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Logarithmic similarity measure of dynamic neutrosophic cubic sets and its application in medical diagnosis

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ABSTRACT

In the medical diagnosis process, the symptoms of patients are always changing with time and relative to dynamical information at different time intervals. However, existing information expression methods usually neglect the temporal correlation of information expression, and hardly depict certain and uncertain information on complex medical diagnosis problems. For the first time, the study presents dynamic neutrosophic cubic set (DNCS) to express the patient's disease symptoms in a time sequence (different time intervals). Then, the logarithmic similarity measure (LSM) of DNCSs is put forward and their properties are verified. After that, a medical diagnosis method is constructed on the basis of the proposed logarithmic similarity measure in DNCS setting, where the disease symptom information collected from different time intervals is given by the form of DNCSs. Lastly, a medical diagnosis example is used to demonstrate its applicability, and then the diagnosis results indicate that the presented method is effective and feasible.

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1. Introduction

With the development of modern detection technology, more effective intelligent medical diagnosis methods are highly demanded due to both the increased volume of the symptom information of diseases and the inherent characteristics of disease symptoms such as the indeterminacy, inconsistency, and dynamic variability at different time intervals. The existing classic fuzzy set (Zadeh, 1965) is described by fuzzy information with only unique membership degree, and then intuitionistic fuzzy sets (Atanassov, 1986) and interval-valued intuitionistic fuzzy sets (Atanassov and Gargov, 1989) are depicted by both membership and non-membership degrees; while neutrosophic sets (NSs) (Smarandache, 1998) can express the incomplete, uncertainty, and inconsistency information depicted by truth, indeterminacy, and falsity degrees. As a subset of NS, simplified NSs (Ye, 2014) contains the concepts of single-valued NSs (SvNSs) (Wang et al., 2010; Broumi et al., 2016a,b) for certain information of truth, indeterminacy, and falsity and interval-valued NSs (IvNSs) (Wang et al., 2005; Broumi et al., 2019) for uncertain information of truth, indeterminacy, and falsity, and then they were successfully applied in the medical diagnosis (Ye, 2015; Abdel-Basset et al., 2019; Liu et al., 2018) or decision-making problems (Ye, 2014). Then, Nguyen et al. (2019) gave a survey of the state-of-the-arts on neutrosophic sets in biomedical diagnoses. Furthermore, to express certain and uncertain information simultaneously, Jun et al. (2012) defined cubic sets, in which the cubic variable consists of a certain degree given by an exact value and an uncertain degree given by an interval value. Ali et al. (2016) and Jun et al. (2017) extended cubic sets to the neutrosophic sets and defined a neutrosophic cubic set (NCS), which consists of a SvNS and an IvNS and can describe the partial certain and partial uncertain information of the truth, indeterminacy and falsity simultaneously. Ye (2018) further presented basic operations and aggregation method of NCSs. Then, the cosine (Lu and Ye, 2017), Jaccard, and Dice similarity measures of NCSs (Tu et al., 2018) and the Dombi (Shi and Ye, 2018), hybrid weighted arithmetic and geometric aggregation operators of NCSs (Shi and Yuan, 2019) were put forward for decision-making applications in NCS setting. Recently, Fu et al. (2018) gave the notion of cubic hesitant fuzzy sets (CHFSS) to express the hybrid information of uncertain and hesitancy fuzzy values and presented the generalized distance and similarity measures of CHFSS for medical diagnosis. After that, Fu et al. (2019) proposed the Dice measure of CHFSS for the initial evaluation of benign prostatic hyperplasia symptoms.

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However, the symptoms of diseases are dynamically changing in the medical diagnosis process regarding different time intervals, so the current expression models of time-invariant information (i.e. single-period information expression models) cannot characterize the dynamical information of the patient's disease symptoms at different time intervals. Ye and Fu (2016) proposed a multi-period medical diagnosis method to deal with medical diagnosis problems with dynamic SvNS information. Ye (2017) further presented dynamic single-valued neutrosophic multi-sets to process dynamic information collected from different time intervals in real decision-making situations. However, there is no research on the dynamic neutrosophic cubic set (DNCS), which consists of a discrete sequence of neutrosophic cubic numbers (NCNs) at different time intervals, to express the partial certain and partial uncertain dynamical information of truth, falsity and indeterminacy in some applications such as medical diagnosis or dynamic target tracing.

To satisfy the dynamic diagnosis demand of the dynamical information regarding different time intervals on medical diagnosis problems, motivated by both NCS (Ali et al., 2016; Jun et al., 2017) and multi-period single valued neutrosophic (dynamic single valued neutrosophic) information expression methods (Ye and Fu, 2016; Ye, 2017), this work proposes the logarithmic similarity measure (LSM) of DNCs and applies it to medical diagnosis. Then, the contributions of this original study are that: (1) the DNCs concept is introduced to describe the complex and dynamic medical diagnosis information which contains the time-variant neutrosophic cubic information at different time intervals, (2) the LSM of DNCs is proposed, and (3) a medical diagnosis method is established based on the proposed LSM in dynamic neutrosophic cubic setting.

The study is constructed below. Section 2 presents the basic concept of NCSs and the definition of the DNCs to depict the time-variant neutrosophic cubic information regarding different time intervals. Then, the logarithmic similarity measure of DNCs is proposed and its properties are proved in Section 3. Furthermore, a medical diagnosis method using LSM is put forward in the time-variant neutrosophic cubic environments in Section 4. A medical diagnosis example is presented to show the application of the proposed medical diagnosis method, its diagnosis robustness is analyzed by the LSM family regarding some parameter values, and then the comparison with existing similar methods is carried out in Section 5. The last section contains conclusions and future study.

2. Neutrosophic cubic sets and dynamic neutrosophic cubic sets

2.1. Neutrosophic cubic sets

As defined by Ali et al. (2016) and Jun et al. (2017), a NCS A in a universal set Z is constructed as the following form:

$$A = \{z, \langle [T^-(z), T^+(z)], [U^-(z), U^+(z)], [F^-(z), F^+(z)] \rangle, \langle \mu(z), \nu(z), \lambda(z) \rangle \mid z \in Z\},$$

where $\langle [T^-(z), T^+(z)], [U^-(z), U^+(z)], [F^-(z), F^+(z)] \rangle$ is an IvNS, depicting the truth, indeterminacy and falsity degrees with the interval values $[T^-(z), T^+(z)], [U^-(z), U^+(z)], [F^-(z), F^+(z)] \subseteq [0, 1]$ for $z \in Z$, and then $\langle \mu(z), \nu(z), \lambda(z) \rangle$ is a SvNS, describing the truth, indeterminacy and falsity degrees with the single values $\mu(z), \nu(z), \lambda(z) \in [0, 1]$ for $z \in Z$. Additionally, the NCS A is called an internal NCS if $\mu(z) \in [T^-(z), T^+(z)], \nu(z) \in [U^-(z), U^+(z)]$, and $\lambda(z) \in [F^-(z), F^+(z)]$ for $z \in Z$; while it is named as an external NCS if $\mu(z) \notin [T^-(z), T^+(z)], \nu(z) \notin [U^-(z), U^+(z)]$, and $\lambda(z) \notin [F^-(z), F^+(z)]$ for $z \in Z$.

For convenient expression, a basic element $(z, \langle [T^-(z), T^+(z)], [U^-(z), U^+(z)], [F^-(z), F^+(z)] \rangle, \langle \mu(z), \nu(z), \lambda(z) \rangle)$ in the NCS A is called a NCN and can be simply denoted as $\langle T, U, F \rangle, \langle \mu, \nu, \lambda \rangle$, where $T = [T^-, T^+]$, $U = [U^-, U^+]$, $F = [F^-, F^+]$ with $T, U, F \subseteq [0, 1]$, and $\mu(z), \nu(z), \lambda(z) \in [0, 1]$ satisfying the conditions of $0 \leq T^+ + U^+ + F^+ \leq 3$ and $0 \leq \mu(z) + \nu(z) + \lambda(z) \leq 3$.

2.2. Dynamic neutrosophic cubic sets

Based on the concept of NCS, DNCs and their relations and operational rules are further presented in this section.

Definition 1. Assume there is a universal set Z and a time sequence set $\tau = \{\tau_1, \tau_2, \dots, \tau_n\}$. A DNCs $B(\tau)$ in Z collected for the time sequence set τ can be expressed as

$$B(\tau) = \left\{ \tau_j, z, \left\langle \begin{matrix} [T^-(\tau_j, z), T^+(\tau_j, z)], \\ [U^-(\tau_j, z), U^+(\tau_j, z)], \\ [F^-(\tau_j, z), F^+(\tau_j, z)] \end{matrix} \right\rangle, \langle \mu(\tau_j, z), \nu(\tau_j, z), \lambda(\tau_j, z) \rangle \mid \tau_j \in \tau, z \in Z \right\}$$

where all variables $[T^-(\tau_j, z), T^+(\tau_j, z)], [U^-(\tau_j, z), U^+(\tau_j, z)], [F^-(\tau_j, z), F^+(\tau_j, z)] \subseteq [0, 1]$ and $\mu(\tau_j, z), \nu(\tau_j, z), \lambda(\tau_j, z) \in [0, 1]$ imply the time τ_j ($j = 1, 2, \dots, n$) for $z \in Z$.

