Multi Criteria Information Fusion for Medical Diagnosis based on m-polar Fuzzy Neutrosophic Generalized Weighted Aggregation Operator

Muhammad Riaz *and Masooma Raza Hashmi †
Faculty of Science, Department of Mathematics
University of the Punjab, Lahore, Pakistan.

Abstract

This manuscript offers a progressive mathematical model for medical diagnosis and recovery of the patient's disease. To manipulate and deal with the hesitations and obscurities of objects and human thinking an innovative notion of m-polar fuzzy neutrosophic set (MPFNS) is inaugurate from the antecedent concepts of fuzzy neutrosophic set (FNS) and m-polar fuzzy set (MPFS). In this article, we introduce various operations on MPFNS and construct its score functions. We propose generalized weighted aggregation operator in the context of MPFNNs. A case study based on data fusion for medical diagnosis is present and scrutinize with the help of MPFN data. By using proposed mathematical modeling on operator, we diagnose the disease of the patient and also discuss about its recovery. Lastly, we present advantages, validity of the proposed algorithm, influence and sensitivity of parameter $\eth$, simplicity, flexibility, the transcendence of the proposed method and comparative analysis with the other approaches.

Keywords: m-polar fuzzy neutrosphic set (MPFNS), score functions, generalized weighted aggregation operator (MPFNGWA), multi criteria decision-making for medical diagnosis, Recovery of patient, comparison analysis.

1 Introduction and background

Multi criteria decision-making (MCDM) process is a sub-field of operations research that explicitly evaluates multiple conflicting criteria in decision-making business, medicine, engineering, artificial intelligence and in daily life. It is regarded as the intellectual process which fallouts the selection of a belief or a class of activity among various alternative possibilities according to diverse standards. If we amass the data and deduce the result without handling ambiguities, then given results will be undefined and equivocal. For this purpose a fuzzy

*mriaz.math@pu.edu.pk
†masoomaraza25@gmail.com

set (FS) was established by Zadeh [31] which is an imperative precise erection to epitomize an assembling of items whose boundary is ambiguous. A FS $\mathfrak{F}$ on $\mathcal{Q}$ is a mathematical mapping $\sigma : \mathcal{Q} \rightarrow [0,1]$. After that, more hybrid models of FS have been presented and investigated such as, intuitionistic fuzzy set (IFS) [2], single valued neutrosophic set (SVNS) [19, 20], m-polar fuzzy set (MPFS) [5] and interval valued fuzzy set (IVFS) [32]. A fuzzy neutrosophic set $\mathfrak{N}$ is defined by $\mathfrak{N} = \{\langle\varsigma, \mathfrak{A}(\varsigma), \mathfrak{S}(\varsigma), \mathfrak{Y}(\varsigma)\rangle, \varsigma \in \mathcal{Q}\}$, where $\mathfrak{A}, \mathfrak{S}, \mathfrak{Y} : \mathcal{Q} \rightarrow]^{-}0, 1^{+}[$ represent truth, falsity and indeterminacy grades respectively and $^{-}0 \leq \mathfrak{A}(\varsigma) + \mathfrak{S}(\varsigma) + \mathfrak{Y}(\varsigma) \leq 3^{+}$ The neutrosophic set yields the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. It is difficult to utilize these values in daily life science and technology problems. Consequently, the neutrosophic set which takes the value from the subset of $[0,1]$ is to be regarded here. An abstraction of bipolar fuzzy set was inaugurated by Chen [5] named as MPFS. An MPFS $\mathfrak{C}$ on a non-empty universal set $\mathcal{Q}$ is a methematical function $\mathfrak{C} : \mathcal{Q} \rightarrow [0,1]^{m}$, symbolized by $\mathfrak{C} = \{\langle\varsigma, P_i o \Lambda(\varsigma)\rangle : \varsigma \in \mathcal{Q}; i = 1, 2, 3, ..., m\}$ where and $P_i : [0,1]^{m} \rightarrow [0,1]$ is the i-th projection mathematical function $(i \in m)$. $\mathfrak{C}_{\phi}(\varsigma) = (0, 0, ..., 0)$ is the smallest value in $[0,1]^{m}$ and $\mathfrak{C}_{\widetilde{X}}(\varsigma) = (1, 1, ..., 1)$ is the greatest value in $[0,1]^{m}$.

Aggregation means the creation of a numeral of things into a cluster or a bunch of things that have come or been taken together. In the past few years, aggregation operators based on FS and its various hybrid structures have made very much attention and become popular because they can easily implement to practical areas of different domains. Xu *et al.* [25, 26, 27] introduced weighted averaging operators, geometric operators and induced generalized operators based on IFNs. Jose and Kuriaskose [10] investigated aggregation operators with the corresponding score function for MCDM in the context of IFNs. Mahmood *et al.* [12] established generalized aggregation operators for cubic hesitant fuzzy numbers (CHFNs) and use it into MCDM. Riaz ans Hashmi [14, 15, 16, 17, 18] investigated certain applications of FPFS-sets, FPFS-metric, FPFS-topology and FPFS-compact spaces. They developed fixed point theorems of FNS-mapping with its decision-making. Ali [1] write a note on soft, rough soft and fuzzy soft sets. Qurashi and Shabir [13] presented generalized approximations of $(\in, \in \vee q)$-fuzzy ideals in quantales. Shabir and Ali [21] established some properties of soft ideals and generalized fuzzy ideals in semigroups. Xueling *et al.* [23] introduced decision-making methods based on various hybrid soft sets. Feng *et al.* [7, 8, 9] introduced properties of soft sets combined with fuzzy and rough sets and MADM models in the environment of generalized IF soft set and fuzzy soft set. Boran *et al.* [4] use TOPSIS decision-making method for the supplier selection in the context of IFS. Liu *et al.* [11] worked on hesitant IF linguistic operators and presented its MAGDM problem. Wei *et al.* [22] established hesitant triangular fuzzy operators in multi attribute group decision-making (MAGDM) problems. A book on HFS was established by Xu [24] with the concept of its various aggregation operations and MCDM. Ye [28, 29, 30] introduced prioritized aggregation operators in the context of interval valued hesitant fuzzy set (IVHFS) and worked on its MAGDM. He also established MCDM methods for interval neutrosophic sets and simplified neutrosophic sets. Zhang *et al.* [33] introduced aggregation operators with MCDM by using interval valued FNS (IVFNS). An extended TOPSIS method for decision-making was developed by Chi and Lui [6] on IVFNS Zhao [34] *et al.* worked on generalized aggregation operators in the context of IFS. Aiwu *et al.* [3] constructed generalized aggregation operator for IVFNS.

The motivation of this extended and hybrid work is given step by step in the whole manuscript. We show that other hybrid structures of fuzzy sets become special cases of MPFNS under some suitable conditions. We discuss about the validity, flexibility, simplicity and superiority of our proposed model. This model is most generalized form and use to collect data at a large scale and applicable in medical, engineering, artificial intelligence, agriculture and other daily life problems. In future this work can be gone easily for other

approaches and different types of aggregated operators. This research will be helpful for researchers in the field of fuzzy and aggregated operators and they can use these ideas in MCDM problems.
The layout of this paper is systematized as follows. Section 2 implies a novel idea of m-polar fuzzy neutrosophic set (MPFNS). We establish some of its operations, score function and improved score function. In section 3, we use MPFNS to establish novel generalized weighted aggregation operator. In section 4, we establish a method for the solution of MCDM problem based on medical diagnosis using defined aggregated operator by the suggested algorithm. This model demonstrates the feasibility and advantages of the proposed approach. This mathematical modeling diagnose the disease and also work on data collection and evaluation history of patient's recovery report. In the sequence, we make a brief comparison analysis, validity of the proposed algorithm, influence and sensitivity of parameter $\eth$, simplicity, flexibility, advantages and superiority of the proposed method with the help of graphs and tables. Finally, the conclusion of this research is summarized in section 5.

2 m-polar Fuzzy Neutrosophic Set (MPFNS)

In this section, we utilize the basic components of NFS and MPFS discussed in above section for the construction of a new hybrid structure called MPFNS. We discuss its score functions and improved form of score function with the addition of various operations based on MPFNSs. In the whole manuscript, we use $\mathcal{Q}$ as a fixed sample space or as the universal circle.

Definition 2.1. An object $\mathfrak{M}_{\mathfrak{N}}$ on the reference set $\mathcal{Q}$ is called MPFNS if it can be scripted as

$$\mathfrak{M}_{\mathfrak{N}} = \{\varsigma, \big(\langle \mathfrak{A}_1(\varsigma), \mathfrak{S}_1(\varsigma), \mathfrak{Y}_1(\varsigma)\rangle, \langle \mathfrak{A}_2(\varsigma), \mathfrak{S}_2(\varsigma), \mathfrak{Y}_2(\varsigma)\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}(\varsigma), \mathfrak{S}_{\mathfrak{M}}(\varsigma), \mathfrak{Y}_{\mathfrak{M}}(\varsigma)\rangle\big) : \varsigma \in \mathcal{Q}\}$$

where $\mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}} : \mathcal{Q} \to [0,1]$ represent truth, falsity and indeterminacy grades respectively and $0 \leq \mathfrak{A}_{\mathfrak{M}}(\varsigma)+\mathfrak{S}_{\mathfrak{M}}(\varsigma)+\mathfrak{Y}_{\mathfrak{M}}(\varsigma) \leq 3;\ \forall\, \mathfrak{M}$. Simply the triplet $\Im = \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}}\rangle = \big(\langle \mathfrak{A}_1, \mathfrak{S}_1, \mathfrak{Y}_1\rangle, \langle \mathfrak{A}_2, \mathfrak{S}_2, \mathfrak{Y}_2\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}}\rangle\big)$ is called m-polar fuzzy neutrosophic number (MPFNN), where $0 \leq \mathfrak{A}_{\mathfrak{M}} + \mathfrak{S}_{\mathfrak{M}} + \mathfrak{Y}_{\mathfrak{M}} \leq 3;\ \ \forall\, \mathfrak{M}$. In tabular form it can be represented as

