Soft Computing (2020) 24:7459–7468
https://doi.org/10.1007/s00500-019-04372-x

METHODOLOGIES AND APPLICATION

Indeterminate Likert scale: feedback based on neutrosophy, its distance measures and clustering algorithm

Ilanthenral Kandasamy[1] · W. B. Vasantha Kandasamy[1] · Jagan M. Obbineni[2] · Florentin Smarandache[3]

Published online: 24 September 2019

Abstract

Likert scale is the most widely used psychometric scale for obtaining feedback. The major disadvantage of Likert scale is information distortion and information loss problem that arise due to its ordinal nature and closed format. Real-world responses are mostly inconsistent, imprecise and indeterminate depending on the customers' emotions. To capture the responses realistically, the concept of neutrosophy (study of neutralities and indeterminacy) is used. Indeterminate Likert scale based on neutrosophy is introduced in this paper. Clustering according to customer feedback is an effective way of classifying customers and targeting them accordingly. Clustering algorithm for feedback obtained using indeterminate Likert scaling is proposed in this paper. While dealing real-world scenarios, indeterminate Likert scaling is better in capturing the responses accurately.

Keywords Likert scale · Star rating · Questionnaire · Survey · Customer feedback · Neutrosophy · Neutrosophic logic · Indeterminacy · Indeterminate Likert scale · Triple refined neutrosophic set · Distance measures · Minimum spanning tree · Clustering algorithm

1 Introduction

Likert scaling introduced by Likert (1932) is the most commonly used psychometric scale for collecting responses from the user/customer in terms of level of agreement. It has been used in several surveys like organizational behaviour in learning institutes (Kiedrowski 2006; Rus et al. 2014), music education (Orr and Ohlsson 2005), prioritization of routine in dental care (Postma 2007), sports for athlete characteristics and outcome (Brown et al. 2007), etc. Likert scaling suffers from several drawbacks like information distortion and information lost problem due to its ordinal nature and closed format (Li 2013).

Zadeh's (1965) fuzzy set theory functions as an important constructive tool that enables soft division of sets. It gives an extension to fuzzy set as intuitionistic fuzzy set (A-IFS) by Atanassov (1986) where each element is given a membership and a non-membership degree in Atanassov (1986). A fuzzy Likert scale was introduced by Li in Li (2013).

To represent inconsistent, imprecise and uncertain information from the real world, indeterminacy membership is represented independently along with truth and falsity membership in neutrosophic set (Smarandache 2000). It generalizes the concept of several sets like classic set, fuzzy set and paradoxist set, and $T_A(x)$, $I_A(x)$ and $F_A(x)$ are membership function which can be real standard or nonstandard subsets.

In this form, it was not possible to apply it in real-world problems of the scientific and engineering areas. Wang et al. (2010) proposed a single-valued neutrosophic set (SVNS), to overcome this. Neutrosophy has found applications in many real-world practical problems like decision-making problems (Liu and Wang 2014; Liu and Shi 2015; Liu and Teng 2017; Liu and Li 2017; Ye 2013, 2014a, b, c), image processing (Sengur and Guo 2011; Cheng and Guo 2008; Zhang et al. 2010), analysis of social network (Salama et al. 2014) and socio-economic and political problems (Vasantha and Smarandache 2003, 2004), etc.

Communicated by V. Loia.

✉ Ilanthenral Kandasamy
ilanthenral.k@vit.ac.in

[1] School of Computer Science and Engineering, Vellore Institute of Technology, Vellore, India

[2] VIT School for Agricultural Innovations and Advanced Learning (VAIAL), Vellore Institute of Technology, Vellore, India

[3] Department of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA

To offer better accuracy and give expression to imprecision in the indeterminacy, the indeterminacy membership existing in the neutrosophic set is categorized as indeterminacy leaning towards truth and towards false memberships. This makes the indeterminacy in the scenario to be more accurate and less imprecise. This was defined as double-valued neutrosophic set (DVNS) by Kandasamy (2018a, 2016a). Distance measure, cross-entropy measure and clustering algorithm of DVNS were introduced in Kandasamy (2018a). Dice measures on DVNS were proposed in Khan et al. (2018).

