RESEARCH ARTICLE | SEPTEMBER 20 2023

An average method for solving MOLPP using different kind of mean techniques in neutrosophic environment FREE

S. N. Mohamed Assarudeen ; N. Suresh Kumar

AIP Conf. Proc. 2831, 030010 (2023)
https://doi.org/10.1063/5.0163130

An Average Method for Solving MOLPP Using Different Kind of Mean Techniques in Neutrosophic Environment

S.N. Mohamed Assarudeen[1,a)] and N. Suresh Kumar[1,b)]

[1)] *Department of Mathematics, Islamiah College, Vaniyambadi, Affiliated to Thiruvalluvar University, Vellore, Tamil Nadu, India.*

[a)] *Corresponding author:assarudeen@gmail.com*

[b)] *Electronic mail: suresh2016msc@gmail.com*

Abstract: This research article proposed three distinct Average Methods with various mean technique is used to determine a solution of the Multi-Objective Linear Programming Problem (MOLPP) in Neutrosophic environment. A Numerical illustration is given to explain the effectiveness of the methods.

Keywords: Multi-objective Linear programming, Simplex technique, Neutrosophic Triangular Number

INTRODUCTION

Zimmerman [17] first applied fuzzy programming to MOLPP. A MOLPP was solved by Chandra Sen [2] by using Mean technique. Mohamed Assarudeen and Junaid Basha [7] solved the Fuzzy MOLPP using various kind of fuzzy mean techniques. The outline of the paper is as follows: The next section is basic tools. The third section is Chandra Sen's method in Neutrosophic environment. The fourth section involves average method for Neutrosophic MOLPP using different kind of mean technique. The fifth section describes the algorithm of Average method for Neutrosophic MOLPP. The sixth section presents some illustrative examples to put on view, how the approach can be applied. The last section summarizes the conclusions.

BASIC TOOLS

Were call some necessary definitions and results to make out the main thought.

Definition A Neutrosophic set $(\tilde{A}^*)$ on an initial universe X is presented by three characterizations namely true value$(\tau_{\tilde{A}^*})$, in-determinant value $(i_{\tilde{A}^*})$ and false value $(\omega_{\tilde{A}^*})$ so that $\tau_{\tilde{A}^*}, i_{\tilde{A}^*}, \omega_{\tilde{A}^*} : X \to]^{-}0,1^{+}[$. Thus X can be designed as : $\left(\{x, \tau_{\tilde{A}^*}(x), i_{\tilde{A}^*}(x), \omega_{\tilde{A}^*}(x)\}: x \in X\right)$ with $-0 < \sup \tau_{\tilde{A}^*}(x) + \sup i_{\tilde{A}^*}(x) + \sup \omega_{\tilde{A}^*}(x) \leq 3+$. Here $1^{+} = 1 + \delta$ where 1 is standard part and δ is non-standard part. Similarly, $0^{-} = 0 - \delta$. Then on-standard set $]^{-}0,1^{+}[$ is basically practiced in philosophical ground and because of the difficulty to adopt it in real field, the standard subset of $]^{-}0,1^{+}[$. i.e., $[0,1]$ is applicable in real neutrosophic environment.

Definition A Neutrosophic set $(\tilde{A}^*)$ over X is called a single valued triangular neutrosophic set when $\tau_{\tilde{A}^*}(x), i_{\tilde{A}^*}(x),$ and $\omega_{\tilde{A}^*}(x)$ are real standard elements of $[0,1]$ only for $x \in X$. Thus, a single valued triangular neutrosophic set X is defined as $:(\{x, \tau_{\tilde{A}^*}(x), i_{\tilde{A}^*}(x), \omega_{\tilde{A}^*}(x)\}: x \in X)$ with $\tau_{\tilde{A}^*}(x), i_{\tilde{A}^*}(x),$ and $\omega_{\tilde{A}^*}(x) \in [0,1]$, $0 < \sup \tau_{\tilde{A}^*}(x) + \sup i_{\tilde{A}^*}(x) + \sup \omega_{\tilde{A}^*}(x) \leq 3$.

Definition A Single Valued Triangular Neutrosophic Number (SVTNN) is defined by $\tilde{A}^* = \{(a_1^l, b_1^m, c_1^u); \tau_a, i_a, \omega_a\}$ whose three membership functions for the truth, indeterminacy, and a falsity of X are given by

International Conference on Smart Technologies and Applications (ICSTA 2022)
AIP Conf. Proc. 2831, 030010-1–030010-10; https://doi.org/10.1063/5.0163130
Published by AIP Publishing. 978-0-7354-4651-9/$30.00