Table 1: m-polar fuzzy neutrosophic set

$\mathfrak{M}_{\mathfrak{N}}$	$MPFNS$
ς_1	$\big(\langle \mathfrak{A}_1(\varsigma_1), \mathfrak{S}_1(\varsigma_1), \mathfrak{Y}_1(\varsigma_1)\rangle, \langle \mathfrak{A}_2(\varsigma_1), \mathfrak{S}_2(\varsigma_1), \mathfrak{Y}_2(\varsigma_1)\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}(\varsigma_1), \mathfrak{S}_{\mathfrak{M}}(\varsigma_1), \mathfrak{Y}_{\mathfrak{M}}(\varsigma_1)\rangle\big)$
ς_2	$\big(\langle \mathfrak{A}_1(\varsigma_2), \mathfrak{S}_1(\varsigma_2), \mathfrak{Y}_1(\varsigma_2)\rangle, \langle \mathfrak{A}_2(\varsigma_2), \mathfrak{S}_2(\varsigma_2), \mathfrak{Y}_2(\varsigma_2)\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}(\varsigma_2), \mathfrak{S}_{\mathfrak{M}}(\varsigma_2), \mathfrak{Y}_{\mathfrak{M}}(\varsigma_2)\rangle\big)$
...	... ...
$\varsigma_{\mathfrak{N}}$	$\big(\langle \mathfrak{A}_1(\varsigma_{\mathfrak{N}}), \mathfrak{S}_1(\varsigma_{\mathfrak{N}}), \mathfrak{Y}_1(\varsigma_{\mathfrak{N}})\rangle, \langle \mathfrak{A}_2(\varsigma_{\mathfrak{N}}), \mathfrak{S}_2(\varsigma_{\mathfrak{N}}), \mathfrak{Y}_2(\varsigma_{\mathfrak{N}})\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}(\varsigma_{\mathfrak{N}}), \mathfrak{S}_{\mathfrak{M}}(\varsigma_{\mathfrak{N}}), \mathfrak{Y}_{\mathfrak{M}}(\varsigma_{\mathfrak{N}})\rangle\big)$

There is a relationship between MPFNS and other hybrid structures of fuzzy set. This relationship can be elaborated in the given flow chart diagram.

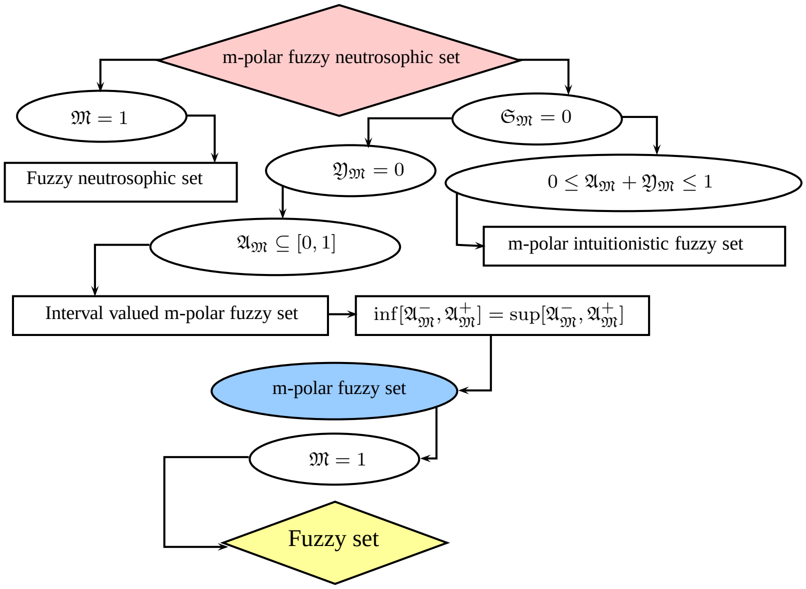


Figure 1: Relationship between MPFNS and other hybrid fuzzy sets

Definition 2.2. An MPFNS $\mathfrak{M}_{\mathfrak{N}}$ is said to be an empty MPFNS if it can be scripted as

$$^0\mathfrak{M}_{\mathfrak{N}} = \{\varsigma, (\langle 0,1,1\rangle, \langle 0,1,1\rangle, ..., \langle 0,1,1\rangle) : \varsigma \in \mathcal{Q}\}$$

and called absolute MPFNS if written as

$$^1\mathfrak{M}_{\mathfrak{N}} = \{\varsigma, (\langle 1,0,0\rangle, \langle 1,0,0\rangle, ..., \langle 1,0,0\rangle) : \varsigma \in \mathcal{Q}\}$$

Definition 2.3. We define some operations for MPFNNs $\Im = \big(\langle \mathfrak{A}_1, \mathfrak{S}_1, \mathfrak{Y}_1\rangle, \langle \mathfrak{A}_2, \mathfrak{S}_2, \mathfrak{Y}_2\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}}\rangle\big)$ and $\Im_{\wp} = \big(\langle {}^{\wp}\mathfrak{A}_1, {}^{\wp}\mathfrak{S}_1, {}^{\wp}\mathfrak{Y}_1\rangle, \langle {}^{\wp}\mathfrak{A}_2, {}^{\wp}\mathfrak{S}_2, {}^{\wp}\mathfrak{Y}_2\rangle, ..., \langle {}^{\wp}\mathfrak{A}_{\mathfrak{M}}, {}^{\wp}\mathfrak{S}_{\mathfrak{M}}, {}^{\wp}\mathfrak{Y}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$ given as:
(i): $\Im^c = \big(\langle \mathfrak{Y}_1, 1-\mathfrak{S}_1, \mathfrak{A}_1\rangle, \langle \mathfrak{Y}_2, 1-\mathfrak{S}_2, \mathfrak{A}_2\rangle, ..., \langle \mathfrak{Y}_{\mathfrak{M}}, 1-\mathfrak{S}_{\mathfrak{M}}, \mathfrak{A}_{\mathfrak{M}}\rangle\big)$
(ii): $\Im_1 = \Im_2 \Leftrightarrow \langle {}^1\mathfrak{Y}_{\mathfrak{M}}, {}^1\mathfrak{S}_{\mathfrak{M}}, {}^1\mathfrak{A}_{\mathfrak{M}}\rangle = \langle {}^2\mathfrak{Y}_{\mathfrak{M}}, {}^2\mathfrak{S}_{\mathfrak{M}}, {}^2\mathfrak{A}_{\mathfrak{M}}\rangle; \quad \forall\, \mathfrak{M}$
(iii): $\Im_1 \subseteq \Im_2 \Leftrightarrow {}^1\mathfrak{A}_{\mathfrak{M}} \leq {}^2\mathfrak{A}_{\mathfrak{M}}, {}^1\mathfrak{S}_{\mathfrak{M}} \geq {}^2\mathfrak{S}_{\mathfrak{M}}, {}^1\mathfrak{Y}_{\mathfrak{M}} \geq {}^2\mathfrak{Y}_{\mathfrak{M}}; \quad \forall\, \mathfrak{M}$
(iv): $\bigcup_{\wp} \Im_{\wp} = \Big(\big\langle \sup_{\wp} {}^{\wp}\mathfrak{A}_1, \inf_{\wp} {}^{\wp}\mathfrak{S}_1, \inf_{\wp} {}^{\wp}\mathfrak{Y}_1\big\rangle, \big\langle \sup_{\wp} {}^{\wp}\mathfrak{A}_2, \inf_{\wp} {}^{\wp}\mathfrak{S}_2, \inf_{\wp} {}^{\wp}\mathfrak{Y}_2\big\rangle, ..., \big\langle \sup_{\wp} {}^{\wp}\mathfrak{A}_{\mathfrak{M}}, \inf_{\wp} {}^{\wp}\mathfrak{S}_{\mathfrak{M}}, \inf_{\wp} {}^{\wp}\mathfrak{Y}_{\mathfrak{M}}\big\rangle\Big)$
(v): $\bigcap_{\wp} \Im_{\wp} = \Big(\big\langle \inf_{\wp} {}^{\wp}\mathfrak{A}_1, \sup_{\wp} {}^{\wp}\mathfrak{S}_1, \sup_{\wp} {}^{\wp}\mathfrak{Y}_1\big\rangle, \big\langle \inf_{\wp} {}^{\wp}\mathfrak{A}_2, \sup_{\wp} {}^{\wp}\mathfrak{S}_2, \sup_{\wp} {}^{\wp}\mathfrak{Y}_2\big\rangle, ..., \big\langle \inf_{\wp} {}^{\wp}\mathfrak{A}_{\mathfrak{M}}, \sup_{\wp} {}^{\wp}\mathfrak{S}_{\mathfrak{M}}, \sup_{\wp} {}^{\wp}\mathfrak{Y}_{\mathfrak{M}}\big\rangle\Big)$

Example 2.4. Consider two 4PFNNs $\Im_1$ and $\Im_2$ given in tabular form as

Table 2: 4PFNNs

$\mathcal{Q}$	$4PFNNs$
$\Im_1$	$(\langle 0.61, 0.11, 0.25\rangle, \langle 0.82, 0.63, 0.11\rangle, \langle 0.72, 0.38, 0.59\rangle, \langle 0.21, 0.32, 0.41\rangle)$
$\Im_2$	$(\langle 0.32, 0.62, 0.51\rangle, \langle 0.83, 0.11, 0.92\rangle, \langle 0.52, 0.43, 0.39\rangle, \langle 0.18, 0.93, 0.82\rangle)$

Now we calculate union and intersection by using Definition 2.3 and results can be seen in tabular form as

Table 3: 4PFNNs

$\mathcal{Q}$	$4PFNNs$
$\Im_1 \cup \Im_2$	$\big(\langle 0.61, 0.11, 0.25\rangle, \langle 0.83, 0.11, 0.11\rangle, \langle 0.72, 0.38, 0.39\rangle, \langle 0.21, 0.32, 0.41\rangle\big)$
$\Im_1 \cap \Im_2$	$\big(\langle 0.32, 0.62, 0.51\rangle, \langle 0.82, 0.63, 0.92\rangle, \langle 0.52, 0.43, 0.59\rangle, \langle 0.18, 0.93, 0.82\rangle\big)$