For instance, a DNCs in the universal set $Z = \{z_1, z_2, z_3\}$ and the time sequence set $\tau = \{\tau_1, \tau_2\}$ is given by

$$B(\tau) = \left\{ \left\{ \left(\tau_1, z_1, \left\langle \begin{matrix} [0.2, 0.3], \\ [0.2, 0.5], \\ [0.5, 0.8] \end{matrix} \right\rangle, \langle 0.3, 0.4, 0.6 \rangle \right), \left(\tau_1, z_2, \left\langle \begin{matrix} [0.1, 0.3], \\ [0.2, 0.4], \\ [0.7, 0.9] \end{matrix} \right\rangle, \langle 0.2, 0.3, 0.8 \rangle \right), \left(\tau_1, z_3, \left\langle \begin{matrix} [0.4, 0.6], \\ [0.4, 0.7], \\ [0.9, 1.0] \end{matrix} \right\rangle, \langle 0.5, 0.5, 0.9 \rangle \right) \right\}, \right. \\ \left. \left\{ \left(\tau_2, z_1, \left\langle \begin{matrix} [0.3, 0.5], \\ [0.1, 0.2], \\ [0.2, 0.5] \end{matrix} \right\rangle, \langle 0.4, 0.1, 0.3 \rangle \right), \left(\tau_2, z_2, \left\langle \begin{matrix} [0.2, 0.5], \\ [0.1, 0.4], \\ [0.4, 0.7] \end{matrix} \right\rangle, \langle 0.4, 0.2, 0.5 \rangle \right), \left(\tau_2, z_3, \left\langle \begin{matrix} [0.2, 0.5], \\ [0.1, 0.2], \\ [0.3, 0.5] \end{matrix} \right\rangle, \langle 0.3, 0.1, 0.4 \rangle \right) \right\} \right\}$$

Definition 2. Assume there exist two DNCs:

$$B(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [T_B^-(\tau_j, z), T_B^+(\tau_j, z)], \\ [U_B^-(\tau_j, z), U_B^+(\tau_j, z)], \\ [F_B^-(\tau_j, z), F_B^+(\tau_j, z)] \end{array} \right\rangle, \left\langle \mu_B(\tau_j, z), \nu_B(\tau_j, z), \lambda_B(\tau_j, z) \right\rangle | \tau_j \in \tau, z \in Z \right\},$$

$$C(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [T_C^-(\tau_j, z), T_C^+(\tau_j, z)], \\ [U_C^-(\tau_j, z), U_C^+(\tau_j, z)], \\ [F_C^-(\tau_j, z), F_C^+(\tau_j, z)] \end{array} \right\rangle, \left\langle \mu_C(\tau_j, z), \nu_C(\tau_j, z), \lambda_C(\tau_j, z) \right\rangle | \tau_j \in \tau, z \in Z \right\} \text{ in } \tau = \{\tau_1, \tau_2, \dots, \tau_n\} \text{ and } Z.$$

Then, there are the following relations:

- (1) Inclusion: $B(\tau) \subseteq C(\tau)$ if and only if $[T_B^-(\tau_j, z), T_B^+(\tau_j, z)] \subseteq [T_C^-(\tau_j, z), T_C^+(\tau_j, z)], [U_B^-(\tau_j, z), U_B^+(\tau_j, z)] \supseteq [U_C^-(\tau_j, z), U_C^+(\tau_j, z)], [F_B^-(\tau_j, z), F_B^+(\tau_j, z)] \supseteq [F_C^-(\tau_j, z), F_C^+(\tau_j, z)], \mu_B(\tau_j, z) \leq \mu_C(\tau_j, z), \nu_B(\tau_j, z) \geq \nu_C(\tau_j, z), \lambda_B(\tau_j, z) \geq \lambda_C(\tau_j, z)$ for $z \in Z, \tau_j \in \tau$, and $j = 1, 2, \dots, n$;
- (2) Equality: $B(\tau) = C(\tau)$ if and only if $B(\tau) \subseteq C(\tau)$ and $C(\tau) \subseteq B(\tau)$;
- (3) Complement:

$$B^c(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [F_B^-(\tau_j, z), F_B^+(\tau_j, z)], \\ [1 - U_B^-(\tau_j, z), 1 - U_B^+(\tau_j, z)], \\ [T_B^-(\tau_j, z), T_B^+(\tau_j, z)] \end{array} \right\rangle, \left\langle \lambda_B(\tau_j, z), 1 - \nu_B(\tau_j, z), \mu_B(\tau_j, z) \right\rangle | \tau_j \in \tau, z \in Z \right\};$$

- (4) Union:

$$B(\tau) \cup C(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [T_B^-(\tau_j, z) \vee T_C^-(\tau_j, z), T_B^+(\tau_j, z) \vee T_C^+(\tau_j, z)], \\ [U_B^-(\tau_j, z) \wedge U_C^-(\tau_j, z), U_B^+(\tau_j, z) \wedge U_C^+(\tau_j, z)], \\ [F_B^-(\tau_j, z) \wedge F_C^-(\tau_j, z), F_B^+(\tau_j, z) \wedge F_C^+(\tau_j, z)] \end{array} \right\rangle, \left\langle \begin{array}{l} \mu_B(\tau_j, z) \vee \mu_C(\tau_j, z), \\ \nu_B(\tau_j, z) \wedge \nu_C(\tau_j, z), \\ \lambda_B(\tau_j, z) \wedge \lambda_C(\tau_j, z) \end{array} \right\rangle | \tau_j \in \tau, z \in Z \right\};$$

- (5) Intersection:

$$B(\tau) \cap C(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [T_B^-(\tau_j, z) \wedge T_C^-(\tau_j, z), T_B^+(\tau_j, z) \wedge T_C^+(\tau_j, z)], \\ [U_B^-(\tau_j, z) \vee U_C^-(\tau_j, z), U_B^+(\tau_j, z) \vee U_C^+(\tau_j, z)], \\ [F_B^-(\tau_j, z) \vee F_C^-(\tau_j, z), F_B^+(\tau_j, z) \vee F_C^+(\tau_j, z)] \end{array} \right\rangle, \left\langle \begin{array}{l} \mu_B(\tau_j, z) \wedge \mu_C(\tau_j, z), \\ \nu_B(\tau_j, z) \vee \nu_C(\tau_j, z), \\ \lambda_B(\tau_j, z) \vee \lambda_C(\tau_j, z) \end{array} \right\rangle | \tau_j \in \tau, z \in Z \right\}.$$

Definition 3. The operational rules of DNCSSs are defined as follows:

- (1) $B(\tau) \oplus C(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [T_B^-(\tau_j, z) + T_C^-(\tau_j, z) - T_B^-(\tau_j, z)T_C^-(\tau_j, z), \\ T_B^+(\tau_j, z) + T_C^+(\tau_j, z) - T_B^+(\tau_j, z)T_C^+(\tau_j, z)], \\ [U_B^-(\tau_j, z)U_C^-(\tau_j, z), U_B^+(\tau_j, z)U_C^+(\tau_j, z)], \\ [F_B^-(\tau_j, z)F_C^-(\tau_j, z), F_B^+(\tau_j, z)F_C^+(\tau_j, z)] \end{array} \right\rangle, \left\langle \begin{array}{l} \mu_B(\tau_j, z) + \mu_C(\tau_j, z) - \mu_B(\tau_j, z)\mu_C(\tau_j, z), \\ \nu_B(\tau_j, z)\nu_C(\tau_j, z), \\ \lambda_B(\tau_j, z)\lambda_C(\tau_j, z) \end{array} \right\rangle | \tau_j \in \tau, z \in Z \right\};$
- (2) $B(\tau) \otimes C(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [T_B^-(\tau_j, z)T_C^-(\tau_j, z), T_B^+(\tau_j, z)T_C^+(\tau_j, z)], \\ [U_B^-(\tau_j, z) + U_C^-(\tau_j, z) - U_B^-(\tau_j, z)U_C^-(\tau_j, z), \\ U_B^+(\tau_j, z) + U_C^+(\tau_j, z) - U_B^+(\tau_j, z)U_C^+(\tau_j, z)], \\ [F_B^-(\tau_j, z) + F_C^-(\tau_j, z) - F_B^-(\tau_j, z)F_C^-(\tau_j, z), \\ F_B^+(\tau_j, z) + F_C^+(\tau_j, z) - F_B^+(\tau_j, z)F_C^+(\tau_j, z)] \end{array} \right\rangle, \left\langle \begin{array}{l} \mu_B(\tau_j, z)\mu_C(\tau_j, z), \\ \nu_B(\tau_j, z) + \nu_C(\tau_j, z) - \nu_B(\tau_j, z)\nu_C(\tau_j, z), \\ \lambda_B(\tau_j, z) + \lambda_C(\tau_j, z) - \lambda_B(\tau_j, z)\lambda_C(\tau_j, z) \end{array} \right\rangle | \tau_j \in \tau, z \in Z \right\}$
- (3) $\rho B(\tau) = \left\{ \tau_j, z, \left\langle \begin{array}{l} [1 - (1 - T_B^-(\tau_j, z))^\rho, 1 - (1 - T_B^+(\tau_j, z))^\rho], \\ [(U_B^-(\tau_j, z))^\rho, (U_B^+(\tau_j, z))^\rho], \\ [(F_B^-(\tau_j, z))^\rho, (F_B^+(\tau_j, z))^\rho] \end{array} \right\rangle, \left\langle \begin{array}{l} 1 - (1 - \mu_B(\tau_j, z))^\rho, \\ (\nu_B(\tau_j, z))^\rho, \\ (\lambda_B(\tau_j, z))^\rho \end{array} \right\rangle | \tau_j \in \tau, z \in Z \right\}$

for $\rho > 0$;

- (4) $(B(\tau))^\rho = \left\{ \tau_j, z, \left\langle \begin{array}{l} [T_B^-(\tau_j, z))^\rho, (T_B^+(\tau_j, z))^\rho], \\ [1 - (1 - U_B^-(\tau_j, z))^\rho, 1 - (1 - U_B^+(\tau_j, z))^\rho], \\ [1 - (1 - F_B^-(\tau_j, z))^\rho, 1 - (1 - F_B^+(\tau_j, z))^\rho] \end{array} \right\rangle, \left\langle \begin{array}{l} (\mu_B(\tau_j, z))^\rho, \\ 1 - (1 - \nu_B(\tau_j, z))^\rho, \\ 1 - (1 - \lambda_B(\tau_j, z))^\rho \end{array} \right\rangle | \tau_j \in \tau, z \in Z \right\}$

for $\rho > 0$.

