Article

Multi-Attribute Decision Making Method Based on Aggregated Neutrosophic Set

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Abstract: Multi-attribute decision-making refers to the decision-making problem of selecting the optimal alternative or sorting the scheme when considering multiple attributes, which is widely used in engineering design, economy, management and military, etc. But in real application, the attribute information of many objects is often inaccurate or uncertain, so it is very important for us to find a useful and efficient method to solve the problem. Neutrosophic set is proposed from philosophical point of view to handle inaccurate information efficiently, and a single-valued neutrosophic set (SVNS) is a special case of neutrosophic set, which is widely used in actual application fields. In this paper, a new method based on single-valued neutrosophic sets aggregation to solve multi-attribute decision making problem is proposed. Firstly, the neutrosophic decision matrix is obtained by expert assessment, a score function of single-valued neutrosophic sets (SVNSs) is defined to obtain the positive ideal solution (PIS) and the negative ideal solution (NIS). Then all alternatives are aggregated based on TOPSIS method to make decision. Finally numerical examples are given to verify the feasibility and rationality of the method.

Keywords: multi-attribute decision making; single-valued neutrosophic set; aggregation; TOPSIS

1. Introduction

With the rapid development of human society, the social environment has become more and more complex, which makes us have difficulties in making actual decision [1–9]. Therefore, in recent years, more and more attention has been paid to multi-attribute decision making [10–16]. In the real world, the attribute information of many objects is often inaccurate or uncertain [17–23], which makes the decision progress difficult. In view of this problem, Zadeh put forward the concept of fuzzy set (FS) in 1965 [24], which made up for the lack of flexibility of classical set theory to some extent. But the traditional FS can not describe the decision information well in some conditions, Atanassov proposed the intuitionistic fuzzy set (IFS) [25] on the basis of FS. An IFS is given by $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle\}$ where $\mu_A(x), \nu_A(x) \in [0,1]$ denote the degree of membership and non-membership of x to A respectively. For instance, A is an IFS in $X = \{1, 2, 3\}$, $A = (0.6, 0.3)/1 + (0.7, 0.1)/2 + (0.5, 0.4)/3$. Because IFSs take into account the information of membership, non-membership and hesitation simultaneously, compared with FSs, IFSs can describe the fuzzy nature [26–29] of the objective world more precisely. IFS attracted attention of many scholars, rich achievements were made in the study of intuitionistic fuzzy sets [30–33]. Torra [34] proposed another generalized form of the fuzzy set, named the hesitant fuzzy set (HFS) in 2009. HFS allowed the membership of each element in the domain to belong to a certain set which was combined by a number of different values. The element of the HFS is named as the hesitant fuzzy element (HFE) by Xia et al. [35], the mathematical symbol of HFS is expressed by Xia as $A = \{\langle x, h_A(x)\rangle | x \in X\}$ where $h_A(x)$ is set of some values in $[0,1]$, denoting the possible membership degrees of element $x \in X$ to the set A. $h_A(x)$ is called a HFE. For example, B is a HFS in $X = \{1, 2, 3\}$

and $B = \{0.6, 07, 0.8\}/1 + \{0.2\}/2 + \{0.3, 0.9\}/3$. The addition and multiplication operations on the hesitant fuzzy element are also given by Xia. The hesitant intuitionistic fuzzy sets (HIFSs) is proposed by Zhou et al. [36]. Zhou proposed that the HIFSs here were the generalization of the IFSs. The group decision-making method under the condition of the uncertain intuitionistic fuzzy priority relation matrix and aggregation operator was also given by Zhou in that paper.

The IFS considers both the truth-membership $T_A(x)$ and the falsity membership $F_A(x)$ with $T_A(x), F_A(x) \in [0,1]$ and $0 \leq T_A(x) + F_A(x) \leq 1$. For IFSs, the indeterminacy is $1 - T_A(x) - F_A(x)$ by default. The IFS can handle incomplete information but can hardly process inaccurate information.