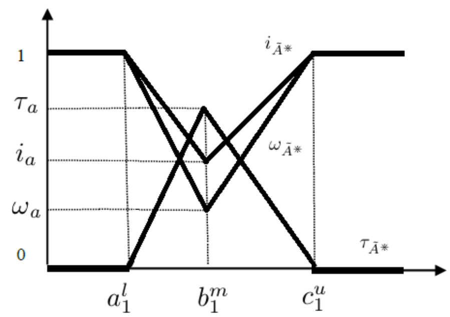

Figure 1: SVTNN

$$\tilde{A}^* = \{(a_1^l, b_1^m, c_1^u); \tau_a, i_a, \omega_a\}$$

$$\tau_{\tilde{A}^*}(x) = \begin{cases} \dfrac{(x - a^l)\tau_a}{b_1^m - a_1^l} & (a_1^l \leq x \leq b_1^m) \\ \tau_a & (x = b_1^m) \\ \dfrac{(c_1^u - x)\tau_a}{c_1^u - b_1^m} & (b_1^m \leq x \leq c_1^u) \\ 0 & otherwise, \end{cases}$$

$$i_{\tilde{A}^*}(x) = \begin{cases} \dfrac{(b_1^m - x)i_a}{b_1^m - a_1^m} & (a_1^l \leq x < b_1^m) \\ i_a & (x = b_1^m) \\ \dfrac{(x - c_1^u)i_a}{c_1^u - b_1^l} & (b_1^m \leq x < c_1^u) \\ 1 & otherwise, \end{cases}$$

$$\omega_{\tilde{A}^*}(x) = \begin{cases} \dfrac{(b_1^m - x)\omega_a}{b_1^m - a_1^l} & (a_1^l \leq x < b_1^m) \\ \omega_a & (x = b_1^m) \\ \dfrac{(x - c_1^u)\omega_a}{c_1^u - b_1^m} & (b_1^m \leq x < c_1^u) \\ 1 & otherwise, \end{cases}$$

where, $0 \leq \tau_{\tilde{A}^*} + i_{\tilde{A}^*} + \omega_{\tilde{A}^*} \leq 3, x \in \tilde{A}^*$. Additionally, when $a_1^l > 0$, $\tilde{A}^*$ is called a non-negative SVTNN. Similarly, when $a_1^l < 0$, $\tilde{A}^*$ becomes a negative SVTNN.

Definition Let $\tilde{a}^n = \{(a_1^l, b_1^m, c_1^u); \tau_a, i_a, \omega_a\}$ and $\tilde{b}^n = \{(a_2^l, b_2^m, c_2^u); \tau_b, i_b, \omega_b\}$ be two SVTNN's and $\gamma \neq 0$. Then

1. $\tilde{a}^n + \tilde{b}^n = \{(a_1^l + a_2^l, b_1^m + b_2^m, c_1^u + c_2^u); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}$
2. $\tilde{a}^n - \tilde{b}^n = \{(a_1^l - c_2^u, b_1^m - b_2^m, c_1^u - a_2^l); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}$
3. $\tilde{a}^n . \tilde{b}^n = \{Min(a_1^l a_2^l, a_1^l c_2^u, c_1^u a_2^l, c_1^u c_2^u), b_1^m b_2^m, Max\,(a_1^l a_2^l, a_1^l c_2^u, c_2^u a_2^l, c_1^u c_2^u); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\}$
4. $\dfrac{\tilde{a}^n}{\tilde{b}^n} = \left\{Min\left(\dfrac{a_1^l}{a_2^l}, \dfrac{a_1^l}{c_2^u}, \dfrac{c_1^u}{a_2^l}, \dfrac{c_1^u}{c_2^u}\right), \dfrac{b_1^m}{b_2^m}, Max\left(\dfrac{a_1^l}{a_2^l}, \dfrac{a_1^l}{c_2^u}, \dfrac{c_1^u}{a_2^l}, \dfrac{c_1^u}{c_2^u}\right); \tau_a \wedge \tau_b, i_a \vee i_b, \omega_a \vee \omega_b\right\}$

5. $\gamma\tilde{a}^n = \begin{cases} \{(\gamma a_1^l, \gamma b_1^m, \gamma c_1^u); \tau_a, i_a, \omega_a\}, (\gamma > 0) \\ \{(\gamma c_1^u, \gamma b_1^m, \gamma a_1^l); \tau_a, i_a, \omega_a\}, (\gamma < 0). \end{cases}$

Neutrosophic Multi-Objective Linear Programming Problem

The following Neutrosophic MOLPP with m constraints and n variables is given as follows:

$$Max(Min)\tilde{Z}_i = (\tilde{c}\tilde{x}), \ \ \forall i = 1,2,...n. \tag{1}$$

Subject to: $A\tilde{x} \leq \tilde{b}$,

where $\tilde{x}$ is the non-negative SVTNN,

$A = [a_{ij}]_{m\times n}$ is the co-efficient matrix,

$\tilde{b} = [\tilde{b}_1, \tilde{b}_2, ..., b_m]^t$ is the neutrosophic available resource vector,

$\tilde{c} = [\tilde{c}_1, \tilde{c}_2, \tilde{c}_3, ...\tilde{c}_n]^t$ is the objective coefficient vector and

$\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, ...\tilde{x}_n]^t$ is the neutrosophic decision variable vector.