3. Logarithmic similarity measure of DNCSSs

Motivated by the LSM of IvFSs (Lu and Ye, 2018), this section introduces a LSM of DNCSSs.

For two NCNs $b_1 = \langle T_1, U_1, F_1 \rangle, \langle \mu_1, \nu_1, \lambda_1 \rangle$ and $b_2 = \langle T_2, U_2, F_2 \rangle, \langle \mu_2, \nu_2, \lambda_2 \rangle$, a function f can be defined as

$$f(b_1, b_2) = \frac{(|T_1^- - T_2^-| + |T_1^+ - T_2^+| + |U_1^- - U_2^-| + |U_1^+ - U_2^+| + |F_1^- - F_2^-| + |F_1^+ - F_2^+| + |\mu_1 - \mu_2| + |\nu_1 - \nu_2| + |\lambda_1 - \lambda_2|)}{9} \quad (1)$$

Let $B(\tau) = \left\{ \tau_j, z_i, \left\langle \begin{matrix} [T_B^-(\tau_j, z_i), T_B^+(\tau_j, z_i)], \\ [U_B^-(\tau_j, z_i), U_B^+(\tau_j, z_i)], \\ [F_B^-(\tau_j, z_i), F_B^+(\tau_j, z_i)] \end{matrix} \right\rangle, \langle \mu_B(\tau_j, z_i), \nu_B(\tau_j, z_i), \lambda_B(\tau_j, z_i) \rangle \mid \tau_j \in \tau, z_i \in Z \right\}$ and $C(\tau) = \left\{ \tau_j, z_i, \left\langle \begin{matrix} [T_C^-(\tau_j, z_i), T_C^+(\tau_j, z_i)], \\ [U_C^-(\tau_j, z_i), U_C^+(\tau_j, z_i)], \\ [F_C^-(\tau_j, z_i), F_C^+(\tau_j, z_i)] \end{matrix} \right\rangle, \langle \mu_C(\tau_j, z_i), \nu_C(\tau_j, z_i), \lambda_C(\tau_j, z_i) \rangle \mid \tau_j \in \tau, z_i \in Z \right\}$ be two DNCSSs in $Z = \{z_1, z_2, \dots, z_m\}$ along with the weight

vector $u = (u(z_1), u(z_2), \dots, u(z_m))$ for $\sum_{i=1}^m u(z_i) = 1$ in $Z = \{z_1, z_2, \dots, z_m\}$ and the weight vector $\omega(\tau) = (\omega(\tau_1), \omega(\tau_2), \dots, \omega(\tau_n))$ for

$\sum_{j=1}^n \omega(\tau_j) = 1$ in $\tau = \{\tau_1, \tau_2, \dots, \tau_n\}$. Then the weighted LSM between two DNCSSs $B(\tau_j)$ and $C(\tau_j)$ at a special time of τ_j can be calculated by

$$L(B(\tau_j), C(\tau_j)) = \sum_{i=1}^m u(z_i) \log_{\alpha} [\alpha - (\alpha - 1) \times f(B(\tau_j, z_i), C(\tau_j, z_i))] \\ = \sum_{i=1}^m u(z_i) \log_{\alpha} \left[\alpha - (\alpha - 1) \frac{\begin{matrix} |T_B^-(\tau_j, z_i) - T_C^-(\tau_j, z_i)| + |T_B^+(\tau_j, z_i) - T_C^+(\tau_j, z_i)| + |U_B^-(\tau_j, z_i) - U_C^-(\tau_j, z_i)| \\ + |U_B^+(\tau_j, z_i) - U_C^+(\tau_j, z_i)| + |F_B^-(\tau_j, z_i) - F_C^-(\tau_j, z_i)| + |F_B^+(\tau_j, z_i) - F_C^+(\tau_j, z_i)| \\ + |\mu_B(\tau_j, z_i) - \mu_C(\tau_j, z_i)| + |\nu_B(\tau_j, z_i) - \nu_C(\tau_j, z_i)| + |\lambda_B(\tau_j, z_i) - \lambda_C(\tau_j, z_i)| \end{matrix}}{9} \right]. \quad (2)$$

for $\alpha \geq 2$.

where α is an integer value, $z_i \in Z (i = 1, 2, \dots, m)$, and $\tau_j \in \tau (j = 1, 2, \dots, n)$. Then, the weighted LSM between $B(\tau)$ and $C(\tau)$ in the time sequence set τ can be calculated by

$$R(B(\tau), C(\tau)) = \sum_{j=1}^n \omega(\tau_j) \times L(B(\tau_j), C(\tau_j)) \text{ for } \alpha \geq 2. \quad (3)$$

Especially, the DNCSSs $B(\tau)$ and $C(\tau)$ become NCSSs with $n=1$, and then the weighted LSM of DNCSSs is reduced to the one of NCSSs.

Then, the weighted LSM of DNCSSs has the following properties:

(P1) $0 \leq R(B(\tau), C(\tau)) \leq 1$;

(P2) $R(B(\tau), C(\tau)) = 1$ if and only if $B(\tau) = C(\tau)$;

(P3) $R(B(\tau), C(\tau)) = R(C(\tau), B(\tau))$;

(P4) if $Q(\tau)$ is a DNCSS in Z and τ , and $B(\tau) \subseteq C(\tau) \subseteq Q(\tau)$, then $R(B(\tau), Q(\tau)) \leq R(B(\tau), C(\tau))$ and $R(B(\tau), Q(\tau)) \leq R(C(\tau), Q(\tau))$.

Proof. (P1) By Eq. (2), $f(B(\tau_j, z_i), C(\tau_j, z_i)) \in [0, 1]$ can be obtained because of $[T^-(\tau_j, z_i), T^+(\tau_j, z_i)]$, $[U^-(\tau_j, z_i), U^+(\tau_j, z_i)]$, $[F^-(\tau_j, z_i), F^+(\tau_j, z_i)] \subseteq [0, 1]$ and $\mu(\tau_j, z_i), \nu(\tau_j, z_i), \lambda(\tau_j, z_i) \in [0, 1]$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. Thus, the logarithm to the base α of the number $\alpha - (\alpha - 1)f(B(\tau_j, z_i), C(\tau_j, z_i))$ ranges from 0 to 1, so the relative LSM $L(B(\tau_j), C(\tau_j))$ will also be in the closed interval of $[0, 1]$

for $j = 1, 2, \dots, n$ with $\sum_{i=1}^m u(z_i) = 1$. Further by Eq. (3), $0 \leq R(B(\tau), C(\tau)) \leq 1$ can be gotten for $\sum_{j=1}^n \omega(\tau_j) = 1$.

(P2) If $B(\tau) = C(\tau)$, there exist $T_B^-(\tau_j, z_i) = T_C^-(\tau_j, z_i)$, $T_B^+(\tau_j, z_i) = T_C^+(\tau_j, z_i)$, $U_B^-(\tau_j, z_i) = U_C^-(\tau_j, z_i)$, $U_B^+(\tau_j, z_i) = U_C^+(\tau_j, z_i)$, $F_B^-(\tau_j, z_i) = F_C^-(\tau_j, z_i)$, $F_B^+(\tau_j, z_i) = F_C^+(\tau_j, z_i)$, $\mu_B(\tau_j, z_i) = \mu_C(\tau_j, z_i)$, $\nu_B(\tau_j, z_i) = \nu_C(\tau_j, z_i)$, and $\lambda_B(\tau_j, z_i) = \lambda_C(\tau_j, z_i)$. Hence, by Eq. (1), there exists $f(B(\tau_j, z_i), C(\tau_j, z_i)) = 0$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. Thus, since the logarithm to the base α of $\alpha - (\alpha - 1)f(B(\tau_j, z_i), C(\tau_j, z_i))$ is equal to 1, $R(B(\tau), C(\tau)) = 1$

can be obtained by Eq. (3) with $\sum_{i=1}^m u(z_i) = 1$ and $\sum_{j=1}^n \omega(\tau_j) = 1$.

