Intuitionistic and Neutrosophic Fuzzy Logic: Basic Concepts and Applications

Amita Jain and Basanti Pal Nandi

Abstract Fuzzy set proposed by Zadeh states that belongingness of an element in a set is a matter of degree unlike classical set where membership is a matter of affirmation or denial. Fuzzy set theory provides more natural representation for real world problems. Intuitionistic fuzzy set (IFS) is the generalization of fuzzy set, proposed by Atanassov, in 1986 (Fuzzy Sets Syst 20(1):87–96, 1986 [1]). It assigns two values called membership degree and a non-membership degree respectively. Later Florentin Smarandache introduced an additional parameter for neutrality which generalise Intuitionistic Fuzzy Set as Neutrosophic Fuzzy Set (NFS). The speciality lies in the 3D Neutrosophic space where each logical statement is evaluated with 3 components namely truth, falsity and indeterminacy. IFS and NFS revolve around these divisions of degree of belongingness to their component structure and so generate different variations. In this chapter we discuss the properties of these two variants of fuzzy set based on their different extension, propositional calculus, predicate calculus, degree of dependence of each component, geometric representation and various application areas of both the sets.

Keywords Intuitionistic fuzzy · Neutrosophic cube · Neutrosophic fuzzy · Predicate calculus · Propositional calculus · Research statistics

1 Introduction

Elements of classical fuzzy set has a membership value assigned to it. In Fuzzy set each element is mapped with some real value between 0 and 1. There are various ways to choose membership functions for classical fuzzy set like triangular function, trapezoid, L-function etc. Intuitionistic fuzzy set (IFS) [2, 3] is an extension to the

A. Jain (✉)
Ambedkar Institute of Advanced Communication Technologies and Research, Delhi, India
e-mail: amitajain@aiactr.ac.in

B. Pal Nandi
Guru Tegh Bahadur Institute of Technology, New Delhi, India
e-mail: basanti_pal@yahoo.com

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classical fuzzy set where each element assigns a membership value and a nonmembership value contrast to the Zahed's fuzzy set [4, 5] where each element assigns only a membership value. Degree of Indeterminacy or degree of Hesitancy is the parameter which is calculated from the two varying parameter mentioned for membership and non-membership category. This concept was extended by Florentin Smarandache [1] which includes a three way decisions. This three way decision provides more natural representation for real world problems. It can be understood by considering some real world examples like for games: win/loss/draw, for voting: pro/cont/blank, for numbers: positive/negative/zero etc. In all these real world problems there are three decisions. To deal with this tri-component logic, Florentin proposed Neutrosophic logic [6–8] which means "The knowledge of neutral thought". The main difference between Intuitionistic fuzzy set (IFS) and Neutrosophic fuzzy set (NFS) is the middle/neutral/indeterminant component. In Neutrosophic Fuzzy set when the summation of truth, indeterminacy and falsity is ≤ 1 then the set is both IFS and NFS. On the other hand when the values of truth, falsity and indeterminacy overlap i.e., the summation of these three membership value is ≥ 1 then the set is NFS not IFS [9].

In this chapter a picture of comparison on the basis of variations of IFS and NFS is represented. Geometrical interpretations as well as the applications of these two logics in different fields are also discussed. IFS and its variants like 'Interval valued Intuitionistic fuzzy set', 'Intuitionistic L-fuzzy set', 'Temporal Intuitionistic fuzzy set' etc. have been applied on various fields like electoral system, medical pattern recognition, medical diagnosis, sociometry, petrochemical farm, pneumatic transportation process etc. [10–12]. On the other hand Neutrosophic Fuzzy logic has been applied in fields like physics, robotics, image segmentation, generating score of a neutral word etc. [13–16]. It gives a precise way to capture inconsistent or imprecise factors of a given problem. It continues to evolve new variations like 'Interval Neutrosophic set', 'Single valued Neutrosophic set', 'Refined Neutrosophic set'. The elements of Intuitionistic fuzzy logics give the relationship between the membership values with the non-membership value. The basic elements of Intuitionistic fuzzy which are covered in this chapter are 'Intuitionistic fuzzy propositional calculus', 'Intuitionistic fuzzy predicate logic'. Neutrosophic elements analogous to Intuitionistic fuzzy set express its propositional calculus and predicate calculus on an instance of neutrosophic theory called 'Interval valued Neutrosophic logic'. To show the differences in these two extensions of classical fuzzy system geometric interpretations give a visual idea which was shown by J. Dezert in 2002 [17] using a Neutrosophic cube. He has shown the difference between absolute and relative neutrosophic values and shown the regional partition for Intuitionistic and Neutrosophic ranges. As the summation of membership and non-membership values are different based on interdependency, in each case the degree of dependency has also been discussed by Florentin [18].