To improve the precision and accuracy of the data analysis and to fit in the Likert's scale that is most frequently used psychometric scale, the indeterminacy concept was subdivided into three: indeterminacy leaning towards truth, indeterminacy and false memberships. This refined neutrosophic set is known as the triple refined indeterminate neutrosophic set (TRINS). TRINS was used recently for personality test and classification based on personality (Kandasamy and Smarandache 2016b). TRINS is redefined here as positive membership, positive indeterminate membership, indeterminate membership, negative indeterminate membership and negative membership, to give the best possible mapping of Likert Scaling.

To conceptualize a real-world example of TRINS, consider the scenario where a customer orders four different food items from the restaurant's menu. He might have immensely enjoyed two of the dishes, regretted ordering a particular dish and be unsure about the other dish thinking it might have been better if it was prepared in a different way. If he is asked to provide feedback using Likert scale, he will obviously give an average/neutral score.

Let TRINS A under consideration be represented by $P_A(x), I_{PA}(x), I_A(x), I_{NA}(x), N_A(x)$, where $P_A(x)$ denotes positive membership, $I_{PA}(x)$ is positive indeterminate membership, $I_A(x)$ is indeterminate, $I_{NA}(x)$ is negative indeterminate and $N_A(x)$ is negative. The scenario is represented as $\langle 0.5, 0.25, 0, 0, 0.25\rangle$ that is giving a value of 0.5 for the two dishes that he enjoyed immensely, 0.25 to the dish he regretted and 0.25 to the dish he was unsure about. The scenario can be captured accurately with needed precision which is vital to the result obtained. All the various choices are captured, thereby evading the preferential choice that is selected in the conventional method of Likert scaling.

Clustering analysis basically exploits the notion of distance measures between any two entities, and based on this clusters are formed. This plays a significant role in research fields in the form of data mining, social networking, pattern recognition and machine learning. Traditionally, clustering analysis has been a hard one, which assigns an item to a particular cluster. Since elements in the given scenario do not have rigid restrictions, it is essential to fragment them softly. In this paper, a clustering algorithm is introduced to handle feedback obtained using indeterminate Likert scaling.

Section one is introductory in nature, and section two recalls some basic concepts about Likert scaling/star rating and neutrosophy. Section three discusses the limitation and problems with Likert scaling and provides justification for using indeterminacy. Indeterminate Likert scaling which maps every degree of agreement individually is introduced in section four. Indeterminacy-based minimum spanning tree (MST) clustering algorithm is proposed in the next section, and an illustrative real-world example is provided. Comparison of indeterminate Likert scaling with existing rating scheme and Likert scaling is carried out in section six. Section seven provides the conclusions and future study.

2 Preliminaries

2.1 Likert scaling

Likert scale is the most often used psychometric scale to collect responses from people in a survey. A typical Likert scale survey does not let its respondents simply select from "yes/no"; it provides specific choices that are degrees of "agreeing" or "disagreeing". The most basic Likert scaling format is a 5-column answer, with choices like: strongly disagree, disagree, neither agree nor disagree (do not know), agree and strongly agree. The neutral option is generally opted by the person who is unsure. A study in Armstrong (1987) found negligible differences between the use of "undecided" and "neutral" as a middle option in a 5-point Likert scale.

A sample of Likert scaling for a simple question "How satisfied are you with our services?" is given in Fig. 1.

The star rating scheme is almost similar to Likert scale. 1 star is taken to be equivalent to the lowest rating while the 5 star is considered as the maximum rating. Stars are used as a common experimental or heuristic element for evaluating quality. A sample questionnaire used to elicit responses from the customers of a restaurant using 5 star rating is given in Table 1. Similarly, a questionnaire that uses Likert scale is given in Table 2. The analysis of Likert scale responses is generally carried out using bar charts to show results, mode in the case of the most common response and range and interquartile ranges in the case of analysing variability.