Similarly, by Eq. (3), if $R(B(\tau), C(\tau)) = 1$, there exists $L(B(\tau_j), C(\tau_j)) = 1$. Then by Eq. (2), the logarithm to the base α of the number $\alpha - (\alpha - 1)f(B(\tau_j, z_i), C(\tau_j, z_i))$ should be 1, and $f(B(\tau_j, z_i), C(\tau_j, z_i))$ should be zero. Regarding the conditions of $T_B^-(\tau_j, z_i) = T_C^-(\tau_j, z_i)$, $T_B^+(\tau_j, z_i) = T_C^+(\tau_j, z_i)$, $U_B^-(\tau_j, z_i) = U_C^-(\tau_j, z_i)$, $U_B^+(\tau_j, z_i) = U_C^+(\tau_j, z_i)$, $F_B^-(\tau_j, z_i) = F_C^-(\tau_j, z_i)$, $F_B^+(\tau_j, z_i) = F_C^+(\tau_j, z_i)$, $\mu_B(\tau_j, z_i) = \mu_C(\tau_j, z_i)$, $\nu_B(\tau_j, z_i) = \nu_C(\tau_j, z_i)$, and $\lambda_B(\tau_j, z_i) = \lambda_C(\tau_j, z_i)$ must be satisfied. That is, $B(\tau) = C(\tau)$.

(P3) $R(B(\tau), C(\tau)) = R(C(\tau), B(\tau))$ can be straightforwardly obtained.

(P4) The condition of $B(\tau) \subseteq C(\tau) \subseteq Q(\tau)$ implies that $T_B^-(\tau_j, z_i) \leq T_C^-(\tau_j, z_i) \leq T_Q^-(\tau_j, z_i)$, $T_B^+(\tau_j, z_i) \leq T_C^+(\tau_j, z_i) \leq T_Q^+(\tau_j, z_i)$, $U_B^-(\tau_j, z_i) \geq U_C^-(\tau_j, z_i) \geq U_Q^-(\tau_j, z_i)$, $U_B^+(\tau_j, z_i) \geq U_C^+(\tau_j, z_i) \geq U_Q^+(\tau_j, z_i)$, $F_B^-(\tau_j, z_i) \geq F_C^-(\tau_j, z_i) \geq F_Q^-(\tau_j, z_i)$, $F_B^+(\tau_j, z_i) \geq F_C^+(\tau_j, z_i) \geq F_Q^+(\tau_j, z_i)$, $\mu_B(\tau_j, z_i) \leq \mu_C(\tau_j, z_i) \leq \mu_Q(\tau_j, z_i)$, $\nu_B(\tau_j, z_i) \geq \nu_C(\tau_j, z_i) \geq \nu_Q(\tau_j, z_i)$, $\lambda_B(\tau_j, z_i) \geq \lambda_C(\tau_j, z_i) \geq \lambda_Q(\tau_j, z_i)$ for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. It follows that

$$\begin{aligned} |T_B^-(\tau_j, z_i) - T_C^-(\tau_j, z_i)| &\leq |T_B^-(\tau_j, z_i) - T_Q^-(\tau_j, z_i)|, |T_C^-(\tau_j, z_i) - T_Q^-(\tau_j, z_i)| \leq |T_B^-(\tau_j, z_i) - T_Q^-(\tau_j, z_i)|, \\ |T_B^+(\tau_j, z_i) - T_C^+(\tau_j, z_i)| &\leq |T_B^+(\tau_j, z_i) - T_Q^+(\tau_j, z_i)|, |T_C^+(\tau_j, z_i) - T_Q^+(\tau_j, z_i)| \leq |T_B^+(\tau_j, z_i) - T_Q^+(\tau_j, z_i)|, \\ |U_B^-(\tau_j, z_i) - U_C^-(\tau_j, z_i)| &\leq |U_B^-(\tau_j, z_i) - U_Q^-(\tau_j, z_i)|, |U_C^-(\tau_j, z_i) - U_Q^-(\tau_j, z_i)| \leq |U_B^-(\tau_j, z_i) - U_Q^-(\tau_j, z_i)|, \\ |U_B^+(\tau_j, z_i) - U_C^+(\tau_j, z_i)| &\leq |U_B^+(\tau_j, z_i) - U_Q^+(\tau_j, z_i)|, |U_C^+(\tau_j, z_i) - U_Q^+(\tau_j, z_i)| \leq |U_B^+(\tau_j, z_i) - U_Q^+(\tau_j, z_i)|, \end{aligned}$$

$$\begin{aligned} |F_B^-(\tau_j, z_i) - F_C^-(\tau_j, z_i)| &\leq |F_B^-(\tau_j, z_i) - F_Q^-(\tau_j, z_i)|, |F_C^-(\tau_j, z_i) - F_Q^-(\tau_j, z_i)| \leq |F_B^-(\tau_j, z_i) - F_Q^-(\tau_j, z_i)|, \\ |F_B^+(\tau_j, z_i) - F_C^+(\tau_j, z_i)| &\leq |F_B^+(\tau_j, z_i) - F_Q^+(\tau_j, z_i)|, |F_C^+(\tau_j, z_i) - F_Q^+(\tau_j, z_i)| \leq |F_B^+(\tau_j, z_i) - F_Q^+(\tau_j, z_i)|, \\ |\mu_B(\tau_j, z_i) - \mu_C(\tau_j, z_i)| &\leq |\mu_B(\tau_j, z_i) - \mu_Q(\tau_j, z_i)|, |\mu_C(\tau_j, z_i) - \mu_Q(\tau_j, z_i)| \leq |\mu_B(\tau_j, z_i) - \mu_Q(\tau_j, z_i)|, \\ |\nu_B(\tau_j, z_i) - \nu_C(\tau_j, z_i)| &\leq |\nu_B(\tau_j, z_i) - \nu_Q(\tau_j, z_i)|, |\nu_C(\tau_j, z_i) - \nu_Q(\tau_j, z_i)| \leq |\nu_B(\tau_j, z_i) - \nu_Q(\tau_j, z_i)|, \\ |\lambda_B(\tau_j, z_i) - \lambda_C(\tau_j, z_i)| &\leq |\lambda_B(\tau_j, z_i) - \lambda_Q(\tau_j, z_i)|, |\lambda_C(\tau_j, z_i) - \lambda_Q(\tau_j, z_i)| \leq |\lambda_B(\tau_j, z_i) - \lambda_Q(\tau_j, z_i)| \end{aligned}$$

Thus, both $f(B(\tau_j, z_i), C(\tau_j, z_i)) \leq f(B(\tau_j, z_i), Q(\tau_j, z_i))$ and $f(C(\tau_j, z_i), Q(\tau_j, z_i)) \leq f(B(\tau_j, z_i), Q(\tau_j, z_i))$ can be obtained for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. Since the logarithm to the base α in Eq. (2) decreases with the increasing of the function f , $R(B, Q) \leq R(B, C)$ and $R(B, Q) \leq R(C, Q)$ can be obtained.

Thus, the proof of the properties is completed.

4. Medical diagnosis method using the logarithmic similarity measure of DNCSS

This section introduces a novel medical diagnosis method applying LSM in dynamic neutrosophic cubic setting.

Assume $D = \{D_1, D_2, \dots, D_q\}$ is a set of the considered diseases, $Z = \{z_1, z_2, \dots, z_m\}$ is a set of relative symptoms, and $\tau = \{\tau_1, \tau_2, \dots, \tau_n\}$ is a set of time sequence. For a patient P_s with multiple symptoms in Z , the related characteristic values between the patient and symptoms are indicated in medical diagnosis problems with time-variant NCSs at different time intervals, as shown in Table 1, where $P_s(\tau_j, z_i)$ denotes the characteristic values of the patient's i th symptom z_i ($i = 1, 2, \dots, m$) at the j th time τ_j ($j = 1, 2, \dots, n$). If $n = 1$, only one-time symptom information of the patient is considered as medical diagnosis problems in the reference Ye (2015). Then, the characteristic values between the considered disease D_k ($k = 1, 2, \dots, q$) and the relative symptom z_i ($i = 1, 2, \dots, m$) are given in Table 2, where $D_k(z_i)$ expresses the characteristic values of the k th considered disease D_k ($k = 1, 2, \dots, q$) on the i th symptom z_i ($i = 1, 2, \dots, m$).

