Introduction to neutrosophic minimal structure

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Abstract

This paper is an introduction to neutrosophic minimal structure and its properties. Extension of indiscrete topology is known as minimal structure. Indiscrete topology contains only empty set and the universal set. Minimal structure contains empty set, universal set and it may also contains any subset of universal set but it should satisfies the first axiom of topology. Neutrosophic set has plenty of application. This motivates us to apply the concept of neutrosophy in minimal structure. The novelty of this notion is introducing neutrosophic minimal structure, closure and interior of a set, subspace. Some properties of neutrosophic minimal structure are also studied. Finally, application of neutrosohpic minimal structure is investigated.

Keywords. Neutrosophic minimal structure, N_m-closure, N_m-interior, connectedness.

1 Introduction and Preliminaries

Zadeh's [15] Fuzzy set laid the foundation of many theories such as intuionistic set and neutrosophic set, rough sets etc. Later, researchers developed K. T. Atonossov's [1] intuitionistic fuzzy set theory in many fields such as differential equations, topology, computer science and so on. F. Smarandache [13,14] found that some attribute may have indeterminacy or neutral other than membership and non-membership. So he coined the notion of neutrosophy. Now researchers [8,9,10,11,12] applying the concept of neutrosophy where they meet inconsistent, incomplete information of an object. The collection contains only X and $\emptyset$ is also a topology (Munkrer [5]). Popa [7] introduced minimal structures and defined seperation axioms using minimal structure. M. Alimohammady, M. Roohi

[2] introduced fuzzy minimal structure in lowen sense. S.Bhattacharya (Halder) and J.Tripathi [3] presented the concept of intuitionistic fuzzy minimal space. In first section of this paper contains the basic definition which are useful for our paper and in second section, we introduced neutrosophic minimal structure in smaradache sense. In further sections some properties of neutrosophic minimal structure is also investigated. Finally, some application of neutrosophic minimal structure is investigated. Note that neutrosophic topological space, neutrosophic supra topological space are neutrosophic minimal structure but converse is not true.

Definition 1.1 *A neutrosophic set(in short NS) A on a set $X \neq \emptyset$ is defined by*

$U = \{\langle a, T_U(a), I_U(a), F_U(a)\rangle : a \in X\}$ *where* $T_U : X \to [0,1]$, $I_U : X \to [0,1]$ *and* $F_U : X \to [0,1]$ *denote the membership of an object, indeterminacy and non-membership of an object for each $a \in X$ to U, respectively and* $0 \leq T_U(a)+I_U(a)+F_U(a) \leq 3$ *for each $a \in X$.*

Definition 1.2 *A neutrosophic topology (NT) in Salama's sense on a nonempty set X is a family τ of NSs in X satisfying three axioms:*

(1) Empty set ($0_\sim$) and universal set($1_\sim$) are members of τ.

(2) Finite intersection of neutrosophic sets U_i are member of τ.

(3) Arbitrary union of neutrosophic sets are member of τ.

Definition 1.3 *(i) A neutrosophic set U is a empty set i.e., $U = 0_\sim$ if 0 as membership of an object and 1 as indeterminacy and non-membership of an object respectively.*

(ii) A neutrosophic set U is a universal set i.e., $U = 1_\sim$ if 1 as membership of an object and 0 as indeterminacy and non-membership of an object respectively

Definition 1.4 *Let NS U in NTS X. Then a neutrosophic interior of U and a neutrosophic closure of U are defined by*

n-int(U) = *max* $\{F : F \text{ is an } NOS \text{ in } X \text{ and } F \leq U\}$ *and*

n-cl(U) = *min* $\{F : F \text{ is an } NCS \text{ in} X \text{ and } F \geq U\}$ *respectively.*

2 Neutrosophic Minimal Structure

Definition 2.1 *Let the neutrosophic minimal structure over a universal set X is denoted by N_m. N_m is said to neutrosophic minimal structure over X if $1_\sim$ and $0_\sim$ are member of N_m.*

Remark 2.1 *Every neutrosophic supra topological spaces are neutrosophic minimal structure but converse not true.*
Similarly, every neutrosophic topological spaces are neutrosophic minimal structure but converse is not true.