2 Intuitionist Fuzzy Set (IFS)

An Intuitionist Fuzzy set A in the domain E is defined according to the following form

$A = \{< x, \mu_a(x), \nu_a(x) > | x \in E\}$, where $(\mu_a(x))$ is the degree of membership and $(\nu_a(x))$ is the degree of non-membership which lies in the range [0, 1] and $0 \le \mu_a(x) + \nu_a(x) \le 1$.

This logic differs from classical fuzzy when the term indeterministic or hesitancy comes and defined in Intuitionistic fuzzy as $\pi_a = 1 - \mu_a(x) - \nu_a(x)$, It is degree of hesitancy of x to A. When the term π_a becomes 0 the set becomes classical fuzzy set [2].

3 Neutrosophic Fuzzy Set (NFS)

In Neutrosophic set three standard and non standard subset T (Truth), I (Indeterminacy), F (False) are defined for each element of the set where the ranges of these components lies in $]^{-}0\ 1^{+}[$. An element x in universe U within a set M has three components with (t, i, f) which interprets as the belongingness of x in M represents with t% truth, i% indeterminacy, and f% falsehood. The components vary from 0 to 1, even can be less than 0 or greater than 1. The components are not necessarily a number but can be a subset of type discrete or continuous set. The other categories can be open, closed, half open, half closed sets and it can also be union or intersection of previous sets [19].

4 Operations on Intuitionistic and Neutrosophic Fuzzy Set

4.1 Some Operations on Intuitionistic Fuzzy Set

A, B are two Intuitionistic Fuzzy Sets of the set E where $\mu(x)$ is the degree of membership and $\nu(x)$ is the degree of non-membership [2] then,

1. $A \subset B\, iff\ (\forall x \in E)(\mu_a(x) \le \mu_b(x)\, and\, \nu_a(x) \ge \nu_b(x))$
2. $A = B\, iff\, (\forall x \in E)(\mu_a(x) = \mu_b(x)\, and\, \nu_a(x) = \nu_b(x))$
3. $A \wedge B = \{< x, \min(\mu_a(x), \mu_b(x)), \max(\nu_a(x), \nu_b(x)) > | x \in E\}$
4. $A \vee B = \{< x, \max(\mu_a(x), \mu_b(x)), \min(\nu_a(x), \nu_b(x)) > | x \in E\}$
5. $A + B = \{< x, (\mu_a(x) + \mu_b(x) - \mu_a(x).\mu_b(x)), \nu_a(x).\nu_b(x) > | x \in E\}$
6. $A\,.\,B = \{x, \mu_a(x)\,.\,\mu_b(x), (\nu_a(x) + \nu_b(x) - \nu_a(x)\,.\,\nu_b(x)) > | x \in E\}$

4.2 Some Operations on Neutrosophic Fuzzy Set

A,B are the sets over Universe U, where $x(T_1, I_1, F_1) \in A$ represents element x in set A having T_1 as neutrosophic membership, I_1 as neutrosophic indeterminacy and F_1 as neutrosophic non-membership values [9].

1. $If\ x(T_1, I_1, F_1) \in A\ and\ x(T_2, I_2, F_2) \in B\ then,$

$$A \cup B = x(T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2)$$

2. $If\ x(T_1, I_1, F_1) \in A\ and\ x(T_2, I_2, F_2) \in B\ then,$

$$A \cap B = x(T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2)$$

3. $If\ x(T_1, I_1, F_1) \in A\ and\ x(T_2, I_2, F_2) \in B\ then,$

$$A/B = x(T_1 \ominus T_1 \odot T_2, I_1 \ominus I_1 \odot I_2, F_1 \ominus F_1 \odot F_2)$$

4. $If\ x(T_1, I_1, F_1) \in A\ and\ y\left(T', I', F'\right) \in B\ then,$

$$A \times B = \left(x(T_1, I_1, F_1), y\left(T', I', F'\right)\right)$$

5 Variants of Intuitionistic and Neutrosophic Fuzzy Set

5.1 Extensions of Intuitionistic Fuzzy Set

There are few extensions of Intuitionistic fuzzy set which change the universe or extent of this fuzzy and sometimes derived from the classical fuzzy.