For a medical diagnosis problem with dynamic neutrosophic cubic information, the characteristic values $P_s(\tau_j, z_i)$ ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) of a patient P_s in the symptom set Z and the time sequence set τ are given by a DNCSS:

$$P_s(\tau_j, z_i) = \left\{ \tau_j, z_i, \left\langle \begin{aligned} &[T_{P_s}^-(\tau_j, z_i), T_{P_s}^+(\tau_j, z_i)], \\ &[U_{P_s}^-(\tau_j, z_i), U_{P_s}^+(\tau_j, z_i)], \\ &[F_{P_s}^-(\tau_j, z_i), F_{P_s}^+(\tau_j, z_i)] \end{aligned} \right\rangle, \langle \mu_{P_s}(\tau_j, z_i), \nu_{P_s}(\tau_j, z_i), \lambda_{P_s}(\tau_j, z_i) \rangle | \tau_j \in \tau, z_i \in Z \right\},$$

and then the characteristic values between the considered disease D_k and the symptom z_i are denoted as a NCS

$$D_k(z_i) = \left\{ z_i, \left\langle \begin{aligned} &[T_{D_k}^-(z_i), T_{D_k}^+(z_i)], \\ &[U_{D_k}^-(z_i), U_{D_k}^+(z_i)], \\ &[F_{D_k}^-(z_i), F_{D_k}^+(z_i)] \end{aligned} \right\rangle, \langle \mu_{D_k}(z_i), \nu_{D_k}(z_i), \lambda_{D_k}(z_i) \rangle | z_i \in Z \right\} \text{ for NCS. Suppose the weight vector in the symptom set } Z \text{ is } u = (u(z_1), u(z_2), \dots, u(z_m)) \text{ satisfying } u(z_i) \in [0, 1] \text{ and } \sum_{i=1}^m u(z_i) = 1 \text{ and the weight vector in the time sequence set } \tau \text{ is } \omega(\tau) = (\omega(\tau_1), \omega(\tau_2), \dots, \omega(\tau_n)) \text{ with } \omega(\tau_j) \in [0, 1] \text{ and } \sum_{j=1}^n \omega(\tau_j) = 1, \text{ where the time sequence weight } \omega(\tau_j) \text{ can be also considered as its forgetting factor (the weight value decreases with the time increasing) in some dynamic medical diagnosis problems.}$$

By applying the LSMs of DNCSSs defined by Eqs. (2) and (3) to the dynamic medical diagnosis problems, the diagnosis steps are listed below:

Step 1: Obtain the weighted LSM between a patient P_s and the considered disease D_k ($k = 1, 2, \dots, q$) in the symptom set $Z = \{z_1, z_2, \dots, z_m\}$ at the time τ_j ($j = 1, 2, \dots, n$) along with the weight vector $u = (u(z_1), u(z_2), \dots, u(z_m))$ by

$$\begin{aligned} L(P_s(\tau_j), D_k) &= \sum_{i=1}^m u(z_i) \log_{\alpha} [\alpha - (\alpha - 1) \times f(P_s(\tau_j, z_i), D_k(z_i))] \\ &= \sum_{i=1}^m u(z_i) \log_{\alpha} \left[\alpha - (\alpha - 1) \frac{\begin{aligned} &|T_{P_s}^-(\tau_j, z_i) - T_{D_k}^-(z_i)| + |T_{P_s}^+(\tau_j, z_i) - T_{D_k}^+(z_i)| + |U_{P_s}^-(\tau_j, z_i) - U_{D_k}^-(z_i)| \\ &+ |U_{P_s}^+(\tau_j, z_i) - U_{D_k}^+(z_i)| + |F_{P_s}^-(\tau_j, z_i) - F_{D_k}^-(z_i)| + |F_{P_s}^+(\tau_j, z_i) - F_{D_k}^+(z_i)| \\ &+ |\mu_{P_s}(\tau_j, z_i) - \mu_{D_k}(z_i)| + |\nu_{P_s}(\tau_j, z_i) - \nu_{D_k}(z_i)| + |\lambda_{P_s}(\tau_j, z_i) - \lambda_{D_k}(z_i)| \end{aligned}}{9} \right] \end{aligned} \quad (4)$$

for $\alpha \geq 2$.

Table 1
Characteristic values between the patient P_s and the symptom z_i .

	τ_j	z_1	z_2	...	z_m
P_s	τ_1	$P_s(\tau_1, z_1)$	$P_s(\tau_1, z_2)$...	$P_s(\tau_1, z_m)$
	τ_2	$P_s(\tau_2, z_1)$	$P_s(\tau_2, z_2)$...	$P_s(\tau_2, z_m)$

	τ_n	$P_s(\tau_n, z_1)$	$P_s(\tau_n, z_2)$...	$P_s(\tau_n, z_m)$

Table 2
Characteristic values between the symptom z_i and the considered disease D_k .

D_k	z_1	z_2	...	z_m
D_1	$D_1(z_1)$	$D_1(z_2)$...	$D_1(z_m)$
D_2	$D_2(z_1)$	$D_2(z_2)$...	$D_2(z_m)$
...
D_q	$D_q(z_1)$	$D_q(z_2)$...	$D_q(z_m)$

Table 3
Characteristic values between the symptom z_i and the considered disease D_k in the form of NCSs.

D_k	z_1 (temperature)	z_2 (headache)	z_3 (stomach pain)	z_4 (cough)	z_5 (chest pain)
D_1 (viral fever)	(<[0.3,0.5], [0.5,0.7], [0.0,0.1]>, <0.4,0.6,0.0>)	(<[0.2,0.4], [0.1,0.3], [0.4,0.6]>, <0.3,0.2,0.5>)	(<[0.0,0.2], [0.2,0.4], [0.6,0.8]>, <0.1,0.3,0.7>)	(<[0.3,0.5], [0.2,0.4], [0.2,0.4]>, <0.4,0.3,0.3>)	(<[0.0,0.2], [0.1,0.3], [0.6,0.8]>, <0.1,0.2,0.7>)
D_2 (malaria)	(<[0.6,0.8], [0.2,0.4], [0.0,0.1]>, <0.7,0.3,0.0>)	(<[0.1,0.3], [0.1,0.3], [0.5,0.7]>, <0.2,0.2,0.6>)	(<[0.0,0.1], [0.0,0.2], [0.8,1.0]>, <0.0,0.1,0.9>)	(<[0.6,0.8], [0.2,0.4], [0.0,0.1]>, <0.7,0.3,0.0>)	(<[0.0,0.2], [0.0,0.2], [0.7,0.9]>, <0.1,0.1,0.8>)
D_3 (typhoid)	(<[0.2,0.4], [0.3,0.5], [0.2,0.4]>, <0.3,0.4,0.3>)	(<[0.5,0.7], [0.2,0.4], [0.0,0.2]>, <0.6,0.3,0.1>)	(<[0.1,0.3], [0.0,0.2], [0.6,0.8]>, <0.2,0.1,0.7>)	(<[0.1,0.3], [0.1,0.3], [0.5,0.7]>, <0.2,0.2,0.6>)	(<[0.0,0.2], [0.0,0.1], [0.8,1.0]>, <0.1,0.0,0.9>)
D_4 (gastritis)	(<[0.0,0.2], [0.1,0.3], [0.6,0.8]>, <0.1,0.2,0.7>)	(<[0.1,0.3], [0.3,0.5], [0.3,0.5]>, <0.2,0.4,0.4>)	(<[0.7,0.9], [0.1,0.3], [0.0,0.1]>, <0.8,0.2,0.0>)	(<[0.1,0.3], [0.0,0.3], [0.6,0.8]>, <0.2,0.1,0.7>)	(<[0.1,0.3], [0.0,0.2], [0.6,0.8]>, <0.2,0.1,0.7>)
D_5 (stenocardia)	(<[0.0,0.2], [0.0,0.2], [0.7,0.9]>, <0.1,0.1,0.8>)	(<[0.0,0.0], [0.1,0.3], [0.7,0.9]>, <0.0,0.2,0.8>)	(<[0.1,0.3], [0.0,0.0], [0.7,0.9]>, <0.2,0.0,0.8>)	(<[0.1,0.3], [0.0,0.0], [0.7,0.9]>, <0.2,0.0,0.8>)	(<[0.7,0.9], [0.0,0.2], [0.0,0.2]>, <0.8,0.1,0.1>)

Step 2: Calculate the weighted LSM between the patient P_s and the considered disease D_k ($k = 1, 2, \dots, q$) in the time sequence set $\tau = \{\tau_1, \tau_2, \dots, \tau_n\}$ along with the weight (forgetting factor) vector $\omega(\tau) = (\omega(\tau_1), \omega(\tau_2), \dots, \omega(\tau_n))$ by

$$R(P_s, D_k) = \sum_{j=1}^n \omega(\tau_j) \times L(P_s(\tau_j), D_k) \quad (5)$$

Step 3: Regarding the maximum weighted LSM value of $R(P_s, D_k)$ for $k = 1, 2, \dots, q$, the proper diagnosis for the patient P_s can be determined.

Step 4: End.

5. Medical diagnosis example

In this section, the proposed medical diagnosis method using the LSM of DNCs is applied to a practical medical diagnosis problem adapted from (Ye, 2015; Ye and Fu, 2016), and both the diagnosis robustness and effectiveness of the proposed method are illustrated by comparative analysis.

Suppose a set of the considered diseases is given as $D = \{D_1, D_2, D_3, D_4, D_5\} = \{\text{viral fever, malaria, typhoid, gastritis, stenocardia}\}$, and a set of the relative symptoms is described by $Z = \{z_1, z_2, z_3, z_4, z_5\} = \{\text{temperature, headache, stomach pain, cough, chest pain}\}$. Then, the characteristic values of the diseases in D and the symptoms in Z are represented by the form of NCSs and are shown in Table 3.

Assume four patients need medical diagnosis. Three samples need to be taken for each patient P_s ($s = 1-4$) at three different time intervals, respectively, and then the information of the patient symptoms is expressed by DNCs. The basic element (NCN) $P_s(\tau_j, z_i)$ in DNCs is used to indicate the characteristic value between the patient P_s and the symptom z_i at the time τ_j . For example, the NCN $P_1(\tau_1, z_1) = \{(<[0.7, 0.9], [0.5, 0.7], [0.4, 0.6]>, <0.8, 0.6, 0.5>)\}$ is the characteristic values of the symptom z_1 for the patient P_1 at the first time τ_1 . Thus, the characteristic values of P_s ($s = 1-4$) on the symptom z_i ($i = 1-5$) at the three times τ_1, τ_2 , and τ_3 can be expressed by DNCs, which is composed of three NCNs for the three times, as shown in Table 4.

