

# Interpretation of Solution of Radial Biquaternion Klein-Gordon Equation and Comparison with EQPET/TSC Model

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## Abstract

In a previous publication,<sup>1</sup> we argued that the biquaternionic extension of the Klein-Gordon equation has numerical solution with sinusoidal form, which differs appreciably from conventional Yukawa potential. In the present article we interpret and compare this result from the viewpoint of the EQPET/TSC (Electronic Quasi-Particle Expansion Theory/Tetrahedral Symmetric Condensate) model described by Takahashi.<sup>2</sup> Further observation is of course recommended in order to refute or verify this proposition.

## Introduction

In the preceding article<sup>1</sup> we argued that biquaternionic extension of the radial Klein-Gordon equation (radialBQKGE) has numerical solution with sinusoidal form, which differs appreciably from conventional Yukawa potential. We also argued that this biquaternionic extension of KGE may be useful in particular to explore new effects in the context of low-energy nuclear reaction (LENR).<sup>3</sup>

Interestingly, Takahashi<sup>2</sup> has discussed key experimental results in condensed matter nuclear effects in light of EQPET/TSC. We argue here that the potential model used in his paper, STTBA (Sudden Tall Thin Barrier Approximation), may be comparable to our derived sinusoidal potential from radial biquaternion KGE.<sup>1</sup> While we can't yet offer numerical prediction, our qualitative comparison may be useful in verifying further experiments.

## Solution of Radial Biquaternionic KGE (radial BQKGE)

In our previous paper,<sup>1</sup> we argued that it is possible to write the biquaternionic extension of the Klein-Gordon equation as follows:

$$(\diamond\diamond + m^2) \varphi(x, t) = 0 \quad (1)$$

Provided we use this definition:<sup>1,3</sup>

$$\begin{aligned} \diamond = \nabla^q + i\nabla^q = & \left( -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \right) \\ & + i \left( -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \right) \end{aligned} \quad (2)$$

where  $e_1, e_2, e_3$  are *quaternion imaginary units* obeying (with ordinary quaternion symbols  $e_1=i, e_2=j, e_3=k$ ):<sup>3,4</sup>

$$\begin{aligned} i^2 = j^2 = k^2 = -1, \quad ij = -ji = k, \\ jk = -kj = i, \quad ki = -ik = j. \end{aligned} \quad (3)$$

And quaternion Nabla operator is defined as:<sup>2</sup>

$$\nabla^q = -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \quad (4)$$

By using polar coordinates transformation,<sup>5</sup> we get this for the one-dimensional situation:

$$\left( \frac{\partial}{\partial r} \left( \frac{\partial}{\partial r} \right) - i \frac{\partial}{\partial r} \left( \frac{\partial}{\partial r} \right) + m^2 \right) \varphi(x, t) = 0 \quad (5)$$

The solution is given by:<sup>1</sup>

$$y = k_1 \cdot \sin \left( \frac{|m|r}{\sqrt{-i-1}} \right) + k_2 \cdot \cos \left( \frac{|m|r}{\sqrt{-i-1}} \right) \quad (6)$$

Therefore, we may conclude that numerical solution of radial biquaternionic extension of Klein-Gordon equation yields different potential compared to the well-known Yukawa potential.<sup>1</sup>

$$u(r) = -\frac{g^2}{r} e^{-mr} \quad (7)$$

In the next section we will discuss an interpretation of this new potential (6) compared to the findings discussed by Takahashi<sup>2</sup> from condensed matter nuclear experiments.

## Comparison with Takahashi's EQPET/TSC/STTBA model

Takahashi<sup>2</sup> reported some findings from condensed matter nuclear experiments, including intense production of helium-4 (<sup>4</sup>He) atoms by electrolysis and laser irradiation experiments.

Takahashi analyzed those experimental results using EQPET formation of TSC were modelled with numerical estimations by STTBA. This STTBA model includes strong interaction with negative potential near the center (where  $r \rightarrow 0$ ). See Figure 1.

Takahashi described that Gamow integral of STTBA is given by:

$$\Gamma_n = 0.218 (\mu^{1/2}) \int_{r_0}^b (V_b - E_d)^{1/2} dr \quad (8)$$

Using  $b=5.6$  fm and  $r_0=5$  fm, he obtained:

$$P_{4d} = 0.77 \quad (9)$$

and

$$V_B = 0.257 \text{ MeV} \quad (10)$$

While his EQPET model gave significant underestimation for 4D fusion rate when rigid constraint of motion in 3D space was attained, introducing different values of  $\lambda_{4d}$  can improve the result.<sup>2</sup> Therefore we may conclude that STTBA can offer good approximation of condensed matter nuclear reactions.<sup>5</sup>

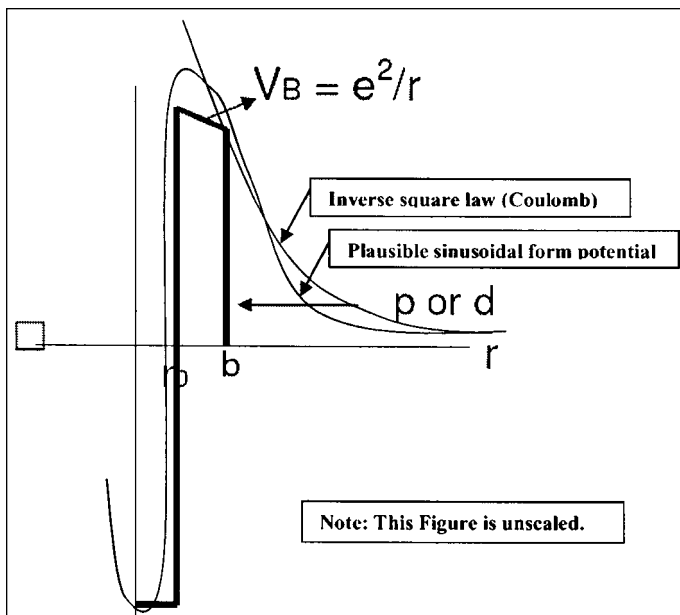


Figure 1. Potential for Coulomb barrier reversal for STTB, from Takahashi.<sup>2</sup>

Interestingly, the STTB lacks sufficient theoretical basis, therefore one can expect that a sinusoidal form (or combined sinusoidal waves such as in Fourier method) may offer better result which agrees with experiments. This will be pursued in a later paper.

Nonetheless, we recommend further observation in order to refute or verify this proposition of a new type of potential derived from the biquaternion radial Klein-Gordon equation.

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