Definition 2.5. If we want to do mathematical modeling with MPFNNs to the decision-making problems or any application to MCDM, then it is necessary to rank these numbers. For this we have to define some score functions corresponding to MPFNN $\Im = \big(\langle \mathfrak{A}_1, \mathfrak{S}_1, \mathfrak{Y}_1\rangle, \langle \mathfrak{A}_2, \mathfrak{S}_2, \mathfrak{Y}_2\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}}\rangle\big)$ given as:

$$\pounds_1(\Im) = \frac{1}{2\mathfrak{M}}\Bigg(\mathfrak{M} + \sum_{\wp'=1}^{\mathfrak{M}} (\mathfrak{A}_{\wp'} - 2\mathfrak{S}_{\wp'} - \mathfrak{Y}_{\wp'})\Bigg); \quad \pounds_1(\Im) \in [0,1]$$

$$\pounds_2(\Im) = \frac{1}{\mathfrak{M}}\sum_{\wp'=1}^{\mathfrak{M}} (\mathfrak{A}_{\wp'} - 2\mathfrak{S}_{\wp'} - \mathfrak{Y}_{\wp'}); \quad \pounds_2(\Im) \in [-1,1]$$

By using above score functions there must be a possibility when score of two MPFNNs will be same. For this purpose we define an improved score function for ranking of MPFNNs scripted as

$$\pounds_3(\Im) = \frac{1}{2\mathfrak{M}}\Bigg(\mathfrak{M} + \sum_{\wp'=1}^{\mathfrak{M}} \big((\mathfrak{A}_{\wp'} - 2\mathfrak{S}_{\wp'} - \mathfrak{Y}_{\wp'})(2 - \mathfrak{A}_{\wp'} - \mathfrak{Y}_{\wp'})\big)\Bigg); \quad \pounds_3(\Im) \in [-1,1]$$

In some cases when $\mathfrak{A}_{\wp'} + \mathfrak{Y}_{\wp'} = 1; \ \ \forall \ \wp' = 1, 2, ..., \mathfrak{M}$ then $\pounds_3(\Im)$ reduces to $\pounds_1(\Im)$.

Definition 2.6. Let $\Im_1$ and $\Im_2$ be two MPFNNs, then by using score function we can define an order relation between these MPFNNs given as:
(a): If $\pounds_1(\Im_1) \succ \pounds_1(\Im_2)$ then $\Im_1 \succ \Im_2$.
(b): If $\pounds_1(\Im_1) = \pounds_1(\Im_2)$ then
(i): If $\pounds_3(\Im_1) \succ \pounds_3(\Im_2)$ then $\Im_1 \succ \Im_2$.
(ii): If $\pounds_3(\Im_1) \prec \pounds_3(\Im_2)$ then $\Im_1 \prec \Im_2$.
(iii): If $\pounds_3(\Im_1) = \pounds_3(\Im_2)$ then $\Im_1 \sim \Im_2$.

Example 2.7. Consider two 2PFNNs $\Im_1$ and $\Im_2$ given in tabular form as

Table 4: 2PFNNs

$\mathcal{Q}$	$2PFNNs$
$\Im_1$	$\big(\langle 0.5, 0.3, 0.4\rangle, \langle 0.5, 0.1, 0.8\rangle\big)$
$\Im_2$	$\big(\langle 0.2, 0.3, 0.1\rangle, \langle 0.2, 0.1, 0.5\rangle\big)$

Then by using Definition 2.5 $\pounds_1(\Im_1) = \frac{1}{2(2)}[2 + 0.5 - 2(0.3) - 0.4 + 0.5 - 2(0.1) - 0.8] = 0.25$. Similarly, $\pounds_1(\Im_2) = 0.25$. This shows that $\pounds_1$ fails to give the ranking between both 2PFNNs. Now we will use second score function $\pounds_2$. From Definition 2.5 we can be easily calculate that $\pounds_2(\Im_1) = -0.5 = \pounds_2(\Im_2)$. This shows that $\pounds_2$ also fails to evaluate the ranking. Now we will use improved score function for the ranking of 2PFNNs. After calculations we get $\pounds_3(\Im_1) = 0.275$ and $\pounds_3(\Im_2) = 0.125$. Hence $\pounds_3(\Im_1) \succ \pounds_3(\Im_2)$, so $\Im_1 \succ \Im_2$.

Remark. • For null MPFNN ${}^{0}\mathfrak{M}_{\mathfrak{N}}$ we have $\pounds_3({}^{0}\mathfrak{M}_{\mathfrak{N}}) = -1$.

• For absolute MPFNN ${}^{1}\mathfrak{M}_{\mathfrak{N}}$ we have $\pounds_3({}^{1}\mathfrak{M}_{\mathfrak{N}}) = 1$.

• For MPFNN of the form $\Im = \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, 1-\mathfrak{A}_{\mathfrak{M}}\rangle;\ \ \forall\ \mathfrak{M}$. We have $\pounds_3(\Im) = \mathfrak{A}_{\mathfrak{M}} - \mathfrak{S}_{\mathfrak{M}};\ \ \forall\ \mathfrak{M}$.

Definition 2.8. Let $\Im = \big(\langle \mathfrak{A}_1, \mathfrak{S}_1, \mathfrak{Y}_1\rangle, \langle \mathfrak{A}_2, \mathfrak{S}_2, \mathfrak{Y}_2\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}}\rangle\big)$ be an arbitrary MPFNN and $\Im_{\wp} = \big(\langle {}^{\wp}\mathfrak{A}_1, {}^{\wp}\mathfrak{S}_1, {}^{\wp}\mathfrak{Y}_1\rangle, \langle {}^{\wp}\mathfrak{A}_2, {}^{\wp}\mathfrak{S}_2, {}^{\wp}\mathfrak{Y}_2\rangle, ..., \langle {}^{\wp}\mathfrak{A}_{\mathfrak{M}}, {}^{\wp}\mathfrak{S}_{\mathfrak{M}}, {}^{\wp}\mathfrak{Y}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$ be an assembling of MPFNNs, then we can define some operations on MPFNNs with an arbitrary real number $\eta > 0$ given as follows:

• $\Im_1 \oplus \Im_2 = \big(\langle {}^{1}\mathfrak{A}_1 + {}^{2}\mathfrak{A}_1 - {}^{1}\mathfrak{A}_1{}^{2}\mathfrak{A}_1, {}^{1}\mathfrak{S}_1{}^{2}\mathfrak{S}_1, {}^{1}\mathfrak{Y}_1{}^{2}\mathfrak{Y}_1\rangle, \langle {}^{1}\mathfrak{A}_2 + {}^{2}\mathfrak{A}_2 - {}^{1}\mathfrak{A}_2{}^{2}\mathfrak{A}_2, {}^{1}\mathfrak{S}_2{}^{2}\mathfrak{S}_2, {}^{1}\mathfrak{Y}_2{}^{2}\mathfrak{Y}_2\rangle, ..., \langle {}^{1}\mathfrak{A}_{\mathfrak{M}} + {}^{2}\mathfrak{A}_{\mathfrak{M}} - {}^{1}\mathfrak{A}_{\mathfrak{M}}{}^{2}\mathfrak{A}_{\mathfrak{M}}, {}^{1}\mathfrak{S}_{\mathfrak{M}}{}^{2}\mathfrak{S}_{\mathfrak{M}}, {}^{1}\mathfrak{Y}_{\mathfrak{M}}{}^{2}\mathfrak{Y}_{\mathfrak{M}}\rangle\big)$.

• $\Im_1 \otimes \Im_2 = \big(\langle {}^{1}\mathfrak{A}_1{}^{2}\mathfrak{A}_1, {}^{1}\mathfrak{S}_1 + {}^{2}\mathfrak{S}_1 - {}^{1}\mathfrak{S}_1{}^{2}\mathfrak{S}_1, {}^{1}\mathfrak{Y}_1 + {}^{2}\mathfrak{Y}_1 - {}^{1}\mathfrak{Y}_1{}^{2}\mathfrak{Y}_1\rangle, \langle {}^{1}\mathfrak{A}_2{}^{2}\mathfrak{A}_2, {}^{1}\mathfrak{S}_2 + {}^{2}\mathfrak{S}_2 - {}^{1}\mathfrak{S}_2{}^{2}\mathfrak{S}_2, {}^{1}\mathfrak{Y}_2 + {}^{2}\mathfrak{Y}_2 - {}^{1}\mathfrak{Y}_2{}^{2}\mathfrak{Y}_2\rangle, ..., \langle {}^{1}\mathfrak{A}_{\mathfrak{M}}{}^{2}\mathfrak{A}_{\mathfrak{M}}, {}^{1}\mathfrak{S}_{\mathfrak{M}} + {}^{2}\mathfrak{S}_{\mathfrak{M}} - {}^{1}\mathfrak{S}_{\mathfrak{M}}{}^{2}\mathfrak{S}_{\mathfrak{M}}, {}^{1}\mathfrak{Y}_{\mathfrak{M}} + {}^{2}\mathfrak{Y}_{\mathfrak{M}} - {}^{1}\mathfrak{Y}_{\mathfrak{M}}{}^{2}\mathfrak{Y}_{\mathfrak{M}}\rangle\big)$.

• $\eta\Im = \big(\langle 1-(1-\mathfrak{A}_1)^{\eta}, (\mathfrak{S}_1)^{\eta}, (\mathfrak{Y}_1)^{\eta}\rangle, \langle 1-(1-\mathfrak{A}_2)^{\eta}, (\mathfrak{S}_2)^{\eta}, (\mathfrak{Y}_2)^{\eta}\rangle, ..., \langle 1-(1-\mathfrak{A}_{\mathfrak{M}})^{\eta}, (\mathfrak{S}_{\mathfrak{M}})^{\eta}, (\mathfrak{Y}_{\mathfrak{M}})^{\eta}\rangle\big)$.