Suppose the weight vector of the five symptoms is given by $u = (1/5, 1/5, 1/5, 1/5, 1/5)$ without considering different weights of the five symptoms and another weight (forgetting factor) vector of the time sequence is indicated by $\omega(\tau) = (0.25, 0.35, 0.4)$. Then the proposed dynamic medical diagnosis method can be applied to the medical diagnosis example.

5.1. Dynamic medical diagnosis based on the logarithmic similarity measure

For the proposed medical diagnosis method, the dynamic medical diagnosis process for the patient P_s ($s = 1-4$) can be given by the following steps:

Step 1: By Eq. (4) with the parameter value $\alpha = 2$, the LSM $L(P_s(\tau_j), D_k)$ between the patient P_s ($s = 1-4$) and the considered disease D_k ($k = 1-5$) at each time τ_j ($j = 1, 2, 3$) can be obtained, as shown in Table 5.

Table 4
Characteristic values between the patient P_s and the symptom z_i in the form of DNCs.

P_s	τ_j	$z_1(\text{temperature})$	$z_2(\text{headache})$	$z_3(\text{stomach pain})$	$z_4(\text{cough})$	$z_5(\text{chest pain})$
P_1	τ_1	($\langle[0.7,0.9],$ $[0.5,0.7],$ $[0.4,0.6]\rangle,$ $\langle 0.8,0.6,0.5\rangle$)	($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.2,0.4]\rangle,$ $\langle 0.5,0.4,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.2,0.4]\rangle,$ $\langle 0.2,0.1,0.3\rangle$)	($\langle[0.6,0.8],$ $[0.5,0.7],$ $[0.2,0.4]\rangle,$ $\langle 0.7,0.6,0.3\rangle$)	($\langle[0.3,0.5],$ $[0.2,0.4],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.3,0.2\rangle$)
		($\langle[0.6,0.8],$ $[0.2,0.4],$ $[0.1,0.3]\rangle,$ $\langle 0.7,0.3,0.2\rangle$)	($\langle[0.5,0.7],$ $[0.2,0.4],$ $[0.1,0.3]\rangle,$ $\langle 0.6,0.3,0.2\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.3,0.5]\rangle,$ $\langle 0.3,0.2,0.4\rangle$)	($\langle[0.5,0.7],$ $[0.4,0.6],$ $[0.1,0.3]\rangle,$ $\langle 0.6,0.5,0.2\rangle$)	($\langle[0.5,0.7],$ $[0.4,0.6],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.5,0.3\rangle$)
		($\langle[0.4,0.6],$ $[0.1,0.3],$ $[0.3,0.5]\rangle,$ $\langle 0.5,0.2,0.4\rangle$)	($\langle[0.5,0.7],$ $[0.2,0.4],$ $[0.3,0.5]\rangle,$ $\langle 0.6,0.3,0.4\rangle$)	($\langle[0.2,0.4],$ $[0.2,0.4],$ $[0.4,0.6]\rangle,$ $\langle 0.3,0.3,0.5\rangle$)	($\langle[0.3,0.5],$ $[0.2,0.4],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.3,0.2\rangle$)	($\langle[0.5,0.7],$ $[0.3,0.5],$ $[0.3,0.5]\rangle,$ $\langle 0.6,0.4,0.4\rangle$)
	τ_2	($\langle[0.5,0.7],$ $[0.5,0.7],$ $[0.0,0.2]\rangle,$ $\langle 0.6,0.6,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.1,0.2,0.6\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.7,0.9]\rangle,$ $\langle 0.3,0.2,0.8\rangle$)	($\langle[0.5,0.7],$ $[0.1,0.3],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.2,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.2,0.4],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.3,0.7\rangle$)
		($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.1,0.3]\rangle,$ $\langle 0.5,0.4,0.2\rangle$)	($\langle[0.1,0.3],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.2,0.2,0.6\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.1,0.7\rangle$)	($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
		($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.4,0.6]\rangle,$ $\langle 0.2,0.1,0.5\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.8,1.0]\rangle,$ $\langle 0.1,0.1,0.9\rangle$)	($\langle[0.6,0.8],$ $[0.1,0.3],$ $[0.0,0.1]\rangle,$ $\langle 0.7,0.2,0.0\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
	τ_3	($\langle[0.2,0.4],$ $[0.0,0.2],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.1,0.2\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.2,0.2\rangle$)	($\langle[0.6,0.8],$ $[0.5,0.7],$ $[0.6,0.8]\rangle,$ $\langle 0.7,0.6,0.7\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.2,0.2\rangle$)	($\langle[0.3,0.5],$ $[0.3,0.5],$ $[0.2,0.4]\rangle,$ $\langle 0.4,0.4,0.3\rangle$)
		($\langle[0.3,0.5],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.2,0.2\rangle$)	($\langle[0.4,0.6],$ $[0.0,0.2],$ $[0.2,0.4]\rangle,$ $\langle 0.5,0.1,0.3\rangle$)	($\langle[0.3,0.5],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.2,0.2\rangle$)	($\langle[0.4,0.6],$ $[0.2,0.4],$ $[0.2,0.4]\rangle,$ $\langle 0.5,0.3,0.3\rangle$)	($\langle[0.5,0.7],$ $[0.2,0.4],$ $[0.1,0.3]\rangle,$ $\langle 0.6,0.3,0.2\rangle$)
		($\langle[0.7,0.9],$ $[0.6,0.8],$ $[0.5,0.7]\rangle,$ $\langle 0.8,0.7,0.6\rangle$)	($\langle[0.6,0.8],$ $[0.4,0.6],$ $[0.4,0.6]\rangle,$ $\langle 0.7,0.5,0.5\rangle$)	($\langle[0.3,0.5],$ $[0.0,0.2],$ $[0.0,0.2]\rangle,$ $\langle 0.4,0.1,0.1\rangle$)	($\langle[0.6,0.8],$ $[0.2,0.4],$ $[0.3,0.5]\rangle,$ $\langle 0.7,0.3,0.4\rangle$)	($\langle[0.6,0.8],$ $[0.3,0.5],$ $[0.4,0.6]\rangle,$ $\langle 0.7,0.4,0.5\rangle$)
P_2	τ_1	($\langle[0.5,0.7],$ $[0.5,0.7],$ $[0.0,0.2]\rangle,$ $\langle 0.6,0.6,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.1,0.2,0.6\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.7,0.9]\rangle,$ $\langle 0.3,0.2,0.8\rangle$)	($\langle[0.5,0.7],$ $[0.1,0.3],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.2,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.2,0.4],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.3,0.7\rangle$)
		($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.1,0.3]\rangle,$ $\langle 0.5,0.4,0.2\rangle$)	($\langle[0.1,0.3],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.2,0.2,0.6\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.1,0.7\rangle$)	($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
		($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.4,0.6]\rangle,$ $\langle 0.2,0.1,0.5\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.8,1.0]\rangle,$ $\langle 0.1,0.1,0.9\rangle$)	($\langle[0.6,0.8],$ $[0.1,0.3],$ $[0.0,0.1]\rangle,$ $\langle 0.7,0.2,0.0\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
	τ_2	($\langle[0.5,0.7],$ $[0.5,0.7],$ $[0.0,0.2]\rangle,$ $\langle 0.6,0.6,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.1,0.2,0.6\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.7,0.9]\rangle,$ $\langle 0.3,0.2,0.8\rangle$)	($\langle[0.5,0.7],$ $[0.1,0.3],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.2,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.2,0.4],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.3,0.7\rangle$)
		($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.1,0.3]\rangle,$ $\langle 0.5,0.4,0.2\rangle$)	($\langle[0.1,0.3],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.2,0.2,0.6\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.1,0.7\rangle$)	($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
		($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.4,0.6]\rangle,$ $\langle 0.2,0.1,0.5\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.8,1.0]\rangle,$ $\langle 0.1,0.1,0.9\rangle$)	($\langle[0.6,0.8],$ $[0.1,0.3],$ $[0.0,0.1]\rangle,$ $\langle 0.7,0.2,0.0\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
	τ_3	($\langle[0.2,0.4],$ $[0.0,0.2],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.1,0.2\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.2,0.2\rangle$)	($\langle[0.6,0.8],$ $[0.5,0.7],$ $[0.6,0.8]\rangle,$ $\langle 0.7,0.6,0.7\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.2,0.2\rangle$)	($\langle[0.3,0.5],$ $[0.3,0.5],$ $[0.2,0.4]\rangle,$ $\langle 0.4,0.4,0.3\rangle$)
		($\langle[0.3,0.5],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.2,0.2\rangle$)	($\langle[0.4,0.6],$ $[0.0,0.2],$ $[0.2,0.4]\rangle,$ $\langle 0.5,0.1,0.3\rangle$)	($\langle[0.3,0.5],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.2,0.2\rangle$)	($\langle[0.4,0.6],$ $[0.2,0.4],$ $[0.2,0.4]\rangle,$ $\langle 0.5,0.3,0.3\rangle$)	($\langle[0.5,0.7],$ $[0.2,0.4],$ $[0.1,0.3]\rangle,$ $\langle 0.6,0.3,0.2\rangle$)
		($\langle[0.7,0.9],$ $[0.6,0.8],$ $[0.5,0.7]\rangle,$ $\langle 0.8,0.7,0.6\rangle$)	($\langle[0.6,0.8],$ $[0.4,0.6],$ $[0.4,0.6]\rangle,$ $\langle 0.7,0.5,0.5\rangle$)	($\langle[0.3,0.5],$ $[0.0,0.2],$ $[0.0,0.2]\rangle,$ $\langle 0.4,0.1,0.1\rangle$)	($\langle[0.6,0.8],$ $[0.2,0.4],$ $[0.3,0.5]\rangle,$ $\langle 0.7,0.3,0.4\rangle$)	($\langle[0.6,0.8],$ $[0.3,0.5],$ $[0.4,0.6]\rangle,$ $\langle 0.7,0.4,0.5\rangle$)
P_3	τ_1	($\langle[0.5,0.7],$ $[0.5,0.7],$ $[0.0,0.2]\rangle,$ $\langle 0.6,0.6,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.1,0.2,0.6\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.7,0.9]\rangle,$ $\langle 0.3,0.2,0.8\rangle$)	($\langle[0.5,0.7],$ $[0.1,0.3],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.2,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.2,0.4],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.3,0.7\rangle$)
		($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.1,0.3]\rangle,$ $\langle 0.5,0.4,0.2\rangle$)	($\langle[0.1,0.3],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.2,0.2,0.6\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.1,0.7\rangle$)	($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
		($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.4,0.6]\rangle,$ $\langle 0.2,0.1,0.5\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.8,1.0]\rangle,$ $\langle 0.1,0.1,0.9\rangle$)	($\langle[0.6,0.8],$ $[0.1,0.3],$ $[0.0,0.1]\rangle,$ $\langle 0.7,0.2,0.0\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
	τ_2	($\langle[0.5,0.7],$ $[0.5,0.7],$ $[0.0,0.2]\rangle,$ $\langle 0.6,0.6,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.1,0.2,0.6\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.7,0.9]\rangle,$ $\langle 0.3,0.2,0.8\rangle$)	($\langle[0.5,0.7],$ $[0.1,0.3],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.2,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.2,0.4],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.3,0.7\rangle$)
		($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.1,0.3]\rangle,$ $\langle 0.5,0.4,0.2\rangle$)	($\langle[0.1,0.3],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.2,0.2,0.6\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.1,0.7\rangle$)	($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
		($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.4,0.6]\rangle,$ $\langle 0.2,0.1,0.5\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.8,1.0]\rangle,$ $\langle 0.1,0.1,0.9\rangle$)	($\langle[0.6,0.8],$ $[0.1,0.3],$ $[0.0,0.1]\rangle,$ $\langle 0.7,0.2,0.0\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
	τ_3	($\langle[0.2,0.4],$ $[0.0,0.2],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.1,0.2\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.2,0.2\rangle$)	($\langle[0.6,0.8],$ $[0.5,0.7],$ $[0.6,0.8]\rangle,$ $\langle 0.7,0.6,0.7\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.3,0.2,0.2\rangle$)	($\langle[0.3,0.5],$ $[0.3,0.5],$ $[0.2,0.4]\rangle,$ $\langle 0.4,0.4,0.3\rangle$)
		($\langle[0.3,0.5],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.2,0.2\rangle$)	($\langle[0.4,0.6],$ $[0.0,0.2],$ $[0.2,0.4]\rangle,$ $\langle 0.5,0.1,0.3\rangle$)	($\langle[0.3,0.5],$ $[0.1,0.3],$ $[0.1,0.3]\rangle,$ $\langle 0.4,0.2,0.2\rangle$)	($\langle[0.4,0.6],$ $[0.2,0.4],$ $[0.2,0.4]\rangle,$ $\langle 0.5,0.3,0.3\rangle$)	($\langle[0.5,0.7],$ $[0.2,0.4],$ $[0.1,0.3]\rangle,$ $\langle 0.6,0.3,0.2\rangle$)
		($\langle[0.7,0.9],$ $[0.6,0.8],$ $[0.5,0.7]\rangle,$ $\langle 0.8,0.7,0.6\rangle$)	($\langle[0.6,0.8],$ $[0.4,0.6],$ $[0.4,0.6]\rangle,$ $\langle 0.7,0.5,0.5\rangle$)	($\langle[0.3,0.5],$ $[0.0,0.2],$ $[0.0,0.2]\rangle,$ $\langle 0.4,0.1,0.1\rangle$)	($\langle[0.6,0.8],$ $[0.2,0.4],$ $[0.3,0.5]\rangle,$ $\langle 0.7,0.3,0.4\rangle$)	($\langle[0.6,0.8],$ $[0.3,0.5],$ $[0.4,0.6]\rangle,$ $\langle 0.7,0.4,0.5\rangle$)
P_4	τ_1	($\langle[0.5,0.7],$ $[0.5,0.7],$ $[0.0,0.2]\rangle,$ $\langle 0.6,0.6,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.1,0.2,0.6\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.7,0.9]\rangle,$ $\langle 0.3,0.2,0.8\rangle$)	($\langle[0.5,0.7],$ $[0.1,0.3],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.2,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.2,0.4],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.3,0.7\rangle$)
		($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.1,0.3]\rangle,$ $\langle 0.5,0.4,0.2\rangle$)	($\langle[0.1,0.3],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.2,0.2,0.6\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.1,0.7\rangle$)	($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
		($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.4,0.6]\rangle,$ $\langle 0.2,0.1,0.5\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.8,1.0]\rangle,$ $\langle 0.1,0.1,0.9\rangle$)	($\langle[0.6,0.8],$ $[0.1,0.3],$ $[0.0,0.1]\rangle,$ $\langle 0.7,0.2,0.0\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
	τ_2	($\langle[0.5,0.7],$ $[0.5,0.7],$ $[0.0,0.2]\rangle,$ $\langle 0.6,0.6,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.1,0.2,0.6\rangle$)	($\langle[0.2,0.4],$ $[0.1,0.3],$ $[0.7,0.9]\rangle,$ $\langle 0.3,0.2,0.8\rangle$)	($\langle[0.5,0.7],$ $[0.1,0.3],$ $[0.2,0.4]\rangle,$ $\langle 0.6,0.2,0.3\rangle$)	($\langle[0.1,0.3],$ $[0.2,0.4],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.3,0.7\rangle$)
		($\langle[0.4,0.6],$ $[0.3,0.5],$ $[0.1,0.3]\rangle,$ $\langle 0.5,0.4,0.2\rangle$)	($\langle[0.1,0.3],$ $[0.1,0.3],$ $[0.5,0.7]\rangle,$ $\langle 0.2,0.2,0.6\rangle$)	($\langle[0.1,0.3],$ $[0.0,0.2],$ $[0.6,0.8]\rangle,$ $\langle 0.2,0.1,0.7\rangle$)	($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)	($\langle[0.0,0.2],$ $[0.0,0.2],$ $[0.7,0.9]\rangle,$ $\langle 0.1,0.1,0.8\rangle$)
		($\langle[0.7,0.9],$ $[0.2,0.4],$ $[0.0,0.2]\rangle,$ $\langle 0.8,0.3,0.1\rangle$)				