5.1.1 Interval Valued Intuitionistic Fuzzy Set

Interval valued Intuitionistic Fuzzy is a combination to both Intuitionistic fuzzy and Interval valued fuzzy set. It is defined on the basic set E where its membership function is M_a and non-membership function is N_a[20],

M_a: $E \to INT([0, 1])$ and N_a: $E \to INT([0, 1])$ and $INT([0, 1])$ is the set of all subsets of the unit interval.

When each of the intervals M_a and N_a contains exactly one element for each element then the set becomes ordinary Intuitionistic fuzzy set. On the other hand if $N_a = \phi$ for each element then the set becomes simple Interval valued fuzzy set.

5.1.2 Intuitionistic L-Fuzzy Set

Intuitionistic L-Fuzzy set is another variation which is derived from L-fuzzy set, where L may be a complete Lattice, complete chain or a complete ordered semi-ring [21]. It is an object in E defined as

$A^* = \{< x, \mu_a(x), \nu_a(x) > | x \epsilon E\}$, where, membership value $\mu_a(x) : E \to L$,

Non-membership value $\nu_a(x) : E \to L$ where $x \epsilon E$ and for every $x \epsilon E$

$0 \le \mu_a(x) \le (\nu_a(x))'$, Where ‘: $L \to L$ is an order reserving operation in $(L, \le)$.

5.1.3 Temporal Intuitionistic Fuzzy Set

An instance of Temporal Intuitionistic Fuzzy Set A(T) is defined over non empty set E and T where elements of T is called ‘Time-moment’ [22].

$$A(T) = \{< x, \ \mu_a(x, t), \ \nu_a(x, t) > | (x, t) \epsilon E \times T\},$$

where:

(a) $A \subset E$ is a fixed set,
(b) $\mu_a(x, t) + \nu_a(x, t) \le 1$ for every $(x, \ t) \epsilon ExT$
(c) $\mu_a(x, t)$ and $\nu_a(x, t)$ are the degree of membership and non-membership value of the element $x \epsilon E$ at the time $t \epsilon T$.

5.1.4 Intuitionistic Fuzzy Set of Second Type

Intuitionistic fuzzy set of second type [1] is another extension of IFS and is defined with varied degree of dependence of membership and non-membership values. Let A be the object of IFS of second type such that

$A = \{< x, \mu_a(x), \nu_a(x) > | x \in E\}$, in which functions $\mu_a(x) : E \to [0, 1]$ and $\nu_a(x) : E \to [0, 1]$ is in the relation then [1],

$0 \le \mu_a(x)^2 + \nu_a(x)^2 \le 1$ and $\pi_a(x) = \sqrt{\left(1 - \mu_a(x)^2 - \nu_a(x)^2\right)}$, here $\pi_a(x)$ is the degree of non-determinacy of the element $x \in E$.

5.2 Extensions of Neutrosophic Fuzzy Set

5.2.1 Interval Neutrosophic Set

Interval Neutrosophic set [23] A in the space X for element $x \in X$ is characterised by Truth-membership function which is denoted by T_a, Indeterminacy-membership function denoted by I_a, Falsity-membership function denoted by F_a. For each element $x \in X$, $T_a(x)$, $I_a(x)$, $F_a(x) \subseteq [0, 1]$.

$$\text{When } X \text{ is continuous} = \int_X < T(x), I(x), F(x) > /x, x \in X$$

$$\text{When } X \text{ is discrete, } A = \sum_{i=1}^{n} < T(x_i), I(x_i), F(x_i) > /x_i,\ x_i \in X$$

For each point x in X we can define T(x) = [inf T(x), sup T(x)], I(x) = [inf I(x), sup I(x)], F(x) = [inf F(x), sup F(x)] ⊆ [0, 1] [24].

5.2.2 Single Valued Neutrosophic Set

Single Valued Neutrosophic set [25, 26] A in the space X for element $x \in X$ is characterised by Truth-membership function T_a, Indeterminacy-membership function I_a, Falsity-membership function F_a. For each element $x \in X$, $T_a(x)$, $I_a(x)$, $F_a(x) \in [0, 1]$. It is a generalization of classical set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set and paraconsistent set.