CHANDRA SEN'S METHOD IN NEUTROSOPHIC ENVIRONMENT

In this section, first all the objective need to be maximized or minimized individually by simplex method in Neutrosophic environment are given. All objective functions are solved then we obtained following equations:

$$\begin{gathered} Max\ \tilde{Z}_1 = \tilde{\varphi}_1 \\ Max\ \tilde{Z}_2 = \tilde{\varphi}_2 \\ \dots \quad \dots \\ \dots \quad \dots \\ Max\ \tilde{Z}_r = \tilde{\varphi}_r \\ Max\ \tilde{Z}_{r+1} = \tilde{\varphi}_{r+1} \\ Max\ \tilde{Z}_{r+2} = \tilde{\varphi}_{r+2} \\ \dots \quad \dots \\ \dots \quad \dots \\ Max\ \tilde{Z}_s = \tilde{\varphi}_s \end{gathered}$$

where $\tilde{\varphi}_1, \tilde{\varphi}_2, ..., \tilde{\varphi}_s$ are the optimal value of the objective function. These functions are used to form single objective using the form

$$Max\ \tilde{Z} = \sum_{i=1}^{r} \frac{\tilde{Z}_i}{|\tilde{\varphi}_i|} - \sum_{i=r+1}^{s} \frac{\tilde{Z}_i}{|\tilde{\varphi}_i|},$$

where $|\tilde{\varphi}_i|$ is the non-negative SVNN subject to the constraints which remain same as in equation (1). Then this single objective linear programming problem is to be optimized.

AVERAGE METHOD FOR NEUTROSOPHIC MOLPP

Here we discussing about the various mean techniques

Different Kind of Mean Techniques:

Let $\widetilde{M}_1 = \max(|\tilde{\varphi}_i|),\ i = 1,2,...,r$ and $\widetilde{M}_2 = \min(|\tilde{\varphi}_i|),\ i = r+1, r+2, ..., s$.

I. Neutrosophic Contra-harmonic Mean (NCHM) Technique:

$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NCHM}$, where $NCHM = \frac{\frac{\widetilde{M_1}^2 + \widetilde{M_2}^2}{2}}{\frac{\widetilde{M_1} + \widetilde{M_2}}{2}}$.

II. Neutrosophic Arithmetic Mean (NAM) Technique:

$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NAM}$, where $NAM = \frac{\widetilde{M}_1 + \widetilde{M}_2}{2}$.

III. Neutrosophic Harmonic Mean (NHM) Technique:

$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NHM}$, where $NHM = \frac{2}{\frac{1}{\widetilde{M}_1} + \frac{1}{\widetilde{M}_2}}$.

ALGORITHM OF AVERAGE FOR NEUTROSOPHIC MOLPP

The Algorithm for Neutrosophic MOLPP for Average Method is constructed as explained below:

Step 1: Using simplex table find the initial basic feasible solution.

Step 2: If feasibility obtained in step 1 then go to step 3, other wise use dual simplex method to remove in-feasibility and go to step 2.

Step 3: Solve the simplex iterations to get the optimal value.

Step 4: Each optimal values of a function should assign with a name, say $(\tilde{\varphi}_i)$, where $i = 1,2,\ldots,r$ for Maximization objective function and $i = r+1, r+2, \ldots, s$ for Minimization objective function.

Step 5: Calculate $\widetilde{M}_1$ and $\widetilde{M}_2$, where $\widetilde{M}_1 = \max(|\tilde{\varphi}_i|)$, $i = 1,2,\ldots,r$ and $\widetilde{M}_2 = \min(|\tilde{\varphi}_i|)$, $i = r+1, r+2, \ldots, s$.

Step 6: Calculate the values NCHM, NAM and NHM as defined in Section (4).

Step 7: Using the Mean technique, determine the combined objective function for each technique.

Step 8: Now using Simplex method solve the each new objective functions using given constraints.

$$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NCHM},$$

$$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NAM},$$

$$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NHM}.$$

Step 9: Then find the optimal solution for the newly formed objective functions.

NUMERICAL ILLUSTRATION

Example: Solve the following Neutrosophic MOLPP

$Max\ \tilde{Z}_1 = \{(1,1,1);.2,.4,.4\}\tilde{x}_1 + \{(1,2,3);.5,.3,.2\}\tilde{x}_2$
$Max\ \tilde{Z}_2 = \{(1,1,1);.3,.5,.2\}\tilde{x}_1$
$Max\ \tilde{Z}_3 = \{(-3,-2,-1);.4,.3,.3\}\tilde{x}_1 + \{(-4,-3,-2);.2,.5,.3\}\tilde{x}_2$
$Max\ \tilde{Z}_4 = \{(-1,-1,-1);.6,.2,.2\}\tilde{x}_2$
subject to $6\tilde{x}_1 + 8\tilde{x}_2 \leq \{(46,47,48);.3,.2,.5\}$
$\tilde{x}_1 + \tilde{x}_2 \geq \{(1,3,5);.4,.4,.2\}$
$\tilde{x}_1 \leq \{(3,4,5);.6,.2,.2\}$
$\tilde{x}_2 \leq \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0);1,0,0\}$

Objective function 1:

$Max\ \tilde{Z}_1 = \{(1,1,1);.2,.4,.4\}\tilde{x}_1 + \{(1,2,3);.5,.3,.2\}\tilde{x}_2$

subject to $6\tilde{x}_1 + 8\tilde{x}_2 \leq \{(46,47,48);.3,.2,.5\}$

$\tilde{x}_1 + \tilde{x}_2 \geq \{(1,3,5);.4,.4,.2\}$

$\tilde{x}_1 \leq \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 \leq \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0);1,0,0\}$

Then $Max\ \tilde{Z}_1 = \{(1,1,1);.2,.4,.4\}\tilde{x}_1 + \{(1,2,3);.5,.3,.2\}\tilde{x}_2 + \{(0,0,0);1,0,0\}\tilde{x}_3 + \{(0,0,0);1,0,0\}\tilde{x}_4$

$+\{(0,0,0);1,0,0\}\tilde{x}_5 + \{(0,0,0);1,0,0\}\tilde{x}_6.$

subject to $6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{x}_3 = \{(46,47,48);.3,.2,.5\}$

$-\ \tilde{x}_1 - \tilde{x}_2 + \tilde{x}_4 = \{(-5,-3,-1);.4,.4,.2\}$

$\tilde{x}_1 + \tilde{x}_5 = \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 + \tilde{x}_6 = \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \{(0,0,0);1,0,0\}.$

The detailed computation is given in Table 1 and Table 2.