Table 5
LSM values of $L(P_s(\tau_i), D_k)$ between the patient P_s and the considered disease D_k at the time τ_i .

$L(P_s(\tau_j), D_k)$	τ_1	τ_2	τ_3
$L(P_1(\tau_j), D_1)$	0.8115	0.8049	0.8525
$L(P_1(\tau_j), D_2)$	0.7872	0.7999	0.7975
$L(P_1(\tau_j), D_3)$	0.7805	0.7939	0.8246
$L(P_1(\tau_j), D_4)$	0.7375	0.7220	0.7850
$L(P_1(\tau_j), D_5)$	0.6969	0.6928	0.7592
$L(P_2(\tau_j), D_1)$	0.9276	0.9119	0.8871
$L(P_2(\tau_j), D_2)$	0.9052	0.9503	0.9740
$L(P_2(\tau_j), D_3)$	0.8266	0.8601	0.8267
$L(P_2(\tau_j), D_4)$	0.7687	0.7627	0.7334
$L(P_2(\tau_j), D_5)$	0.7392	0.7337	0.7078
$L(P_3(\tau_j), D_1)$	0.8372	0.8361	0.7768
$L(P_3(\tau_j), D_2)$	0.7684	0.7794	0.7517
$L(P_3(\tau_j), D_3)$	0.8089	0.7970	0.7486
$L(P_3(\tau_j), D_4)$	0.7903	0.7908	0.7618
$L(P_3(\tau_j), D_5)$	0.7536	0.7528	0.6882
$L(P_4(\tau_j), D_1)$	0.7784	0.7653	0.7223
$L(P_4(\tau_j), D_2)$	0.7284	0.7480	0.7165
$L(P_4(\tau_j), D_3)$	0.7926	0.7802	0.7435
$L(P_4(\tau_j), D_4)$	0.8249	0.8010	0.7879
$L(P_4(\tau_j), D_5)$	0.9458	0.9593	0.9505

Table 6
Diagnosis results of the four patients.