A Single valued neutrosophic set A over a finite domain X is represented as

$$\text{When } X \text{ is continuous } A = \int_X < T(x), I(x), F(x) > /x, x \in X$$

$$\text{When } X \text{ is discrete } A = \sum_{i=1}^{n} < T(x_i), I(x_i), F(x_i) > /x_i, x_i \in X$$

Both of these variants of Neutrosophic set differs from the ordinary Neutrosophic set as the truth-membership value, Indeterminacy-membership value and the False-membership value for Interval Neutrosophic Set (INS) and Single Valued Neutrosophic Set (SVNS) lies in the range of [0, 1] but for Neutrosophic set it is non-standard value $]^{-}0, 1^{+}[$.

5.2.3 Refined Neutrosophic Set

n-valued refined neutrosophic set [16] introduced by Florentin [27] has a general definition where the Truth, Indeterminacy and Falsehood are subdivided into parts of the same components. An element x of A is comprised of $x\left(T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s\right) \in A$

$$where\, p, r, s \geq 1\, and\; p + r + s \geq 3.$$

$T_1, T_2, \ldots, T_p; I_1, I_2, \ldots, I_r; F_1, F_2, \ldots, F_s$ are sub components of membership degrees, Indeterminacy degrees and non-membership degrees respectively.

6 Propositional and Predicate Calculus for Intuitionistic and Neutrosophic Fuzzy Set

6.1 Propositional and Predicate Calculus Defined Over Intuitionistic Fuzzy Logic

6.1.1 Propositional Calculus for Intuitionistic Fuzzy

For each proposition in Intuitionistic Fuzzy logic [28] a “truth-degree” and a “falsity-degree” is assigned to $\mu(p)$ and $\nu(p)$, here p is the proposition having a relation $\mu(p) + \nu(p) \leq 1$. Assignment of the proposition p to a function $V(p)$ can be defined as

$$V(p) = <\mu(p), \nu(p)>$$

The evaluation function over propositional logic is discussed below:

1. The evaluation of negation of proposition p i.e. $\sim p$ can be written as:

$$V(\sim p) = <\nu(p), \mu(p)>$$

2. The evaluation function for AND operator on proposition p and q

$$V(p \wedge q) = <\min(\mu(p), \mu(q)), \max(\nu(p), \nu(q))>$$

3. The evaluation function for OR operator on proposition p and q

$$V(p \vee q) = <\max(\mu(p), \mu(q)), \min(\nu(p), \nu(q))>$$

4. The evaluation of $p \supset q$ defined as

$$V(p \supset q) = <\max(\nu(p), \mu(q)), \min(\mu(p), \nu(q))>$$

6.1.2 Predicate Calculus for Intuitionistic Fuzzy

Predicate Calculus is the application of propositional operators over quantifiers (Existential and Universal). Predicate logic formulae are the applications of quantifiers using propositional operations "∼","⋀"," ∨","⊃". If A is a formula and x-A are variables then Ɐ × A and ∋ × A are formulae. The function V defines on x ranges over E:

$$\mathrm{V}(\forall \mathrm{xA}) = < \min\ \mu(A(i(x) = a)), \max\ \nu(A(i(x) = a))> \textit{ where } a \in E$$
$$\mathrm{V}(\exists \mathrm{xA}) = < \max\ \mu(A(i(x) = a)), \min\ \nu(A(i(x) = a))> \textit{ where } a \in E$$

6.2 Propositional and Predicate Calculus Defined Over Neutrosophic Fuzzy Logic

In the case of Neutrosophic Fuzzy set propositional and predicate calculus [23] are defined over its variant Interval Neutrosophic set. We have already discussed the basics of Interval Neutrosophic set. In this section we will discuss only the Propositional Calculus and Predicate Calculus on Interval Neutrosophic Logic (INL). The proposition p in Interval Neutrosophic Fuzzy consists of $<t(p), i(p), f(p)>$ where $t(p), i(p), f(p) \in [0, 1]$ and it comprises of a syntax and a semantics to define the well-formed formulae.

6.2.1 Propositional Calculus for Neutrosophic Fuzzy

The set of formulae (well-formed formulae) on Interval Neutrosophic Propositional Calculus defined by its semantics are as follows in Table 1.