2.2 Neutrosophy and refined neutrosophic set

Neutrosophy, familiarized by Smarandache (2000), studies a perception or event or entity, "A" in relation to its opposite, "Anti-A" and not A, "Non-A", and as neither "A" nor "Anti-A", denoted by "Neut-A".

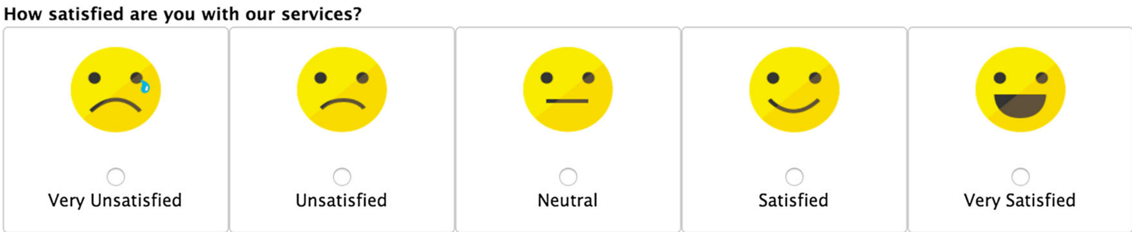


Fig. 1 Sample Likert scale

Table 1 Sample questionnaire using five-star rating scheme for restaurant

Question	Scale
Quality of Food	★★★★☆
Service	★★★★☆
Hygiene	★★★☆☆
Value for money	★★☆☆☆
Ambience	★★★☆☆
Overall Experience	★★★☆☆

Let X be a space of points (objects) with elementary elements in X represented by x. A single-valued neutrosophic set (SVNS) A in X is characterized by truth $T_A(x)$, indeterminacy $I_A(x)$ and falsity $F_A(x)$ membership functions. For each point x in X, there are $T_A(x), I_A(x), F_A(x) \in [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. A is denoted by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$. The refined neutrosophic logic defined by Smarandache (2013) is as follows:

Definition 1 The truth T is divided into several types of truths: $T_1, T_2, \ldots, T_p$, and I into various indeterminacies: $I_1, I_2, \ldots, I_r$, and F into various falsities: $F_1, F_2, \ldots, F_s$, where all $p, r, s \geq 1$ are integers, and $p + r + s = n$.

Triple refined indeterminate neutrosophic sets have the indeterminacy concept divided into three memberships: indeterminacy favouring positive opinion, indeterminacy favouring negative opinion and indeterminacy. This division helps in improving the accuracy and precision and fits the Likert scale. TRINS (Kandasamy and Smarandache 2016b) has been used to classify personality. In double-valued neutrosophic set (DVNS), the indeterminacy concept is divided into two.

Definition 2 A triple refined indeterminate neutrosophic set (TRINS) A in X as given above is characterized by positive $P_A(x)$, indeterminacy $I_A(x)$, negative $N_A(x)$, positive indeterminacy $I_{PA}(x)$ and negative indeterminacy $I_{NA}(x)$ membership functions. Each has a weight $w_m \in [0, 5]$ associated with it. For each $x \in X$, there are

$$P_A(x), I_{PA}(x), I_A(x), I_{NA}(x), N_A(x) \in [0, 1],$$
$$w_P^m(P_A(x)), w_{I_P}^m(I_{PA}(x)), w_I^m(I_A(x)),$$
$$w_{I_N}^m(I_{NA}(x)), w_N^m(N_A(x)) \in [0, 5]$$

and $0 \leq P_A(x) + I_{PA}(x) + I_A(x) + I_{NA}(x) + N_A(x) \leq 5$. Therefore, a TRINS A can be represented by

$$A = \{\langle x, P_A(x), I_{PA}(x), I_A(x), I_{NA}(x), N_A(x)\rangle \mid x \in X\}.$$