In order to better describe uncertain information, Smarandache introduced Neutrosophy in 1995 [37,38]. Neutrosophy is a kind of philosophy which studies the nature, scope, and origin of neutralities, as well as their joint parts with different ideational spectra [37]. Neutrosophic set (NS) was also propose by Smarandache. NS is a very powerful tool which generalizes the concept of the classic set, fuzzy set, interval-valued fuzzy set, IFS, interval-valued intuitionistic fuzzy set [39], dialetheist set, paraconsistent set, and tautological set [37]. A NS A is defined on a universe of discourse u. An element x in set A is expressed as $x = x(t,i,f) \in A$, where t is a truth-membership function, i is an indeterminacy-membership function, and f is a falsity-membership function, t, i, and f are the real standard or non-standard subsets of $]0^-, 1^+[$. For a NS, the indeterminacy is denoted explicitly in contrast of that of IFSs. The indeterminacy can be divided into more parts so as to better express the inaccurate information [37]. However neutrosophic sets are hard to use in actual occasions. So Wang et al. [40] proposed the concept of single-valued neutrosophic sets (SVNSs). For instance, $C = (0.7, 0.5, 0.2)$ is a SVNS, in which the truth-membership $t = 0.7$, the indeterminacy $i = 0.5$ and the falsity-membership $f = 0.2$. Because SVNS are easy to express the inaccurate information, SVNSs are widely used in actual situations, such as in medicine [41], image processing [42], multi-criteria decision-making [43–47], fault diagnosis [41,48,49], etc.

Neutrosophic set has many advantages in handling uncertain information. A lot of researches are conducted on it. In [50], SVNS is used to express the decision information, weighted average operator, TOPSIS method is adopted to propose the multi-criteria decision making method. Peng introduced relevant concepts about the interval neutrosophic set [51], and the multi-criteria decision making problem is also analyzed by combining the ranking method. Jiang proposed a new method to measure the similarity between SVNSs using Dempster-Shafer evidence theory [52]. Ye proposed a method to make decision based on the weighted correlation coefficient of SVNSs in [44], and proposed another method to make decision in multi-criteria environment based on single valued neutrosophic cross-entropy in [45]. Ye utilized cross entropy between the ideal solution and an alternative to get the rank of all alternatives according to the values of cross entropy and to choose the most suitable one(s). A decision method for the interval neutrosophic set is proposed based on cross entropy by Tian et al. [53]. Deli applied bipolar neutrosophic sets to multi-criteria decision situations [54]. The TOPSIS method can be effectively combined with SVNSs to accomplish multi-attribute decision making problems. By the way, the key point of TOPSIS is that the ideal alternative should have the shortest distance from the PIS and the farthest distance from the NIS. The standard TOPSIS to new multi-attribute decision-making called simplified-TOPSIS is proposed by Elhassouny [55], which simplifies the process of the classical TOPSIS and get the same result.

This paper mainly introduces a new multi-attribute decision-making method based on SVNSs. First, based on the given decision matrix, use score function to get the PIS and the NIS. Second, aggregate all alternatives to get aggregated neutrosophic set. Last, use TOPSIS method to rank all alternatives to make decision.

The paper is organized as follow. In Section 2, we present some preliminaries. Section 3 will focus on the proposed multi-criteria decision method based on aggregated neutrosophic set. Afterwards, two illustrative examples are introduced in Section 4. In the final section, conclusions are drawn.

2. Preliminaries

A brief review of some preliminaries will be given in the following part.

2.1. Neutrosophic Sets

Neutrosophic set is an efficient tool to deal with the uncertain information. In [37], Smarandache gave the definitions about a NS as follow:

Definition 1. *Y is a universal space of point (objects) with a generic element of Y denoted by y. A neutrosophic set $N \subset Y$ is consist of a truth-membership function $T_N(y)$, an indeterminacy-membership function $I_N(y)$ and a falsity-membership function $F_N(y)$. $T_A(y)$, These three components are real standard or real nonstandard subset of $[0^-, 1^+]$. So that all three components*

$$T_N(y) \rightarrow [0^-, 1^+]$$

$$I_N(y) \rightarrow [0^-, 1^+]$$

$$F_N(y) \rightarrow [0^-, 1^+]$$

$T_N(y), I_N(y)$ and $F_N(y)$ are related as follow:

$$0^- \leq \sup T_N(y) + \sup I_N(y) + \sup F_N(y) \leq 3^+.$$

Definition 2. *A neutrosophic set N has its' complement which is characterized by N^c and is characterized as*

$$T_N^c(y) = 1^+ - T_N(y)$$

$$I_N^c(y) = 1^+ - I_N(y)$$

$$F_N^c(y) = 1^+ - F_N(y)$$

for every y in Y.