Table 1 Propositional calculus for Neutrosophic Fuzzy

Connectives	Semantics
$INL(\sim p)$	$<f(p), 1 - i(p), t(p)>$
$INL(p \wedge q)$	$< \min(t(p), t(q)), \max(i(p), i(q)), \max(f(p), f(q))>$
$INL(p \vee q)$	$< \max(t(p), t(q)), \min(i(p), i(q)), \min(f(p), f(q))>$
$INL(p \rightarrow q)$	$< \min(1, 1 - t(p) + t(q)), \max(0, i(q) - i(p)), \max(0, f(q) - f(p)) >$

Table 2 Predicate calculus for neutrosophic fuzzy

Connectives	Semantics
$INP(\forall x F)$	$< \min t(F(E(x))), \min i(F(E(x))), \max f(F(E(x))) >, E(x) \in D$
$INP(\exists x F)$	$< \max t(F(E(x))), \max i(F(E(x))), \min f(F(E(x))) >, E(x) \in D$

6.2.2 Predicate Calculus for Neutrosophic Fuzzy

The semantics in first order predicate logic on Interval Neutrosophic set gives the meaning of well formed formulae. The predicate calculus semantics are same as propositional calculus for the four connectives "~", "∧", "∨", "→". The semantics of qualifiers on the propositions of Interval Neutrosophic Predicate Logic (INP) are given below. Here the interpretation function (or interpretation) of a formula F in the first order interval neutrosophic predicate logic consists of a nonempty domain D (Table 2).

7 Degree of Dependence of Each Component of Neutrosophic and Intuitionistic Fuzzy

For Single valued neutrosophic set the sum of the components $(T + I + F)$ lies between 0 to 3 [18] when all three components are independent i.e, $0 \leq T+I+F \leq 3$.

When two components are dependent and the third component is independent with respect to the other two then the summation of the three lies between 0 to 2 i.e, $0 \leq T + I + F \leq 2$.

If each component of the three is dependent on each other, then the sum is between 0 to 1 i.e, $0 \leq T + I + F \leq 1$.

When the summation is greater than 1 i.e, when three or two components among T, I, F are independent then the information may be incomplete information $(sum < 1)$ or para-consistent or contradictory information $(sum > 1)$ or complete information $(sum = 1)$.

If T, I, F are dependent then the information is either incomplete $(sum < 1)$ or complete $(sum = 1)$.

Three sources of T, I, F are independent if they do not influence each other and gives a max summation value 3. If they are fully dependent then it gives a max summation value 1.

In the case of Intuitionistic Fuzzy set the properties of dependence or independence for the two valued fuzzy set is applicable. In Intuitionistic fuzzy two components $\mu(x)$ and $\nu(x)$ that vary in the unit interval [0, 1] may be dependent or independent with a dependence degree $d^{\circ}(\mu(x), \nu(x))$ with a relation: $0 \leq \mu(x) + \nu(x) \leq 2-d^{\circ}(\mu(x), \nu(x))$ where $d^{\circ}(\mu(x), \nu(x))$ is called "degree of dependence". It is 0 when $\mu(x)$ and $\nu(x)$ are completely independent and 1 if both of them are completely dependent. The values of $d^{\circ}(\mu(x), \nu(x))$ lie between [0, 1].

Therefore $2 - d^{\circ}(\mu(x), \nu(x))$ is degree of independence between $\mu(x)$ and $\nu(x)$.

8 Geometrical Interpretation of Neutrosophic and Intuitionistic Fuzzy

A Neutrosophic cube introduced by J. Desert in 2002 [17] is a useful tool to visualize the concept of relative and absolute Neutrosophic set [29] and has the ability to differentiate between Intuitionistic and Neutrosophic ranges. It is drawn in the 3D Cartesian coordinate system where T is the truth axis value ranges from $]^{-}0\ 1^{+}[$, I is the indeterminacy axis value ranges from $]^{-}0\ 1^{+}[$ and F is the false axis value ranges from $]^{-}0\ 1^{+}[$.

In Fig. 1 we have taken a Neutrosophic cube with ranges [0, 1]. The cube can be extended in the more positive and more negative directions to get the range $]^{-}0\ 1^{+}[$.

The triangle ACH has the side of $\sqrt{2}$ units and has the summation of locus is 1 for any point situated on it. So, for any point p on or inside the triangle ACH gives $t_p + i_p + f_p = 1$ and it represents Atanassov-Intuitionistic fuzzy set.

Whereas points inside the pyramid ADCH including its side ADH,ADC and DCH but excluding the side ACH gives summation of locus value less than 1.