Consider $Q = [q_1, q_2]$ where q_1 is question 1 (quality) and q_2 is question 2 (service) from Table 2. The values of q_1 and q_2 are in [0, 1], and the weight of the membership is applied the values are in [0, 5]. Take the same scenario where a customer orders 4 different items from the menu. He might have immensely enjoyed two of the dishes, regretted ordering a particular dish and may be undecided about the other dish being good. If he is asked to provide feedback using Likert

Table 2 Sample questionnaire using Likert scaling for restaurant

Question	Terrible	Bad	Average	Good	Excellent
Quality of food	☐	☐	☐	☑	☐
Service	☐	☐	☐	☑	☐
Hygiene	☐	☐	☑	☐	☐
Value for money	☐	☑	☐	☐	☐
Ambience	☐	☐	☑	☐	☐
Overall experience	☐	☐	☑	☐	☐

scale, he will obviously give a average/neutral score. This is mapped to TRINS as follows:

Option for "quality" would be a degree of excellent food that is the dishes he enjoyed immensely, a degree of indeterminacy choice towards "good food" will be the dish he was undecided about but thought it was a little good but is unsure, a degree of uncertain and indeterminate combination of good food and not so good food, a degree of indeterminate choice bordering close to bad food and a degree of poor quality food which will be the food that he regretted ordering, instead of a forced single choice. Similarly, the service will vary and can be marked accordingly to different degrees.

This can be represented by TRINS A of X as

$$A = \langle 0.5, 0.25, 0, 0, 0.25\rangle/x_1 + \langle 0.5, 0.1, 0.1, 0.1, 0.2\rangle/x_2.$$

Operators related to set theory like associativity, distributivity, commutativity, idempotency, absorption and the DeMorgan's laws were defined over TRINS (Kandasamy and Smarandache 2016b).

3 Justification for applying indeterminacy-based scaling

Generally in Likert scaling, the user is forced to select the most dominant choice. For example, the normal five-level Likert item would be

- Strongly disagree
- Disagree
- Neither agree or disagree
- Agree
- Strongly agree

Any user will have feelings/options which actually vary from strongly agree to strongly disagree and which are not definite; they are always a mixture of feelings. A small amount of disagreement might bring down the option from "strongly agree" to "agree", whereas a different person might choose to go with the dominant choice of "strongly agree" ignoring the small/meagre amount of disagreement. Some other person might mark the option "neither agree nor disagree" due to the same negative experience. However, it is very obvious and clear that people react differently to the same experience while answering the same question in the questionnaire. The questionnaire using a Likert scale will fail to capture the feelings/exact degree of strong agreement, degrees of weak agreement, degrees of neither agreement nor disagreement, degrees of weak disagreement and the degree of strong disagreement. The respondent/person is generally forced to go with the dominant choice or the choice which he feels at that time or the choice which may be only a shade dominant than the other choice; thereby, the degree of the memberships with other choices is completely lost.

Only a measure of coarse ordinal scale with closed format is used by Likert method. It fails in approximating interval data, and a substantial amount of information is gone and distorted due to the built-in limitations of Likert scaling as said by Russell and Bobko (1992).

A person who opts for "strongly agree" option might not be 100% agree with the statement. There might have been some amount of disagreement which the user was forced to override or only a small difference in mind between any two of the 5 attributes. To exactly capture the various degrees of membership TRINS is used to represent the choices. Using of TRINS and creating a Likert type scale for questionnaire will result in capturing the uncertainty, incomplete and indeterminate nature of the persons opinion in the collected data.

Every option will be given a degree of membership, and the person need not be forced to go with the dominant choice. The various degrees and choices will be captured more accurately with good precision, in fact in a sensitive, accurate and realistic way and not in an approximate way. This will eventually aid in better understanding of the customers and their needs; thereby, better marketing can be carried out.

