Journal of Intelligent & Fuzzy Systems 37 (2019) 2403–2417
DOI:10.3233/JIFS-182723
IOS Press

Multi-criteria decision making method based on the single valued neutrosophic sets[1]

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Abstract. In this paper, a new tangent similarity between single valued neutrosophic sets is given, which contain tangent similarity [23] as a special case. A best-worst multi-criteria decision making method based on the single valued neutrosophic sets is proposed. To achieve this goal, we design an algorithm to identify the best and worst criteria through computing the out-degrees and in-degrees of the collective single valued neutrosophic preference relation directed network, and then calculate the optimal weight vector of attributes. Moreover, a mathematical model corresponding to the definitions of consistent single valued neutrosophic preference relation is contracted. Finaly, the evaluation values of all alternatives are calculated by the proposed new tangent similarity. This approach can make the process of decision making more vivid, intuitive and dynamic in the decision-making process. To illustrate the effectiveness of the proposed method, we used it to solve appointment registration systems problems.

Keywords: Single Valued Neutrosophic Set, Multi-criteria Decision-making, Consistency Ratio, similarity

1. Introduction

Analytic hierarchy process (AHP) was first introduced by Saaty [33], it has become an important decision making method. The AHP can help the decision maker to describe the general decision operation by decomposing a complex problem into a multi-level hierarchic structure of objectives, criteria, sub-criteria and alternatives. And it has been applied comprehensively to solve various decision making problems, such as the the marketing investment [17], the evaluation of information and communication technology (ICT) business alternatives [10], and so on. However, in some realistic situations, people find that it can not solve the inherent uncertainty and fuzziness effectively.

In order to deal with the inherent uncertainty and fuzziness information of classical AHP, Lai proposed fuzzy AHP (FAHP) [11] by combining fuzzy set theory [26] with classical AHP. Due to the traditional fuzzy set uses one real value $\mu_A(x) \in [0, 1]$ to represent the grade of membership of fuzzy set A. Sometimes it is uncertain and hard to be defined by a crisp value.

In AHP and FAHP methods, the computational complexity of pairwise comparison is relatively high, it needs $n(n-1)/2$ comparisons. Rezaei proposed the Best-worst method (BWM) [21], which can be taken as an enhancement of the traditional AHP and FAHP methods, it needs $2n-3$ comparisons, the computational complexity is reduced. With the Best-worst method, the decision maker does not need to conduct pairwise comparisons between all criteria but only needs to identify the best criteria and the worst criteria, and then makes pairwise comparisons between the best/worst criteria and the other criteria. However, it is not easy to determine which criteria

[1]This work is supported by the National Natural Science Foundation of China (Grant Nos.61773019 and 11701540).

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is the best or worst one when the number of criteria is very large, and the approach is improper under uncertain circumstances. Therefore, the reference comparisons of BWM can be executed by employing fuzzy number other than crisp value in some practical issues, which may be more in line with the actual situations. Guo et al. [32] proposed fuzzy best-worst multi-criteria decision-making method, which the reference comparisons were executed by using the fuzzy comparing judgments.

Due to the complexity and uncertainty of decision environment, it is sometimes difficult for decision makers to provide exact opinions. As a novel extension of the fuzzy set, Torra [34] defined the hesitant fuzzy set by allowing the membership degree of an object to have a set of possible values. In the framework of decision making based on preference relations, Zhu and Xu [2] extended the concept of hesitant fuzzy sets to fuzzy preference relations and defined the hesitant fuzzy preference relation (HFPR). Zhang el at [37–40] studied consistency analysis, consensus reaching process, additive consistency and multiplicative consistency analysis for hesitant fuzzy preference relations.

As another extension of the fuzzy set, intuitionistic fuzzy set [25] use intuitionistic fuzzy values to represent membership degrees, non-membership degrees and hesitancy degrees, it can be seen as a particular case of type-2 fuzzy set. Xu proposed intuitionistic fuzzy analytic hierarchy process (IFAHP) [36] by combining intuitionistic fuzzy sets with classical AHP. Mou et al. [29] proposed intuitionistic fuzzy best-worst method (BWM) through extending the Best-worst method to accommodate intuitionistic fuzzy circumstances, and combine the advantages of the directed network [19] to determine the best/worst criteria. This approach can make the process of decision making more intuitive than other decision making methods. Garg et al. [15] present a novel multi-attribute decision making method under interval-valued intuitionistic fuzzy set environment by integrating a technique for order preference by similarity to ideal solution method.

Although intuitionistic fuzzy set can deal with fuzzy information better, it is very difficult to process indeterminate and inconsistent information. In order to deal with this case, Smarandache [6] proposed a neutrosophic set (NS). However, neutrosophic set needs to be specified from a technical point of view. To this effect, Smarandache [6] and Wang et al. [18] proposed a case of neutrosophic set, called the single valued neutrosophic set (SVNS). Here, the truth-membership, indeterminacy-membership and falsitymembership degree are real number in unit interval [0,1]. Single valued neutrosophic set can be considered as a generalization of classical set, fuzzy set and intuitionistic fuzzy set, which is suitable for solving many real-world decision-making problems, especially decision-making problems related to the use of incomplete and imprecise information, uncertainties, predictions and so on.

Since the concept of the single valued neutrosophic set has been put forward, many decision methods based on single valued intelligence set have been studied. Peng et al. [20] developed an outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. Harish et al. [12] proposes the decision making approach for solving the multiattribute decision making problems with SVNS information. Mariya et al. [31] developed an aggregation method for solving group multi-criteriadecision-making problems with single-valued neutrosophic sets. Harish el at [13] proposed two algorithms for possibility linguistic single-valued neutrosophic decision-making based on complex proportional assessment and aggregation operators with new information measures. Nancy [27, 28] proposed an axiomatic definition of divergence measure for single-valued neutrosophic sets, and develop a novel technique for order preference by similarity to ideal solution method for solving single-valued neutrosophic multi-criteria decision-making with incomplete weight information. Harish el at [16] introduced some new linguistic prioritized aggregation operators which simultaneously considers the priority among the attributes and the uncertainty in linguistic terms under the linguistic single-valued neutrosophic set. Mohamed et al. [1] proposed a novel model of three-way decisions based on neutrosophic sets, and apply AHP in neutrosophic environment.

However, the computational complexity of Neutrosophic AHP is very high. In order to reduce the computational complexity, we propose single valued neutrosophic best-worst method through extending the Best-worst method to accommodate single valued neutrosophic sets circumstances, and combine the advantages of the directed network to determine the best/worst criteria.

The rest of this paper is organized as follows: in Section 2, we review some basic definitions for the single valued neutrosophic set related to this paper. In Section 3, a new tangent similarity based on single valued neutrosophic set is given. In Section 4, the best-worst multi-criteria decision making method based on single valued neutrosophic sets is constructed, while in Section 5, an example is considered with the aim to explain in detail the proposed methodology.

2. Preliminary

Definition 2.1. ([5]) Let X be the universe of discourse, with a generic element in X denoted by x. Then, the Neutrosophic Set (NS) A in X is as follows:

$$A = \{x < T_A(x), I_A(x), F_A(x) > |x \in X\},$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the indeterminacy-membership function and the falsity-membership function, respectively,
$T_A, I_A, F_A : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.2. ([6, 18]) Let X be the universe of discourse. The Single Valued Neutrosophic Set (SVNS) A over X is an object having the form:

$$A = \{< T_A(x), I_A(x), F_A(x) > |x \in X\},$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the truth-membership function, the intermediacy-membership function and the falsity-membership function, respectively,
$T_A, I_A, F_A : X \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3. ([5]) For an SVNS A in X, the triple $A_i =< T_i, I_i, F_i >$ is called the single valued neutrosophic number (SVNN).

Definition 2.4. ([3]) Let $A_1 =< T_1, I_1, F_1 >$ and $A_2 =< T_2, I_2, F_2 >$ be two SVNNs and $\lambda > 0$; then, the basic operations are defined as follows:

(1) $A_1 + A_2 =< T_1 + T_2 - T_1T_2, I_1I_2, F_1F_2 >$.
(2) $A_1 \cdot A_2 =< T_1T_2, I_1 + I_2 - I_1I_2, F_1 + F_2 - F_1F_2 >$.
(3) $\lambda A_1 =< 1 - (1 - T_1)^\lambda, I_1^\lambda, F_1^\lambda >$.
(4) $A_1^\lambda =< T_1^\lambda, I_1^\lambda, 1 - (1 - F_1)^\lambda >$.