Table 1: Objective function – 1 – Iteration - 1

$\tilde{C}_j$	$\{(1,1,1);.2,.4,.4\}$	$\{(1,2,3);.5,.3,.2\}$	$\{(0,0,0);1,0,0\}$	$\{(0,0,0);1,0,0\}$	$\{(0,0,0);1,0,0\}$	$\{(0,0,0);1,0,0\}$	
B.V.	$\tilde{x}_1$	$\tilde{x}_2\downarrow$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	Solution
$\tilde{x}_3$	6	8	1	0	0	0	$\{(46,47,48);.3,.2,.5\}$
$\leftarrow\tilde{x}_4$	-1	-1	0	1	0	0	$\{(-5,-3,-1);.4,.4,.2\}$
$\tilde{x}_5$	1	0	0	0	1	0	$\{(3,4,5);.6,.2,.2\}$
$\tilde{x}_6$	0	1	0	0	0	1	$\{(2,3,4);.4,.3,.3\}$
$\tilde{Z}_j - \tilde{C}_j$	$\{(-1,-1,-1);.2,.4,.4\}$	$\{(-3,-2,-1);.5,.3,.2\}$	$\{(0,0,0);1,0,0\}$	$\{(0,0,0);1,0,0\}$	$\{(0,0,0);1,0,0\}$	$\{(0,0,0);1,0,0\}$	
B.V.	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4\downarrow$	$\tilde{x}_5$	$\tilde{x}_6$	Solution
$\tilde{x}_3$	-2	0	1	8	0	0	$\{(6,23,40);.3,.4,.5\}$
$\tilde{x}_2$	1	1	0	-1	0	0	$\{(1,3,5);.4,.4,.2\}$
$\tilde{x}_5$	1	0	0	0	1	0	$\{(3,4,5);.6,.2,.2\}$
$\leftarrow\tilde{x}_6$	-1	0	0	1	0	1	$\{(-3,0,3);.4,.4,.3\}$
$\tilde{Z}_j - \tilde{C}_j$	$\{(0,1,2);.5,.4,.4\}$	$\{(0,0,0);.5,.3,.2\}$	$\{(0,0,0);.5,.3,.2\}$	$\{(-3,-2,-1);.5,.3,.2\}$	$\{(0,0,0);.5,.3,.2\}$	$\{(0,0,0);.5,.3,.2\}$	
B.V.	$\tilde{x}_1\downarrow$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	Solution
$\leftarrow\tilde{x}_3$	6	0	1	0	0	-1	$\{(-18,23,64);.3,.4,.5\}$
$\tilde{x}_2$	0	1	0	0	0	1	$\{(-2,3,8);.4,.4,.2\}$
$\tilde{x}_5$	1	0	0	0	1	0	$\{(3,4,5);.6,.2,.2\}$
$\tilde{x}_6$	-1	0	0	1	0	1	$\{(-3,0,3);.4,.4,.3\}$
$\tilde{Z}_j - \tilde{C}_j$	$\{(-1,-1,-1);.2,.4,.4\}$	$\{(0,0,0);.5,.3,.2\}$	$\{(0,0,0);.5,.3,.2\}$	$\{(0,0,0);.5,.3,.2\}$	$\{(0,0,0);.5,.3,.2\}$	$\{(1,2,3);.5,.3,.2\}$	

Table 2: Objective function – 1 – Iteration – 2

$\tilde{C}_j$	$\left\{\begin{matrix}(1,1,1); \\ .2,.4,.4\end{matrix}\right\}$	$\left\{\begin{matrix}(1,2,3); \\ .5,.3,.2\end{matrix}\right\}$	$\left\{\begin{matrix}(0,0,0); \\ 1,0,0\end{matrix}\right\}$	$\left\{\begin{matrix}(0,0,0); \\ 1,0,0\end{matrix}\right\}$	$\left\{\begin{matrix}(0,0,0); \\ 1,0,0\end{matrix}\right\}$	$\left\{\begin{matrix}(0,0,0); \\ 1,0,0\end{matrix}\right\}$	
B.V.	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	Solution
$\tilde{x}_1$	1	0	$\frac{1}{6}$	0	0	$-\frac{8}{6}$	$\left\{\begin{matrix}(-3,3.83.8,10.6); \\ .3,.4,.5\end{matrix}\right\}$
$\tilde{x}_2$	0	1	0	0	0	1	$\left\{\begin{matrix}(-2,3,8); \\ .4,.4,.3\end{matrix}\right\}$
$\tilde{x}_5$	0	0	$-\frac{1}{6}$	0	1	$\frac{8}{6}$	$\left\{\begin{matrix}(-7.6,0.2,8); \\ .6,.2,.2\end{matrix}\right\}$
$\tilde{x}_4$	0	0	$\frac{1}{6}$	1	0	$-\frac{1}{3}$	$\left\{\begin{matrix}(-3,0,3); \\ .4,.4,.3\end{matrix}\right\}$
$\tilde{Z}_j - \tilde{C}_j$	$\left\{\begin{matrix}(0,0,0); \\ .5,.3,.2\end{matrix}\right\}$	$\left\{\begin{matrix}(0,0,0); \\ .5,.3,.2\end{matrix}\right\}$	$\left\{\begin{matrix}\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right); \\ .5,.3,.2\end{matrix}\right\}$	$\left\{\begin{matrix}(0,0,0); \\ .5,.3,.2\end{matrix}\right\}$	$\left\{\begin{matrix}(0,0,0); \\ .5,.3,.2\end{matrix}\right\}$	$\left\{\begin{matrix}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3}\right); \\ .5,.3,.2\end{matrix}\right\}$	