Generally, the Likert scale is a bipolar scaling technique, determining either positive or negative response to a statement, whereas the TRINS- or DVNS-based Likert type scaling will be measuring both/all responses to a statement, thereby collecting the indeterminate/incomplete details about the options of the persons. This will provide a clear and more detailed view of the various degrees of membership. In the Likert scale, sometimes even point scale is used, where the middle option of "neither agree nor disagree" is removed. This is known as the forced choice method. This can be appropriately represented by DVNS. However, the neutral option is generally opted by the person who is unsure. A study in Armstrong (1987) found insignificant differences between the usage of "undecided" and "neutral" as a central option in a 5-point Likert scale.

There is actually a lot of difference between someone who is undecided and someone who is neutral; in a TRINS-based Likert scale, there can be a separate option for undecided, since equal amount of agreement and disagreement can be represented in degree of weak agreement and degree of weak disagreement, individually.

Indeterminate Likert scaling will remove the necessity to go with the dominant choice or a forced option which cannot always be true if it is varying from the other option only be a small or a shade of difference. The users exact feelings/thinking/option cannot be captured very realistically by Likert scaling, but certainly indeterminate Likert scaling based on TRINS can do this very accurately.

4 Indeterminate Likert scale

The normal five-level Likert scale items are

- Strongly disagree
- Disagree
- Neither agree or disagree
- Agree
- Strongly agree

They will get mapped in indeterminate Likert scale as follows:

- Negative membership
- Indeterminacy leaning towards negative membership
- Indeterminate membership
- Indeterminacy leaning towards positive membership
- Positive membership

While using a five-star rating, this will be mapped from one star to five stars. An indeterminacy-based Likert scale will have a negative membership which will capture the degree of strongly disagree of usual Likert scaling and degree of one-star rating in the star rating scheme. Similarly, the membership of indeterminacy leaning towards negative will capture the degree of disagree or two-star rating. Neural/degree of neither agree not disagree/do not know of the usual Likert scale or the three-star rating will be captured by the indeterminacy membership. Similarly, for degree of agree and degree of strongly agree will be mapped to indeterminacy or neutrality leaning towards positive membership and positive membership, respectively.

An indeterminate Likert scale will be given a representation as shown in Fig. 2: very unsatisfied, unsatisfied, neutral, satisfied, very satisfied with individual scales for grading.

A five-star rating will be like the one represented in Fig. 3. If a user is asked to rate the service provided in the restaurant, the user might have several different types of emotions about the service. The service of the waiters might have been excellent; he will give a 0.5 to "very satisfied". He might have waited for a long time for the food he ordered to arrive, hence a 0.25 to very unsatisfied. Regarding the politeness of the waiters/staff he might not be in a position to make up his mind, he might nevertheless be unable to map it as good or bad, hence a 0.25 for the indeterminate/neural options.

Such a case is given as example in Fig. 2. The user basically has option to slide using the slider provided in each level of agreement. Similarly, in a five-star rating scheme the user can fill the star to provide the degree of membership for each level as shown in Fig. 3. This can be easily implemented in mobile applications. As soon as a negative feedback is obtained, the user can be asked to provide more details by asking particular questions and making the feedback interactive. Due to the nature of indeterminate Likert scale, identifying and isolating a negative experience of the customer become easy. Table 3 gives the input received from the user using a indeterminate Likert scaling-based questionnaire.

This indeterminate Likert scale can be extended to 7-point Likert scale, or any multipoint Likert scale. In fact, it can be altered to the needs of researchers. Truth, indeterminate and Falsity memberships can be divided according to the researchers. These are known as multipoint indeterminate Likert scale. Studies in this direction is left open.

