MODIFIED METHOD FOR SOLVING NON-LINEAR PROGRAMMING FOR MULTI-CRITERIA DECISION MAKING PROBLEMS UNDER INTERVAL NEUTROSOPHIC SET ENVIRONMENT

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Abstract Garg and Nancy (Applied Intelligence, 2017, https://doi.org/10.1007/s10489-017-1070-5) proposed a non-linear programming (NLP) method for solving interval neutrosophic multi-criteria decision making (INMCDM) problems (decision making problems in which rating value of each alternative over each criteria is represented by an interval neutrosophic set). In future, other researchers may use the same method in their research work as well as for solving real life INMCDM problems. However, after a deep study, it is observed that Garg and Nancy have used some mathematical incorrect assumptions in their proposed method. Therefore, it is scientifically incorrect to use this method in its present form. Keeping the same in mind, the method, proposed by Garg and Nancy, is modified.

Keywords: Neutrosophic set, Interval neutrosophic set, Non-linear programming model, Multi criteria decision-making, TOPSIS.

1. Introduction: In daily life problems, a process is followed by an individual or group of persons to finalize a decision. This process is called decision making (DM). For example, to find the best student of a class the arithmetic mean of the marks, obtained by each student in different subject, is calculated and then the obtained average marks is used to find the best student of the class. One of the important steps of DM is to collect the information/data regarding the problem. It is pertinent to mention that it is not always possible to represent the collected data/information as a real number. For example, the cost to hire a cab between two fixed places cannot be represented by a real number as it varies from time to time depending on the traffic/weather-conditions/route etc. Similarly, the rating of a movie review cannot be presented by a real number instead it can be expressed in linguistic terms such as poor, average, good, excellent etc.

In the literature, different ways have been introduced to handle these types of data. One of the way, used by researchers to handle the same is to express the data as fuzzy set (FS) [38] and its extensions (intuitionistic fuzzy set (IFS) [1], interval-valued intuitionistic fuzzy set (IVIFS) [2], Pythagorean fuzzy set (PFS) [37], interval-valued Pythagorean fuzzy set (IVPFS) [26], soft set (SS) [21], neutrosophic set (NS) [31], interval neutrosophic set (INS) [34] etc.).

In the last few years, a lot of researchers have proposed various methods for solving different types of DM problems under fuzzy environment and its extensions.

The methods for solving DM problems under fuzzy environment and its extensions can be categorized into several different categories. Some of these are as follows:

1. **Intuitionistic Fuzzy Environment:** This category contains all those methods for solving DM problems in which some or all the collected information/data is expressed as intuitionistic fuzzy sets.
2. **Interval-Valued Intuitionistic Fuzzy Environment:** This category contains all those methods for solving DM problems in which some or all the collected information/data is expressed as interval-valued intuitionistic fuzzy sets.
3. **Interval-Valued Intuitionistic Fuzzy Soft Environment:** This category contains all those methods for solving DM problems in which some or all the collected information/data is expressed as interval-valued intuitionistic fuzzy soft sets.
4. **Pythagorean Fuzzy Environment:** This category contains all those methods for solving DM problems in which some or all the collected information/data is expressed as Pythagorean fuzzy sets.
5. **Interval-Valued Pythagorean Fuzzy Environment:** This category contains all those methods for solving DM problems in which some or all the collected information/data is expressed as interval-valued Pythagorean fuzzy sets.
6. **Neutrosophic Environment:** This category contains all those methods for solving DM problems in which some or all the collected information/data is expressed as neutrosophic sets.
7. **Interval-Valued Neutrosophic Environment:** This category contains all those methods for solving DM problems in which some or all those collected information/data is expressed as interval-valued neutrosophic sets.

Kumar and Garg [20] pointed out that several researchers [4]-[5], [15], [28], [33], [36] have used the connection number (CN) [39] for solving multi-attribute decision making (MADM) problems under crisp environment as well as MADM problems under fuzzy environment [38]. However, till now no one has used the same for solving MADM problems under intuitionistic fuzzy environment [1]. Kumar and Garg [20] also considered some intuitionistic fuzzy multi-attribute decision making (IFMADM) problems and shown that the existing IFMADM method [3] fails to rank the alternatives of the considered IFMADM problems. Kumar and Garg [20] pointed out that this drawback of the existing IFMADM method [3] can be resolved with the help of a CN. Since to fill this gap, there was need to transform each intuitionistic fuzzy number (IFN) [1] of the intuitionistic fuzzy decision matrix into a CN as well as to compare the connection numbers (CNs). But, neither there was any method in the literature to transform an IFN into a CN nor any method to compare two CNs. Therefore, Kumar and Garg [20] firstly proposed a method to transform an IFN into a CN and a method to compare CNs. Then using these methods, Kumar and Garg [20] proposed a method to solve IFMADM problems.