Definition 2.5. ([30]) $A_j =< T_j, I_j, F_j >$ be a collection of SVNSs and $W = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be an associated weighting vector. Then the Single Valued Neutrosophic Weighted Average ($SVNWA$) operator of A_j is as follows:

$$SVNWA(A_1, A_2, \cdots, A_n) = \sum_{j=1}^{n} \omega_j A_j$$
$$= (1 - \prod_{j=1}^{n}(1 - T_j)^{\omega_j}, \prod_{j=1}^{n}(I_j)^{\omega_j}, \prod_{j=1}^{n}(F_j)^{\omega_j})',$$ where ω_j is the weight of the decision maker e_j, $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

Definition 2.6. ([23]) Let A and B be any two SVNSs on $X = \{x_1, x_2, \cdots, x_n\}$. Based on the tangent function, define the following similarity measures between A and B:

$$S_1(A, B) = 1 - \frac{1}{n}\sum_{j=1}^{n} \tan[\frac{\pi}{4}\max(|T_A(x_j) - T_B(x_j)|, |I_A(x_j) - I_B(x_j)|, |F_A(x_j) - F_B(x_j)|) \quad (1)$$

$$S_2(A, B) = 1 - \frac{1}{n}\sum_{j=1}^{n} \tan[\frac{\pi}{12}(|T_A(x_j) - T_B(x_j)| + |I_A(x_j) - I_B(x_j)| + |F_A(x_j) - F_B(x_j)|)] \quad (2)$$

Definition 2.7. ([23]) For any two SVNSs A and B on the $X = \{x_1, x_2, \cdots, x_n\}$, Let the weight of the element x_j be $\omega_j (j = 1, 2, \cdots, n)$, with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, then weighted tangent similarity measures between SVNSs A and B :

$$S_{W1}(A, B) = 1 - (\sum_{j=1}^{n} \omega_j \tan[\frac{\pi}{4}\max(|T_A(x_j) - T_B(x_j)|, |I_A(x_j) - I_B(x_j)|, |F_A(x_j) - F_B(x_j)|)]) \quad (3)$$

$$S_{W2}(A, B) = 1 - (\sum_{j=1}^{n} \omega_j \tan[\frac{\pi}{12}(|T_A(x_j) - T_B(x_j)| + |I_A(x_j) - I_B(x_j)| + |F_A(x_j) - F_B(x_j)|)]) \quad (4)$$

Especially, when $\omega_j = \frac{1}{n}(j = 1, 2, \cdots, n)$, (3) and (4) can reduce to (1) and (2) respectively.

Obviously, the tangent similarity measures $S_k(k = 1, 2)$ and the weighted tangent similarity measures $S_{Wk}(k = 1, 2)$ satisfy the following four properties:

(P1) $0 \leq S_k(A, B) \leq 1$;
(P2) $S_k(A, B) = 1$ if and only if $A = B$;
(P3) $S_k(A, B) = S_k(B, A)$;
(P4) If C is a SVNS in X and $A \subseteq B \subseteq C$, then $S_k(A, C) \leq S_k(A, B)$, $S_k(A, C) \leq S_k(B, C)$.

For any two SVNSs A and B on the $X = \{x_1, x_2, \cdots, x_n\}$, some existed similarity measure for SVNSs are as following:

(i) The cosine similarities [24] : $SC_1(A, B)$

$$= \frac{1}{n}\sum_{j=1}^{n} \cos[\frac{\pi}{2} \max(|T_A(x_j) - T_B(x_j)|, |I_A(x_j) - I_B(x_j)|, |F_A(x_j) - F_B(x_j)|) \quad (5)$$

(ii) The cosine similarity[24] : $SC_2(A, B)$

$$= \frac{1}{n}\sum_{j=1}^{n} \cos[\frac{\pi}{6}(|T_A(x_j) - T_B(x_j)|+ |I_A(x_j) - I_B(x_j)| + |F_A(x_j) - F_B(x_j)|)] \quad (6)$$

(iii) The vetor similarity[22] : $S_C(A, B)$

$$= \frac{1}{n}\sum_{j=1}^{n} [(T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j)+ F_A(x_j)F_B(x_j))/(\sqrt{T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)} \sqrt{T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)})] \quad (7)$$

(iv) The similarity measure [9] : $S_{1SVNS}(A, B)$

$$= \frac{1}{n}\sum_{j=1}^{n} [(min(T_A(x_j), T_B(x_j)) + min(I_A(x_j), I_B(x_j)) + min(F_A(x_j), F_B(x_j)))/(max(T_A(x_j), T_B(x_j)) + max(I_A(x_j), I_B(x_j)) +max(F_A(x_j), F_B(x_j)))] \quad (8)$$

(v) The similarity measure[9] : $S_{2SVNS}(A, B)$

$$= \frac{1}{n}[\sum_{j=1}^{n}(1 - \frac{1}{3}(|T_A(x_j) - T_B(x_j)| + |I_A(x_j)- I_B(x_j)| + |F_A(x_j) - F_B(x_j)|)] \quad (9)$$

(vi) The similarity measure[9] : $S_{3SVNS}(A, B)$

$$=1 - [\sum_{j=1}^{n}[|T_A(x_j) - T_B(x_j)| + |I_A(x_j) - I_B(x_j)| +|F_A(x_j) - F_B(x_j)|]/ \sum_{j=1}^{n}[|T_A(x_j) + T_B(x_j)|+ |I_A(x_j) + I_B(x_j)| + |F_A(x_j) + F_B(x_j)|]] \quad (10)$$

(vii) The similarity measure[9] : $S_{4SVNS}(A, B)$

$$= \sum_{j=1}^{n}[min(T_A(x_j), T_B(x_j)) + min(I_A(x_j), I_B(x_j)) + min(F_A(x_j), F_B(x_j))]/ \sum_{j=1}^{n}[max (T_A(x_j), T_B(x_j)) + max(I_A(x_j), I_B(x_j))+ max(F_A(x_j), F_B(x_j))] \quad (11)$$

3. A new tangent function similarity

In this section, we introduce a new similarty measures between two SNVSs based on tangent function, which contain the similarity measures (Definition 2.6 and 2.7) [23] as special case.

Definition 3.1. Let A and B be any two SVNSs in $X = \{x_1, x_2, \cdots, x_n\}$. Based on the tangent function, we define the following similarity measures between A and B:

$$S_3(A, B) = 1 - (\frac{1}{n}\sum_{j=1}^{n} \tan[\frac{\pi}{4} \max(|T_A(x_j) - T_B(x_j)|^p, |I_A(x_j) - I_B(x_j)|^p, |F_A(x_j) - F_B(x_j)|^p)])^{\frac{1}{p}} \quad (12)$$

$$S_4(A, B) = 1 - (\frac{1}{n}\sum_{j=1}^{n} \tan[\frac{\pi}{12}(|T_A(x_j) - T_B(x_j)|^p+ |I_A(x_j) - I_B(x_j)|^p + |F_A(x_j) - F_B(x_j)|^p)])^{\frac{1}{p}} \quad (13)$$

where $1\leq p \leq\infty$. Especially, when $p = 1$, (12) and (13) reduce to (1) and (2) respectively.

Definition 3.2. If we consider the weight of each element x_j for $x_j \in X = \{x_1, x_2, \cdots, x_n\}$ and assume that the weight of an element x_j is $\omega_j(j = 1, 2, \cdots, n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$, then we can introduce the following weighted tangent similarity measures between SVNNs:

$$S_{W3}(A, B) = 1 - (\sum_{j=1}^{n} \omega_j \tan[\frac{\pi}{4} \max(|T_A(x_j) - T_B(x_j)|^p, |I_A(x_j) - I_B(x_j)|^p, |F_A(x_j) - F_B(x_j)|^p)])^{\frac{1}{p}}, \quad (14)$$

$$S_{W4}(A, B) = 1 - (\sum_{j=1}^{n} \omega_j \tan[\frac{\pi}{12}(|T_A(x_j) - T_B(x_j)|^p+ |I_A(x_j) - I_B(x_j)|^p + |F_A(x_j) - F_B(x_j)|^p)])^{\frac{1}{p}} \quad (15)$$

where $1\leq p \leq\infty$. Especially, when $P = 1$, (14) and (15) reduce to (3) and (4). When $\omega_j = \frac{1}{n}(j = 1, 2, \cdots, n)$, (14) and (15) can reduce to (12) and (13) respectively.