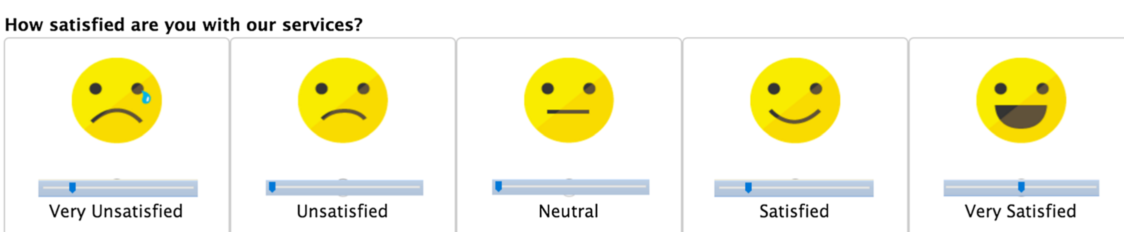


Fig. 2 Indeterminate Likert scale

Fig. 3 Indeterminate rating scale

Table 3 Sample questionnaire using indeterminate Likert scaling for restaurant

Question	P(A)	IP(A)	I(A)	IN(A)	N(A)
Quality of food	0.9	0.03	0.05	0.02	0
Service	0.8	0.05	0.05	0.1	0
Hygiene	0.7	0	0.1	0.1	0.1
Value for money	0.8	0.1	0	0	0.1
Ambience	0.7	0.1	0.1	0.05	0.05
Overall experience	0.75	0	0.05	0.1	0

Input: TVNS $A_1, \ldots\ldots, A_m$,
Output: Distance matrix D with elements d_{ij}
begin
  for $i \leftarrow 1, m$ **do**
    for $j \leftarrow 1, m$ **do**
      if $i = j$ **then**
        $d_{ij} \leftarrow 0$
      else
        $d_{ij} \leftarrow \{d_\lambda\ (A_i, A_j)\}$
      `// Using Equation 3`

Algorithm 1: TRINS weighted distance matrix D

5 Indeterminate MST clustering algorithm using distance measures

5.1 Distance measures of TRINS

The distance measures and its related algorithm of TRINS are defined in the following:

Consider two TRINS A and B in a universe of discourse, $X = x_l, x_2, \ldots, x_n$, which are denoted by

$$A = \{\langle x_i, P_A(x_i), I_{PA}(x_i), I_A(x_i), I_{NA}(x_i), N_A(x_i)\rangle \mid x_i \in X\}, and B = \{\langle x_i, P_B(x_i), I_{PB}(x_i), I_B(x_i), I_{NB}(x_i), N_B(x_i)\rangle \mid x_i \in X\},$$

where $P_A(x_i)$, $I_{PA}(x_i)$, $I_A(x_i)$, $I_{NA}(x_i)$, $N_A(x_i)$, $P_B(x_i)$, $I_{PB}(x_i)$, $I_B(x_i)$, $I_{NB}(x_i)$, $N_B(x_i) \in [0, 5]$ for every $x_i \in X$.

Let $w_i (i = 1, 2, \ldots, n)$ be the weight of an element $x_i (i = 1, 2, \ldots, n)$, with $w_i \geq 0$; $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$.

Then, the generalized TRINS weighted distance is as follows:

$$d_\lambda(A, B) = \left\{\frac{1}{5}\sum_{i=1}^{n} w_i[\mid P_A(x_i) - P_B(x_i)\mid^\lambda + \mid I_{PA}(x_i) - I_{PB}(x_i)\mid^\lambda + \mid I_A(x_i) - I_B(x_i)\mid^\lambda + \mid I_{NA}(x_i) - I_{NB}(x_i)\mid^\lambda + \mid N_A(x_i) - N_B(x_i)\mid^\lambda]\right\}^{1/\lambda} \tag{1}$$

where $\lambda > 0$.