The optimal solution as

$$\tilde{x}_1 = \{(-3,3.8,10.6);.2,.4,.4\}$$
$$\tilde{x}_2 = \{(-2,3,8);.4,.4,.3\}$$
$$Max\ \widetilde{Z_1} = \{(-5,9.8,34.6);.2,.4,.4\}.$$

Objective function 2:

$$Max\ \tilde{Z}_1 = \{(1,1,1);.3,.5,.2\}\tilde{x}_1$$

subject to $6\tilde{x}_1 + 8\tilde{x}_2 \leq \{(46,47,48);.3,.2,.5\}$

$$\tilde{x}_1 + \tilde{x}_2 \geq \{(1,3,5);.4,.4,.2\}$$
$$\tilde{x}_1 \leq \{(3,4,5);.6,.2,.2\}$$
$$\tilde{x}_2 \leq \{(2,3,4);.4,.3,.3\}$$
$$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0);1,0,0\}$$

Then $Max\ \tilde{Z}_2 = \{(1,1,1);.3,.2,.5\}\tilde{x}_1 + \{(0,0,0);1,0,0\}\tilde{x}_2 + \{(0,0,0);1,0,0\}\tilde{x}_3 + \{(0,0,0);1,0,0\}\tilde{x}_4$

$$+\{(0,0,0);1,0,0\}\tilde{x}_5 + \{(0,0,0);1,0,0\}\tilde{x}_6.$$

subject to $6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{x}_3 = \{(46,47,48);.3,.2,.5\}$

$$-\ \tilde{x}_1 - \tilde{x}_2 + \tilde{x}_4 = \{(-5,-3,-1);.4,.4,.2\}$$
$$\tilde{x}_1 + \tilde{x}_5 = \{(3,4,5);.6,.2,.2\}$$
$$\tilde{x}_2 + \tilde{x}_6 = \{(2,3,4);.4,.3,.3\}$$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \{(0,0,0);1,0,0\}$.

After solving it, we get the optimal solution as

$$\tilde{x}_1 = \{(-1,4,9);.4,.4,.2\}$$
$$Max\ \widetilde{Z_2} = \{(-1,4,9);.3,.5,.2\}.$$

Objective function 3:

$$Max\ \tilde{Z}_3 = \{(-3,-2,-1);.4,.3,.3\}\tilde{x}_1 + \{(-4,-3,-2);.2,.5,.3\}\tilde{x}_2$$

Subject to $6\tilde{x}_1 + 8\tilde{x}_2 \leq \{(46,47,48);.3,.2,.5\}$

$$\tilde{x}_1 + \tilde{x}_2 \geq \{(1,3,5);.4,.4,.2\}$$
$$\tilde{x}_1 \leq \{(3,4,5);.6,.2,.2\}$$
$$\tilde{x}_2 \leq \{(2,3,4);.4,.3,.3\}$$

$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0);1,0,0\}$

Then $Max\ \tilde{Z}_3 = \{(-3,-2,-1);.4,.3,.3\}\tilde{x}_1 + \{(-4,-3,-2);.2,.5,.3\}\tilde{x}_2 + \{(0,0,0);1,0,0\}\tilde{x}_3 + \{(0,0,0);1,0,0\}\tilde{x}_4 + \{(0,0,0);1,0,0\}\tilde{x}_5 + \{(0,0,0);1,0,0\}\tilde{x}_6.$

Subject to $6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{x}_3 = \{(46,47,48);.3,.2,.5\}$

$-\tilde{x}_1 - \tilde{x}_2 + \tilde{x}_4 = \{(-5,-3,-1);.4,.4,.2\}$

$\tilde{x}_1 + \tilde{x}_5 = \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 + \tilde{x}_6 = \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \{(0,0,0);1,0,0\}$.

After solving it, we get the optimal solution as

$\tilde{x}_1 = \{(-3,3.8,10.6);.3,.4,.5\}$

$\tilde{x}_2 = \{(-2,3,8);.4,.4,.3\}$

$Max\ \widetilde{Z_3} = \{(-7,16.6,63.8);.2,.5,.5\}$.