Proposition 3.1. *For two SVNSs A and B in* $X = \{x_1, x_2, \cdots, x_n\}$, *the similarity measures* $S_k(A, B)$ $S_{Wk}(A, B)$ $(k = 3, 4)$ *satisfy the following properties,*

(P1) $0 \leq S_k(A, B) \leq 1$;

(P2) $S_k(A, B) = 1$ *if and only if* $A = B$;

(P3) $S_k(A, B) = S_k(B, A)$;

(P4) If C is a SVNS in X and $A \subseteq B \subseteq C$, *then* $S_k(A, C) \leq S_k(A, B)$, $S_k(A, C) \leq S_k(B, C)$.

Proof. We only prove the case $S_k(A, B)(k = 3, 4)$. The Proof of the weighted tangent similarity measures $S_{Wk}(A, B)(k = 3, 4)$ is similar.

(P1) Due to the value of the tangent function $tan(x)$ is within [0,1] in $x \in [0, \frac{\pi}{4}]$, the similarity measure value based on the tangent function also is within [0,1] according to Eqs. (7) and Eqs. (8). Hence, there is $0 \leq S_k(A, B) \leq 1 (k = 3, 4.)$

(P2) When $A = B$, there is $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$, $j = 1, 2, \cdots, n$. Then $T_A(x_j) - T_B(x_j) = 0$, $I_A(x_j) - I_B(x_j) = 0$, $F_A(x_j) - F_B(x_j) = 0$. So $\tan(0) = 0$, Thus $S_k(A, B) = 1, (k = 3, 4)$.

When $S_k(A, B) = 1$, then $\tan(0) = 0$. This implies $T_A(x_j) - T_B(x_j) = 0$, $I_A(x_j) - I_B(x_j) = 0$, $F_A(x_j) - F_B(x_j) = 0$. Then $T_A(x_j) = T_B(x_j)$, $I_A(x_j) = I_B(x_j)$, $F_A(x_j) = F_B(x_j)$, So $A = B$.

(P3) Obviously.

(P4) If $A \subseteq B \subseteq C$, then $T_A(x_j) \leq T_B(x_j) \leq T_C(x_j)$, $I_A(x_j) \geq I_B(x_j) \geq I_C(x_j)$, and $F_A(x_j) \geq F_B(x_j) \geq F_C(x_j)$. Thus, we have the following inequalities:

$|T_A(x_j) - T_B(x_j)| \leq |T_A(x_j) - T_C(x_j)|$,
$|T_B(x_j) - T_C(x_j)| \leq |T_A(x_j) - T_C(x_j)|$,
$|I_A(x_j) - I_B(x_j)| \leq |I_A(x_j) - I_C(x_j)|$,
$|I_B(x_j) - I_C(x_j)| \leq |I_A(x_j) - I_C(x_j)|$,
$|F_A(x_j) - F_B(x_j)| \leq |F_A(x_j) - F_C(x_j)|$,
$|F_B(x_j) - F_C(x_j)| \leq |F_A(x_j) - F_C(x_j)|$,

Thus $S_k(A, C) \leq S_k(A, B)$ and $S_k(A, C) \leq S_k(B, C)$.

4. The Best-worst multi-criteria decision making method based on single valued neutrosophic sets

Definition 4.1. Let $G = (N, O, Q)$ be a directed network, where N is a node set, i.e., a collection of all criteria, O is an arc set, i.e., a collection of all arcs a_{ik} $(i, k \in N)$, and Q is a weight set, associated to all preference information. The weight of the arc $a_{ik} = (T_{ik}, I_{ik}, F_{ik}) \in Q$ indicates the relative preference degree of the criterion C_i to the criterion C_k.

Definition 4.2. ([19]) The out-degree of the node i is explained as the number of all arcs whose arrow tails are the node i (denoted as D_i^{out}). The in-degree of the node i is defined as the number of all arcs whose arrow heads are the node i (denoted as in D_i^{in}).

Definition 4.3. Select all arcs which meet the condition $T_{ij} \geq T_{ji}$, where arrow heads points to node j, arrow tails points to node i. Then calculate the number of out-degrees and in-degrees of each node. If $D_i^{out} > D_j^{out}$, or $D_i^{out} = D_j^{out}$ and $D_i^{in} < D_j^{in}$, then called C_i is preferred to C_j.

Definition 4.4. Let $C = \{C_1, C_2, \cdots, C_n\}$ be a criterion set, then an single valued neutrosophic set preference relation is represented as $A = (a_{ik})_{n\times n}$, where $a_{ik} = (T_{ik}, I_{ik}, F_{ik})$ $(i, k \in N)$ are SVNs. T_{ik} indicating the degree to which the alternative C_i is preferred to C_k, I_{ik} indicating the intermediacy degree to which the alternative C_i is preferred to C_k, F_{ik} indicating the degree to which the alternative C_i is not preferred to C_k. T_{ik}, I_{ik} and F_{ik} satisfy the conditions $T_{ik} = F_{ki}$, $T_{ik} + I_{ik} + F_{ik} \leq 3$, and $0 \leq T_{ik}, I_{ik}, F_{ik} \leq 1$.

Definition 4.5. An single valued neutrosophic set preference relation $A = (a_{ik})_{n\times n}$ with $a_{ik} = (T_{ik}, I_{ik}, F_{ik})$ is called multiplicatively consistent if satisfied the following conditions:

$$T_{ik} = \begin{cases} 0, & \text{if } T_{ij} = 0, T_{jk} = 1 \\ & \text{or } T_{ij} = 1, T_{jk} = 0; \\ \frac{T_{ij}T_{jk}}{T_{ij}T_{jk}+(1-T_{ij})(1-T_{jk})}, & \text{otherwise,} \\ & \text{for all } i \leq j \leq k. \end{cases}$$

$$I_{ik} = \begin{cases} 0, & \text{if } I_{ij} = 0, I_{jk} = 1 \\ & \text{or } I_{ij} = 1, I_{jk} = 0 \\ \frac{I_{ij}I_{jk}}{I_{ij}I_{jk}+(1-I_{ij})(1-I_{jk})}, & \text{otherwise,} \\ & \text{for all } i \leq j \leq k. \end{cases}$$

$$F_{ik} = \begin{cases} 0, & \text{if } F_{ij} = 0, F_{jk} = 1 \\ & \text{or } F_{ij} = 0, F_{jk} = 1 \\ \frac{F_{ij}F_{jk}}{F_{ij}F_{jk}+(1-F_{ij})(1-F_{jk})}, & \text{otherwise,} \\ & \text{for all } i \leq j \leq k. \end{cases}$$

The Best-Worst multi criteria decision-making method based on the single valued neutrosophic sets:

Step 1. Determine the best and worst criteria.

For a multi-criteria decision-making problem with n criteria, the decision makers provide single valued neutrosophic preference relations via pairwise comparisons over the criteria $C_j (j = 1, 2, \cdots, n)$. Let an single valued neutrosophic preference relation A be

$$A = \begin{pmatrix} (T_{11}, I_{11}, F_{11}) \cdots (T_{1n}, I_{1n}, F_{1n}) \\ (T_{21}, I_{21}, F_{21}) \cdots (T_{2n}, I_{2n}, F_{2n}) \\ \vdots \quad \vdots \quad \vdots \\ (T_{n1}, I_{n1}, F_{n1}) \cdots (T_{nn}, I_{nn}, F_{nn}) \end{pmatrix}$$

where (T_{ij}, I_{ij}, F_{ij}) indicates the relative preference of the criterion C_i to the criterion C_j with the conditions, $T_{ij}, I_{ij}, F_{ij} \in [0, 1]$.

According to Definition 4.3, We can determine the best criterion C_{best} and the worst criterion C_{worst}.

Step 2. Calculate the optimal weight vector of attributes $(\omega_1^*, \omega_2^*, \cdots, \omega_n^*)$.

For convenience, we suppose that the optimal weight vector is $W^* = (\omega_1^*, \omega_2^*, \cdots, \omega_n^*)'$, where $\omega_j^* = (\rho_j^*, \tau_j^*, \sigma_j^*)$ $(j \in N)$, ρ_j^*, τ_j^* and σ_j^* are the truth-membership, intermediacy-membership and falsity-membership degrees of importance respectively.