• $\Im^{\eta} = \big(\langle (\mathfrak{A}_1)^{\eta}, 1-(1-\mathfrak{S}_1)^{\eta}, 1-(1-\mathfrak{Y}_1)^{\eta}\rangle, \langle (\mathfrak{A}_2)^{\eta}, 1-(1-\mathfrak{S}_2)^{\eta}, 1-(1-\mathfrak{Y}_2)^{\eta}\rangle, ..., \langle (\mathfrak{A}_{\mathfrak{M}})^{\eta}, 1-(1-\mathfrak{S}_{\mathfrak{M}})^{\eta}, 1-(1-\mathfrak{Y}_{\mathfrak{M}})^{\eta}\rangle\big)$.

Remark. $\Im_1 \oplus \Im_2, \Im_1 \otimes \Im_2, \eta\Im$ and $\Im^{\eta}$ are also MPFNNs.

Theorem 2.9. *Let $\Im = \big(\langle \mathfrak{A}_1, \mathfrak{S}_1, \mathfrak{Y}_1\rangle, \langle \mathfrak{A}_2, \mathfrak{S}_2, \mathfrak{Y}_2\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}}\rangle\big)$ be an arbitrary MPFNN and $\Im_{\wp} = \big(\langle {}^{\wp}\mathfrak{A}_1, {}^{\wp}\mathfrak{S}_1, {}^{\wp}\mathfrak{Y}_1\rangle, \langle {}^{\wp}\mathfrak{A}_2, {}^{\wp}\mathfrak{S}_2, {}^{\wp}\mathfrak{Y}_2\rangle, ..., \langle {}^{\wp}\mathfrak{A}_{\mathfrak{M}}, {}^{\wp}\mathfrak{S}_{\mathfrak{M}}, {}^{\wp}\mathfrak{Y}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$ be an assembling of MPFNNs, then for real numbers $\eta > 0, \eta_1 > 0$ and $\eta_2 > 0$ following results are true.*

(i): $\Im_1 \oplus \Im_2 = \Im_2 \oplus \Im_1$

(ii): $\Im_1 \otimes \Im_2 = \Im_2 \otimes \Im_1$

(iii): $\eta(\Im_1 \oplus \Im_2) = \eta\Im_2 \oplus \eta\Im_1$

(iv): $(\Im_1 \otimes \Im_2)^{\eta} = \Im_2^{\eta} \otimes \Im_1^{\eta}$

(v): $\eta_1\Im \oplus \eta_2\Im = (\eta_1 \oplus \eta_2)\Im$

(vi): $\Im^{\eta_1} \otimes \Im^{\eta_2} = \Im^{\eta_1+\eta_2}$

(vii): $(\eta_1 \oplus \eta_2) \oplus \eta_3 = \eta_1 \oplus (\eta_2 \oplus \eta_3)$

(viii): $(\eta_1 \otimes \eta_2) \otimes \eta_3 = \eta_1 \otimes (\eta_2 \otimes \eta_3)$

Proof. The proof of above results can be easily done by using Definition 2.8. □

3 m-polar Fuzzy Neutrosophic Generalized Weighted Aggregation Operator (MPFNGWA)

Definition 3.1. Let $\Im_{\wp} = \big(\langle {}^{\wp}\mathfrak{A}_1, {}^{\wp}\mathfrak{S}_1, {}^{\wp}\mathfrak{Y}_1\rangle, \langle {}^{\wp}\mathfrak{A}_2, {}^{\wp}\mathfrak{S}_2, {}^{\wp}\mathfrak{Y}_2\rangle, ..., \langle {}^{\wp}\mathfrak{A}_{\mathfrak{M}}, {}^{\wp}\mathfrak{S}_{\mathfrak{M}}, {}^{\wp}\mathfrak{Y}_{\mathfrak{M}}\rangle\big)$, $(\wp = 1, 2, 3, ..., \mathfrak{N})$ be an assembling of MPFNNs and $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ is the weighted vector of $\Im_{\wp}$ such that $\zeta_{\wp} > 0$ with $\sum_{\wp=1}^{\mathfrak{N}} \zeta_{\wp} = 1$. Then m-polar fuzzy neutrosophic generalized weighted aggregation operator is a mapping MPFNGWA : $\mho^{\mathfrak{N}} \to \mho$ and defined as follows:

$$\text{MPFNGWA}(\Im_1, \Im_2, ..., \Im_{\mathfrak{N}}) = \Big(\sum_{\wp=1}^{\mathfrak{N}} \zeta_{\wp}\Im_{\wp}^{\eth}\Big)^{1/\eth};\ \ \eth\ \text{ is parameter.}$$

$\mathrm{MPFNGWA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \Big(\sum\limits_{\wp=1}^{\mathfrak{N}} \zeta_\wp \big[\big\langle ({}^{\wp}\mathfrak{A}_1)^{\eth}, 1-(1-{}^{\wp}\mathfrak{S}_1)^{\eth}, 1-(1-{}^{\wp}\mathfrak{Y}_1)^{\eth}\big\rangle, \big\langle ({}^{\wp}\mathfrak{A}_2)^{\eth}, 1-(1-{}^{\wp}\mathfrak{S}_2)^{\eth}, 1-(1-{}^{\wp}\mathfrak{Y}_2)^{\eth}\big\rangle, ..., \big\langle ({}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\eth}, 1-(1-{}^{\wp}\mathfrak{S}_{\mathfrak{M}})^{\eth}, 1-(1-{}^{\wp}\mathfrak{Y}_{\mathfrak{M}})^{\eth}\big\rangle\big]\Big)^{1/\eth}$

$\mathrm{MPFNGWA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \Big(\sum\limits_{\wp=1}^{\mathfrak{N}} \big[\big\langle 1-(1-({}^{\wp}\mathfrak{A}_1)^{\eth})^{\zeta_\wp}, (1-(1-{}^{\wp}\mathfrak{S}_1)^{\eth})^{\zeta_\wp}, (1-(1-{}^{\wp}\mathfrak{Y}_1)^{\eth})^{\zeta_\wp}\big\rangle, \big\langle 1-(1-({}^{\wp}\mathfrak{A}_2)^{\eth})^{\zeta_\wp}, (1-(1-{}^{\wp}\mathfrak{S}_2)^{\eth})^{\zeta_\wp}, (1-(1-{}^{\wp}\mathfrak{Y}_2)^{\eth})^{\zeta_\wp}\big\rangle, ..., \big\langle 1-(1-({}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\eth})^{\zeta_\wp}, (1-(1-{}^{\wp}\mathfrak{S}_{\mathfrak{M}})^{\eth})^{\zeta_\wp}, (1-(1-{}^{\wp}\mathfrak{Y}_{\mathfrak{M}})^{\eth})^{\zeta_\wp}\big\rangle\big]\Big)^{1/\eth}$

$\mathrm{MPFNGWA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \Big(\big\langle 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-({}^{\wp}\mathfrak{A}_1)^{\eth})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{S}_1)^{\eth})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{Y}_1)^{\eth})^{\zeta_\wp}\big\rangle, \big\langle 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-({}^{\wp}\mathfrak{A}_2)^{\eth})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{S}_2)^{\eth})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{Y}_2)^{\eth})^{\zeta_\wp}\big\rangle, ..., \big\langle 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-({}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\eth})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{S}_{\mathfrak{M}})^{\eth})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{Y}_{\mathfrak{M}})^{\eth})^{\zeta_\wp}\big\rangle\Big)^{1/\eth}$

$\mathrm{MPFNGWA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \Big(\big\langle (1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-({}^{\wp}\mathfrak{A}_1)^{\eth})^{\zeta_\wp})^{1/\eth}, 1-(1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{S}_1)^{\eth})^{\zeta_\wp})^{1/\eth}, 1-(1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{Y}_1)^{\eth})^{\zeta_\wp})^{1/\eth}\big\rangle, \big\langle (1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-({}^{\wp}\mathfrak{A}_2)^{\eth})^{\zeta_\wp})^{1/\eth}, 1-(1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{S}_2)^{\eth})^{\zeta_\wp})^{1/\eth}, 1-(1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{Y}_2)^{\eth})^{\zeta_\wp})^{1/\eth}\big\rangle, ..., \big\langle (1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-({}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\eth})^{\zeta_\wp})^{1/\eth}, 1-(1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{S}_{\mathfrak{M}})^{\eth})^{\zeta_\wp})^{1/\eth}, 1-(1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-(1-{}^{\wp}\mathfrak{Y}_{\mathfrak{M}})^{\eth})^{\zeta_\wp})^{1/\eth}\big\rangle\Big) \qquad (A)$

Remark. • When $\eth \to 0$ then MPFNGWA operator reduces to m-polar fuzzy neutrosophic weighted geometric aggregation operator (MPFNWGA) defined as:

$\mathrm{MPFNWGA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \prod\limits_{\wp=1}^{\mathfrak{N}} \Im_\wp^{\zeta_\wp}$

$\mathrm{MPFNWGA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \Big(\big\langle \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{A}_1)^{\zeta_\wp}, 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{S}_1)^{\zeta_\wp}, 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{Y}_1)^{\zeta_\wp}\big\rangle, \big\langle \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{A}_2)^{\zeta_\wp}, 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{S}_2)^{\zeta_\wp}, 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{Y}_2)^{\zeta_\wp}\big\rangle, ..., \big\langle \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\zeta_\wp}, 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{S}_{\mathfrak{M}})^{\zeta_\wp}, 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{Y}_{\mathfrak{M}})^{\zeta_\wp}\big\rangle\Big) \qquad (B)$

• When $\eth = 1$ then MPFNGWA operator reduces to m-polar fuzzy neutrosophic weighted arithmetic aggregation operator (MPFNWAA) defined as:

$\mathrm{MPFNWAA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \sum\limits_{\wp=1}^{\mathfrak{N}} \zeta_\wp \Im_\wp$

$\mathrm{MPFNWAA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}}) = \Big(\big\langle 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{A}_1)^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{S}_1)^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{Y}_1)^{\zeta_\wp}\big\rangle,$
$\big\langle 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{A}_2)^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{S}_2)^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{Y}_2)^{\zeta_\wp}\big\rangle, ..., \big\langle 1-\prod\limits_{\wp=1}^{\mathfrak{N}}(1-{}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{S}_{\mathfrak{M}})^{\zeta_\wp}, \prod\limits_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{Y}_{\mathfrak{M}})^{\zeta_\wp}\big\rangle\Big) \qquad (C)$

• Let $\mathfrak{X} = \mathrm{MPFNGWA}(\Im_1,\Im_2,...,\Im_{\mathfrak{N}})$. When $\mathfrak{X} = \Big(\sum\limits_{\wp=1}^{\mathfrak{N}} \zeta_\wp \Im_\wp^{\eth}\Big)^{1/\eth}$ the value of $\sum\limits_{\wp=1}^{\mathfrak{N}} \zeta_\wp(\mathfrak{X}-\Im_\wp^{\eth})^2$ is at its minimum value. Therefore, MPFNWGA operator is the better approximation than others.