Kumar and Garg [19] pointed out that several researchers [3]-[5], [15], [28], [33], [35]-[36] have used the CN [39] for solving MADM problems under crisp environment, fuzzy environment [38], interval-valued fuzzy environment [22] and intuitionistic fuzzy environment [1]. However, till now no one has used the same for solving MADM problems under interval-valued intuitionistic fuzzy environment [2]. Since to fill this gap, there was need to propose a method for transforming an IVIFS into a CN as well as a ranking method for comparing CNs. Therefore, Kumar and Garg [19] firstly proposed the methods for the same and then using these methods, Kumar and Garg [19] proposed an interval-valued intuitionistic fuzzy multi-attribute decision making (IVIFMADM) method for solving IVIFMADM problems.

Garg and Arora [9] claimed that there is no method in the literature to solve interval-valued intuitionistic fuzzy soft multi-attribute decision making (IVIFSMADM) problems and hence, proposed a non-linear programming (NLP) method for solving IVIFSMADM problems.

Garg [7] pointed out the flaws of the existing methods for comparing interval-valued Pythagorean fuzzy numbers (IVPFNs). To resolve the flaws, Garg [7] proposed a new method for the same. Furthermore, Garg [7] used this method for comparing IVPFNs, to propose a method for solving DM problems under interval-valued Pythagorean fuzzy (IVPF) environment.

Garg [8] chosen some counter examples to show that the existing score function and the accuracy function defined to rank the IVPFNs fails to rank correctly. Also, to resolve this flaw, Garg [8] proposed an improved accuracy function for ranking IVPFNs. Furthermore, Garg [8] used this method for comparing IVPFNs, to propose a method for solving DM problems under IVPF environment.

Thamaraiselvi and Santhi [32] pointed out that the neutrosophic set [31] is used in different research areas. However, till now no one have used the neutrosophic set in transportation problems. While, several researchers have used fuzzy numbers for representing various parameters of transportation problems [6], [12]-[14], [16]-[18], [24]-[25], [27], [30]. Therefore, Thamaraiselvi and Santhi [32] proposed the approaches for solving neutrosophic transportation problem of Type-I (transportation problem in which cost for transporting unit quantity of the product is represented as trapezoidal neutrosophic number, whereas availability and demands are represented as real numbers) and neutrosophic transportation problem of Type-II (transportation problem in which cost for transporting unit quantity of the product, availability of a product and demand of the product are represented as trapezoidal neutrosophic numbers).

Nancy and Garg [23] pointed out the shortcomings of the existing method [29] for the ranking of single valued neutrosophic sets (SVNS) as well for the ranking of interval neutrosophic sets (INS). Also, to resolve the shortcomings, Nancy and Garg [23] proposed new methods for the same. Furthermore, Nancy and Garg [23] used these methods for comparing SVNS and INS, to propose a method for solving such DM problems under neutrosophic sets environment.

Garg and Nancy [10] claimed that there is no method in the literature to solve interval neutrosophic multi-criteria decision making (INMCDM) problems and hence, proposed a NLP method for solving INMCDM problems. Since, it is only method for solving INMCDM problems so the other researchers may be attracted to use this method for solving real life INMCDM problems. However, after a deep study, it is observed that some mathematical incorrect assumptions have been considered in this method. Therefore, it is scientifically incorrect to use this method for solving real life INMCDM problems. Keeping the same, in mind the method, proposed by Garg and Nancy [10], is modified.

This paper is organized as follows: In Section 2, the existing method [10] is presented in a brief manner. In Section 3, the mathematical incorrect assumptions, considered in the existing method [10], are pointed out. In Section 4, the impact of the mathematical incorrect assumptions on the solution of real life problems is discussed. In Section 5, the required modification in the existing method [10], to resolve it flaws, are suggested. In Section 6, existing method [10] with suggested modification is used to find the exact solution of an existing problem. Section 7 concludes the paper.

2. A Brief Review of Nancy & Garg's Method: The aim of this paper is to point out the mathematical incorrect assumptions considered in the existing method [10, Section 3.5] as well as to propose a modified method. Since, to do the same there is need to discuss the existing method [10, Section 3.5]. Therefore, in this section, the existing method [10, Section 3.5] is presented in a brief manner.