2.2. SVNS

For NS is difficult to be used in practical occasions, Wang [40] proposed the concept of single-valued neutrosophic sets (SVNSs), which can be easily used in actual situations. The definition of SVNSs are introduced as follow:

Definition 3. *Let Y be a space of points (objects) which generic elements in Y denoted by y. A SVNS N is characterized by a truth-membership function T_N, an indeterminacy-membership function I_N, and a falsity-membership function F_N with $T_N, I_N, F_N \in [0, 1]$.*

When Y is continuous, an SVNS N can be expressed as:

$$N = \int_X \langle T_N(y), I_N(y), F_N(y) \rangle | y, \quad y \in Y. \tag{1}$$

When Y is discrete, an SVNS N can be characterized as:

$$N = \sum_y \langle T_N(y), I_N(y), F_N(y) \rangle | y, \quad y \in Y \tag{2}$$

For convenience, a SVNS is usually denoted by its' simplified symbol $N = \langle T_N(Y), I_N(Y), F_N(Y) \rangle$ for all $y \in Y$.

Definition 4. *A SVNS N has its' complement N^c which is characterized as*

$$T_N^c(y) = F_N(y)$$

$$I_N^c(y) = 1 - I_N(y)$$

$$F_N^c(y) = T_N(y)$$

for all y in Y.

Definition 5. *A SVNS N is contained in the other SVNS M, $N \subseteq M$, if and only if*

$$T_N(y) \leq T_M(y)$$

$$I_N(y) \geq I_M(y)$$

$$F_N(y) \geq F_M(y)$$

for all y in Y.

Definition 6. *Two SVNSs N and M are equal, written as $N = M$, if and only if $N \subseteq M$ and $N \subseteq M$.*

2.3. Score Function

Definition 7. *Assume $A = (T_A, I_A, F_A)$ be a single valued neutrosophic number, then the score function [56] is defined as*

$$S(A) = \frac{T_A - I_A - F_A}{3}. \tag{3}$$

Score function is a very useful tool to illustrate which neutrosophic number is better. For example, $A_1 = (0.5, 0.3, 0.4)$, $A_2 = (0.6, 0.4, 0.5)$, use the score function and get $S(A_1) = -\frac{2}{15}$, $S(A_2) = -\frac{1}{6}$, $S(A_1) > S(A_2)$, this result is in line with intuition, in this case, A_1 is better than A_2.

2.4. Distance between Two Neutrosophic Sets

Assume that there two NSs, shows as follow:

$$M = \langle t_M(x_i), i_M(x_i), f_M(x_i) \rangle \quad i = 1, 2, \dots, n \tag{4}$$

$$N = \langle t_N(x_i), i_N(x_i), f_N(x_i) \rangle \quad i = 1, 2, \dots, n \tag{5}$$

Then the Hamming distance between M and N is defined as follow:

$$d_1(M, N) = \sum_{i=1}^{n} \{|t_M(x_i) - t_N(x_i)| + |i_M(x_i) - i_N(x_i)| + |f_M(x_i) - f_N(x_i)|\} \tag{6}$$

the standard Hamming distance between M and N is defined as follow:

$$d_2(M, N) = \frac{1}{3n} \sum_{i=1}^{n} \{|t_M(x_i) - t_N(x_i)| + |i_M(x_i) - i_N(x_i)| + |f_M(x_i) - f_N(x_i)|\} \tag{7}$$

the Euclid distance between M and N is defined as follow:

$$d_3(M, N) = \sqrt{\sum_{i=1}^{n} \{(t_M(x_i) - t_N(x_i))^2 + (i_M(x_i) - i_N(x_i))^2 + (f_M(x_i) - f_N(x_i))^2\}} \tag{8}$$

the standard Euclid distance between M and N is defined as follow:

$$d_4(M, N) = \sqrt{\frac{1}{3n} \sum_{i=1}^{n} \{(t_M(x_i) - t_N(x_i))^2 + (i_M(x_i) - i_N(x_i))^2 + (f_M(x_i) - f_N(x_i))^2\}} \qquad (9)$$

3. An Improved Multi-Criteria Decision Making Method

Multi-criteria decision making often faces complex environments. At present, multi-criteria decision making methods mostly handle multiple attributes of an alternative separately for decision making. Few studies try to aggregate all neutrosophic sets of one alternative before decision making to simplify the decision progress. Based on the idea of information fusion, the NSs can be aggregated before using TOPSIS method to make a decision. In this way, decision progress can be simplified. A multi-attribute decision making method based on aggregated SVNSs is introduced in this part.

step1: Construct neutrosophic decision matrix.