Example 3.2. Consider three 3PFNNs $\Im_1, \Im_2$ and $\Im_3$ with $\zeta = (0.3, 0.4, 0.3)^T$ as $\sum\limits_{\wp=1}^{3} \zeta_\wp = 1$. In tabular form 3PFNNs can be represented as:

Table 5: 3PFNNs

$\mathcal{Q}$	$3PFNNs$
$\Im_1$	$\big(\langle 0.81, 0.24, 0.31\rangle, \langle 0.56, 0.43, 0.28\rangle, \langle 0.61, 0.71, 0.38\rangle\big)$
$\Im_2$	$\big(\langle 0.91, 0.32, 0.41\rangle, \langle 0.73, 0.15, 0.23\rangle, \langle 0.34, 0.25, 0.61\rangle\big)$
$\Im_3$	$\big(\langle 0.36, 0.21, 0.41\rangle, \langle 0.91, 0.85, 0.34\rangle, \langle 0.73, 0.35, 0.25\rangle\big)$

Then for $\eth = 1$, we have

$$\prod_{\wp=1}^{3}(1 - {}^{\wp}\mathfrak{A}_1)^{\zeta_\wp} = (1 - 0.81)^{0.3} \times (1 - 0.91)^{0.4} \times (1 - 0.36)^{0.3} = 0.2028$$

$$\prod_{\wp=1}^{3}({}^{\wp}\mathfrak{S}_1)^{\zeta_\wp} = (0.24)^{0.3} \times (0.32)^{0.4} \times (0.21)^{0.3} = 0.2586$$

$$\prod_{\wp=1}^{3}({}^{\wp}\mathfrak{Y}_1)^{\zeta_\wp} = (0.31)^{0.3} \times (0.41)^{0.4} \times (0.41)^{0.3} = 0.3770$$

$$\prod_{\wp=1}^{3}(1 - {}^{\wp}\mathfrak{A}_2)^{\zeta_\wp} = (1 - 0.56)^{0.3} \times (1 - 0.73)^{0.4} \times (1 - 0.91)^{0.3} = 0.2248$$

$$\prod_{\wp=1}^{3}({}^{\wp}\mathfrak{S}_2)^{\zeta_\wp} = (0.43)^{0.3} \times (0.15)^{0.4} \times (0.85)^{0.3} = 0.3461$$

$$\prod_{\wp=1}^{3}({}^{\wp}\mathfrak{Y}_2)^{\zeta_\wp} = (0.28)^{0.3} \times (0.23)^{0.4} \times (0.34)^{0.3} = 0.2743$$

$$\prod_{\wp=1}^{3}(1 - {}^{\wp}\mathfrak{A}_3)^{\zeta_\wp} = (1 - 0.61)^{0.3} \times (1 - 0.34)^{0.4} \times (1 - 0.73)^{0.3} = 0.4310$$

$$\prod_{\wp=1}^{3}({}^{\wp}\mathfrak{S}_3)^{\zeta_\wp} = (0.71)^{0.3} \times (0.25)^{0.4} \times (0.35)^{0.3} = 0.4049$$

$$\prod_{\wp=1}^{3}({}^{\wp}\mathfrak{Y}_3)^{\zeta_\wp} = (0.38)^{0.3} \times (0.61)^{0.4} \times (0.25)^{0.3} = 0.3770$$

Then by using equation (C) we get
$\text{MPFNWGA}(\Im_1, \Im_2, \Im_3) = \big(\langle 0.7972, 0.2586, 0.3770\rangle, \langle 0.7752, 0.3461, 0.2743\rangle, \langle 0.569, 0.3782, 0.4049\rangle\big)$

4 Multi criteria decision-making for medical diagnosis

For a short time interval the infectious disease which spread quickly among the inhabitants is called epidemic disease. In that respect are diverse epidemic models for infectious diseases, but we argue here about the SIR model for the given decision-making problem. The SIR model is a mathematical model of infectious diseases, where we have three compartments given as;
S= Susceptible.
I= Infected or infectious.
R= Recover or removed.
For the construction of such types of models we follow some assumptions according to the model and situations. For very simple and basic model with no death and birth rates we have the following Figure. 2. The variation in population of every compartment with the rates **b** and **r** can be seen graphically as Figure. 3:

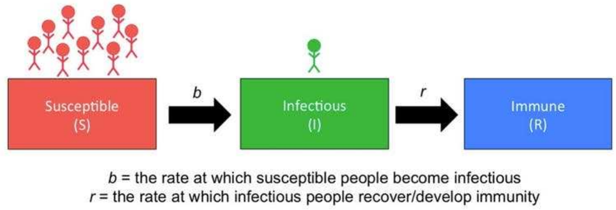


Figure 2: SIR model for epidemic diseases

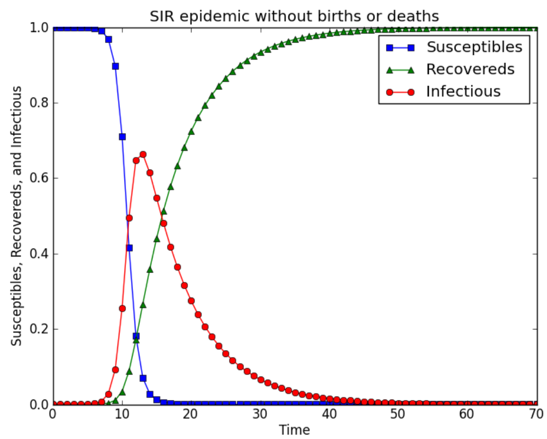


Figure 3: Graphical representation of SIR model

4.1 Case Study

A man visits a doctor and told him about his health problems which he was facing since last three months. He stated that he was suffering from tiredness and fever with loss of appetite. He mentions that he has unintentional weight loss with inadequate immune system. He also feels muscle and joint pain. Granting to the doctor all the symptoms lead to three diseases, Hepatitis C, Tuberculosis and Typhoid fever. It is challenging for a physician to diagnose the exact disease of this patient without any medical test, because on that point is an overlapping between the symptoms of above named diseases. We present a novel algorithm with a modified model of MPFNGWA operator to diagnose the disease of the patient and we also discuss about the recovery of the patient by using this mathematical modeling.

Proposed Technique:

In this part of our manuscript, we establish the technique of MPFNGWA operator to detect the disease of the patient in the environment of MPFN data.

Input:

Step 1: The following $\mathcal{Q} = \{\Im_1, \Im_2, ..., \Im'_{\mathfrak{N}}\}$ be the assembling of alternatives and $\mathfrak{P} = \{\mathcal{J}_1, \mathcal{J}_2, ..., \mathcal{J}_{\mathfrak{N}}\}$ be the collection of attributes or criteria. The weighted vector according to the choice of decision-maker is given by $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ with the condition $\sum_{\wp=1}^{\mathfrak{N}} \zeta_\wp = 1$. We further assume that $\mathcal{M} = [\Im^{\wp}_{\wp'}]_{\wp\times\wp'}$, for

$\wp = \{1, 2, ..., \mathfrak{N}\}$ and $\wp' = \{1, 2, ..., \mathfrak{N}'\}$ be an assembling of decision matrix provided by experts or decision-maker, where each $\Im^{\wp}_{\wp'}$ be a MPFNN.

Step 2: In business term we mostly consider two main attribute terms including, benefit and cost. In MCDM the greatest value of benefit attribute and lower value of cost attribute leads us to success. The value of loss attribute case can be converted into value of benefit attribute by normalizing the input data $\mathcal{M} = [\Im^{\wp}_{\wp'}]_{\wp \times \wp'}$. It is necessary to normalize the input information before further calculations to obtain the best and precise solutions. Therefore the MPFN evaluation can be normalized by

$$\widetilde{\Im}^{\wp}_{\wp'} = \begin{cases} \big(\langle \mathfrak{A}_1, \mathfrak{S}_1, \mathfrak{Y}_1\rangle, \langle \mathfrak{A}_2, \mathfrak{S}_2, \mathfrak{Y}_2\rangle, ..., \langle \mathfrak{A}_{\mathfrak{M}}, \mathfrak{S}_{\mathfrak{M}}, \mathfrak{Y}_{\mathfrak{M}}\rangle\big); & \text{for same type} \\ \big(\langle \mathfrak{Y}_1, 1-\mathfrak{S}_1, \mathfrak{A}_1\rangle, \langle \mathfrak{Y}_2, 1-\mathfrak{S}_2, \mathfrak{A}_2\rangle, ..., \langle \mathfrak{Y}_{\mathfrak{M}}, 1-\mathfrak{S}_{\mathfrak{M}}, \mathfrak{A}_{\mathfrak{M}}\rangle\big); & \text{for different type} \end{cases}$$

If the type is same for all attributes, then there is no need to normalize the information.

Calculations:

Step 3: Compute the aggregated values of alternatives $\Im_{\wp'}; (\wp' = 1, 2, 3, ..., \mathfrak{N}')$ corresponding to the different criteria $\mathcal{J}_{\wp}; (\wp = 1, 2, 3, ..., \mathfrak{N})$ by using MPFNGWA operator given in equation (A) for different values of parameter $\eth$ and hence the evaluated aggregated values are given by $\mathcal{O}_{\wp'}; (\wp' = 1, 2, 3, ..., \mathfrak{N}')$.