We first consider the truth-membership function only. An single valued neutrosophic set preference relation is consistent when Definition 4.5 is satisfied. For the inconsistent cases, according the idea of reference [29], minimize the maximum absolute differences:

$$\mid (\frac{\rho_B}{\rho_j} \cdot \frac{\rho_j}{\rho_k})/[\frac{\rho_B}{\rho_j} \cdot \frac{\rho_j}{\rho_k} + (1 - \frac{\rho_B}{\rho_j}) \cdot (1 - \frac{\rho_j}{\rho_k})] - T_{Bk} \mid$$

and

$$\mid (\frac{\rho_j}{\rho_k} \cdot \frac{\rho_k}{\rho_W})/[\frac{\rho_j}{\rho_k} \cdot \frac{\rho_k}{\rho_W} + (1 - \frac{\rho_j}{\rho_k}) \cdot (1 - \frac{\rho_k}{\rho_W})] - T_{jW} \mid$$

for all best$\leq j \leq k \leq$worst, $j, k \in N$. Thus, the following mathematical models can be established

$$\min \xi$$

s.t.

$$\begin{aligned} &\mid \frac{\rho_B \cdot \rho_j}{\rho_B \cdot \rho_j + \rho_j \cdot \rho_W - (\rho_j)^2 - \rho_B \cdot \rho_W + \rho_B \cdot \rho_j} - T_{BW} \mid \leq \xi \\ &\mid \frac{\rho_B \cdot \rho_j}{\rho_B \cdot \rho_j + \rho_j \cdot \rho_k - (\rho_j)^2 - \rho_B \cdot \rho_k + \rho_B \cdot \rho_j} - T_{Bk} \mid \leq \xi \\ &\mid \frac{\rho_j \cdot \rho_k}{\rho_j \cdot \rho_k + \rho_k \cdot \rho_W - (\rho_k)^2 - \rho_j \cdot \rho_W + \rho_j \cdot \rho_k} - T_{jW} \mid \leq \xi \end{aligned} \quad (16)$$

$\sum_{j=1}^n \rho_j = 1$, $\rho_B \geq \cdots \geq \rho_j \geq \cdots \rho_k \cdots \geq \rho_W$, $\rho_j \geq 0, \rho_k \geq 0$, for all $j, k \in N, \xi \geq 0$.

For the indeterminacy-membership function, similar to the truth-membership function, we have

$$\min \zeta$$

s.t.

$$\begin{aligned} &\mid \frac{\tau_B \cdot \tau_j}{\tau_B \cdot \tau_j + \tau_j \cdot \tau_W - (\tau_j)^2 - \tau_B \cdot \tau_W + \tau_B \cdot \tau_j} - I_{BW} \mid \leq \zeta \\ &\mid \frac{\tau_B \cdot \tau_j}{\tau_B \cdot \tau_j + \tau_j \cdot \tau_k - (\tau_j)^2 - \tau_B \cdot \tau_k + \tau_B \cdot \tau_j} - I_{Bk} \mid \leq \zeta \\ &\mid \frac{\tau_j \cdot \tau_k}{\tau_j \cdot \tau_k + \tau_k \cdot \tau_W - (\tau_k)^2 - \tau_j \cdot \tau_W + \tau_j \cdot \tau_k} - I_{jW} \mid \leq \zeta \end{aligned} \quad (17)$$

$\sum_{j=1}^n \tau_j = 1$, $\tau_B \geq \cdots \geq \tau_j \geq \cdots \tau_k \cdots \geq \tau_W$, $\tau_j \geq 0, \tau_k \geq 0$, for all $j, k \in N, \zeta \geq 0$.

By the same reason, for the falsity-membership function, we have

$$\min \eta$$

s.t.

$$\begin{aligned} &\mid \frac{\sigma_B \cdot \sigma_j}{\sigma_B \cdot \sigma_j + \sigma_j \cdot \sigma_W - (\sigma_j)^2 - \sigma_B \cdot \sigma_W + \sigma_B \cdot \sigma_j} - F_{BW} \mid \leq \eta \\ &\mid \frac{\sigma_B \cdot \sigma_j}{\sigma_B \cdot \sigma_j + \sigma_j \cdot \sigma_k - (\sigma_j)^2 - \sigma_B \cdot \varrho_k + \sigma_B \cdot \sigma_j} - F_{Bk} \mid \leq \eta \\ &\mid \frac{\sigma_j \cdot \sigma_k}{\sigma_j \cdot \sigma_k + \sigma_k \cdot \sigma_W - (\sigma_k)^2 - \sigma_j \cdot \sigma_W + \sigma_j \cdot \sigma_k} - F_{jW} \mid \leq \eta \end{aligned} \quad (18)$$

$\sum_{j=1}^n \sigma_j = 1$, $\sigma_B \geq \cdots \geq \sigma_j \geq \cdots \sigma_k \cdots \geq \sigma_W$, $\sigma_j \geq 0, \sigma_k \geq 0$, for all $j, k \in N, \eta \geq 0$.

Solving models $(*_1)$, $(*_2)$ and $(*_3)$, we obtain the optimal solutions $(\rho_1^*, \rho_2^*, \cdots, \rho_n^*)'$, $(\tau_1^*, \tau_2^*, \cdots, \tau_n^*)'$, $(\sigma_1^*, \sigma_2^*, \cdots, \sigma_n^*)'$ and ξ^*, ζ^*, η^*(which is satisfied with all constraints in the models, the difference between the optimal solution ξ^* in Model $(*_1)$, ζ^* in Model $(*_2)$, η^* in Model $(*_3)$. Then, we derive the optimal weight vector of the criteria:

$$\begin{aligned} W^* &= (\omega_1^*, \omega_2^*, \cdots, \omega_n^*)' \\ &= (< \rho_1^*, \tau_1^*, \sigma_1^* >, < \rho_2^*, \tau_2^*, \sigma_2^* >, \\ &\quad \cdots, < \rho_n^*, \tau_n^*, \sigma_n^* >)' \end{aligned}$$

Step 3. Calculate the consistency index values ($CI1$, $CI2$ and $CI3$) and the consistency ratio (CR).

The corresponding single valued neutrosophic set preference relation is not consistent when the Definition 4.5 is not satisfied. That is to say

$$T_{BW} \neq \frac{T_{Bj} \cdot T_{jW}}{T_{Bj} \cdot T_{jW} + (1 - T_{Bj}) \cdot (1 - T_{jW})},$$

or

$$I_{BW} \neq \frac{I_{Bj} \cdot I_{jW}}{I_{Bj} \cdot I_{jW} + (1 - I_{Bj}) \cdot (1 - I_{jW})},$$

or

$$F_{BW} \neq \frac{F_{Bj} \cdot F_{jW}}{F_{Bj} \cdot F_{jW} + (1 - F_{Bj}) \cdot (1 - F_{jW})}.$$

According the idea of reference [29], we introduce three new deviation variables δ, ε and ϵ, we can transform the above inequalities into equalities and obtain:

$$\begin{cases} T_{BW}+\delta \\ =\frac{(T_{Bj}-\delta)\cdot(T_{jW}-\delta)}{(T_{Bj}-\delta)\cdot(T_{jW}-\delta)+(1-(T_{Bj}-\delta))\cdot(1-(T_{jW}-\delta))} \\ I_{BW}+\varepsilon \\ =\frac{(I_{Bj}-\varepsilon)\cdot(I_{jW}-\varepsilon)}{(I_{Bj}-\varepsilon)\cdot(I_{jW}-\varepsilon)+(1-(I_{Bj}-\varepsilon))\cdot(1-(I_{jW}-\varepsilon))} \\ F_{BW}+\epsilon \\ =\frac{(F_{Bj}-\epsilon)\cdot(F_{jW}-\epsilon)}{(F_{Bj}-\epsilon)\cdot(F_{jW}-\epsilon)+(1-(F_{Bj}-\epsilon))\cdot(1-(F_{jW}-\epsilon))} \end{cases}$$