The steps of the existing method [10, Section 3.5] are as follows:

Step 1: Write the crisp linear fractional programming problems (LFPPs) (P_1) [10, Section 3.5, Eq. 16] and (P_2) [10, Section 3.5, Eq. 18] corresponding to the i^{th} alternative of the considered INMCDM problem.

$$K_i^L = min\left\{\frac{\sum_{j=1}^{n}\left\{\omega_j\,\mu_{ij}^L + \xi_j\left(1-\rho_{ij}^U\right) + \eta_j\left(1-\nu_{ij}^U\right)\right\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}\right\}$$

$$s.t.\begin{cases}\omega_j^L \le \omega_j \le \omega_j^U \quad ; \quad j = 1,2,\dots,n,\\ \xi_j^L \le \xi_j \le \xi_j^U \quad ; \quad j = 1,2,\dots,n,\\ \eta_j^L \le \eta_j \le \eta_j^U \quad ; \quad j = 1,2,\dots,n.\end{cases} \qquad (P_1)$$

$$K_i^U = max\left\{\frac{\sum_{j=1}^{n}\left\{\omega_j\,\mu_{ij}^U + \xi_j\left(1-\rho_{ij}^L\right) + \eta_j\left(1-\nu_{ij}^L\right)\right\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}\right\}$$

s.t. $\qquad (P_2)$

Constraints of the problem (P_1).

Step 2: Using Charnes and Cooper's transformations [11], the crisp linear fractional programming problem (LFPP) (P_1) and the crisp LFPP (P_2) can be transformed into its equivalent crisp linear programming problem (LPP) (P_3) [10, Section 3.5, Eqn. 19] and crisp LPP (P_4) [10, Section 3.5, Eqn. 20] respectively.

$$K_i^L = min\left\{\sum_{j=1}^{n}\left\{t_j\,\mu_{ij}^L + r_j\left(1-\rho_{ij}^U\right) + y_j\left(1-\nu_{ij}^U\right)\right\}\right\}$$

$$s.t.\begin{cases}z\omega_j^L \le t_j \le z\omega_j^U\,; \quad j = 1,2,\dots,n,\\ z\xi_j^L \le r_j \le z\xi_j^U\,; \quad j = 1,2,\dots,n,\\ z\eta_j^L \le y_j \le z\eta_j^U\,; \quad j = 1,2,\dots,n,\\ \sum_{j=1}^{n}\left(t_j + r_j + y_j\right) = 1,\\ z \ge 0.\end{cases} \qquad (P_3)$$

$$K_i^U = max\left\{\sum_{j=1}^{n}\left\{t_j\,\mu_{ij}^U + r_j\left(1-\rho_{ij}^L\right) + y_j\left(1-\nu_{ij}^L\right)\right\}\right\}$$

$s.t.$ $\qquad (P_4)$

Constraints of the problem (P_3).

Step 3: Using the optimal values K_i^L and K_i^U of the crisp LPP (P_3) and the crisp LPP (P_4), obtain $K_i = [K_i^L, K_i^U]$ $(i = 1,2,\dots,m)$.

Step 4: Using the values of $K_i = [K_i^L, K_i^U]$ $(i = 1,2,\dots,m)$, obtained in Step 3, construct a $m \times m$ matrix $P = [p_{ik}]_{m\times m}$, where,

$$p_{ik} = \begin{cases} max\left\{1 - max\left(\frac{K_k^U - K_i^L}{K_i^U - K_i^L + K_k^U - K_k^L}, 0\right), 0\right\}; & if\ \ i \ne k\\ \frac{1}{2} & ;\ if\ \ i = k\end{cases}$$

Step 5: Find the value of $\theta_i = \frac{\sum_{j=1}^{n} p_{ik} + \frac{n}{2} - 1}{n(n-1)}$, $(i = 1,2,\dots,m;\ k = 1,2,\dots,m)$ and check that $\theta_i > \theta_k$ or $\theta_i < \theta_k$ or $\theta_i = \theta_k$.

Case (i) If $\theta_i > \theta_k$ then $A_i > A_k$

Case (ii) If $\theta_i < \theta_k$ then $A_i < A_k$

Case (iii) If $\theta_i = \theta_k$ then $A_i = A_k$.

3. Mathematical Incorrect Assumptions Considered in the Existing Method: The objective of the crisp LFPP (P_1) and the crisp LFPP (P_2) is to find such values of ω_j, ξ_j, η_j $(j = 1,2,\dots,n)$ where $0 \le$

$\omega_j, \xi_j, \eta_j \leq 1$ corresponding to which the value of $\frac{\sum_{j=1}^{n}\left\{\omega_j\, \mu_{ij}^{L} + \xi_j\left(1-\rho_{ij}^{U}\right) + \eta_j\left(1-\nu_{ij}^{U}\right)\right\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}$ will be minimum and the value of $\frac{\sum_{j=1}^{n}\left\{\omega_j\, \mu_{ij}^{U} + \xi_j\left(1-\rho_{ij}^{L}\right) + \eta_j\left(1-\nu_{ij}^{L}\right)\right\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}$ will be maximum.