Output:

Step 5: Using $\mathcal{O}_{\wp'}; (\wp' = 1, 2, 3, ..., \mathfrak{N}')$ calculate score values by using Definition 2.5.

Step 6: We rank these alternative on the basis of score values according to the Definition 2.6.

Step 7: Choose the alternative with the maximum score calculated through the purposed method.

Mathematical modeling:

Algorithm 1 Algorithm for application of MPFNGWA operator

procedure APPLY(MPFNGWA)
    Input: collection of input MPFN data in decision matrix $[\Im^{\wp}_{\wp'}]_{\wp \times \wp'}$ of patient's disease given by doctor and experts.
    Output: collection of input MPFN data $\widetilde{\Im}^{\wp}_{\wp'}$ after normalizing.
    for $\wp$=1 to $\mathfrak{N}$
        for $\wp'$=1 to $\mathfrak{N}'$
            if $\Im^{\wp}_{\wp'}$ is an entry in cost attribute **then**
                $\widetilde{\Im}^{\wp}_{\wp'} = (\Im^{\wp}_{\wp'})^c$
            else
                $\widetilde{\Im}^{\wp}_{\wp'} = \Im^{\wp}_{\wp'}$
            end if
        end for
    end for
    for $\wp$=1 to $\mathfrak{N}$
        for $\wp'$=1 to $\mathfrak{N}'$
            for $\eth$=1 to 10000
                Compute MPFNGWA$({}^{\eth}\widetilde{\Im}^{\wp}_{\wp'}) = {}^{\eth}\hat{\Im}_{\wp'} = {}^{\eth}\mathcal{O}_{\wp'}$ ▷ Where ${}^{\eth}\mathcal{O}_{\wp'} = [{}^{\eth}\hat{\Im}_{\wp'}]$ aggregated matrix
            end for
        end for
    end for
    for $\wp'$=1 to $\mathfrak{N}'$
        Compute $\pounds_3({}^{\eth}\mathcal{O}_{\wp'})$
    end for
    Rank the alternatives
end procedure

For the given case study we have a set of alternatives consists of three diseases $\mathcal{Q} = \{\Im_1, \Im_2, \Im_3\}$ and six symptoms according to the patient given as $\mathfrak{P} = \{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, \mathcal{J}_4, \mathcal{J}_5, \mathcal{J}_6\}$ where,

$\Im_1$ = Tuberculosis

$\Im_2 =$ Hepatitis C
$\Im_3 =$ Typhoid fever
$\mathcal{J}_1 =$ Fatigue
$\mathcal{J}_2 =$ Fever
$\mathcal{J}_3 =$ Loss of appetite
$\mathcal{J}_4 =$ Unintentional weight loss
$\mathcal{J}_5 =$ Muscle and joint pain
$\mathcal{J}_6 =$ poor immune system.
Granting to the patient's description about his health problem, the doctor can place a weighted vector according to the diseases and symptoms given as $\zeta = (0.2, 0.1, 0.1, 0.1, 0.2, 0.3)^T$. We choose $\mathfrak{M} = 3$, which shows that for input we take the data of last three months. Now we input 3PFN data for the set of diseases and symptoms. This input data can be constructed with the help of an expert. Expert must know to relate the physical conditions into mathematical terms and fuzzy logics. The tabular form of input data is given as;

Table 6: 3PFN-data

Order	$\mathcal{Q}$	3PFNNs
1	$\Im_1$	$\mathcal{J}_1 : \big(\langle 0.52, 0.23, 0.67\rangle, \langle 0.61, 0.33, 0.68\rangle, \langle 0.68, 0.41, 0.72\rangle\big)$
2	$\Im_1$	$\mathcal{J}_2 : \big(\langle 0.47, 0.31, 0.21\rangle, \langle 0.52, 0.38, 0.41\rangle, \langle 0.67, 0.41, 0.27\rangle\big)$
3	$\Im_1$	$\mathcal{J}_3 : \big(\langle 0.53, 0.34, 0.18\rangle, \langle 0.61, 0.19, 0.23\rangle, \langle 0.71, 0.31, 0.11\rangle\big)$
4	$\Im_1$	$\mathcal{J}_4 : \big(\langle 0.61, 0.41, 0.24\rangle, \langle 0.56, 0.32, 0.13\rangle, \langle 0.73, 0.11, 0.17\rangle\big)$
5	$\Im_1$	$\mathcal{J}_5 : \big(\langle 0.38, 0.13, 0.27\rangle, \langle 0.47, 0.23, 0.17\rangle, \langle 0.59, 0.41, 0.37\rangle\big)$
6	$\Im_1$	$\mathcal{J}_6 : \big(\langle 0.45, 0.18, 0.21\rangle, \langle 0.53, 0.23, 0.34\rangle, \langle 0.67, 0.11, 0.17\rangle\big)$
1	$\Im_2$	$\mathcal{J}_1 : \big(\langle 0.73, 0.17, 0.23\rangle, \langle 0.83, 0.11, 0.22\rangle, \langle 0.89, 0.13, 0.21\rangle\big)$
2	$\Im_2$	$\mathcal{J}_2 : \big(\langle 0.79, 0.23, 0.34\rangle, \langle 0.87, 0.25, 0.37\rangle, \langle 0.91, 0.23, 0.31\rangle\big)$
3	$\Im_2$	$\mathcal{J}_3 : \big(\langle 0.83, 0.19, 0.23\rangle, \langle 0.89, 0.15, 0.25\rangle, \langle 0.95, 0.23, 0.17\rangle\big)$
4	$\Im_2$	$\mathcal{J}_4 : \big(\langle 0.72, 0.18, 0.31\rangle, \langle 0.73, 0.15, 0.27\rangle, \langle 0.84, 0.25, 0.27\rangle\big)$
5	$\Im_2$	$\mathcal{J}_5 : \big(\langle 0.67, 0.15, 0.24\rangle, \langle 0.75, 0.17, 0.25\rangle, \langle 0.78, 0.27, 0.29\rangle\big)$
6	$\Im_1$	$\mathcal{J}_6 : \big(\langle 0.87, 0.12, 0.36\rangle, \langle 0.92, 0.31, 0.27\rangle, \langle 0.97, 0.23, 0.31\rangle\big)$
1	$\Im_3$	$\mathcal{J}_1 : \big(\langle 0.43, 0.51, 0.21\rangle, \langle 0.31, 0.56, 0.37\rangle, \langle 0.53, 0.41, 0.45\rangle\big)$
2	$\Im_3$	$\mathcal{J}_2 : \big(\langle 0.37, 0.61, 0.71\rangle, \langle 0.39, 0.67, 0.68\rangle, \langle 0.43, 0.21, 0.14\rangle\big)$
3	$\Im_3$	$\mathcal{J}_3 : \big(\langle 0.28, 0.63, 0.81\rangle, \langle 0.35, 0.65, 0.71\rangle, \langle 0.41, 0.63, 0.53\rangle\big)$
4	$\Im_3$	$\mathcal{J}_4 : \big(\langle 0.27, 0.53, 0.61\rangle, \langle 0.37, 0.25, 0.61\rangle, \langle 0.45, 0.63, 0.58\rangle\big)$
5	$\Im_3$	$\mathcal{J}_5 : \big(\langle 0.31, 0.61, 0.27\rangle, \langle 0.43, 0.71, 0.35\rangle, \langle 0.52, 0.35, 0.19\rangle\big)$
6	$\Im_1$	$\mathcal{J}_6 : \big(\langle 0.21, 0.71, 0.38\rangle, \langle 0.51, 0.31, 0.39\rangle, \langle 0.61, 0.51, 0.41\rangle\big)$

By using 3PFNGWA operator from equation (A) for $\eth = 1$ which is equivalent to 3PFNWAA operator from equation (C) we get,
$\mathcal{O}_1 = \big(\langle 0.4805, 0.2164, 0.2779\rangle, \langle 0.5570, 0.2635, 0.3026\rangle, \langle 0.6685, 0.2355, 0.2658\rangle\big)$
$\mathcal{O}_2 = \big(\langle 0.7890, 0.1565, 0.2842\rangle, \langle 0.8570, 0.1891, 0.2613\rangle, \langle 0.9195, 0.2136, 0.2628\rangle\big)$
$\mathcal{O}_3 = \big(\langle 0.3077, 0.6093, 0.3794\rangle, \langle 0.4168, 0.4688, 0.4432\rangle, \langle 0.5271, 0.4322, 0.3416\rangle\big)$.
We use improved score function $\pounds_3$ to calculate score values because it gives better an accurate results as compared to $\pounds_1$ and $\pounds_2$. Hence the score values of above aggregated 3PFNNs can be obtained by using Definition 2.5 given as,
$\pounds_3(\mathcal{O}_1) = 0.3883, \pounds_3(\mathcal{O}_2) = 0.5928, \pounds_3(\mathcal{O}_3) = -0.0934$. These score values shows that $\Im_2 \succ \Im_3 \succ \Im_1$. Which shows that patient should get serious about his health, because he is suffering from hepatitis C.