So for p in ADCH we have $t_p + i_p + f_p \leq 1$ and gives incomplete information.

The solid on the opposite side of pyramid ADCH with respect to the side ACH gives partially or fully independent information with $t_p + i_p + f_p \geq 1$.

9 Applications of Intuitionistic and Neutrosophic Fuzzy

IFS and NFS differ from classical fuzzy by incorporating undecidable and non-membership factors in the element of the set. The systems where these uncertain and non-membership elements lies underlying there the application of such fuzzy systems are successful. Liu and Wang [30] has applied IFS in Multi-criteria decision making problem. They have divided the uncertain portion of IFS into affirmative, dissent and abstention part in order to get decision by an evaluation function based on

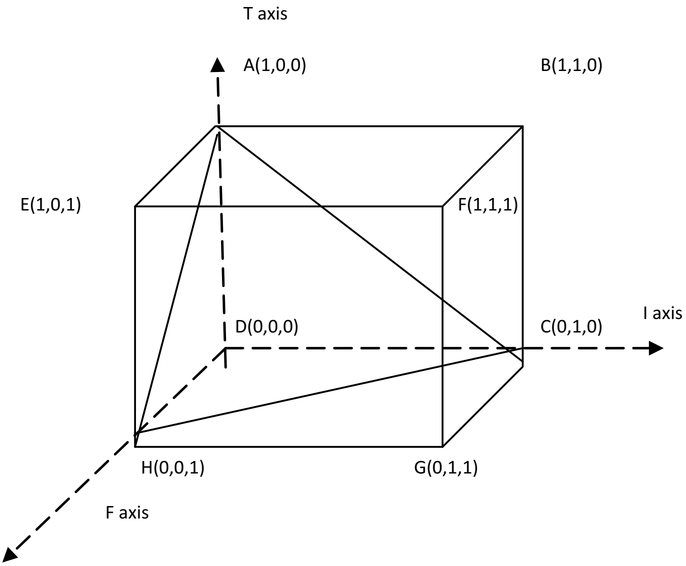

Fig. 1 Absolute Neutrosophic cube

Intuitionistic fuzzy logic. De et al. [10] determined the disease on the basis of symptoms by evaluation of max-min-max composition of IFS element with Intuitionistic Fuzzy relation. In this decision making problem non-membership function has more importance than membership function as there always exists a non-zero hesitant part. Fuzzy methods have been successfully applied for image segmentation. Huang et al. [31] applied Intuitionistic fuzzy to segment MRI images using C-means clustering techniques. Dengfeng and Chuntian [11] applied a new similarity measure between two Intuitionistic fuzzy set, as the previously proposed methods for similarity measure of a classical fuzzy is not applicable to IFS. They have used this measure for pattern recognition where maximum degree of similarity decides in which pattern class the sample belongs to. As introduction to the hesitant part gives better prediction level, the application of IFS in Financial interference is quite successful. This has been shown by Hajek and Olej [32], who have introduced a new de-fuzzification method (MOM) and applied in Intuitionistic neuro-fuzzy network trained with Particle swarm optimization for a financial inference system. They have shown that this method outperforms other Neuro-fuzzy inference techniques. In the network system also IFS has been applied by Dutta and Sait [12] for routing. The system is for both dynamic and static routing. The resource management is less costly in the system with overall gain of the performance of the network. Expert system is another application of Intuitionistic fuzzy. Degree of hesitancy can simulate better result for the decision of expert system as shown by Atanassov.

