}$ be a N$^*T_{\frac{1}{2}}$ space and let H be an $NGCS$ of $(\mathcal{Z},\Gamma)$. By hypothesis H is an NCS. Since every NCS in $(\mathcal{Z},\Gamma)$ is a NG^*CS of $(\mathcal{Z},\Gamma)$. Hence $\mathcal{Z}$ is a N$^*T_{\frac{1}{2}}$ space. □

Theorem 5.12. *A neutrosophic topological space $(\mathcal{Z},\Gamma)$ is an N$T_{\frac{1}{2}}$ space iff it is both N$^*T_{\frac{1}{2}}$ space and N$T^*_{\frac{1}{2}}$ space.*

Proof. Necessity: Follows from Theorem 5.3 and 5.11.
Sufficiency: Suppose $(\mathcal{Z},\Gamma)$ is both N$^*T_{\frac{1}{2}}$ space and N$T^*_{\frac{1}{2}}$ space. Let H be a $NGCS$ of $(\mathcal{Z},\Gamma)$. Since $(\mathcal{Z},\Gamma)$ is an $^*T_{\frac{1}{2}}$ space, then H is NG^*CS of $(\mathcal{Z},\Gamma)$. Since $(\mathcal{Z},\Gamma)$ is a N$T^*_{\frac{1}{2}}$ space, then $H$is NCS of $(\mathcal{Z},\Gamma)$. Thus $(\mathcal{Z},\Gamma)$ is a N$T_{\frac{1}{2}}$ space. □

Theorem 5.13. *A neutrosophic topological space $(\mathcal{Z},\Gamma)$ is a N$^*T_{\frac{1}{2}}$ space iff $NG^*OS(\mathcal{Z}) = NGOS(\mathcal{Z})$*

Proof. Necessity: Assume H be a $NGOS(\mathcal{Z}) \in \mathcal{Z}$, then H^c is a $NGCS \in \mathcal{Z}$. However H^c is a NG^*CS in $\mathcal{Z}$, and this implies H is an NG^*OS in $\mathcal{Z}$. Hence $NGOS(\mathcal{Z}) = NG^*OS(\mathcal{Z})$.
Sufficiency: Assume H be a $NGCS$ of $\mathcal{Z}$, the H^c is an $NGOS$ in $\mathcal{Z}$. By Hypothesis H^c is a NG^*OS in $\mathcal{Z}$, which implies H is a NG^*CS in $\mathcal{Z}$. Hence $(\mathcal{Z},\Gamma)$ is a N$^*T_{\frac{1}{2}}$ space. □

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6 Conclusion

In this write up, we established inequalities and introduce the notion of Neutrosophic g^* set, Neutrosophic $^*T_{\frac{1}{2}}$ space by using Ng^* set in Neutrosophic topological spaces. Furthermore, we investigate and define NG^*CS, NG^*OS. Then we introduce and study the relationships between NG^*CS, NG^*OS and other generalization closed and open sets respectively, and also present some properties and applications of Neutrosophic g^* set in Neutrosophic topological space. This work can easily be extended in various structures like Neutrosophic Pg^*, Neutrosophic Sg^*, Neutrosophic αg^* sets. The other properties of these types of sets can be found and one can introduce some other relations to these types of sets to develop the skills of learning mathematics.

7 Author's contributions

This article was written in collaboration by all of the contributors. The final manuscript was read and approved by all writers.

8 Conflicts of interest

There are no competing interests declared by the authors.

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