To achieve this objective, Garg and Nancy [10, Section 3.5] have solved the crisp LFPP (P_1) and the crisp LFPP (P_2) independently by transforming the crisp LFPP (P_1) and the crisp LFPP (P_2) into the crisp LPP (P_3) and the crisp LPP (P_4) respectively. However, it is mathematically incorrect to solve the crisp LPP (P_3) and the crisp LPP (P_4) independently due to the following reasons:

On solving the crisp LPP (P_3) and the crisp LPP (P_4) independently, the obtained values of t_j, r_j, y_j $(j = 1,2,\ldots,n)$ will not necessarily be equal. While, as t_j, r_j, y_j $(j = 1,2,\ldots,n)$ are real numbers so the values of t_j, r_j, y_j $(j = 1,2,\ldots,n)$, obtained on solving the crisp LPP (P_3) and the crisp LPP (P_4), should be equal.

For example, to find the solution of the existing problem [10, Section 5.1], the crisp LFPP (P_5) and the crisp LFPP (P_6) are solved independently by transforming the crisp LFPP (P_5) and the crisp LFPP (P_6) into the crisp LPP (P_7) [10, Section 5.1, Eqn. 37] and the crisp LPP (P_8) [10, Section 5.1, Eqn. 38] with the help of Charnes and Cooper's transformation [11].

$$K_1^L = min\left\{\frac{\begin{array}{c}0.7\,\omega_1 + 0.6\,\omega_2 + 0.8\,\omega_3 + 0.7\,\omega_4 + \\ 0.3\,\xi_1 + 0.5\,\xi_2 + 0.4\,\xi_3 + 0.6\,\xi_4 + \\ 0.8\,\eta_1 + 0.7\,\eta_2 + 0.8\,\eta_3 + 0.8\,\eta_4\end{array}}{\omega_1+\omega_2+\omega_3+\omega_4+\xi_1+\xi_2+\xi_3+\xi_4+\eta_1+\eta_2+\eta_3+\eta_4}\right\}$$

$$s.t.\begin{cases}0.10 \leq \omega_1 \leq 0.3; 0.10 \leq \xi_1 \leq 0.2; \\ 0.2 \leq \eta_1 \leq 0.4; \\ 0.20 \leq \omega_2 \leq 0.5; 0.1 \leq \xi_2 \leq 0.2; \\ 0.15 \leq \eta_2 \leq 0.25; \\ 0.25 \leq \omega_3 \leq 0.4; 0.2 \leq \xi_3 \leq 0.3; \\ 0.15 \leq \eta_3 \leq 0.3; \\ 0.15 \leq \omega_4 \leq 0.3; 0.1 \leq \xi_4 \leq 0.3; \\ 0.3 \leq \eta_4 \leq 0.4.\end{cases} \qquad (P_5)$$

$$K_1^U = max\left\{\frac{\begin{array}{c}0.8\,\omega_1 + 0.8\,\omega_2 + 0.8\,\omega_3 + 0.9\,\omega_4 + \\ 0.5\,\xi_1 + 0.6\,\xi_2 + 0.6\,\xi_3 + 0.7\,\xi_4 + \\ 0.9\,\eta_1 + 0.7\,\eta_2 + 0.9\,\eta_3 + 0.8\,\eta_4\end{array}}{\omega_1+\omega_2+\omega_3+\omega_4+\xi_1+\xi_2+\xi_3+\xi_4+\eta_1+\eta_2+\eta_3+\eta_4}\right\}$$

$$s.t. \qquad (P_6)$$

Constraints of the problem (P_5).

$$K_1^L = min(0.7t_1 + 0.6t_2 + 0.8\,t_3 + 0.7t_4 + 0.3\,r_1 + 0.5\,r_2 + 0.4r_3 + 0.6r_4 + 0.8\,y_1 + 0.7\,y_2 + 0.8y_3 + 0.8\,y_4)$$

$$s.t.\begin{cases} 0.10z \leq t_1 \leq 0.3z\,; 0.10z \leq r_1 \leq 0.2z; \\ 0.2z \leq y_1 \leq 0.4z; \\ 0.20z \leq t_2 \leq 0.5z; 0.1z \leq r_2 \leq 0.2z; \\ 0.15z \leq y_2 \leq 0.25z; \\ 0.25z \leq t_3 \leq 0.4z; 0.2z \leq r_3 \leq 0.3z; \\ 0.15z \leq y_3 \leq 0.3z; \\ 0.15z \leq t_4 \leq 0.3z; 0.1z \leq r_4 \leq 0.3z; \\ 0.3z \leq y_4 \leq 0.4z; \\ \sum_{j=1}^{n}(t_j + r_j + y_j) = 1; \\ z \geq 0. \end{cases} \qquad (P_7)$$

$K_1^U = max(0.8t_1 + 0.8t_2 + 0.8\,t_3 + 0.9t_4 + 0.5\,r_1 + 0.6\,r_2 + 0.6r_3 + 0.7r_4 + 0.9\,y_1 + 0.7\,y_2 + 0.9y_3 + 0.8\,y_4)$
$s.t.$ (P_8)

Constraints of the problem (P_7).