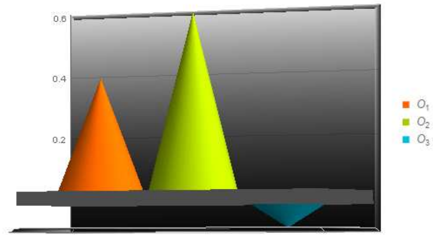


Figure 4: Ranking of 3PFNNs $\mathcal{O}_1, \mathcal{O}_2$ and $\mathcal{O}_3$

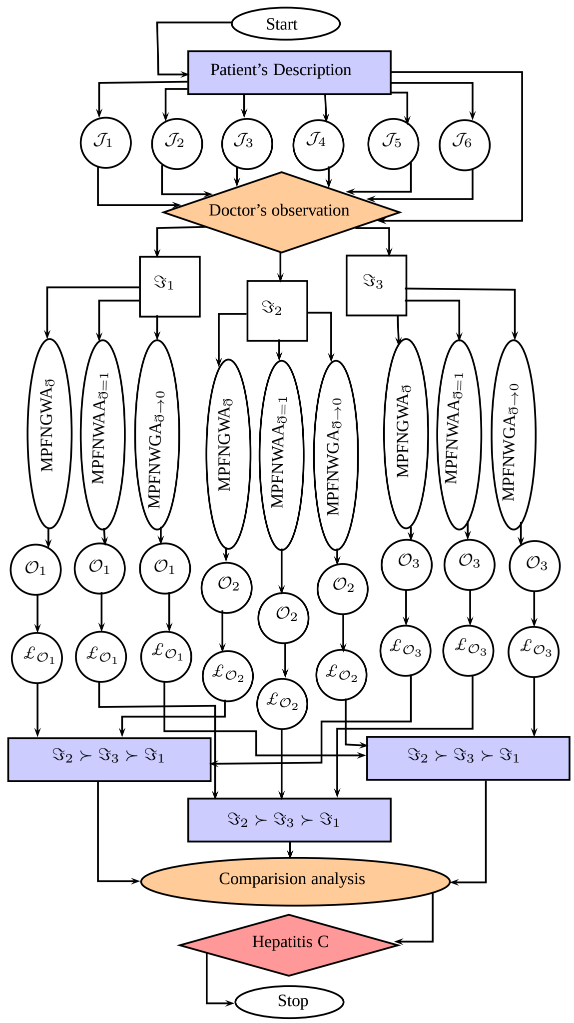


Figure 5: Flow chart diagram of proposed algorithm

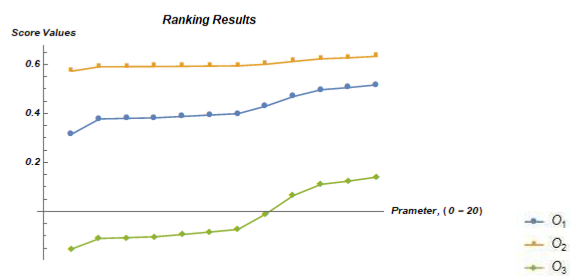
The Influence and Sensitivity of Parameter $\eth$:

We calculate the aggregated 3PFNNs for different values of parameter $\eth$ from the input 3PFN-data and will see the behavior of the operator by analyzing their score values. The values are given in tabular form as

Table 7: Ranking order of MPFNGWA for $\eth$

$\eth$	Type of operator	$\pounds_3(\mathcal{O}_1), \pounds_3(\mathcal{O}_2), \pounds_3(\mathcal{O}_3)$	Ranking Order	Result
$\to 0$	MPFNWGA	$0.3164, 0.5731, -0.1527$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
0.1	MPFNGWA	$0.3781, 0.5911, -0.1097$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
0.3	MPFNGWA	$0.3804, 0.5914, -0.1063$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
0.5	MPFNGWA	$0.3826, 0.5918, -0.1028$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
1	MPFNWAA	$0.3883, 0.5928, -0.0934$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
1.5	MPFNGWA	$0.3940, 0.5937, -0.0833$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
2	MPFNGWA	$0.3996, 0.5948, -0.0727$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
5	MPFNGWA	$0.4304, 0.6012, -0.0097$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
10	MPFNGWA	$0.4697, 0.6126, 0.0658$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
15	MPFNGWA	$0.4974, 0.6235, 0.1114$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
17	MPFNGWA	$0.5061, 0.6275, 0.1246$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$
20	MPFNGWA	$0.5172, 0.6330, 0.1407$	$\Im_2 \succ \Im_3 \succ \Im_1$	$\Im_2$

The parameter $\eth$ has no consequence on the ranking results of MPFNGWA operator. This signifies that the obtained ranking results from MPFNGWA operator are not sensitive to the parameter $\eth$. For different values of parameter $\eth$ there will be no variations in the process and results remains constant. The parameter $\eth$ represents the behavior of decision-makers or any criteria related to MCDM. Smaller $\eth$ indicate more conservative behavior and larger $\eth$ represent more optimistic behavior. The answers show that MPFNGWA is more elastic and desirable for the MCDM problems.



(a) Ranking of MPFNNs for MPFNGWA operator

Convergence in Recovery of Patient:
All the previous process shows that how to diagnose the disease of a patient with mathematical modeling under the environment of MPFN data. In this subsection, we use above modeling to determine that how much time and factors are postulated for a patient to recuperate from that disease. From above discussion, we know that decision goes for Hepatitis C. For initial and smaller values of parameter $\eth$ we see the behavior of aggregated MPFNN and its score value. It is clear from the calculations and graphical representation that initially its score valued increases means that disease is uncured and patient is infected and its infection is increasing day by day. After diagnosis, he starts his treatment, according to the doctor's prescription. He used medicines suggested by a doctor and stick with his complete diet plan. Then by increasing time period

his infection reduces and score values decreases for the larger values of parameter. We relate the parameter ð with the time and treatment, so as ð increases all the aggregated values goes to null MPFNNs. This proves that the patient is recovering from the disease. After just about specific time and treatment score value goes to minimum which is -1 and after that no changes occur in score value with the changing of parameter. This stands for that patient is entirely cured and it moves towards recovering population from the infected population. Smaller ð indicate more conservative behavior and larger ð represent more optimistic behavior. The graphical view clearly expresses all the history of patient disease from start to stop. Starting values show that he in infected and diagnosed and after diagnosis and treatment with the passage of time he get cured and last values shows that he runs to the box of recovered population and he is nowadays out of peril. This mathematical modeling helps us to examine the perfect story of patient from infected to regain. This mannequin can be offered for various diseases and for a great number of patients. Our proposed model is a more abstracted form of fuzzy set and utilize to diagnose disease, development of patient's history and gather data at a very big plate.

Table 8: Results depending on ð for Recovery of patient

ð	Score values	ð	Score values	ð	Score values	ð	Score values	ð	Score values
$\to 0$	0.5731	20	0.6330	136	0.6897	260	0.4755	1000	-0.5105
0.1	0.5911	25	0.6411	137	0.6900	270	0.2760	1100	-0.5104
0.3	0.5914	30	0.6479	138	0.6250	275	0.2763	1150	-0.5104
0.5	0.5918	40	0.6583	139	0.6254	280	0.11276311	1190	-0.5102
1	0.5928	50	0.6658	140	0.6250	285	-0.0159	1192	-0.5102
2	0.5748	70	0.6757	150	0.5577	300	-0.0158	1193	-1
5	0.6012	100	0.6841	160	0.5582	400	-0.0398	1200	-1
10	0.6126	120	0.6876	180	0.5587	500	-0.5109	1500	-1
15	0.6235	130	0.6891	200	0.4849	700	-0.5107	5000	-1
17	0.6275	135	0.6892	250	0.4852	900	-0.5105	10000	-1

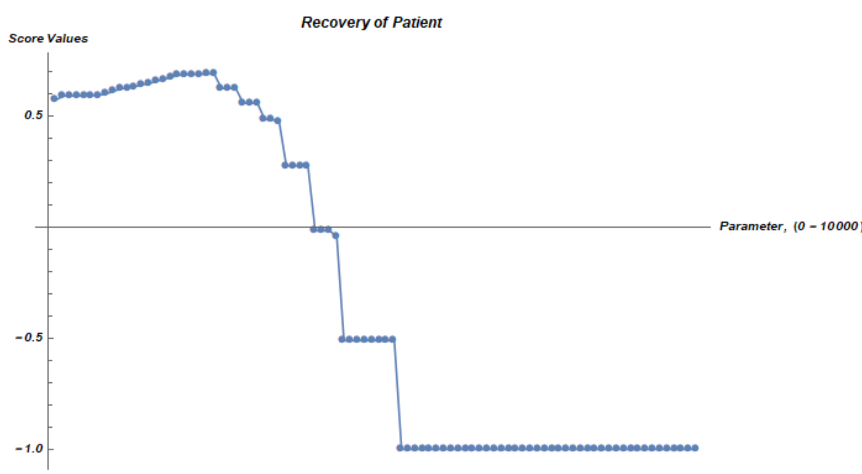


Figure 6: Recovery graph of patient from Hepatitis C

Advantages of Proposed Approach:

In this part, we discuss about some advantages of proposed approach based on MPFNSs.

(i) Validity of the method:
The suggested method is valid and suitable for all types of input data. These operators easily deal with the flaws appears in the input data and handle the ambiguities and uncertainties. As we can see in Figure. 3 that MPFNGWA operator covers all the hybrid defined operators, so this is a most generalized model and used to collect information on a big plate with multiple criteria of alternatives.
(ii) Simplicity dealing with different criteria:
In MAGDM problems we experience different types of criteria and input data according to the given situations. The proposed MPFNGWA operators are simple and easy to understand which can be applied easily on whatever type of alternatives and measures.
(iii) Flexibility of aggregation with different inputs and outputs:
The suggested algorithm is flexible and easily altered according to the different situations, inputs and outputs. There is a slightly difference between the ranking of the proposed operators because different operators have different ordering strategies so they can afford the slightly different effect according to their deliberations.
(iv) Superiority of proposed method:
From all above discussion we observe that our proposed model and MPFNGWA operators are superior to others. fuzzy neutrosophic operators, m-polar intuitionistic fuzzy operators, interval valued m-polar fuzzy operators, m-polar fuzzy operators and fuzzy operators become the special cases of MPFNGWA operator with the addition of some suitable conditions. So our method is valid, flexible, simple and superior to other hybrid structures of fuzzy set and operators defined in [3, 10, 12, 25, 26, 34].
Comparison Analysis and Discussion:
In our proposed research, we defined generalized aggregated operator by using the advanced concept of MPFNS. The impressive point of this model is that we can use it for mathematical modeling at a large scale or $\mathfrak{M}$ numbers of degrees with its truth, falsity and indeterminacy part. These $\mathfrak{M}$-degrees basically show the corresponding properties or any set criteria about the alternative ψ. As in given problem we use it for $\mathfrak{M} = 3$ means we analyze data for last three months we can extend this period according to our requirements. This $\mathfrak{M}$ can be taken as for different type of criteria which is not possible to use for other sets in a whole model like FS, IFS, NFS, etc. This item proves that it is a hybrid and more generalized model of other approaches. Other sets such as FS, MPIFS, MPFS, IVMPFS etc. become the special case of MPFNS with the addition of some suitable conditions (see Figure. 1). On the same form all the operators corresponding to the given sets become the particular cases of our purposed operator for MPFNS.