	D_1 (viral fever)	D_2 (malaria)	D_3 (typhoid)	D_4 (gastritis)	D_5 (stenocardia)	Diagnosis result
$R(P_1, D_k)$	0.8256	0.7958	0.8028	0.7511	0.7204	viral fever
$R(P_2, D_k)$	0.9059	0.9485	0.8383	0.7525	0.7247	malaria
$R(P_3, D_k)$	0.8126	0.7656	0.7806	0.7791	0.7272	viral fever
$R(P_4, D_k)$	0.7514	0.7305	0.7686	0.8018	0.9524	stenocardia

Step 2: By Eq. (5) with the parameter value $\alpha = 2$, calculate the weighted LSM $R(P_s, D_k)$ between the patient P_s ($s = 1-4$) and the considered disease D_k ($k = 1-5$) in the time sequence set τ along with the relative weighted vector $\omega(\tau)$. The obtained results are shown in Table 6. For instance, the LSM results between the patient P_1 and the considered disease D_k ($k = 1-5$) are given as $R(P_1, D_1) = 0.8256$, $R(P_1, D_2) = 0.7958$, $R(P_1, D_3) = 0.8028$, $R(P_1, D_4) = 0.7511$, and $R(P_1, D_5) = 0.7204$.

Step 3: According to the maximum weighted LSM values $R(P_1, D_1) = 0.8256$, $R(P_2, D_2) = 0.9485$, $R(P_3, D_3) = 0.8126$, and $R(P_4, D_4) = 0.9524$ among all weighted LSM values, patients P_1 and P_3 suffer from viral fever, P_2 suffers from malaria, and P_4 suffers from stenocardia.

5.2. Diagnosis robust analysis

To illustrate the robustness of the proposed medical diagnosis method, the proposed method is verified by the different parameter values of α in Eqs. (4) and (5) in the range of [2, 100]. Regarding the diagnosis steps listed in Section 5.1, the LSM values of $R(P_s, D_k)$ are obtained, as shown in Fig. 1. Obviously, for each patient P_s , although the relative LSM values of $R(P_s, D_k)$ regarding the considered disease D_k ($k = 1-5$) increase with the increasing of the parameter values of α , the ranking orders and diagnosis results of the possible suffering diseases keep identical. Thus, the proposed medical diagnosis method is robust for the practical application regarding the LSM family.

5.3. Comparative analysis

To illustrate the effectiveness of the proposed medical diagnosis method, some comparison with existing similar diagnosis method (Ye and Fu, 2016) was carried out under the environment of multi-period SvNSs. To apply the existing diagnosis method (Ye and Fu, 2016) with

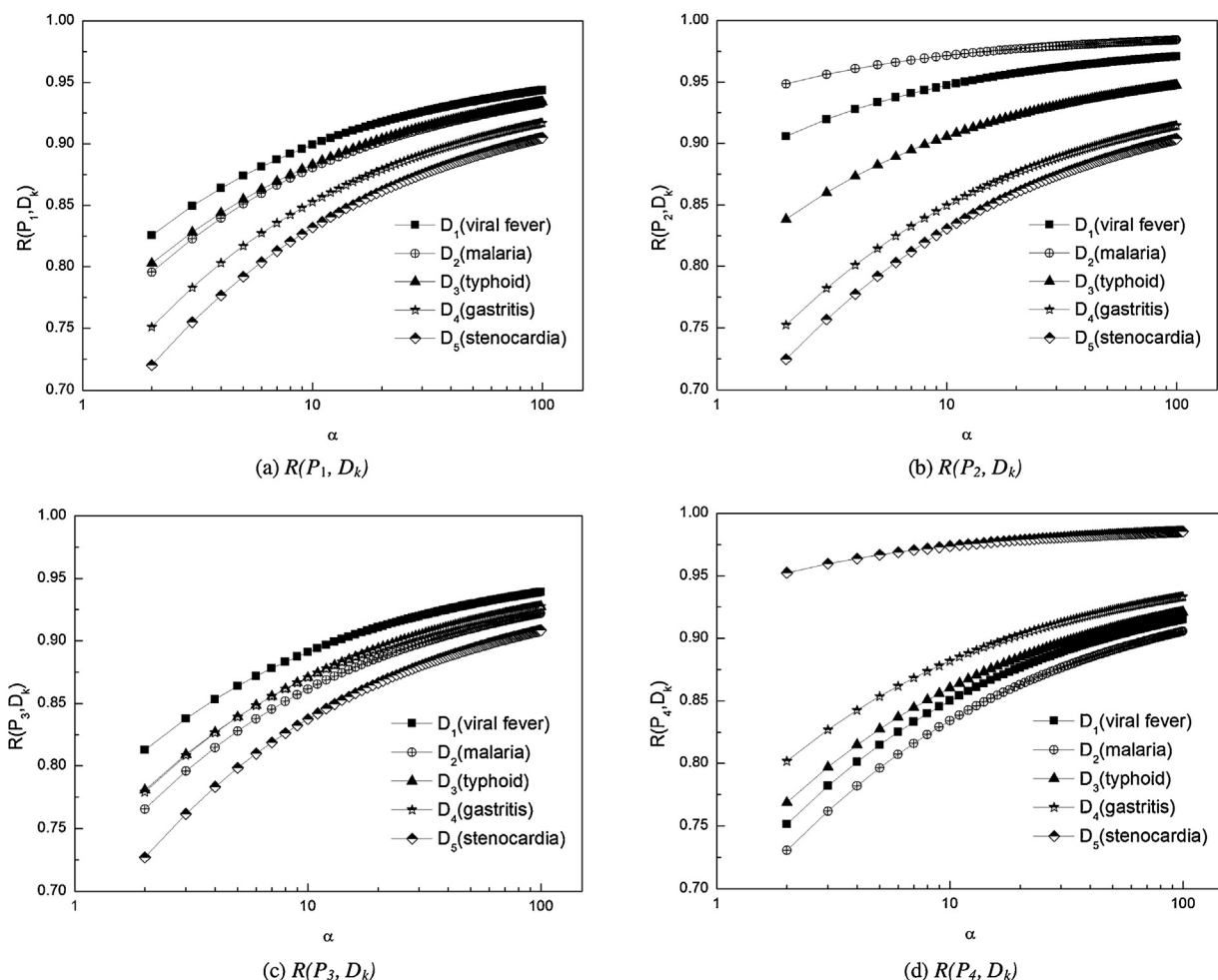


Fig. 1. LSM values of $R(P_s, D_k)$ between the patient P_s and the disease D_k for $\alpha \in [2, 100]$.

the multi-period SvNSs to the medical diagnosis problem with DNCs information, a special case of DNCs can be considered at the condition that there are no IvNSs in DNCs, in other words, DNCs are reduced to the dynamical SvNSs. Thus, the diagnosis results obtained from the reference (Ye and Fu, 2016) are the same as the diagnosis results obtained by the proposed new diagnosis method in the medical diagnosis example. It is obvious that the proposed medical diagnosis method using the LSM of DNCs is effective and reasonable. And also the algorithms in the proposed method are easily implemented in computer software, which can be used to establish the computer-aided diagnosis and decision system.

5.4. Discussion

The above results and analysis indicate that the diagnosis method applying the LSM of DNCs is robust and effective. The presented DNCs provides a novel way to represent the dynamic certain and uncertain information which widely exists in clinical medical diagnosis situations. Then, the current multi-period SvNS (dynamic SvNS) can only depict dynamic certain information at different time intervals and the NCS can only express certain and uncertain information at a specific time interval. However, the multi-cycle SvNS or NCS cannot express the DNCs information and is a special case of DNCs. It is obvious that the presented DNCs is superior to the multi-cycle SvNS or NCS in the information representation.

Moreover, it is the first time that DNCs have been introduced to medical diagnosis in this work. The LSM of DNCs can handle medical diagnosis problems with DNCs information. The advantage of the proposed diagnosis method is that it can express NCNs at different time intervals (i.e. DNCs) and deal with the medical diagnosis problems with dynamic neutrosophic cubic information, which existing medical diagnosis methods cannot do. Although the base $\alpha \geq 2$ in the LSM of DNCs is limited as the insufficiency, the proposed medical diagnosis method based on the LSM of DNCs is effective and reasonable and shows the robustness of the diagnosis results to some extent. Therefore, this work provides basic theory and algorithms for the computer-aided diagnosis and decision system.

6. Conclusion

This study presented the concept of DNCs for the first time to describe the hybrid form of a dynamic IvNS and a dynamic SvNS along with the truth, indeterminacy, and falsity degrees in a time consequence (at different time intervals), and then gave the basic relations and operations of DNCs. Next, the study proposed the LSM of DNCs and investigated its properties. Then, a medical diagnosis method was proposed based on the weighted LSM of DNCs. Lastly, a practical medical diagnosis example with dynamic neutrosophic cubic information was presented to illustrate the application of the developed medical diagnosis method and to verify the robustness and effectiveness by comparative analysis. The diagnosis results showed that the proposed dynamic medical diagnosis method can indicate a proper diagnosis result from dynamic medical diagnosis information. However, it should be noted that the LSM of the DNCs proposed in this work is valid only when the base $\alpha \geq 2$ in the LSM of DNCs is limited in this condition. Therefore, in the future research, novel similarity measures for DNCs should be further developed to overcome the limitation and applied to the diagnosis/evaluation of benign prostatic hyperplasia, prostate cancer, renal cancer, and so on.

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