Example 2.1 *Let $A = \{< 0.6, 0.4, 0.3 >: a\}$, $B = \{< 0.6, 0.3, 0.3 >: a\}$ are neutrosophic sets over the universal set $X = \{a\}$. Then the neutrosophic minimal structure is $N_m = \{0, 1, A, B\}$. But N_m is not a neutrosophic topology and not a neutrosophic supra topology.*

Definition 2.2 *A is N_m-closed if and only if $N_m cl(A) = A$.*
Similarly, A is a N_m-open if and only if $N_m int(A) = A$.

Definition 2.3 *Let N_m be any neutrosophic minimal structure and A be any neutrosophic set. Then*

1. *Every $A \in N_m$ is open and its complement is closed.*
2. *N_m-closure of A is minimum of all closed sets which is greater than or equal to A and its denoted by $N_m cl(A)$.*
3. *N_m-interior of A is maximum of all open set which is less than or equal to A and it is denoted by $N_m int(A)$.*

In general $N_m int(A)$ is less than or equal to A and A is less than or equal to $N_m cl(A)$.

Proposition 2.1 *Suppose A and B are any subset of neutrosophic minimal structure N_m over X. Then*

i. $N_m^C = \{0, 1, A_i^C\}$ *where A_i^C is a neutrosophic complement of a set A_i.*

ii. $X - N_m int(B) = N_m cl(X - B)$.

iii. $X - N_m cl(B) = N_m int(X - B)$.

iv. $N_m cl(A^C) = (N_m cl(A))^C = N_m int(A)$.

v. N_m *closure of empty set is a empty set and* N_m *closure of universal set is a universal set. Similarly,* N_m *interior of empty set and universal set respectively empty and universal set.*

vi. *If* B *is a subset of* A *then* $N_m cl(B) \leq N_m cl(A)$ *and* $N_m int(B) \leq N_m int(A)$.

vii. $N_m cl(N_m cl(A)) = N_m cl(A)$ *and* $N_m int(N_m int(A)) = N_m int(A)$.

viii. $N_m cl(A \vee B) = N_m cl(A) \vee N_m cl(B)$

ix. $N_m cl(A \wedge B) = N_m cl(A) \wedge N_m cl(B)$

Proof:
(i) We know that $A^C = X - A$. Then $N_m cl(X - A) = N_m cl(A^C) = (N_m cl(A))^C = N_m int(A)$, from (iv).
Similarly for (ii).
(vi) Let $B \leq A$. We know that $B \leq N_m cl(B)$ and $A \leq N_m cl(B)$. So $B \leq N_m cl(B) \leq A \leq N_m cl(A)$. Therefore $N_m cl(B) \leq N_m cl(A)$.
Proof of (vii) is straight forward.
(viii) We know that $A \leq A \vee B$ and $B \leq A \vee B$. $N_m cl(A) \leq N_m cl(A \vee B)$ and $N_m cl(B) \leq N_m cl(A \vee B)$ this implies $N_m cl(A) \vee N_m cl(B) \leq N_m cl(A \vee B)$.$\longrightarrow (*)$
Also $A \leq N_m cl(A) and B \leq N_m cl(B) \Rightarrow A \vee B \leq N_m cl(A) \vee N_m cl(B)$. $N_m(A \vee B) \leq N_m(N_m cl(A) \vee N_m cl(B)) = N_m cl(A) \vee n_m cl(B) \longrightarrow (**)$
. From (*) and (**), we have $N_m cl(A \vee B) = N_m cl(A) \vee N_m cl(B)$.

Example 2.2 *Consider Example 2.1, the complement of* N_m *is* $\{0, 1, A^C, B^C\}$ *where* $A^C = \{< 1 - 0.6, 1 - 0.4, 1 - 0.3 > /a \in X\} = \{< 0.4, 0.6, 0.7 > /a\}$ *and* $B^C = \{< 1 - 0.6, 1 - 0.3, 1 - 0.3 > /a \in X\} = \{< 0.4, 0.7, 0.7 > /a\}$.

Definition 2.4 *A function* $f : (X, \tau_X) \rightarrow (Y, \tau_Y)$ *is called neutrosophic minimal continuous function if and only if* $f^{-1}(V) \in \tau_X$ *whenever* $V \in \tau_Y$.