Objective function 4:

$$Max\ \tilde{Z}_4 = \{(-1,-1,-1);.6,.2,.2\}\tilde{x}_2$$

Subject to $6\tilde{x}_1 + 8\tilde{x}_2 \leq \{(46,47,48);.3,.2,.5\}$

$\tilde{x}_1 + \tilde{x}_2 \geq \{(1,3,5);.4,.4,.2\}$

$\tilde{x}_1 \leq \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 \leq \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2 \geq \{(0,0,0);1,0,0\}$

Then $Max\ \tilde{Z}_4 = \{(0,0,0);1,0,0\}\tilde{x}_1 + \{(1,1,1);.6,.2,.2\}\tilde{x}_2 + \{(0,0,0);1,0,0\}\tilde{x}_3 + \{(0,0,0);1,0,0\}\tilde{x}_4 + \{(0,0,0);1,0,0\}\tilde{x}_5 + \{(0,0,0);1,0,0\}\tilde{x}_6.$

Subject to $6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{x}_3 = \{(46,47,48);.3,.2,.5\}$

$-\tilde{x}_1 - \tilde{x}_2 + \tilde{x}_4 = \{(-5,-3,-1);.4,.4,.2\}$

$\tilde{x}_1 + \tilde{x}_5 = \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 + \tilde{x}_6 = \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \{(0,0,0);1,0,0\}$.

After solving it, we get the optimal solution as

$\tilde{x}_2 = \{(-2,3,8);.4,.3,.3\}$,

$Max\ \widetilde{Z_4} = \{(-1,3,7);.4,.3,.3\}$.

After solving the objective functions, we get the optimal solutions. And then take absolute values of the each objective functions. Next, we find the value of $\widetilde{M}_1$ is the maximum value of the two maximum objective function, and we find the value of $\widetilde{M}_2$ is the minimum value of the two minimum objective function. Finally, we formed the combined solution table.

Table 3: Combined Solution Table

i	$\widetilde{\varphi}_i$	$\lvert\widetilde{\varphi}_i\rvert$	Values of $\widetilde{M}_1$ and $\widetilde{M}_2$
1	$\{(-5,9.8,34.6);.2,.4,.4\}$	$\{(5,9.8,34.6);.2,.4,.4\}$	$\widetilde{M}_1 = \{(5,9.8,34.6);.2,.5,.4\}$
2	$\{(-1,4,9);.3,.5,.2\}$	$\{(1,4,9);.3,.5,.2\}$	
3	$\{(-7,16.6,63.8);.2,.5,.5\}$	$\{(7,16.6,63.8);.2,.5,.5\}$	$\widetilde{M}_2 = \{(1,3,7);.2,.5,.5\}$
4	$\{(-1,3,7);.4,.4,.3\}$	$\{(1,3,7);.4,.4,.3\}$	

Type 1: Neutrosophic Contra-Harmonic Mean Technique

$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NCHM}$, where $NCHM = \left(\frac{\frac{\tilde{M}_1^{\ 2} + \tilde{M}_2^{\ 2}}{2}}{\frac{\tilde{M}_1 + \tilde{M}_2}{2}}\right)$.

$\tilde{M}_1^{\ 2} + \tilde{M}_2^{\ 2} = \{(26,105.04,1246.16);.2,.5,.5\}$, $\tilde{M}_1 + \tilde{M}_2 = \{(6,12.8,41.6);.2,.5,.5\}$

$$NCHM = \left(\frac{\frac{\tilde{M}_1^{\ 2} + \tilde{M}_2^{\ 2}}{2}}{\frac{\tilde{M}_1 + \tilde{M}_2}{2}}\right) = \frac{\tilde{M}_1^{\ 2} + \tilde{M}_2^{\ 2}}{\tilde{M}_1 + \tilde{M}_2} = \{(0.6,8.2,207.6);.2,.5,.5\}$$

$$\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i = \{(3,4,5);.2,.5,.5\}\tilde{x}_1 + \{(4,6,8);.2,.5,.5\}\tilde{x}_2$$

$$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NCHM} = \left[\frac{\{(3,4,5);.2,.5,.5\}\tilde{x}_1 + \{(4,6,8);.2,.5,.5\}\tilde{x}_2}{\{(0.6,8.2,207.6);.2,.5,.5\}}\right]$$

$$= \{(0.01,0.48,8.33);.2,.5,.5\}\tilde{x}_1 + \{(0.01,0.73,13.33);.2,.5,.5\}\tilde{x}_2$$

$$= \{(0.01,0.48,8.33);.2,.5,.5\}\tilde{x}_1 + \{(0.01,0.73,13.33);.2,.5,.5\}\tilde{x}_2 + \{(0,0,0);1,0,0\}\tilde{x}_3 + \{(0,0,0);1,0,0\}\tilde{x}_4 + \{(0,0,0);1,0,0\}\tilde{x}_5 + \{(0,0,0);1,0,0\}\tilde{x}_6.$$

Subject to $6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{x}_3 = \{(46,47,48);.3,.2,.5\}$

$-\ \tilde{x}_1 - \tilde{x}_2 + \tilde{x}_4 = \{(-5,-3,-1);.4,.4,.2\}$

$\tilde{x}_1 + \tilde{x}_5 = \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 + \tilde{x}_6 = \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \{(0,0,0);1,0,0\}$.