The single-valued neutrosophic decision matrix is constructed based on expert assessments. For instance, when an expert is asked the opinion of an alternative A_1 with respect to a criterion C_1, the expert may say that possibility in which the alternative is good is 0.5 and false degree is 0.4 and the expert is not sure is 0.2. For the NS, it can be expressed as $d_{11} = \langle 0.5, 0.2, 0.4 \rangle$.

Assume there are m alternatives and n criteria. The neutrosophic decision matrix D is expressed as follow:

$$\begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix}$$

where $d_{ij} = (T_{ij}, I_{ij}, F_{ij})$, $1 \leq i \leq m$, $1 \leq j \leq n$, T_{ij}, I_{ij}, F_{ij} are the truth-membership degree, the indeterminacy-membership degree, the falsity-membership degree of alternative A_i with respect to criterion j.

step2: Determine the PIS and the NIS.

In this step, score function mentioned above is utilized to get the PIS A^* and the NIS A^{-*}.

Assume $A^* = (d_1^*, d_2^*, \dots, d_n^*)$, among them $S(d_j^*) = \max_i \{S(d_{ij})\}$, $j = 1, 2, \dots, n$, and $A^{-*} = (d_1^{-*}, d_2^{-*}, \dots, d_n^{-*})$ among them $S(d_j^{-*}) = \min_i \{S(d_{ij})\}$, $j = 1, 2, \dots, n$, the score function can be used to illustrate that how good or bad a neutrosophic set is. A simple example is given as follow:

Example 1. *Assume there is a decision matrix which is obtained from expert assessments showing as follow:*

$$\begin{bmatrix} (0.4, 0.2, 0.3) & (0.4, 0.2, 0.4) & (0.5, 0.2, 0.2) \\ (0.6, 0.1, 0.2) & (0.6, 0.1, 0.2) & (0.2, 0.2, 0.5) \\ (0.7, 0.1, 0.1) & (0.5, 0.3, 0.3) & (0.4, 0.3, 0.2) \end{bmatrix}$$

Specifically, let us consider the neutrosophic set $(0.4, 0.2, 0.3)$, $T_{11} = 0.4$, $I_{11} = 0.2$, $F_{11} = 0.3$, *use Equation (3) to obtain the score function as follows:*

$$S(d_{11}) = \frac{0.4 - 0.2 - 0.3}{3} = -\frac{1}{30}$$

In this way, scores of all neutrosophic sets can be obtained, neutrosophic sets which have the highest or the lowest scores will be choose to constitute the PIS(A^*) and the NIS(A^{-*}). For example, in the first column of the matrix:

$$S(d_{11}) = -\frac{1}{30}, S(d_{21}) = \frac{1}{10}, S(d_{31}) = \frac{1}{6}$$

In this case, d_{31} and d_{11} are chosen to constitute the first column of A^* and A^{-*} respectively. A^* and A^{-*} can be obtained as follow:

$$A^* = ((0.7, 0.1, 0.1), (0.6, 0.1, 0.2), (0.5, 0.2, 0.2))$$

$$A^{-*} = ((0.4, 0.2, 0.3), (0.4, 0.2, 0.4), (0.2, 0.2, 0.5))$$

step3: Aggregate the SVNSs.

In this step, all the attributes of a neutrosophic set are aggregated. Arithmetic average method is used to combine neutrosophic set. $A_j^a = (t_j^a, i_j^a, f_j^a)$ is the aggregated neutrosophic set, $1 \le j \le m$, where

$$t_j^a = \frac{t_{j1} + t_{j2} + \ldots + t_{jn}}{n}$$

$$i_j^a = \frac{i_{j1} + i_{j2} + \ldots + i_{jn}}{n}$$

$$f_j^a = \frac{f_{j1} + f_{j2} + \ldots + f_{jn}}{n}$$

n is the number of attributes.