When $\lambda = 1$, Eq. 1 reduces to TRINS weighted Hamming distance; when $\lambda = 2$, it reduces to TRINS weighted Euclidean distance and is given as

$$d_\lambda(A, B) = \left\{\frac{1}{5}\sum_{i=1}^{n} w_i[\mid P_A(x_i) - P_B(x_i)\mid^2 + \mid I_{PA}(x_i) - I_{PB}(x_i)\mid^2 + \mid I_A(x_i) - I_B(x_i)\mid^2 + \mid I_{NA}(x_i) - I_{NB}(x_i)\mid^2 + \mid N_A(x_i) - N_B(x_i)\mid^2]\right\}^{1/2} \tag{2}$$

where $\lambda = 2$ in Eq. 1.

The TRINS distance matrix D is as follows:

Definition 3 Let $A_j (j = 1, 2, \ldots, m)$ be a collection of m TRINS, then we define the TRINS distance matrix $D = (d_{ij})_{m\times m}$, where $d_{ij} = d_\lambda(A_i, A_j)$ is the generalized TRINS distance between A_i and A_j and satisfies the following:

1. $d_{ij} \in [0, 5], \forall i, j = 1, 2, \ldots, m$;
2. $d_{ij} = 0$ if and only if $A_i = A_j$;
3. $d_{ij} = d_{ji}$ for all $i, j = 1, 2, \ldots, m$.

The algorithm to calculate the TRINS weighted distance matrix D is given in Algorithm 1.

5.2 Indeterminate MST clustering algorithm

Indeterminate minimum spanning tree (MST) clustering algorithm is proposed as a generalization of the IFMST, SVN-MST and DVN-MST clustering algorithms here.

Consider $X = \{x_1, x_2, \ldots, x_n\}$ to be an attribution space and the weight vector of an element $x_i (i = 1, 2, \ldots, n)$ be $w = \{w_1, w_2, \ldots, w_n\}$, where $w_i \geq 0 (i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$. Let the m samples that need to be clustered be represented as $F_j (j = 1, 2, \ldots, m)$, a collection of m TRINSs. It is $F_j = \{\langle x_j, P_{F_j}(x_j), I_{PF_j}(x_j), I_{F_j}(x_j), I_{NF_j}(x_j), N_{F_j}(x_j)\rangle \mid x_j \in X\}$.

The triple refined indeterminate neutrosophic minimum spanning tree (TRIN-MST) clustering algorithm is provided in Algorithm 2. The description of the algorithm is done along with an example.

Input: $D = (d_{ij})_{m\times m}$
Output: Minimum Spanning Tree S and Clusters
begin
Step 1: Calculate D distance matrix $(F_1, \ldots, F_m)$ `// Using Algo 2`
Step 2: Create TRINS graph $G(V, E)$
for $i \leftarrow 1, m$ **do**
  for $j \leftarrow 1, m$ **do**
    if i != j **then**
      Insert edge from F_i to F_j with d_{ij}
Step 3: Compute MST of G: `// by use of Kruskal's algorithm`
Sort the edges in order (increasing) of weight in E.
while *No. of edges in subgraph S of G* $< (V - 1)$ **do**
  Select edge (v_i, v_j) with minimum weight.
  Delete (v_i, v_j) from E
  if (v_i, v_j) *creates a cycle in S* **then**
    Discard edge v_i, v_j
  else
    Include the edge v_i, v_j in S
S is the MST of $G(V, E)$.
Step 4: Clustering S with threshold r
for $i \leftarrow 1, m$ **do**
  for $j \leftarrow 1, m$ **do**
    if $d_{ij} \geq r$ **then**
      Remove edge
Clusters are created automatically

Algorithm 2: Indeterminate Minimum Spanning Tree (MST) Clustering algorithm

5.3 Illustrative examples

To demonstrate the effectiveness of the proposed TRIN-MST clustering algorithm in the real-world applications, a descriptive example is presented. The results of the indeterminate feedback of ten different people which are represented by TRINS are clustered using the indeterminate MST clustering algorithm.