The neutrality component of Neutrosophic fuzzy logic gives information about the indeterminacy which can be captured into a mathematical formula to make certain decision. Deli et al. [16] makes feasible to use this uncertain information to get a medical diagnosis. This type of medical diagnosis was done on IFS earlier but here they have used Neutrosophic fuzzy logic. Hamming distance, Normalized hamming distance, Euclidean distance and Normalized Euclidean distance used on Neutrosophic set of symptoms to determine the type of disease. Liu et al. [30] suggested some new operator on Neutrosophic set and showed their result in multiple attribute group decision making problem. They proposed the generalized neutrosophic number Hamacher weighted averaging (GNNHWA) operator, generalized neutrosophic number Hamacher ordered weighted averaging (GNNHOWA) operator, and generalized neutrosophic number Hamacher hybrid averaging (GNNHHA) operator and studied their properties like union, intersection, t-norm, t-conorm etc. They showed these multi-attribute group decision making logic over air-quality ranking with more flexible results on single valued Neutrosophic information. Zhang et al. [24] used Interval Neutrosophic set which is an extension of SVNS and SNS, for multi-criteria decision making using some aggregation operator. NS has application to image processing also. Zhang et al. [13] decided the homogeneity of a image by using Neutrosophic logic which gives the value of degree of a pixel being a object pixel or edges or a background pixel. Kavitha et al. [33] have reduced the uncertainty of Intrusion detection system using Neutrosophic logic. They have used KDDcup'99 dataset and used Neutrosophic logic classifier to classify the dataset and used Genetic algorithm to make more precise Neutrosophic rules. Smarandache and Vladareanu [14] applied Neutrosophic logic to control a robot. A robot uses fusion of information from various sensors. The fuzziness or conflicting information optimized by Neutrosophic logic controls the kinematics of the robot. Ansari et al. [34] proposed a Neutrosophic classifier to classify Iris dataset. Besides the above mentioned application Neutrosophic logic has applied for detecting the neutrality of a word in a corpus also. Colhon et al. [15] determined the degree of neutrality of a neutral word. He classified the neutral word in three classes namely "pure neutral word", "half positive half negative word" and "Positive negative balanced word" using Neutrosophic logic.

10 Statistics of Research on Intuitionistic Fuzzy

As compared to Intuitionistic Fuzzy the effort of research has put into Neutrosophic Fuzzy are very less till today. So the picture we depict on the comparison of research publication are mainly based on Intuitionistic Fuzzy. We have tried to give a statistics of research work done in Intuitionistic Fuzzy on the basis of area of research, countries of the interest in Intuitionistic Fuzzy, fields where applied and the journals of publications.

Out of 1762 publications it shows that India is the second leading country after China to move ahead with the research in this topic. The following graph shows top 10 countries of the research publication in Intuitionistic Fuzzy (Fig. 2).

Statistics shows that Intuitionistic Fuzzy is the most explored area in the field of Computer Science. The other top four fields are Mathematics, Engineering, Operations research management science and Science. The application of Intuitionistic