To the maximum extent, there may be $T_{Bj}=T_{jW}=T_{BW}, I_{Bj}=I_{jW}=I_{BW}, F_{Bj}=F_{jW}=F_{BW}$. Thus, the upper equation can be converted to

$$\begin{cases} T_{BW}+\delta \\ =\frac{(T_{BW}-\delta)\cdot(T_{BW}-\delta)}{(T_{BW}-\delta)\cdot(T_{BW}-\delta)+(1-(T_{BW}-\delta))\cdot(1-(T_{BW}-\delta))} \\ I_{BW}+\varepsilon \\ =\frac{(I_{BW}-\varepsilon)\cdot(I_{BW}-\varepsilon)}{(I_{BW}-\varepsilon)\cdot(I_{BW}-\varepsilon)+(1-(I_{BW}-\varepsilon))\cdot(1-(I_{BW}-\varepsilon))} \\ F_{BW}+\epsilon \\ =\frac{(F_{BW}-\epsilon)\cdot(F_{BW}-\epsilon)}{(F_{BW}-\epsilon)\cdot(F_{BW}-\epsilon)+(1-(F_{BW}-\epsilon))\cdot(1-(F_{BW}-\epsilon))} \end{cases}$$

It is equivalent to the following equations:

$$\begin{cases} 2\delta^3+(1-2T_{BW})\delta^2+[-2(T_{BW})^2+2T_{BW} \\ +1]\delta+[2(T_{BW})^3-3(T_{BW})^2+T_{BW}]=0 \\ 2\varepsilon^3+(1-2I_{BW})\varepsilon^2+[-2(I_{BW})^2+2I_{BW} \\ +1]\varepsilon+[2(I_{BW})^3-3(I_{BW})^2+I_{BW}]=0 \\ 2\epsilon^3+(1-2F_{BW})\epsilon^2+[-2(F_{BW})^2+2F_{BW} \\ +1]\epsilon+[2(F_{BW})^3-3(F_{BW})^2+F_{BW}]=0 \end{cases}$$

Solving for different values of $T_{BW}\in[0,1]$, $I_{BW}\in[0,1]$, $F_{BW}\in[0,1]$, we can find the corresponding maximum possible δ, ε and ϵ. We use these maximum values as consistency index(see Table1). We then calculate the consistency ratio, using ξ^*, ζ^*, η^* and the corresponding consistency index, as follows:

Consistency Ratio $(CR)=\max\{\frac{\xi^*}{CI1},\frac{\zeta^*}{CI2},\frac{\eta^*}{CI3}\}$ The value of CR is a measure to check if the weights are reliable or not. The smaller CR is, the better the consistency would be. Specially, when $CR=1$, the consistency of single valued neutrosophic preference relation is the worst one. If the single valued neutrosophic preference relation are unacceptable consistency, then return the inconsistent single valued neutrosophic preference relation to the decision makers for re-evaluation or amend partly preference information until they are acceptable.

Step 4. Calculate the evaluation values of all alternatives.

Let the decision matrix be $D=(d_{ij})_{m\times n}$. Based on the optimal weights and the operational rules (1) and (2) in Definition 2.4, we can calculate the evaluation values of all alternatives:

$$\begin{aligned} U(a_i) &= \sum_{j=1}^{n}(\omega_j\cdot d_{ij}) \\ &= \sum_{j=1}^{n}((\rho_j,\tau_j,\sigma_j)\cdot(T_{ij},I_{ij},F_{ij})) \\ &= \sum_{j=1}^{n}(\rho_j T_{ij},\tau_j+I_{ij}-\tau_j I_{ij},\sigma_j+F_{ij}-\sigma_j F_{ij}) \end{aligned}$$

$(i=1,2,\cdots,m.)$

Step 5. Give the ranking of all alternatives.

Calculating the tangent similarity measures between alternative $U(a_i)$ and the ideal alternative value $(1,0,0)$ by using Definition 3.1, then give the ranking of all alternatives according to the evaluation values $S_{(a_i)}$.

5. A numerical illustration

5.1. Numerical example

With the quick development of science and technology, especially in internet and telecommunication, more and more new digital products and electronic equipment bring many conveniences to us and change human life. Hence, in this digital era, hospitals can take advantage of these information technologies to develop the diversified appointment registration systems in order to assist patients and increase hospital efficiency. Usually, the adopted technologies have 6 types:

A_1: Automatic Terminal Information Service;

A_2: 114 Telephone Appointment;

A_3: the Official Hospital Website;

A_4: WeChat Public Platform;

A_5: the Bank's Self-service Terminal Appointment;

A_6: the Registration Window at the hospital (a traditional queuing method to register and on-site appointments were made).

However, the hospitals have expressed no knowledge of patients' opinions regarding those novel healthcare appointment registration systems. Therefore, the purpose of this example is to investigate

Table 1
Consistency index

T_{BW}	0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1
$CI1(\max\rho)$	0	0.0166	0.0325	0.0472	0.0599	0.0694	0.0746	0.0735	0.0633	0.0403	0
I_{BW}	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$CI2(\max\tau)$	-	0.0403	0.0633	0.0735	0.0746	0.0694	0.0599	0.0472	0.0325	0.0166	0
F_{BW}	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$CI3(\max\sigma)$	-	0.0403	0.0633	0.0735	0.0746	0.0694	0.0599	0.0472	0.0325	0.0166	0

the effectiveness of those healthcare appointment registration systems through exploring the factors influencing the patients'registration system choices.

Yu et al. [35] carried out a survey study using a questionnaire which was conducted in West China Hospital in February 2012, and analyzed available outpatients randomly selected from different hospital departments and the questionnaire was distributed and collected on-site. We apply the single valued neutrosophic best-worst method to evaluate the satisfaction of patients' different outpatient appointment registration systems of six ways in an upper first-class hospital. We invite four decision makers to determine four criteria as follows:

C_1: spend less time;

C_2: easy to operation;

C_3: convenient to pay;

C_4: queueing time or waiting time.

The weight vector of four decision maker $e_k(k = 1, 2, 3, 4)$ is $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})'$. They provide their single valued neutrosophic preference relations via pairwise comparison among the four criteria as:

$$R^{(1)} = \begin{pmatrix} <0.5000, 0.5000, 0.5000> & <0.3500, 0.6000, 0.6000> \\ <0.6000, 0.3500, 0.3500> & <0.5000, 0.5000, 0.5000> \\ <0.4000, 0.6000, 0.6000> & <0.0500, 0.9000, 0.9000> \\ <0.5000, 0.5000, 0.5000> & <0.3500, 0.6000, 0.6000> \end{pmatrix.$$

$$\begin{matrix} <0.9100, 0.0200, 0.0200> & <0.8000, 0.5000, 0.5000> \\ <0.9000, 0.0500, 0.0500> & <0.6000, 0.3500, 0.3500> \\ <0.5000, 0.5000, 0.5000> & <0.4000, 0.6000, 0.6000> \\ <0.6000, 0.4000, 0.4000> & <0.5000, 0.5000, 0.5000> \end{pmatrix}$$

$$R^{(2)} = \begin{pmatrix} <0.5000, 0.5000, 0.5000> & <0.5800, 0.3500, 0.3500> \\ <0.3500, 0.5800, 0.5800> & <0.5000, 0.5000, 0.5000> \\ <0.0500, 0.8000, 0.8000> & <0.4500, 0.5300, 0.5300> \\ <0.2000, 0.6000, 0.6000> & <0.4500, 0.5300, 0.5300> \end{matrix}$$

$$\begin{matrix} <0.9500, 0.0200, 0.0200> & <0.6000, 0.2000, 0.2000> \\ <0.4100, 0.4500, 0.4500> & <0.5300, 0.4500, 0.4500> \\ <0.5000, 0.5000, 0.5000> & <0.4000, 0.5800, 0.5800> \\ <0.5800, 0.4000, 0.4000> & <0.5000, 0.5000, 0.5000> \end{pmatrix}$$

$$R^{(3)} = \begin{pmatrix} <0.5000, 0.5000, 0.5000> & <0.5300, 0.2000, 0.2000> & <0.9000, 0.0200, 0.0200> & <0.6500, 0.3500, 0.3500> \\ <0.2000, 0.5300, 0.5300> & <0.5000, 0.5000, 0.5000> & <0.7000, 0.3000, 0.3000> & <0.5900, 0.3000, 0.3000> \\ <0.1000, 0.8000, 0.8000> & <0.3000, 0.7000, 0.7000> & <0.5000, 0.5000, 0.5000> & <0.3600, 0.5400, 0.5400> \\ <0.3500, 0.6500, 0.6500> & <0.3000, 0.5900, 0.5900> & <0.5400, 0.3600, 0.3600> & <0.5000, 0.5000, 0.5000> \end{pmatrix}$$

$$R^{(4)} = \begin{pmatrix} <0.5000, 0.5000, 0.5000> & <0.6000, 0.2500, 0.2500> & <0.9700, 0.0200, 0.0200> & <0.7900, 0.3000, 0.3000> \\ <0.2500, 0.6000, 0.6000> & <0.5000, 0.5000, 0.5000> & <0.6000, 0.4000, 0.4000> & <0.4500, 0.5500, 0.5500> \\ <0.0500, 0.9500, 0.9500> & <0.4000, 0.6000, 0.6000> & <0.5000, 0.5000, 0.5000> & <0.4000, 0.5900, 0.5900> \\ <0.3000, 0.6500, 0.6500> & <0.5500, 0.4500, 0.4500> & <0.5900, 0.4000, 0.4000> & <0.5000, 0.5000, 0.5000> \end{pmatrix}$$

Step 1. Determine the best and worst criteria.