Definition 2.5 *Boundary of a neutrosophic set A of neutrosophic minimal structure (X, τ_X) is the intersection of closure of the set A and closure of $X - A$.*

Theorem 2.1 *If $(X, \tau_X)$and (Y, τ_Y) are neutrosophic minimal structure. Then*

1. *Identity map from (X, τ_X) to (Y, τ_Y) is a neutrosophic minimal continuous function.*

2. *Any constant function which maps from (X, τ_X) to (Y, τ_Y) is a neutrosophic minimal continuous function.*

Theorem 2.2 *Let the map f from neutrosophic minimal structure space (X, τ_X) to neutrosophic minimal structure space (Y, τ_Y). Then the following are equivalent,*

1. *The map f is a neutrosophic minimal continuous function.*

2. *$f^{-1}(V)$ is a neutrosophic minimal closed set for each neutrosophic minimal closed set $V \in \tau_Y$. .*

3. *$N_m cl(f^{-1}(V)) \leq f^{-1}(N_m cl(V))$, for each $V \in \tau_Y$.*

4. *$N_m cl(f(A)) \geq f(N_m cl(A))$, for each $A \in \tau_X$.*

5. *$N_m int(f^{-1}(V)) \geq f^{-1}(N_m int(V))$, for each $V \in \tau_Y$.*

Proof.
(1) $\rightarrow$ (2): Let A be a N_m-closed in Y. Then $f^{-1}(A)^C = f^{-1}(A^C) \in \tau_X$.
(2) $\rightarrow$ (3):$N_m cl(f^{-1}(A) = \wedge\{D : f^{-1}(A) \leq D, D^C \in \tau_X\} \leq \wedge\{f^{-1}(D) : A \leq D, D^C \in \tau_Y\} = f^{-1}(\{D : A \leq D^C \in \tau_Y\}) = F^{-1}(N_m cl(A))$.
(3) $\rightarrow$ (4): Since $A \leq f^{-1}(f(A))$, then $N_m cl(A) \leq N_m cl f^{-1}(f(A)) \leq f^{-1}(N_m cl(f(A)))$.
Therefore$f(N_m cl(A)) \leq N_m cl(f(A))$.
(4) $\rightarrow$ (5): $f(N_m int f^{-1}(A))^C = f(N_m cl(f^{-1}(A))^C) = f(N_m cl(f(A)^C)) \leq N_m cl(f(f^{-1}(A^C))) \leq N_m cl(A^C) = (N_m int(A))^C$. This implies that $N_m int(f_{-1}(B))^C \leq f^{-1}(N_m int(A))^C = (f^{-1}(N_m int(A)))^C$.
Taking complement on both sides, $f^{-1}(N_m int(A)) \leq N_m int f^{-1}(B)$.

Definition 2.6 *Let (X, τ_X) be neutrosophic minimal structure space. Arbitrary union of neutrosophic minimal open sets in (X, τ_X) is neutrosophic minimal open. (Union Property) Finite intersection of neutrosophic minimal open sets in (X, τ_X) is neutrosophic minimal open. (intersection Property)*

3 Neutrosophic Minimal Subspace

In this section, we introduced the neutrosophic minimal subspace and investigate some properties of subspace.

Definition 3.1 *Let A be a neutrosophic set in neutrosophic minimal structure space (X, τ_X). Then Y is said to be neutrosophic minimal subspace if $(Y, \tau_Y) = \{A \cap U : U \in \tau_Y\}$*

Lemma 3.1 *If neutrosophic set b in the basis B for neutrosophic minimal structure space X. Then the collection $B_Y = \{b \cap Y : Y \subset X\}$ is a basis for neutrosophic minimal subspace on Y.*

Proof.
Given a neutrosophic set A in X and C is a neutrosophic set in both A and subset Y of X. Consider a basis element b of B such that C in b and in Y. Then $C \in B \cap Y \subset U \cap Y$. Hence B_Y is a basis for the neutrosophic minimal subspace on the set Y.

Lemma 3.2 *Let (Y, τ_Y) be a subspace of (X, τ_X). If A is a neutrosophic set in Y and $Y \subset X$. Then A is in (X, τ_X).*

Proof:
Given that neutrosophic set A in (Y, τ_Y). $A = Y \cap B$ for some neutrosophic set $B \in X$. Since Y and B in X. Then A is in X.