Table 9: Comparison of different methods

Methods	Operators	Ranking of alternatives
Aiwu [3]	IVNSGWA	$\Im_2 \succ \Im_3 \succ \Im_1$
Jose [10]	IVIFWA	$\Im_2 \succ \Im_3 \succ \Im_1$
Mahmood [12]	GCHFWA	$\Im_2 \succ \Im_3 \succ \Im_1$
Xu [25, 26]	IFWA,IFWG	$\Im_2 \succ \Im_3 \succ \Im_1$
Zhao [34]	GIFWA,GIVIFWA	$\Im_2 \succ \Im_3 \succ \Im_1$
Proposed method	MPFNGWA	$\Im_2 \succ \Im_3 \succ \Im_1$
Proposed method $\eth = 1$	MPFNWAA	$\Im_2 \succ \Im_3 \succ \Im_1$
Proposed method $\eth \to 0$	MPFNWGA	$\Im_2 \succ \Im_3 \succ \Im_1$

From the above table, we can insure that results obtained from different aggregation operators are similar to proposed method. These results affirm that our proposed algorithm is authentic and correct. But the question turns out here that if we bring these resolutions from other operators then why we need to specify a novel algorithm based on this novel structure? There are many arguments which show that proposed operator is modified and most generalized form of others. Foremost of all we understand that due to the behavior of parameter $\eth$ we can as well examine the recovery of the patient and its complete graph history from beginning to stop. But other operators such as IVIFWA [10] and IFWA, IFWG [25, 26] only diagnose the disease, but not covers the convergence of recovery of patients. Secondly, when we are dealing with [3, 10, 12, 25, 26, 34] operators then we face difficulties to collect the input data for all three months of the patient and see no flexibility to deal with the various numbers of criteria with truth, falsity and indeterminacy degrees and all these ingredients make the calculations very difficult. Only when we are handling with the MPFN data, then due to $\mathfrak{M}$ criteria and hybrid property of our model we deal easily with the input and output information and ensure the recovery convergence graph of the patient. Comparative analysis showed that this modified operator for hybrid and most generalized model can easily deal with the real life glitches and decision-making problems and check the properties of flexibility, simplicity and superiority to others.

5 Conclusion

In this manuscript, we have established MPFNS with the compounding of the FNS and MPFS. A generalized weighted aggregated operator in the context of MPFNNs has been found by using the MPFN operations. Score function and improved score function have been manifested in the comparison of MPFNNs. In the late years, many aggregation operators corresponding to numerous hybrid fuzzy sets have been instituted to deal with the MCDM problems. We have developed most hybrid generalized weighted aggregation operator based on MPFNS and use them into MCDM for medical diagnosis. Other bands such as FS, MPIFS, MPFS, IVMPFS etc. become the exceptional case of MPFNS with the add-on of some suitable weather. On the same form all the operators corresponding to the given sets become the particular cases of our purposed operator for MPFNS. Comparative analysis showed that these modified operators can easily share with the real life glitches and decision-making problems and use to collect information on a large scale for $\mathfrak{M}$ criteria. There are slightly different between the conclusion of different operators due to their setting up strategies and calculations but mostly conclude the same result. This overture is more efficient and feasible as compared to other approaches, (see Table. 9) because this is based on most generalized set. In future, this work can be gone easily for other approaches and different types of manipulators. Researchers will get beneficial results by exploring and investing through these concepts in the field of MCDM by using numerous aggregation operators.

References

[1] M. I. Ali, *A note on soft sets, rough soft sets and fuzzy soft sets*, Applied Soft Computing, **11**(2011), 3329-3332.

[2] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets ans Systems, **20**(1)(1986), 87-96.

[3] Z. Aiwu, D. Jianguo and G. Hongjun, *Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregated operator*, Journal of Intelligent and Fuzzy Systems, **29**(2015), 2697-2706.

[4] F. E. Boran, S. Genc, M. Kurt and D. Akay, *A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method*, Expert Systems with Applications, **36**(8)(2009), 11363-11368.

[5] J. Chen, S. Li, S. Ma, and X. Wang, *m-Polar Fuzzy Sets: An Extension of Bipolar Fuzzy Sets*, The Scientific World Journal, (2014).

[6] P. P. Chi and P. D. Lui, *An extended TOPSIS method for the multiple ttribute decision making problems based on interval neutrosophic set*, Neutrosophic Sets and Systems, **1**(2013) 63-70.

[7] F. Feng, Y.B. Jun, X. Liu and L. Li, *An adjustable approach to fuzzy soft set based decision making*, Journal of Computational and Applied Mathematics, **234**(1)(2010), 10-20.

[8] F. Feng, C. Li, B. Davvaz, M.I. Ali, *Soft sets combined with fuzzy sets and rough sets*, a tentative approach, Soft Computing, **14** (9) (2010) 899-911.

[9] F. Feng, H. Fujita, M. I. Ali, R. R. Yager, Life Fellow, IEEE and Xiaoyan Liu, *Another view on generalized intuitionistic fuzzy soft sets and related multiattribute decision making methods* IEEE Transactions On Fuzzy Systems, (2018), 1-15.

[10] S. Jose and S. Kuriaskose, *Aggregation operators, score function and accuracy function for multi criteria decision making in intuitionistic fuzzy context*, Notes on Intuitionistic Fuzzy Sets, **20**(1)(2014),40-44.

[11] X. Liu, Y. Ju and S. Yang, *Hesitant intuitionistic fuzzy linguistic aggregation operators and their applications to multi attribute decision making*, Journal of Intelligent and Fuzzy Systems, **26**(3)(2014), 1187-1201.

[12] T. Mahmood, F. Mehmood and Q. Khan, *Some generalized aggregation operators for cubic hesitant fuzzy sets and their application to multi criteria decision making*, Punjab University Journal of Mathematics, **49**(1)(2017), 31-49.

[13] S. M. Qurashi and M. Shabir, *Generalized approximations of* $(\in, \in \vee q)$*-fuzzy ideals in quantales*, Computational and Applied Mathematics, (2018) 1-17.

[14] M. Riaz and M. R. Hashmi, *Certain applications of fuzzy parameterized fuzzy soft sets in decision-making problems*, International Journal of Algebra and Statistics, **5**(2)(2016), 135-146.

[15] M. Riaz and M. R. Hashmi, *Fuzzy parameterized fuzzy soft topology with applications*, Annals of Fuzzy Mathematics and Informatics, **13**(5)(2017), 593-613.

[16] M. Riaz, M. R. Hashmi, *Fuzzy parameterized fuzzy soft compact spaces with decision-making*, Punjab University Journal of Mathematics, 50(2)(2018), 131-145.

[17] M. Riaz, M. R. Hashmi, *Fixed points of fuzzy neutrosophic soft mapping with decision-making*, Fixed point theory and applications, **7**(2018), 1-10.

[18] M. Riaz, M. R. Hashmi, *Fuzzy parameterized fuzzy soft metric spaces*, Journal of mathematical analysis, **9**(2)(2018), 25-36.

[19] F. Smarandache, *Neutrosophy Neutrosophic Probability*, Set and Logic, American Research Press,(1998) Rehoboth, USA.

[20] H. Wang, F. Smarandache, Y.Q. Zhang, and R. Sunderraman, *Single valued neutrosophic sets*, Multispace and Multistructure, **4**(2010), 410-413.

[21] M. Shabir and M. I. Ali, *Soft ideals and generalized fuzzy ideals in semigroups*, New Math. Nat. Comput. **5**(2009), 599-615.

[22] G. Wei, H. Wang, X. Zhao and R. Lin, *Hesitant triangular fuzzy information aggregation in multiple attribute decision making*, Journal of Intelligence and Fuzzy Systems, **26**(3)(2014), 1201-1209.

[23] Xueling Ma, Q. Liu and J. Zhan, *A survey of decision making methods based on certain hybrid soft set models*, Artif Intell Rev **47** (2017), 507-530

[24] Z. S. Xu, *Hesitant fuzzy set theory*, Studies in Fuzziness and Soft Computing, **314**(2014).

[25] Z. S. Xu, *Intuitionistic fuzzy aggregation operators*, IEEE Transections on Fuzzy Systems, **15**(2007), 1179-1187.

[26] Z. S. Xu and R. R. Yager, *Some geometric aggregation operators based on intuitionistic fuzzy sets*, International Journal of General Systems, **35**(2006) 417-433.

[27] Z. S. Xu and M. M. Xia, *Induced generalized intuitionitic fuzzy operators*, Knowledge based Systems, **24**(2011) 197-209.

[28] J. Ye, *Interval-valued hesitant fuzzy prioritized weighted aggregation operators for multi attribute decision making*, Journal of Algorithms and Computational Technology, **8**(2)(2013), 179-192.

[29] J. Ye, *Multicriteria decision-making method using the correlation coefficient under single-value neutrosophic enviornment*, International Journal of General Systems, **42**(2013) 386-394.

[30] J. Ye, *A multicriteria decison-making method using aggregation operators for simplified neutrosophic sets*, Journal of Intelligent and Fuzzy Systems, **26**(2014) 2459-2466.

[31] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8**(1965) 338-353.

[32] L. A. Zadeh, *The concept of a linguistic variable and its application to approximate reasoning-I*, Information Sciences, **8**(3)(1975), 199249.

[33] H. Y. Zhang, J. Q, Wang and X. H. Chen *Interval neutrosophic sets and their applications in multi-criteria decision making problems*, The Scientific World Journal, (2014) 1-15.

[34] H. Zhao, Z. S. Xu, M. F. Ni and S. S. Lui, *Generalized aggregation operators for intuitionistic fuzzy sets*, International Journal of Intelligent Systems, **25**(2010) 1-30.