This method is used to aggregate all the alternatives, the PIS and the NIS. So $A_j^a = (t_j^a, i_j^a, f_j^a)$, $1 \le j \le m$, $A^{*a} = (t^{*a}, i^{*a}, f^{*a})$ and $A^{-*a} = (t^{-*a}, i^{-*a}, f^{-*a})$ are obtained.

step4: Multi-criteria decision making based on TOPSIS method

In this step, TOPSIS method is utilized to finish a multi-attribute neutrosophic decision making. In step 3, the aggregated SVNSs, the aggregated PIS and NIS are obtained. Then the distance for each aggregated SVNS between A^{*a} and between A^{-*a} can be calculated by Euclid distance Equation (8). The distance between A_j^a and A^{*a} is calculated as follows:

$$D_j^* = \sqrt{(t_j^a - t^{*a})^2 + (i_j^a - i^{*a})^2 + (f_j^a - f^{*a})^2} \quad j = 1, 2, \ldots, m \tag{10}$$

The distance between A_j^a and A^{-*a} is got as follows:

$$D_j^{-*} = \sqrt{(t_j^a - t^{-*a})^2 + (i_j^a - i^{-*a})^2 + (f_j^a - f^{-*a})^2} \quad j = 1, 2, \ldots, m \tag{11}$$

The relative closeness T_j based on TOPSIS is got as follows:

$$T_j = \frac{D_j^{-*}}{(D_j^{-*} + D_j^*)} \quad j = 1, 2, \ldots, m \tag{12}$$

Obviously, the bigger the value of T_j is, the farther the alternative is from the NIS, and the closer the alternative is to the PIS, and vice versa. The ranked of all the alternatives is obtained in the descending order of the value of T_j.

4. Illustrative Example

In this section, two examples are used to demonstrate the application of the proposed method.

Example 2. *Let us consider the decision-making problem originated from [56]. There is an investment company which wants to invest some money in a schools. There are four possible schools to invest the money: $\{A_1, A_2, A_3, A_4\}$. The investment company is going to choose one school to invest with respect to the following four criteria: $\{C_1, C_2, C_3, C_4\}$.*

Step1: *Experts evaluate all the four possible schools according to the four attributes. According to the evaluation results, the following SVNS decision matrix is shown in Table 1:*

Table 1. Solution matrix for SVNS.

	C_1	C_2	C_3	C_4
A_1	$(0.6, 0.3, 0.5)$	$(0.5, 0.7, 0.6)$	$(0.7, 0.6, 0.5)$	$(0.5, 0.5, 0.6)$
A_2	$(0.6, 0.4, 0.5)$	$(0.4, 0.5, 0.6)$	$(0.3, 0.5, 0.6)$	$(0.4, 0.5, 0.6)$
A_3	$(0.5, 0.6, 0.7)$	$(0.7, 0.2, 0.8)$	$(0.7, 0.6, 0.3)$	$(0.4, 0.4, 0.5)$
A_4	$(0.4, 0.3, 0.2)$	$(0.5, 0.4, 0.3)$	$(0.6, 0.7, 0.2)$	$(0.4, 0.3, 0.2)$

Step2: *Then the score function is utilized to get A^* and A^{-*}. The calculation results are expressed in Table 2:*

Table 2. Scores matrix for SVNS.

	C_1	C_2	C_3	C_4
A_1	$-\frac{1}{15}$	$-\frac{4}{15}$	$-\frac{2}{15}$	$-\frac{1}{5}$
A_2	$-\frac{1}{10}$	$-\frac{7}{30}$	$-\frac{4}{15}$	$-\frac{7}{30}$
A_3	$-\frac{4}{15}$	$-\frac{1}{10}$	$-\frac{1}{15}$	$-\frac{1}{6}$
A_4	$-\frac{1}{30}$	$-\frac{1}{15}$	$-\frac{1}{10}$	$-\frac{1}{30}$