The optimal values of $t_1, t_2, t_3, t_4, r_1, r_2, r_3, r_4, y_1, y_2, y_3$ and y_4, obtained on solving the crisp LPP (P_7) and the crisp LPP (P_8), are shown in Table 1.

It is obvious from the result, shown in Table 1, that the values of the variables $t_1, t_2, t_3, t_4, r_1, r_2, r_3, r_4, y_1, y_2, y_3$ and y_4, obtained on solving the existing crisp LPP (P_7) [10, Section 5.1, eq. 37] and the existing crisp LPP (P_8) [10, Section 5.1, eq. 38], are not equal, which is mathematically incorrect.

Table 1: Optimal Values of the Variables

Variables →	t_1	t_2	t_3	t_4	r_1	r_2	r_3	r_4	y_1	y_2	y_3	y_4	z	Value of the objective function
Min K_1^L	$\frac{1}{28}$	$\frac{5}{28}$	$\frac{5}{56}$	$\frac{3}{56}$	$\frac{1}{14}$	$\frac{1}{14}$	$\frac{3}{28}$	$\frac{3}{28}$	$\frac{1}{14}$	$\frac{3}{56}$	$\frac{3}{56}$	$\frac{3}{28}$	$\frac{5}{14}$	$\frac{22}{35}$
Max K_1^U	$\frac{6}{65}$	$\frac{2}{13}$	$\frac{8}{65}$	$\frac{6}{65}$	$\frac{2}{65}$	$\frac{2}{65}$	$\frac{4}{65}$	$\frac{2}{65}$	$\frac{8}{65}$	$\frac{3}{65}$	$\frac{6}{65}$	$\frac{8}{65}$	$\frac{4}{13}$	$\frac{517}{650}$

4. Impact of the Mathematical Incorrect Assumptions: It is obvious from Section 3 that on applying the existing method [10, Section 3.5] the obtained optimal value will be an interval but the values of the variables will not be real numbers. This is like a situation that one is saying that the profit is rupees 20 (assumed) but it is not possible to find a strategy corresponding to which this profit will be achieved. In actual case, on applying the existing method [10, Section 3.5], there doesn't exist any such strategy corresponding to which the profit is rupees 20. Hence, it is not appropriate to apply the existing method [10, Section 3.5] for solving real life problems.

5. Suggested Modifications: The flaws of the existing method [10, Section 3.5], pointed out in Section 3, can be resolved if to achieve the objective i.e., to maximize $\frac{\sum_{j=1}^{n}\left\{\omega_j\,\mu_{ij}^U + \xi_j\left(1-\rho_{ij}^L\right) + \eta_j\left(1-\nu_{ij}^L\right)\right\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}$ and to minimize $\frac{\sum_{j=1}^{n}\{\omega_j \mu_{ij}^{L} + \xi_j (1-\rho_{ij}^{U}) + \eta_j (1-\nu_{ij}^{U})\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}$, the crisp LFPP (P_9) is solved instead of the crisp LFPP (P_1) and the crisp LFPP (P_2) independently.

$$K_i = max\left\{\frac{\sum_{j=1}^{n}\left\{\begin{matrix}\omega_j\ \mu_{ij}^{U} + \xi_j (1-\rho_{ij}^{L}) + \\ \eta_j (1-\nu_{ij}^{L})\end{matrix}\right\} - \sum_{j=1}^{n}\left\{\begin{matrix}\omega_j\ \mu_{ij}^{L} + \xi_j (1-\rho_{ij}^{U}) + \\ \eta_j (1-\nu_{ij}^{U})\end{matrix}\right\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}\right\}$$

$$= max\left\{\frac{\sum_{j=1}^{n}\left\{\begin{matrix}\omega_j(\mu_{ij}^{U} - \mu_{ij}^{L}) + \xi_j\left((1-\rho_{ij}^{L}) - (1-\rho_{ij}^{U})\right) + \\ \eta_j\left((1-\nu_{ij}^{L}) - (1-\nu_{ij}^{U})\right)\end{matrix}\right\}}{\sum_{j=1}^{n}(\omega_j + \xi_j + \nu_j)}\right\}$$

$s.t.$ (P_9)

Constraints of the problem (P_1).