After solving it, we get the optimal solution as

$\tilde{x}_1 = \{(-3,3.8,10.6);.3,.4,.5\}$

$\tilde{x}_2 = \{(-2,3,8);.4,.4,.3\}$

$Max\ \tilde{Z} = \{(-0.05,4.014,194.938);.2,.5,.5\}$.

Type 2: Neutrosophic Arithmetic Mean Technique

$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NAM}$, where $NAM = \frac{\tilde{M}_1 + \tilde{M}_2}{2}$.

$\tilde{M}_1 + \tilde{M}_2 = \{(5,9.8,34.6);.2,.5,.4\} + \{(1,3,7);.2,.5,.5\} = \{(6,12.8,41.6);.2,.5,.5\}$

$$NAM = \frac{\{(6,12.8,41.6);.2,.5,.5\}}{2} = \{(3,4.26,20.8);.2,.5,.5\}$$

$$\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i = \{(3,4,5);.2,.5,.5\}\tilde{x}_1 + \{(4,6,8);.2,.5,.5\}\tilde{x}_2$$

$$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r}\tilde{Z}_i - \sum_{i=r+1}^{s}\tilde{Z}_i}{NAM} = \left[\frac{\{(3,4,5);.2,.5,.5\}\tilde{x}_1 + \{(4,6,8);.2,.5,.5\}\tilde{x}_2}{\{(3,4.26,20.8);.2,.5,.5\}}\right]$$

$$= \{(0.1,0.9,1.6);.2,.5,.5\}\tilde{x}_1 + \{(0.1,1.4,2.6);.2,.5,.5\}\tilde{x}_2$$

$$= \{(0.1,0.9,1.6);.2,.5,.5\}\tilde{x}_1 + \{(0.1,1.4,2.6);.2,.5,.5\}\tilde{x}_2 + \{(0,0,0);1,0,0\}\tilde{x}_3 + \{(0,0,0);1,0,0\}\tilde{x}_4 + \{(0,0,0);1,0,0\}\tilde{x}_5 + \{(0,0,0);1,0,0\}\tilde{x}_6.$$

Subject to $6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{x}_3 = \{(46,47,48);.3,.2,.5\}$

$- \tilde{x}_1 - \tilde{x}_2 + \tilde{x}_4 = \{(-5,-3,-1);.4,.4,.2\}$

$\tilde{x}_1 + \tilde{x}_5 = \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 + \tilde{x}_6 = \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \{(0,0,0);1,0,0\}$.

After solving it, we get the optimal solution as

$\tilde{x}_1 = \{(-3,3.8,10.6);.3,.4,.5\}$

$\tilde{x}_2 = \{(-2,3,8);.4,.4,.3\}$

$Max\ \tilde{Z} = \{(-0.5,7.62,37.76);.2,.5,.5\}$.

Type 3: Neutrosophic Harmonic Mean Technique

$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r} \tilde{Z}_i - \sum_{i=r+1}^{s} \tilde{Z}_i}{NHM}$, where $NHM = \frac{2}{\frac{1}{\widetilde{M}_1} + \frac{1}{\widetilde{M}_2}} = \frac{2\widetilde{M}_1\widetilde{M}_2}{\widetilde{M}_1 + \widetilde{M}_2}$.

$\widetilde{M}_1 + \widetilde{M}_2 = \{(6,12.8,41.6);.2,.5,.5\}$, $2\widetilde{M}_1\widetilde{M}_2 = \{(10,58.8,484.4);.2,.5,.5\}$

$$NHM = \frac{\{(10,58.8,484.4);.2,.5,.5\}}{\{(6,12.8,41.6);.2,.5,.5\}} = \{(0.2,4.5,80.7);.2,.5,.5\}$$

$$\sum_{i=1}^{r} \tilde{Z}_i - \sum_{i=r+1}^{s} \tilde{Z}_i = \{(3,4,5);.2,.5,.5\}\tilde{x}_1 + \{(4,6,8);.2,.5,.5\}\tilde{x}_2$$

$$Max\ \tilde{Z} = \frac{\sum_{i=1}^{r} \tilde{Z}_i - \sum_{i=r+1}^{s} \tilde{Z}_i}{NAM} = \left[\frac{\{(3,4,5);.2,.5,.5\}\tilde{x}_1 + \{(4,6,8);.2,.5,.5\}\tilde{x}_2}{\{(0.2,4.5,80.7);.2,.5,.5\}}\right]$$

$$= \{(0.03,0.8,25);.2,.5,.5\}\tilde{x}_1 + \{(0.04,1.3,40);.2,.5,.5\}\tilde{x}_2$$

$$= \{(0.03,0.8,25);.2,.5,.5\}\tilde{x}_1 + \{(0.04,1.3,40);.2,.5,.5\}\tilde{x}_2 + \{(0,0,0);1,0,0\}\tilde{x}_3 + \{(0,0,0);1,0,0\}\tilde{x}_4 + \{(0,0,0);1,0,0\}\tilde{x}_5 + \{(0,0,0);1,0,0\}\tilde{x}_6.$$

Subject to $6\tilde{x}_1 + 8\tilde{x}_2 + \tilde{x}_3 = \{(46,47,48);.3,.2,.5\}$

$- \tilde{x}_1 - \tilde{x}_2 + \tilde{x}_4 = \{(-5,-3,-1);.4,.4,.2\}$

$\tilde{x}_1 + \tilde{x}_5 = \{(3,4,5);.6,.2,.2\}$

$\tilde{x}_2 + \tilde{x}_6 = \{(2,3,4);.4,.3,.3\}$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq \{(0,0,0);1,0,0\}$.