From Table 2, $S(d_{41}) = -\frac{1}{30}$ is the maximum in the first column, $S(d_{42})$, $S(d_{33})$, $S(d_{44})$ is the maximum in column $2, 3, 4$ respectively. The neutrosophic PIS A^* is composed of $d_{41}, d_{42}, d_{33}, d_{44}$. Similarly, the d_{ij} which has the minimum score in each column is chosen to constitute the neutrosophic NIS A^{-*}. The PIS A^* and the NIS A^{-*} can be obtained as follow:

$$A^* = ((0.4, 0.3, 0.2), (0.5, 0.4, 0.3), (0.7, 0.6, 0.3), (0.4, 0.3, 0.2))$$

$$A^{-*} = ((0.5, 0.6, 0.7), (0.5, 0.7, 0.6), (0.3, 0.5, 0.6), (0.4, 0.5, 0.6))$$

Step3: *Aggregate all the alternatives, A^* and A^{-*}. For example:*

$$A^* = ((0.4, 0.3, 0.2), (0.5, 0.4, 0.3), (0.7, 0.6, 0.3), (0.4, 0.3, 0.2))$$

The $T_{A^*}, I_{A^*}, F_{A^*}$ of A_c^* can be obtained as follow:

$$T_{A^*} = \frac{0.4 + 0.5 + 0.7 + 0.4}{4} = 0.500$$

$$I_{A^*} = \frac{0.3 + 0.4 + 0.6 + 0.3}{4} = 0.400$$

$$F_{A^*} = \frac{0.2 + 0.3 + 0.3 + 0.2}{4} = 0.250$$

$$A_c^* = (T_{A^*}, I_{A^*}, F_{A^*}) = (0.500, 0.400, 0.250)$$

All aggregated neutrosophic sets are obtained in the same way and shown as follow:

$$A_{1c} = (0.575, 0.525, 0.550)$$

$$A_{2c} = (0.425, 0.475, 0.575)$$

$$A_{3c} = (0.575, 0.450, 0.575)$$

$$A_{4c} = (0.475, 0.425, 0.225)$$

$$A_c^* = (0.500, 0.400, 0.250)$$

$$A_c^{-*} = (0.425, 0.575, 0.625)$$

Step4: *All alternatives are ranked by the TOPSIS method. Euclid distances between alternatives and the PIS are calculated as follow:*

$$d(A_{1c}, A_c^*) = \sqrt{(T_{A_{1c}} - T_{A_c^*})^2 + (I_{A_{1c}} - I_{A_c^*})^2 + (F_{A_{1c}} - F_{A_c^*})^2} = 0.334$$

$$d(A_{2c}, A_c^*) = \sqrt{(T_{A_{2c}} - T_{A_c^*})^2 + (I_{A_{2c}} - I_{A_c^*})^2 + (F_{A_{2c}} - F_{A_c^*})^2} = 0.342$$

$$d(A_{3c}, A_c^*) = \sqrt{(T_{A_{3c}} - T_{A_c^*})^2 + (I_{A_{3c}} - I_{A_c^*})^2 + (F_{A_{3c}} - F_{A_c^*})^2} = 0.337$$

$$d(A_{4c}, A_c^*) = \sqrt{(T_{A_{4c}} - T_{A_c^*})^2 + (I_{A_{4c}} - I_{A_c^*})^2 + (F_{A_{4c}} - F_{A_c^*})^2} = 0.043$$

Euclid distances between alternatives and the NIS are calculated as follow:

$$d(A_{1c}, A_c^{-*}) = \sqrt{(T_{A_{1c}} - T_{A_c^{-*}})^2 + (I_{A_{1c}} - I_{A_c^{-*}})^2 + (F_{A_{1c}} - F_{A_c^{-*}})^2} = 0.175$$

$$d(A_{2c}, A_c^{-*}) = \sqrt{(T_{A_{2c}} - T_{A_c^{-*}})^2 + (I_{A_{2c}} - I_{A_c^{-*}})^2 + (F_{A_{2c}} - F_{A_c^{-*}})^2} = 0.112$$

$$d(A_{3c}, A_c^{-*}) = \sqrt{(T_{A_{3c}} - T_{A_c^{-*}})^2 + (I_{A_{3c}} - I_{A_c^{-*}})^2 + (F_{A_{3c}} - F_{A_c^{-*}})^2} = 0.202$$

$$d(A_{4c}, A_c^{-*}) = \sqrt{(T_{A_{4c}} - T_{A_c^{-*}})^2 + (I_{A_{4c}} - I_{A_c^{-*}})^2 + (F_{A_{4c}} - F_{A_c^{-*}})^2} = 0.430$$