The crisp LFPP (P_9) can be solved as follows:

Step 1: Using Charnes and Cooper's transformation [11], the crisp LFPP (P_9) can be transformed into its equivalent crisp LPP (P_{10}).

$$K_i = max\left\{\sum_{j=1}^{n}\left\{t_j\left(\mu_{ij}^{U} - \mu_{ij}^{L}\right) + r_j\left((1-\rho_{ij}^{L}) - (1-\rho_{ij}^{U})\right) + y_j\left((1-\nu_{ij}^{L}) - (1-\nu_{ij}^{U})\right)\right\}\right\}$$

$s.t.$ (P_{10})

Constraints of the problem (P_7).

Step 2: Find the optimal solution $\{t_j, r_j, y_j;\ j = 1,2,\dots,n\}$ of the crisp LPP (P_{10}).

Step 3: Using the optimal solution, obtained in Step 4, find

$$K_i^L = \sum_{j=1}^{n}\{t_j\,\mu_{ij}^{L} + r_j(1-\rho_{ij}^{U}) + y_j(1-\nu_{ij}^{U})\}$$

and

$$K_i^U = \sum_{j=1}^{n}\{t_j\,\mu_{ij}^{U} + r_j(1-\rho_{ij}^{L}) + y_j(1-\nu_{ij}^{L})\}.$$

Step 4: Construct a $m \times m$ matrix $P = [p_{ik}]_{m\times m}$, where,

$$p_{ik} = \begin{cases} max\left\{1 - max\left(\frac{K_k^U - K_i^L}{K_i^U - K_i^L + K_k^U - K_k^L}, 0\right), 0\right\}; & if\ \ i \neq k \\ \frac{1}{2} & ;\ if\ \ i = k \end{cases}$$

Step 5: Find the value of $\theta_i = \frac{\sum_{j=1}^{n} p_{ik} + \frac{n}{2} - 1}{n(n-1)}$, $(i = 1,2,\dots,m;\ k = 1,2,\dots,m)$ and check that $\theta_i > \theta_k$ or $\theta_i < \theta_k$ or $\theta_i = \theta_k$.

Case (i) If $\theta_i > \theta_k$ then $A_i > A_k$

Case (ii) If $\theta_i < \theta_k$ then $A_i < A_k$

Case (iii) If $\theta_i = \theta_k$ then $A_i = A_k$.

6. Exact Solution of the Existing Problem: Garg and Nancy [10, Section 5.1] solved a real life problem to illustrate their proposed method. However as discussed in Section 2 that Garg and Nancy [10, Section 3.5] have used some mathematical incorrect assumptions in their proposed method, therefore the results of the real life problem, obtained by Garg and Nancy [10, Section 5.1], are not exact. In this section, the exact result of the same problem is obtained by the modified method.

Using the modified method, the exact results of the INMCDM problem [10, Section 5.1] can be obtained as follows:

Step 1: Using Step 1 of the modified method, the crisp LFPP (P_{11}), the crisp LFPP (P_{12}), the crisp LFPP (P_{13}), the crisp LFPP (P_{14}) and the crisp LFPP (P_{15}) can be obtained.

$$K_1 = max\left\{\frac{\begin{array}{c}0.1\,\omega_1 + 0.2\,\omega_2 + 0\,\omega_3 + 0.2\,\omega_4 + \\ 0.2\,\xi_1 + 0.1\,\xi_2 + 0.2\,\xi_3 + 0.1\,\xi_4 + \\ 0.1\,\eta_1 + 0\,\eta_2 + 0.1\,\eta_3 + 0\,\eta_4\end{array}}{\omega_1+\omega_2+\omega_3+\omega_4+\xi_1+\xi_2+\xi_3+\xi_4+\eta_1+\eta_2+\eta_3+\eta_4}\right\}$$

$s.t.$ (P_{11})

Constraints of the problem (P_5).

$$K_2 = max\left\{\frac{\begin{array}{c}0.2\,\omega_1 + 0.2\,\omega_2 + 0\,\omega_3 + 0.2\,\omega_4 + \\ 0.2\,\xi_1 + 0.2\,\xi_2 + 0.1\,\xi_3 + 0\,\xi_4 + \\ 0.2\,\eta_1 + 0.2\,\eta_2 + 0.1\,\eta_3 + 0.2\,\eta_4\end{array}}{\omega_1+\omega_2+\omega_3+\omega_4+\xi_1+\xi_2+\xi_3+\xi_4+\eta_1+\eta_2+\eta_3+\eta_4}\right\}$$

$s.t.$ (P_{12})

Constraints of the problem (P_5).