Utilizing the Definition 2.5 *SVNWA* operator, we can aggregate four single valued neutrosophic preference relations into a group one:

$$R = \begin{pmatrix} <0.5000, 0.5000, 0.5000> & <0.5240, 0.3201, 0.3201> & <0.9318, 0.0200, 0.0200> & <0.7044, 0.3201, 0.3201> \\ <0.4267, 0.5041, 0.5041> & <0.5000, 0.5000, 0.5000> & <0.6560, 0.2280, 0.2280> & <0.5462, 0.4015, 0.4015> \\ <0.1645, 0.7772, 0.7772> & <0.3156, 0.6690, 0.6690> & <0.5000, 0.5000, 0.5000> & <0.4448, 0.5770, 0.5770> \\ <0.3468, 0.5967, 0.5967> & <0.4201, 0.5390, 0.5390> & <0.6631, 0.3276, 0.3276> & <0.5000, 0.5000, 0.5000> \end{pmatrix}$$

According to the collective single valued neutrosophic preference relation above, we can draw the directed network as shown in Fig.1 and Fig.2, and derive the ranking of the criteria.

We select those SVNs whose membership degrees $T_{ij} \geq 0.5 \ (i, j = 1, 2, 3, 4)$. Calculating the out-degrees of all arcs, we get the out-degrees of the criteria as: $D_1^{out} = 3,\ D_2^{out} = 2,\ D_3^{out} = 0,\ D_4^{out} = 1$. Thus, the ranking of criteria is $D_1^{out} > D_2^{out} > D_4^{out} > D_3^{out}$. That is to say, the best criterion is C_1, the worst one is C_3.

Step 2. Calculate the optimal weight vector of four attributes

$$\begin{aligned} W^* &= (\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)' \\ &= (<\rho_1^*, \tau_1^*, \sigma_1^*>, <\rho_2^*, \tau_2^*, \sigma_2^*>, \\ &\quad <\rho_3^*, \tau_3^*, \sigma_3^*>, <\rho_4^*, \tau_4^*, \sigma_4^*>)'. \end{aligned}$$

In order to derive the optimal weight vector of the ordered criterion set using the SVN-BWM, we can constitute the following model:

$$\min \ \xi$$

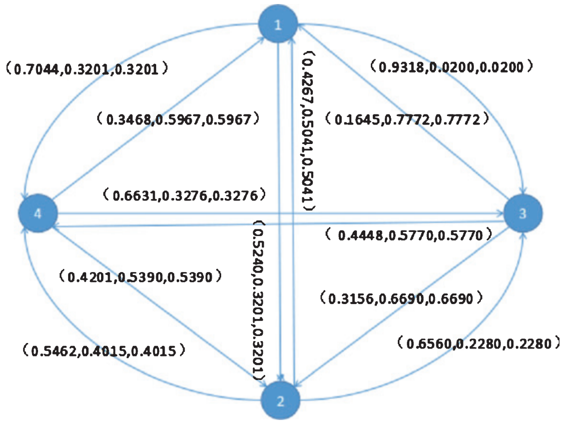

Fig. 1. The single valued neutrosophic preference relations of four criteria

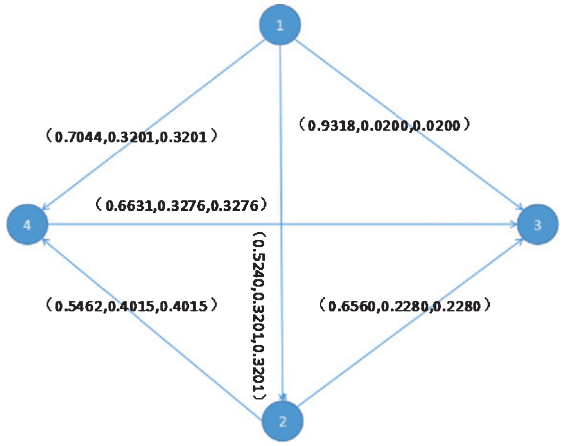

Fig. 2. The directed network of the preference relations of four criteria

s.t.

$$\begin{aligned}
&\left| \frac{\rho_2 \cdot \rho_4}{2\rho_2 \cdot \rho_4 + \rho_4 \cdot \rho_3 - (\rho_4)^2 - \rho_2 \cdot \rho_3} - T_{23} \right| \leq \xi \\
&\left| \frac{\rho_1 \cdot \rho_2}{2\rho_1 \cdot \rho_2 + \rho_2 \cdot \rho_4 - (\rho_2)^2 - \rho_1 \cdot \rho_4} - T_{14} \right| \leq \xi \\
&\left| \frac{\rho_1 \cdot \rho_2}{2\rho_1 \cdot \rho_2 + \rho_2 \cdot \rho_3 - (\rho_2)^2 - \rho_1 \cdot \rho_3} - T_{13} \right| \leq \xi \\
&\left| \frac{\rho_1 \cdot \rho_4}{2\rho_1 \cdot \rho_4 + \rho_4 \cdot \rho_3 - (\rho_4)^2 - \rho_1 \cdot \rho_3} - T_{13} \right| \leq \xi
\end{aligned} \tag{19}$$

$\rho_1 + \rho_2 + \rho_3 + \rho_4 = 1$, $\rho_1, \rho_2, \rho_3, \rho_4 \geq 0$, where $T_{23} = 0.6560$, $T_{14} = 0.7044$, $T_{13} = 0.9318$. Then we obtain the optimal solution of model (4),

$(\rho_1^*, \rho_2^*, \rho_3^*, \rho_4^*)' =$
$(0.5650, 0.4126, 0.0088, 0.0136)'$.
$\xi^* = 0.0888$.

In analogous, if we consider the degrees of indeterm-inacy-membership, the following model can be build:

$$\min\ \zeta$$

s.t.

$$\begin{aligned}
&\left| \frac{\tau_2 \cdot \tau_4}{2\tau_2 \cdot \tau_4 + \tau_4 \cdot \tau_3 - (\tau_4)^2 - \tau_2 \cdot \tau_3} - I_{23} \right| \leq \zeta \\
&\left| \frac{\tau_1 \cdot \tau_2}{2\tau_1 \cdot \tau_2 + \tau_2 \cdot \tau_4 - (\tau_2)^2 - \tau_1 \cdot \tau_4} - I_{14} \right| \leq \zeta \\
&\left| \frac{\tau_1 \cdot \tau_2}{2\tau_1 \cdot \tau_2 + \tau_2 \cdot \tau_3 - (\tau_2)^2 - \tau_1 \cdot \tau_3} - I_{13} \right| \leq \zeta \\
&\left| \frac{\tau_1 \cdot \tau_4}{2\tau_1 \cdot \tau_4 + \tau_4 \cdot \tau_3 - (\tau_4)^2 - \tau_1 \cdot \tau_3} - I_{13} \right| \leq \zeta
\end{aligned} \tag{20}$$

$\tau_1 + \tau_2 + \tau_3 + \tau_4 = 1$, $\tau_1, \tau_2, \tau_3, \tau_4 \geq 0$, where $I_{23} = 0.2280$, $I_{14} = 0.3201$, $I_{13} = 0.0200$. Then we obtain the optimal solution of model (5),

$(\tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*)' =$
$(0.0128, 0.0668, 0.8187, 0.1017)'$
$\zeta^* = 0$.