The relative closeness are calculated as follow:

$$T_1 = \frac{d(A_{1c}, A_c^{-*})}{d(A_{1c}, A_c^{-*}) + d(A_{1c}, A_c^*)} = 0.344$$

$$T_2 = \frac{d(A_{2c}, A_c^{-*})}{d(A_{2c}, A_c^{-*}) + d(A_{2c}, A_c^*)} = 0.247$$

$$T_3 = \frac{d(A_{3c}, A_c^{-*})}{d(A_{3c}, A_c^{-*}) + d(A_{3c}, A_c^*)} = 0.375$$

$$T_4 = \frac{d(A_{4c}, A_c^{-*})}{d(A_{4c}, A_c^{-*}) + d(A_{4c}, A_c^*)} = 0.909$$

Rank the T_i, $i = 1, 2, 3, 4$ in descending order, and get $T_4 > T_3 > T_1 > T_2$. In this case, A_4 is chosen as the ideal solution. This example shows that by using the proposed method decision results can be easily obtained.

Example 3. *For a further understanding and comparison of our approach, another problem is considered. The data in this example originated from [57]. Let us suppose that decision makers intend to select the most suitable tablet from the four chosen tablets (A_1, A_2, A_3, A_4) by considering six attributes namely: Features C_1, Hardware C_2, Display C_3, Communication C_4, Affordable Price C_5, Customer care C_6. By using the presented method, the problem is handled by the following steps.*

　　Step1: *Assume that $A_i(A_1, A_2, A_3, A_4)$ are alternatives with respect to six criteria $(C_1, C_2, C_3, C_4, C_5, C_6)$. Neutrosophic decision matrix are shown in Table 3:*

Table 3. Neutrosophic decision matrix.

	C_1	C_2	C_3
A_1	$(0.864, 0.136, 0.081)$	$(0.853, 0.147, 0.092)$	$(0.800, 0.200, 0.150)$
A_2	$(0.667, 0.333, 0.277)$	$(0.727, 0.273, 0.219)$	$(0.667, 0.333, 0.277)$
A_3	$(0.880, 0.120, 0.067)$	$(0.887, 0.113, 0.064)$	$(0.834, 0.166, 0.112)$
A_4	$(0.667, 0.333, 0.277)$	$(0.735, 0.265, 0.195)$	$(0.768, 0.232, 0.180)$
	C_4	C_5	C_6
A_1	$(0.704, 0.296, 0.241)$	$(0.823, 0.177, 0.123)$	$(0.864, 0.136, 0.081)$
A_2	$(0.744, 0.256, 0.204)$	$(0.652, 0.348, 0.293)$	$(0.608, 0.392, 0.336)$
A_3	$(0.779, 0.256, 0.204)$	$(0.811, 0.189, 0.109)$	$(0.850, 0.150, 0.092)$
A_4	$(0.727, 0.273, 0.221)$	$(0.791, 0.209, 0.148)$	$(0.808, 0.192, 0.127)$

Step2: *Score function are used to calculate and get the score matrix in Table 4.*

Table 4. Score matrix.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.216	0.205	0.150	0.056	0.174	0.216
A_2	0.019	0.078	0.019	0.095	0.004	-0.040
A_3	0.231	0.237	0.185	0.106	0.171	0.203
A_4	0.019	0.092	0.119	0.078	0.145	0.163

From Table 4, the neutrosophic PIS can be got as follow:

$$A^* = ((0.880, 0.120, 0.067), (0.887, 0.113, 0.064), (0.834, 0.166, 0.112),$$

$$(0.779, 0.256, 0.204), (0.823, 0.177, 0.123), (0.864, 0.136, 0.081))$$

and the neutrosophic NIS is shown as follow:

$$A^{-*} = ((0.667, 0.333, 0.277), (0.727, 0.273, 0.219), (0.667, 0.333, 0.277),$$

$$(0.704, 0.296, 0.241), (0.652, 0.348, 0.293), (0.608, 0.392, 0.336)).$$

Step3: *Neutrosophic sets aggregation is done in this step. After calculation, results are shown as follows:*