$$K_3 = max\left\{\frac{\begin{array}{c}0.2\,\omega_1 + 0.1\,\omega_2 + 0.1\,\omega_3 + 0.1\,\omega_4 + \\ 0\,\xi_1 + 0.2\,\xi_2 + 0.1\,\xi_3 + 0.1\,\xi_4 + \\ 0.2\,\eta_1 + 0.1\,\eta_2 + 0.1\,\eta_3 + 0.1\,\eta_4\end{array}}{\omega_1+\omega_2+\omega_3+\omega_4+\xi_1+\xi_2+\xi_3+\xi_4+\eta_1+\eta_2+\eta_3+\eta_4}\right\}$$

$s.t.$ (P_{13})

Constraints of the problem (P_5).

$$K_4 = max\left\{\frac{\begin{array}{c}0.1\,\omega_1 + 0.1\,\omega_2 + 0.1\,\omega_3 + 0.1\,\omega_4 + \\ 0.1\,\xi_1 + 0.1\,\xi_2 + 0.1\,\xi_3 + 0.1\,\xi_4 + \\ 0\,\eta_1 + 0.1\,\eta_2 + 0.1\,\eta_3 + 0.1\,\eta_4\end{array}}{\omega_1+\omega_2+\omega_3+\omega_4+\xi_1+\xi_2+\xi_3+\xi_4+\eta_1+\eta_2+\eta_3+\eta_4}\right\}$$

$s.t.$ (P_{14})

Constraints of the problem (P_5).

$$K_5 = max\left\{\frac{\begin{array}{c}0.1\,\omega_1 + 0.1\,\omega_2 + 0.1\,\omega_3 + 0.2\,\omega_4 + \\ 0.1\,\xi_1 + 0.1\,\xi_2 + 0.1\,\xi_3 + 0\,\xi_4 + \\ 0.1\,\eta_1 + 0.1\,\eta_2 + 0.1\,\eta_3 + 0.1\,\eta_4\end{array}}{\omega_1+\omega_2+\omega_3+\omega_4+\xi_1+\xi_2+\xi_3+\xi_4+\eta_1+\eta_2+\eta_3+\eta_4}\right\}$$

$s.t.$ (P_{15})

Constraints of the problem (P_5).

Step 2: Using Step 2 of the modified method the crisp LFPP (P_{11}),(P_{12}), (P_{13}), (P_{14}) and (P_{15}) can be transformed into the crisp LPP (P_{16}), (P_{17}), (P_{18}), (P_{19}) and (P_{20}) respectively.

$$K_1 = max\,\{0.1t_1 + 0.2t_2 + 0t_3 + 0.2t_4 + 0.2r_1 + 0.1r_2 + 0.2r_3 + 0.1r_4 + 0.1\,y_1 + 0y_2 + 0.1y_3 + 0y_4\}$$

$$s.t.\begin{cases} 0.10z \le t_1 \le 0.3z\,;\, 0.10z \le r_1 \le 0.2z; \\ 0.2z \le y_1 \le 0.4z; \\ 0.20z \le t_2 \le 0.5z\,;\, 0.1z \le r_2 \le 0.2z; \\ 0.15z \le y_2 \le 0.25z; \\ 0.25z \le t_3 \le 0.4z\,;\, 0.2z \le r_3 \le 0.3z; \\ 0.15z \le y_3 \le 0.3z; \\ 0.15z \le t_4 \le 0.3z\,;\, 0.1z \le r_4 \le 0.3z; \\ 0.3z \le y_4 \le 0.4z; \\ \sum_{j=1}^{n}(t_j + r_j + y_j) = 1; \\ z \ge 0. \end{cases} \qquad (P_{16})$$

$K_2 = max\,\{0.2t_1 + 0.2t_2 + 0\,t_3 + 0.2t_4 + 0.2r_1 + 0.2\,r_2 + 0.1r_3 + 0r_4 + 0.2y_1 + 0.2\,y_2 + 0.1y_3 + 0.2\,y_4\}$

$s.t.$ (P_{17})
Constraints of the problem (P_{16}).
$K_3 = max\,\{0.2t_1 + 0.1\,t_2 + 0.1t_3 + 0.1t_4 + 0r_1 + 0.2r_2 + 0.1\,r_3 + 0.1r_4 + 0.2\,y_1 + 0.1y_2 + 0.1y_3 + 0.1y_4\}$

$s.t.$ (P_{18})
Constraints of the problem (P_{16}).
$K_4 = max\,\{0.1t_1 + 0.1t_2 + 0.1t_3 + 0.1t_4 + 0.1r_1 + 0.1\,r_2 + 0.1\,r_3 + 0.1\,r_4 + 0\,y_1 + 0.1y_2 + 0.1y_3 + 0.1y_4\}$

$s.t.$ (P_{19})
Constraints of the problem (P_{16}).
$K_5 = max\,\{0.1t_1 + 0.1t_2 + 0.1t_3 + 0.2t_4 + 0.1r_1 + 0.1r_2 + 0.1r_3 + 0r_4 + 0.1\,y_1 + 0\,.1y_2 + 0.1y_3 + 0.1y_4\}$

$s.t.$ (P_{20})
Constraints of the problem (P_{16}).