In analogous, if we consider the degrees of falsity-membership, the following model can be build:

$$\min\ \eta$$

s.t.

$$\begin{aligned}
&\left| \frac{\sigma_2 \cdot \sigma_4}{2\sigma_2 \cdot \sigma_4 + \sigma_4 \cdot \sigma_3 - (\sigma_4)^2 - \sigma_2 \cdot \sigma_3} - F_{23} \right| \leq \eta \\
&\left| \frac{\sigma_1 \cdot \sigma_2}{2\sigma_1 \cdot \sigma_2 + \sigma_2 \cdot \sigma_4 - (\sigma_2)^2 - \sigma_1 \cdot \sigma_4} - F_{14} \right| \leq \eta \\
&\left| \frac{\sigma_1 \cdot \sigma_2}{2\sigma_1 \cdot \sigma_2 + \sigma_2 \cdot \sigma_3 - (\sigma_2)^2 - \sigma_1 \cdot \sigma_3} - F_{13} \right| \leq \eta \\
&\left| \frac{\sigma_1 \cdot \sigma_4}{2\sigma_1 \cdot \sigma_4 + \sigma_4 \cdot \sigma_3 - (\sigma_4)^2 - \sigma_1 \cdot \sigma_3} - F_{13} \right| \leq \eta
\end{aligned} \tag{21}$$

$\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4 = 1$, $\sigma_1, \sigma_2, \sigma_3, \sigma_4 \geq 0$, where $F_{23} = 0.2280$, $F_{14} = 0.3201$, $F_{13} = 0.0200$. Then we obtain the optimal solution of model (6) is $(\sigma_1^*, \sigma_2^*, \sigma_3^*, \sigma_4^*)' = (0.0128, 0.0668, 0.8187, 0.1017)'$, $\eta^* = 0$.

Combining models (4), (5) and (6), we obtain the optimal weight of the criterion set:

$$W^* = (\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)' = \begin{pmatrix} < 0.5650,\ 0.0128,\ 0.0128 > \\ < 0.4126,\ 0.0668,\ 0.0668 > \\ < 0.0088,\ 0.8187,\ 0.8187 > \\ < 0.0136,\ 0.1017,\ 0.1017 > \end{pmatrix}$$

Step 3. Calculate the consistency index and the consistency ratio.

For the consistency ratio, consistency ratio = $\max\{\frac{\xi^*}{CI1}, \frac{\zeta^*}{CI2}, \frac{\eta^*}{CI3}\} = \max\{\frac{0.0888}{0.0633}, \frac{0}{0.0403}, \frac{0}{0.0403}\}$, which implies a very good consistency. If the comparisons are not fully consistent, for problems with more than three criteria, multiple optimal solutions might be founded, one of which can be selected by the decision-maker.

Step 4. Calculate the evaluation values of six alternatives.

There are six choices concerning the clinical appointment systems in a hospital. The four decision maker give their evaluations and constructs four single valued neutrosophic decision matrix:

$$D^{(1)} = \left(\begin{matrix} < 0.3500, 0.1500, 0.7000 > & < 0.8000, 0.2500, 0.2900 > \\ < 0.8500, 0.1400, 0.1900 > & < 0.6500, 0.3500, 0.5000 > \\ < 0.7400, 0.1500, 0.1400 > & < 0.5500, 0.2500, 0.1900 > \\ < 0.4500, 0.2400, 0.2500 > & < 0.3500, 0.7000, 0.8500 > \\ < 0.5000, 0.4500, 0.4500 > & < 0.7500, 0.1500, 0.2100 > \\ < 0.8500, 0.1100, 0.1700 > & < 0.1000, 0.6500, 0.6000 > \end{matrix} \right.$$

$$\left. \begin{matrix} < 0.5000, 0.4300, 0.4300 > & < 0.3500, 0.7000, 0.8500 > \\ < 0.4500, 0.2900, 0.4200 > & < 0.6500, 0.2700, 0.3500 > \\ < 0.3500, 0.7000, 0.7100 > & < 0.5500, 0.2500, 0.3500 > \\ < 0.8000, 0.2500, 0.2900 > & < 0.0500, 0.7500, 0.8500 > \\ < 0.2100, 0.6000, 0.6500 > & < 0.1100, 0.5000, 0.5700 > \\ < 0.6100, 0.2500, 0.2500 > & < 0.6100, 0.2500, 0.2500 > \end{matrix} \right)$$

$$D^{(2)} = \left(\begin{matrix} < 0.1500, 0.3500, 0.8500 > & < 0.7000, 0.2900, 0.1900 > \\ < 0.7900, 0.1500, 0.1500 > & < 0.5000, 0.4000, 0.5800 > \\ < 0.7500, 0.1400, 0.1500 > & < 0.6000, 0.2900, 0.2900 > \\ < 0.5500, 0.2500, 0.2400 > & < 0.1500, 0.8500, 0.7500 > \\ < 0.8800, 0.1000, 0.1500 > & < 0.1000, 0.7000, 0.6500 > \\ < 0.6500, 0.4300, 0.4300 > & < 0.8400, 0.2900, 0.1300 > \end{matrix} \right.$$

$$\left. \begin{matrix} < 0.6700, 0.4200, 0.2900 > & < 0.8200, 0.2800, 0.1500 > \\ < 0.2000, 0.7100, 0.8500 > & < 0.6500, 0.3000, 0.3500 > \\ < 0.7000, 0.2900, 0.1900 > & < 0.0700, 0.8000, 0.8400 > \\ < 0.5000, 0.4500, 0.4500 > & < 0.7900, 0.2500, 0.2000 > \\ < 0.2100, 0.6500, 0.5800 > & < 0.1100, 0.5500, 0.5000 > \\ < 0.6100, 0.2500, 0.2900 > & < 0.6100, 0.2500, 0.2900 > \end{matrix} \right)$$

$$D^{(3)} = \left(\begin{matrix} < 0.2000, 0.2800, 0.7100 > & < 0.6500, 0.2000, 0.2500 > \\ < 0.7500, 0.2100, 0.1400 > & < 0.5700, 0.5800, 0.3500 > \\ < 0.8000, 0.1900, 0.2100 > & < 0.6200, 0.2000, 0.2500 > \\ < 0.5000, 0.3000, 0.3000 > & < 0.2000, 0.7100, 0.7100 > \\ < 0.8000, 0.1500, 0.1100 > & < 0.1000, 0.5800, 0.7000 > \\ < 0.4500, 0.4300, 0.4300 > & < 0.8400, 0.1900, 0.1300 > \end{matrix} \right.$$

$$\left.\begin{array}{ll}
< 0.6000, 0.3200, 0.3800 > & < 0.7900, 0.1500, 0.2800 > \\
< 0.2800, 0.7500, 0.7000 > & < 0.7000, 0.3500, 0.2500 > \\
< 0.6500, 0.2000, 0.2500 > & < 0.1500, 0.8500, 0.8000 > \\
< 0.5000, 0.4500, 0.4500 > & < 0.8000, 0.2100, 0.2500 > \\
< 0.2100, 0.5800, 0.6000 > & < 0.1100, 0.5700, 0.5000 > \\
< 0.6100, 0.2500, 0.1900 > & < 0.6100, 0.2500, 0.1900 >
\end{array}\right)$$

$$D^{(4)} = \left(\begin{array}{ll}
< 0.2800, 0.2700, 0.7500 > & < 0.6700, 0.1900, 0.2000 > \\
< 0.8300, 0.1900, 0.2100 > & < 0.4500, 0.5000, 0.4000 > \\
< 0.8500, 0.2100, 0.1900 > & < 0.5500, 0.1900, 0.2000 > \\
< 0.6100, 0.3000, 0.3000 > & < 0.2800, 0.7500, 0.7000 > \\
< 0.9000, 0.1700, 0.1000 > & < 0.1000, 0.6000, 0.5800 > \\
< 0.5700, 0.4300, 0.4300 > & < 0.8400, 0.2000, 0.1300 >
\end{array}\right.$$

$$\left.\begin{array}{ll}
< 0.5300, 0.3800, 0.3200 > & < 0.6000, 0.3500, 0.2700 > \\
< 0.1500, 0.8500, 0.7500 > & < 0.4500, 0.3500, 0.3000 > \\
< 0.6700, 0.1900, 0.2000 > & < 0.2000, 0.8400, 0.7500 > \\
< 0.5000, 0.4500, 0.4500 > & < 0.8500, 0.2000, 0.1500 > \\
< 0.2100, 0.7000, 0.7000 > & < 0.1100, 0.5000, 0.5500 > \\
< 0.6100, 0.2500, 0.2000 > & < 0.6100, 0.2500, 0.2000 >
\end{array}\right)$$