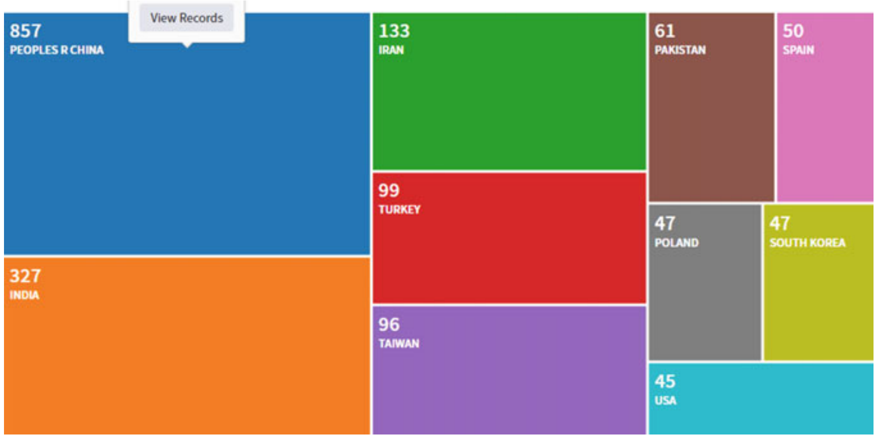

Fig. 2 Top 10 countries on the research interest in Intuitionistic fuzzy

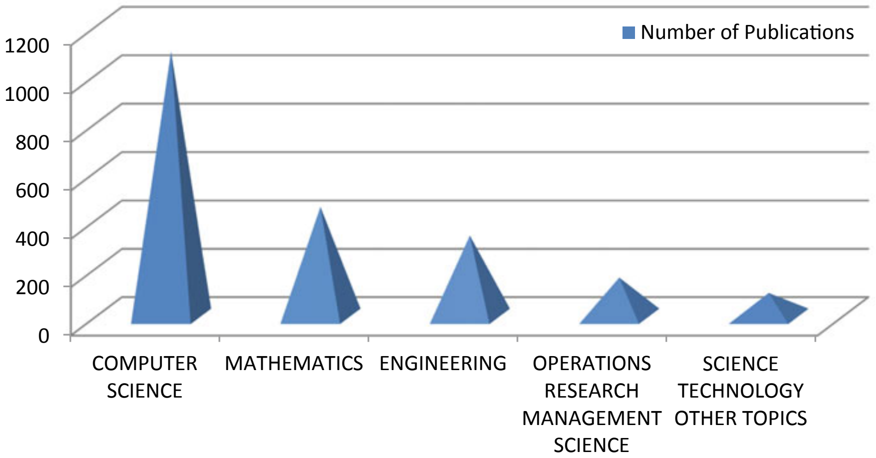

Fig. 3 Top 5 research areas for Intuitionistic fuzzy

Fuzzy in various Computer Science fields shows a new direction to handle uncertainty of data in corresponding domain (Figs. 3, 4 and 5; Table 3).

11 Conclusion

The differences of Neutrosophic and Intuitionistic fuzzy set gives the idea that both have their own importance in the field of mathematical logic, graphical, lattice, ring, field and other application areas. The implementation has changed diversely

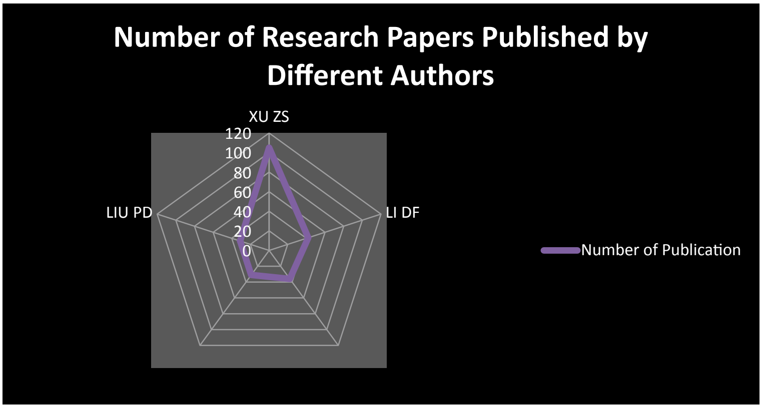

Fig. 4 Top 5 authors for Intuitionistic fuzzy research

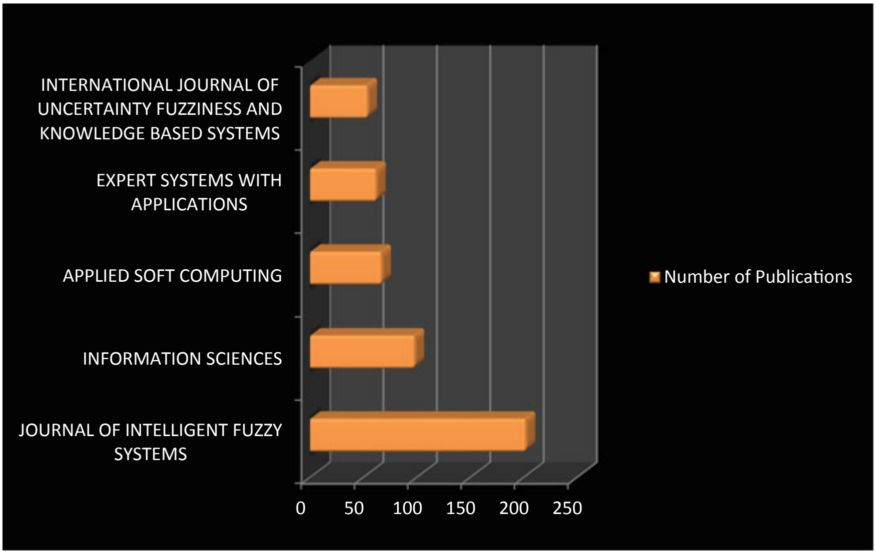

Fig. 5 Top 5 journals for publications of Intuitionistic fuzzy research

Table 3 Number of publications on Top 5 research areas

Research area	Number of publications
Computer Science	1096
Mathematics	454
Engineering	335
Operations Research Management Science	161
Science Technology and Other Topics	97

with the generalization or specialization of these two fuzzy sets. As the philosophy has changed so the application of these two types of fuzzy has also been changed. Neutrosophic fuzzy can be made fit to analyze fields where uncertainty plays a vital role. The undetermined factors give a mathematical notion to be included in the calculation. Specified degree of dependence and independence gives the idea of complete or incomplete or contradictory information. So in the age of information technology the data analysis can give new directions with the study of these fuzzy techniques over huge, uncertain and incomplete information.

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