After solving it, we get the optimal solution as

$\tilde{x}_1 = \{(-3,3.8,10.6);.3,.4,.5\}$

$\tilde{x}_2 = \{(-2,3,8);.4,.4,.3\}$

$Max\ \tilde{Z} = \{(-0.17,6.94,583);.2,.5,.5\}$

.

The optimal value $Max\ \tilde{Z}$ for different mean techniques are given below:

Table 4: Final Table

Mean Techniques	$\tilde{x}_i;\ \ \forall i = 1,2.$	Values of $Max\ \tilde{Z}$
NCHM	$(\{(-3,3.8,10.6);.3,.4,.5\},\{(-2,3,8);.4,.4,.3\})$	$\{(-0.05,4.014,194.938);.2,.5,.5\}$.
NAM	$(\{(-3,3.8,10.6);.3,.4,.5\},\{(-2,3,8);.4,.4,.3\})$	$\{(-0.5,7.62,37.76);.2,.5,.5\}$.
NHM	$(\{(-3,3.8,10.6);.3,.4,.5\},\{(-2,3,8);.4,.4,.3\})$	$\{(-0.17,6.94,583);.2,.5,.5\}$.

CONCLUSION

In this article, we have introduced three distinct average methods to obtain a solution to the Neutrosophic MOLPP. The Simplex Algorithm in Neutrosophic Environment is approached for determining the optimal value of a LPP. Here we obtain the optimal solution to satisfy the given constraints. Numerical examples are given to illustrate different types of mean techniques.

REFERENCES

1. R. E. Bellman, L. A. Zadeh, Decision making in a fuzzy environment, (Management Sciences, 1970), 17(4), B – 141.
2. Chandra Sen, A New Approach for Multi-Objective Rural development planning, (The Indian Economic Journal, 1983), 30 (4), pp.91 – 96.
3. Chandra Sen, Sen's Multi-Objective Programming Method and its Comparison with Other Techniques, (American Journal of Operational Research, 2018), 8 (1), pp.10 – 13.
4. Chandra Sen, Improved Scalarizing Techniques for Solving Multi-Objective Optimization Problems, (American Journal of Operational Research, 2019), 9 (1), pp.8 – 11.
5. H. R. Maleki, Ranking function and their Application to Fuzzy Linear Programming, (Far East Journal of Mathematical Sciences, 2002), 4 (3), pp.283 – 301.
6. Mehmet Sahin, Abdullah Kargin, and Florentin Smarandache, Generalized Single Valued Triangular Neutrosophic Numbers and Aggregation Operators for Application to Multi-attribute Group Decision Making, (New Trends in Neutrosophic Theory and Applications, 2018), II, pp.51 – 84.
7. S. N. Mohamed Assarudeen, and M. Junaid Basha, A Proposed Average Method for Solving Fuzzy Multi-Objective Linear Programming Problem Using Various Kinds of Fuzzy Mean Techniques, (International Journal of Research and Analytical Reviews (IJRAR), 2019), 6 (2), pp.673 – 683.
8. Mohamed, Mai., Mohamed Abdel-Basset, Abdel Nasser Zaied, and Florentin Smarandache, Neutrosophic Integer Programming Problem, (Neutrosophic Sets and Systems, 2017), 15 (1), pp.3 – 7.
9. Mohamed Abdel-Baset, Mai Mohamed, Abdel-Nasser Hessian, Florentin Smarandache, Neutrosophic Integer Programming Problem, (Neutrosophic Operational Research, 2016), 1, pp.49 – 61.
10. Nahar Samsun and Alim Md. Abdul, A new geometric average technique to solve multi-objective linear fractional programming problem and comparison with new arithmetic average technique, (IOSR Journal of Mathematics, 2017), 13(3), pp.39 – 52.
11. Nahar Samsun and Alim Md. Abdul, A Statistical averaging method to solve multi-objective linear programming problem, (International Journal of Science and Research, 2017), 6 (8), pp.623 – 629.
12. N. A. Sulaiman, And G. W. Sadiq, Solving the linear Multi objective programming problems Using mean and median values, (Al-Rafiden Journal of Computer Sciences and Mathematics, 2006), 13 (1), pp.69 – 83.
13. N. A. Sulaiman, and R. B. Mustafa, Using harmonic mean to solve multi-objective linear programming problems, (American Journal of Operations Research, 2016), 6 (1), pp.25 – 30.
14. Tuhin Bera, and Nirmal Kumar Mahapatra, Neutrosophic linear programming problem and its application to real life, (Afrika Matematika, 2020), 31, pp.709 – 726.
15. R. Vidhya, R. Irene Hepzibah, and A. Nagoorgani, Neutrosophic Multi Objective Linear Programming Problems, (Global Journal of Pure and Applied Mathematics, 2017), 13, pp.265 – 280.
16. H. J. Zimmermann, Using Fuzzy sets in operational research, (European Journal of Operations Research, 1983), 13, pp.201 – 216.