$$A_{1c} = (0.818, 0.182, 0.128)$$

$$A_{2c} = (0.677, 0.323, 0.268)$$

$$A_{3c} = (0.840, 0.166, 0.108)$$

$$A_{4c} = (0.749, 0.251, 0.191)$$

$$A_c^* = (0.844, 0.161, 0.109)$$

$$A_c^{-*} = (0.671, 0.329, 0.274)$$

Step4: *In the last step, Euclid distances between $A_{ic}, i = 1, 2, 3, 4$ and A_c^* and between $A_{ic}, i = 1, 2, 3, 4$ and A_c^{-*} are obtained, and the relative closeness of each alternative $T_i(i = 1, 2, 3, 4)$ is obtained. Euclid distances between alternatives and the neutrosophic PIS are as follow:*

$$d(A_{1c}, A_c^*) = 0.038 \quad d(A_{2c}, A_c^*) = 0.282$$

$$d(A_{3c}, A_c^*) = 0.007 \quad d(A_{4c}, A_c^*) = 0.154$$

Euclid distances between alternatives and the neutrosophic NIS are below:

$$d(A_{1c}, A^{-*}) = 0.254 \quad d(A_{2c}, A^{-*}) = 0.011$$

$$d(A_{3c}, A^{-*}) = 0.288 \quad d(A_{4c}, A^{-*}) = 0.138$$

So the relative closeness of each alternative to the ideal solution can be easily obtained as follow:

$$T_1 = 0.870 \quad T_2 = 0.038$$

$$T_3 = 0.976 \quad T_4 = 0.473$$

the rank of them can be obtained: $T_3 > T_1 > T_4 > T_2$. Obviously, A_3 is the best solution.

 According to the results, method which uses aggregated SVNSs in this paper is more simplified than that proposed by Pramanik [57]. Additionally, the Pramanik's results are shown as follow:

$$T_1 = 0.8190 \quad T_2 = 0,1158$$

$$T_3 = 0.8605 \quad T_4 = 0.4801$$

The same rank of alternatives are got, but in our approach the difference between the data is greater and the results are clearer than that in Pramanik's. The presented method is different with Pramanik's method in handling the attributes. In Pramanik's method, attribute weight is used to revise the neutrosophic set. In the presented method neutrosophic sets of different attributes are aggregated to one neutrosophic set, which simplifies the decision-making process and makes the process more efficient. In this way, multi-criteria decision making can be easily made.

 To verify the rationality and usefulness of the presented method, correlation coefficient method [44] and cross-entropy method [45] are used to calculate the same decision problem in Table 3. In Table 3, B_i refers to the correlation between alternative A_i and the PIS A^*, and E_i refers to the distance between A_i and the PIS A^*. The comparison of presented method with these three methods are shown in Table 5.

Table 5. Comparison with other methods.

Methods	Indexes for Decision Making	Rank of Indexes	The Chosen Alternative
Elhassouny's [57]	$T_1 = 0.8190, T_2 = 0,1158, T_3 = 0.8605, T_4 = 0.4801$	$T_3 > T_1 > T_4 > T_2$	A_3
correlation coefficient [44]	$B_1 = 0.959, B_2 = 0.846, B_3 = 0.970, B_4 = 0.917$	$B_3 > B_1 > B_4 > B_2$	A_3
cross-entropy [45]	$E_1 = 0.530, E_2 = 0.966, E_3 = 0.466, E_4 = 0.763$	$E_3 < E_1 < E_4 < E_2$	A_3
Presented method	$T_1 = 0.870, T_2 = 0.038, T_3 = 0.976, T_4 = 0.473$	$T_3 > T_1 > T_4 > T_2$	A_3

5. Conclusions

 In this paper, a multi-attribute decision method used aggregated neutrosophic set and TOPSIS is proposed. In our method, arithmetic average method is used to aggregate neutrosophic sets, and aggregated sets can be used by TOPSIS method to get a final rank. The proposed approach could reduce the computation since it firstly aggregate the neutrosophic sets before ranking, so it is useful in real application with high requirements of real-time. Two examples have demonstrated the rationality and feasibility of our approach. However, this method also has some limitations, averaging the validity of the criteria may lead to the loss of some criteria information. If the criteria information is especially emphasized in the decision process, this presented method may be not so appropriate to be used. In the future, more work will be done on the score function to make the decision making process more accurate, and the geometric average method is also need to be checked. Continuous work in the application of complex decision-making problems such as group decision-making problems and other domains such as fuzzy system is also need to be done.

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