Step 3: The optimal solutions $\{t_j, r_j, y_j;\ j = 1,2,\dots,n\}$ of the crisp LPP (P_{16}), (P_{17}), (P_{18}), (P_{19}) and (P_{20}) are,

$\left\{t_1 = \frac{2}{53}, t_2 = \frac{10}{53}, t_3 = \frac{5}{53}, t_4 = \frac{6}{53}, r_1 = \frac{4}{53}, r_2 = \frac{2}{53}, r_3 = \frac{6}{53}, r_4 = \frac{2}{53}, y_1 = \frac{4}{53}, y_2 = \frac{3}{53}, y_3 = \frac{3}{53}, y_4 = \frac{6}{53}\right\}$,
$\left\{t_1 = \frac{6}{65}, t_2 = \frac{2}{13}, t_3 = \frac{1}{13}, t_4 = \frac{6}{65}, r_1 = \frac{4}{65}, r_2 = \frac{4}{65}, r_3 = \frac{4}{65}, r_4 = \frac{2}{65}, y_1 = \frac{8}{65}, y_2 = \frac{1}{13}, y_3 = \frac{3}{65}, y_4 = \frac{8}{65}\right\}$,
$\left\{t_1 = \frac{3}{25}, t_2 = \frac{2}{25}, t_3 = \frac{1}{10}, t_4 = \frac{3}{50}, r_1 = \frac{1}{25}, r_2 = \frac{2}{25}, r_3 = \frac{2}{25}, r_4 = \frac{1}{25}, y_1 = \frac{4}{25}, y_2 = \frac{3}{50}, y_3 = \frac{3}{50}, y_4 = \frac{3}{25}\right\}$,
$\left\{t_1 = \frac{6}{73}, t_2 = \frac{10}{73}, t_3 = \frac{8}{73}, t_4 = \frac{6}{73}, r_1 = \frac{4}{73}, r_2 = \frac{4}{73}, r_3 = \frac{6}{73}, r_4 = \frac{6}{73}, y_1 = \frac{4}{73}, y_2 = \frac{5}{73}, y_3 = \frac{6}{73}, y_4 = \frac{8}{73}\right\}$
and
$\left\{t_1 = \frac{2}{43}, t_2 = \frac{4}{43}, t_3 = \frac{5}{43}, t_4 = \frac{6}{43}, r_1 = \frac{2}{43}, r_2 = \frac{2}{43}, r_3 = \frac{4}{43}, r_4 = \frac{2}{43}, y_1 = \frac{4}{43}, y_2 = \frac{3}{43}, y_3 = \frac{3}{43}, y_4 = \frac{6}{43}\right\}$
respectively.

Step 4: Using the optimal solutions $\{t_j, r_j, y_j;\ j = 1,2,\dots,n\}$, obtained in Step 3, $K_1^L = \frac{339}{530}$, $K_1^U = \frac{202}{265}$; $K_2^L = \frac{191}{325}$, $K_2^U = \frac{491}{650}$; $K_3^L = \frac{141}{250}$, $K_3^U = \frac{87}{125}$; $K_4^L = \frac{413}{730}$, $K_4^U = \frac{241}{365}$; $K_5^L = \frac{267}{430}$, $K_5^U = \frac{157}{215}$.

Step 5: Using Step 5 of the modified method,

$$P = \begin{bmatrix} 0.5 & 0.60127 & 0.74536 & 0.78284 & 0.76806 \\ 0.39872 & 0.5 & 0.63860 & 0.72319 & 0.57499 \\ 0.22139 & 0.36139 & 0.5 & 0.57499 & 0.31110 \\ 0.09509 & 0.27680 & 0.42501 & 0.5 & 0.19301 \\ 0.3906 & 0.51459 & 0.68889 & 0.73532 & 0.5 \end{bmatrix}$$

Step 6: Using Step 6 of the modified method, $\theta_1 = 0.2448765$, $\theta_2 = 0.216775$, $\theta_3 = 0.1734435$, $\theta_4 = 0.1494955$ and $\theta_5 = 0.2164725$ respectively. Furthermore, since, $\theta_1 > \theta_2 > \theta_5 > \theta_3 > \theta_4$ so $A_1 > A_2 > A_5 > A_3 > A_4$.

Conclusion: The mathematical incorrect assumptions considered in the existing method [10, Section 3.5] are pointed out. Also, the impact of these mathematical incorrect assumptions on the solutions of real life problems is discussed. Furthermore, the required modifications in the existing method [10, Section 5.1], to resolve its flaws, are suggested.

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