Proposition 3.1 *Suppose (Y, τ_Y) is a neutrosophic minimal subspace of (X, τ_X).*

1. *If the neutrosophic minimal structure space (X, τ_X) has the union property, then the subspace (Y, τ_Y) also has union property.*

2. *If the neutrosophic minimal structure space (X, τ_X) has the intersection property, then the subspace (Y, τ_Y) also has union property.*

Proof.
Suppose the family of open set $V_i : i \in Y$ in neutrosophic minimal subspace(Y, τ_Y) then there exist a family of open sets $\{U_j : j \in X\}$ in neutrosophic minimal structure space (X, τ_X) such that $V_i = U_j \cap A, \forall i \in Y$ where $A \in \tau_Y$. $\bigcup_(i \in Y)V_i = \bigcup_(j \in X)(U_j \cap A) = \bigcup_(i \in Y)U_j \cap A$. Since (X, τ_X) has union property then (Y, τ_Y) also has union property. The proof of (ii) is similarly to (i).

Definition 3.2 *Suppose* (B, τ_B) *and* (C, τ_C) *are neutrosophic minimal subspaces of neutrosophic minimal structure spaces* (Y, τ_Y) *and* (Z, τ_Z) *respectively. Also, suppose that* f *is a mapping from* (Y, τ_Y) *to* (Z, τ_Z) *is a mapping. We say that* f *is a mapping from* (B, τ_B) *into* (C, τ_C) *if the image of* B *under* f *is a subset of* C.

Definition 3.3 *Suppose* (A, τ_A) *and* (B, τ_B) *are neutrosophic minimal subspaces of neutrosophic minimal structure spaces* (Y, τ_Y) *and* (Z, τ_Z) *respectively. The mapping* f *from* (A, τ_A) *into* (B, τ_B) *is called a*

1. *comparative neutrosophic minimal continuous, if* $f^1(W) \wedge A \in \tau_A$ *for every neutrosophic minimal structure set* W *in* B,
2. *comparative neutrosophic minimal open, if* $f(V) \in \tau_B$ *for every fuzzy set* $V \in \tau_A$.

4 Application of neutrosophic minimal structure space

The application of neutrosophic minimal structure space is based on the minimal element and maximal element. In neutrosophic minimal structure space, $0_\sim$ is the minimal element and $1_\sim$ is the maximal element. The application of neutrosophic minimal structure space used in consumer theory where the customer has only two objective. In consumer theory, the customer has either minimize purchase cost and maximize the quantity or maximize the durability.
Let the set of variety of cars be $X = \{C_1, C_2, C_3\}$ and the parameter set $E = \{a = costofthecar, b = safety, c = maintenance\}$. A customer assign minimum value $0_\sim$ to bad features, $1_\sim$ to the best features of the product. If any car in the universal set receives $0_\sim$ with respect to the parameter then it is subject to reject from wish list. Similarly, if any of the car in the list receive $1_\sim$ with respect to the parameter then it is good to buy. Membership, indeterminacy and non-membership values taken from customer's review rating. Membership referred to cost of the car is worth to the model, safe and low maintenance cost. Non-membership referred to cost of the car is not worth to the model, not safe due to break failure or some other reason and high maintenance cost. Indeterminacy referred to neutrality of cost of the car, safe if drive safe and maintenance is neutral. Let us assume the following values are taken from customer review ratting for the models A, B and C. Say $C_1 = \{\frac{a}{0.6,0.2,0.4}, \frac{b}{0.7,0.3,0.4}, \frac{c}{0.6,0.3,0.4}\}$, $C_2 = \{\frac{a}{0.6,0.3,0.4}, \frac{b}{0.6,0.3,0.4}, \frac{c}{0.5,0.2,0.4}\}$,

$C_3 = \{\frac{a}{0.7,0.3,0.4}, \frac{b}{0.8,0.2,0.2}, \frac{c}{0.6,0.2,0.3}\}$. The neutrosophic minimal structure is defined as $N_m = \{0 \sim, C_1, C_2, C_3, 1 \sim\}$. Among the given values A and C got better rating but it doesnot meet 1. But the customer can choose the car based on the weightage given to each parameter.

Conclusions:

This paper carried the following work: Neutrosophic minimal structure is introduced and some of its properties discussed along with this. Neutrosophic minimal continuous and subspace also investigated with few properties. Finally, application of neutrosophic minimal structure is provided. This paper motivates to investigate various open sets and separation axioms in neutrosophic minimal structure space. Also the application part discussed in this work leads to analyze this problem in weak structure.

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