Utilizing the Definition 2.4 *SVNWA* operator, we can aggregate four single valued neutrosophic matrix into a group one:

$$D = \left(\begin{array}{ll}
< 0.2489, 0.2510, 0.7503 > & < 0.7114, 0.2291, 0.2291 > \\
< 0.8087, 0.1701, 0.1701 > & < 0.5490, 0.4489, 0.4489 > \\
< 0.7899, 0.1701, 0.1701 > & < 0.5811, 0.2291, 0.2291 > \\
< 0.5313, 0.2711, 0.2711 > & < 0.2489, 0.7503, 0.7503 > \\
< 0.8622, 0.1294, 0.1294 > & < 0.1000, 0.6308, 0.6308 > \\
< 0.5490, 0.4300, 0.4300 > & < 0.8400, 0.2291, 0.1300 >
\end{array}\right.$$

$$\left.\begin{array}{ll}
< 0.5702, 0.3489, 0.3489 > & < 0.7303, 0.2510, 0.2510 > \\
< 0.2489, 0.7503, 0.7503 > & < 0.5985, 0.3096, 0.3096 > \\
< 0.7114, 0.2291, 0.2291 > & < 0.1196, 0.8090, 0.8090 > \\
< 0.5000, 0.4500, 0.4500 > & < 0.8008, 0.1992, 0.1992 > \\
< 0.2100, 0.6308, 0.6308 > & < 0.1100, 0.5291, 0.5291 > \\
< 0.6100, 0.2500, 0.2291 > & < 0.6100, 0.2500, 0.2291 >
\end{array}\right)$$

Table 2
The results of similarity measures and decision making results

Similarities	A_1	A_2	A_3	A_4	A_5	A_6	Best Alternative
S_1[23]	0.4925	0.6614	0.6560	0.4721	0.5949	0.6359	A_2
S_2[23]	0.8200	0.8739	0.8710	0.8073	0.8445	0.8681	A_2
SC_1[24]	0.5902	0.7943	0.7925	0.5642	0.7180	0.7659	A_2
SC_2[24]	0.9372	0.9687	0.9673	0.9284	0.9528	0.9658	A_2
S_C[22]	0.7042	0.9970	0.9962	0.9803	0.9907	0.9973	A_4
S_{1SVNS}[9]	0.3714	0.5494	0.5431	0.3442	0.4639	0.5257	A_2
S_{2SVNS}[9]	0.7733	0.8402	0.8367	0.7577	0.8036	0.8330	A_2
S_{3SVNS}[9]	0.5416	0.7091	0.7039	0.5121	0.6338	0.6892	A_2
S_{4SVNS}[9]	0.3714	0.5494	0.5431	0.3442	0.4639	0.5257	A_2
$S_3(P=2)$	0.7114	0.8634	0.8622	0.6903	0.8091	0.8435	A_2
$S_4(P=2)$	0.9050	0.9542	0.9536	0.8980	0.9357	0.9478	A_2
$S_3(P=3)$	0.8304	0.9435	0.9427	0.8121	0.9073	0.9309	A_2
$S_4(P=3)$	0.9439	0.9812	0.9809	0.9379	0.9691	0.9770	A_2

We get the overall values of all patients:

$$U(A_i) = \sum_{j=1}^{4}(\omega_j \cdot d_{ij})$$

$$= \sum_{j=1}^{4}((\rho_j, \tau_j, \sigma_j) \cdot (T_{ij}, I_{ij}, F_{ij}))$$

$$= \sum_{j=1}^{4}(\rho_j T_{ij}, \tau_j + I_{ij} - \tau_j I_{ij}, \sigma_j + F_{ij} - \sigma_j F_{ij})$$

$(i = 1, 2, \cdots, 6)$

$U(A_1) = < 0.4019, 0.0211, 0.0610 >$,

$U(A_2) = < 0.5843, 0.0318, 0.0318 >$,

$U(A_3) = < 0.5824, 0.0362, 0.0362 >$,

$U(A_4) = < 0.3816, 0.0543, 0.0543 >$,

$U(A_5) = < 0.5099, 0.0496, 0.0496 >$,

$U(A_6) = < 0.5554, 0.0346, 0.0218 >$.

Step 5. Give the ranking of six alternatives.

Take $p = 2$, the tangent similarity measures between alternative A_i and the ideal alternative value $A = (1, 0, 0)$ are calculated by using the formula (13) in Definition 3.1,

$S(A_1, A) = 0.9050$, $S(A_2, A) = 0.9542$,
$S(A_3, A) = 0.9536$, $S(A_4, A) = 0.8980$,
$S(A_5, A) = 0.9357$, $S(A_6, A) = 0.9478$.

Thus, the ranking order of the assessments of six appointment registration systems: $A_2 > A_3 > A_6 > A_5 > A_1 > A_4$. Therefore, the most effective alternative is A_2.

5.2. Comparison analysis and discussion

In order to show the validity of the similarity measure proposed in this paper, the comparison analysis for some similarity measures is summarized in Table 2. For the data in the example, the best alternative is A_2 using the proposed similarity measure $S_2(p = 2, 3)$ and $S_3(p = 2, 3)$ in this paper. Therefore, the choice of the optimal scheme is independent of the parameter p. Meanwhile, these results are the same A_2 using eight exits similarity measures ([9, 23, 24]). The best alternative is A_4 using the similarity S_C([22]). Therefore, the most effective alternative is A_2. However, there is some shortcomings for the proposed similarity measure in this paper. The amount of calculation will increase when the parameter p is large.

Most of young people would select some convenient and effective methods, such as the Official Hospital Website and WeChat Public Platform. Because they can make appointment by mobilephone at anywhere with the internet connection. From this point of view, we agree with the opinion in Ref. [35]. Their conclusions are: convenience was a major motivation for patients' use of appointment registration systems. So Hospitals must improve the design and promotion of healthcare appointment registration systems to better facilitate their use.

6. Conclusions

A new tangent similarity based on single valued neutrosophic set is propose, which contain tangent similarity [23] as a special case. We extend best-worst method to single valued neutrosophic set circumstances and combine the advantages the directed network, proposed the single valued neutrosophic set best-worst method to solve multi-criteria decision-making problems. We first aggregate the individual single valued neutrosophic preference relation provided by the decision makers into a collective single valued neutrosophic preference relation by the single valued neutrosophic weighted average operator. Afterwards, we draw the directed network according to the collective single valued neutrosophic preference relation, then design an algorithm to identify the best and the worst criteria through computing the out-degrees and in-degrees of the directed network. The optimal weight vector of attributes are calculated using some mathematical models. A consistency ratio is proposed for the best-worst multi-criteria decision making method based on the single valued neutrosophic sets to check the reliability of the comparisons. The final score of an alternative is obtained by aggregating different criteria of the alternative. The best alternative is selected by the propose tangent similarity measures between the alternative and the ideal alternative. We used the proposed method to solve real-word decision-making problems with indeterminate and inconsistent information.

Some new methods for multi-criteria decision making problems based on interval neutrosophic sets and type-2 neutrosophic sets have been proposed. D. Rani et al. [4] gave some modified results of the subtraction and division operations on interval neutrosophic sets. H. Garg el at [14] proposed non-linear programming method for multi-criteria decision making problems under interval neutrosophic set environment. F. Smarandache et al. [7] proposed an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. Nagarajan et al. [8] proposed a new perspective on traffic control management using triangular interval type-2 fuzzy sets and interval neutrosophic sets. Because advantage of interval neutrosophic sets, we will extending the best-worst multi-criteria decision making method based on the single valued neutrosophic sets to interval neutrosophic sets circumstances in our further work.

Acknowledgments

The authors would like to thank Editor-in-Chief Reza Langari PhD., Associate Editor Harish Garg Dr. and anonymous reviewers for their valuable comments and suggestions.

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