Example 1 The real-world problem of feedback given by customers of a restaurant (restaurant name is kept anonymous) was taken. The six evaluation questions based on Table 2 were considered and transformed to indeterminate questionnaire as given in Table 3. The answers of the indeterminate feedback of ten different people $F_j (j = 1, 2, \ldots, 10)$ are taken for clustering. The questionnaire has been changed accordingly so as to ensure the use of distance measures. The responses collected from 10 people are given in Kandasamy (2018b).

The weight vector $w_i = 0.167$ is taken uniformly for the attribute $x_i (i = 1, 2, \ldots, 6)$. The TRIN-MST clustering algorithm provided in Algorithm 2 is used to group the ten people of $F_j (j = 1, 2, \ldots, 10)$ into clusters.

The stepwise working of the TRINS-MST clustering algorithm is as follows:

Step 1 The distance matrix $D = d_{ij} = d_\lambda(F_i, F_j)$ is calculated by using Algorithm 1 (taking $\lambda = 2$). $D = (d_{ij})_{m\times m}$ is obtained as given in Fig. 4:

Step 2 Based on D the TRINS graph $G(V, E)$ is constructed where every edge between F_i and $F_j (i, j = 1, 2, \ldots, 10)$ is assigned the TRINS weighted distance d_{ij} that represents the degree of dissimilarity between the elements F_i and F_j.

Step 3 Construction of the MST of the TRINS graph $G(V, E)$ is done as follows:

1. The distances of edges of G sorted in increasing order by weights.
2. A subgraph (empty) S of G is taken and the edge e with minimum weight is added to S, if the end points of e are not connected in S. Here the smallest edge is between F_1 and F_4; $d_{14} = 0.08456$ is added to S and removed from the sorted list.
3. The next minimum weight edge is selected from G; if no cycle is created in S, it is deleted from the list and added to S.
4. Process (3) is repeated until the obtained subgraph S spans all the ten nodes.

The MST S of the TRINS graph so obtained is illustrated in Fig. 5.

Step 4 A threshold r is selected, and all the edges with weights more than r are disconnected to get the subtrees (clusters), as listed in Table 4.

The clusters that are formed when the threshold value r is taken as 0.2928 are given in Fig. 6. It can be clearly seen that there are three different clusters grouped based on their feedback as satisfied customers (F_2, F_5, F_7), unsatisfied customers $(F_1, F_4, F_3, F_8, F_9)$ and indeterminate customers (F_6, F_{10}). Based on the clusters, targeted and interactive marketing can be carried out. This type of clustering is not possible with Likert scaling.

Clustering of customer feedback can be carried only on the basis of particular questions, and from these clusters and other information several insights can be gained.

6 Comparison and discussions

6.1 Comparison with Likert scale

It is known that Likert scaling has drawbacks like information distortion and information loss. These problems are over-

0	0.456	0.1087	0.08456	0.4006	0.3568	0.3865	0.1645	0.2271	0.3834
0.456	0	0.4278	0.507	0.1263	0.3938	0.139	0.4213	0.4649	0.4599
0.1087	0.4278	0	0.1554	0.3684	0.3267	0.3531	0.1686	0.2435	0.3776
0.08456	0.507	0.1554	0	0.453	0.3944	0.4394	0.186	0.2404	0.4157
0.4006	0.1263	0.3684	0.453	0	0.3155	0.134	0.3558	0.4007	0.3928
0.3568	0.3938	0.3267	0.3944	0.3155	0	0.3258	0.3238	0.3618	0.258
0.3865	0.139	0.3531	0.4394	0.134	0.3258	0	0.3375	0.3837	0.3753
0.1645	0.4213	0.1686	0.186	0.3558	0.3238	0.3375	0	0.1412	0.3059
0.2271	0.4649	0.2435	0.2404	0.4007	0.3618	0.3837	0.1412	0	0.2928
0.3834	0.4599	0.3776	0.4157	0.3928	0.258	0.3753	0.3059	0.2928	0

Fig. 4 Distance matrix

Fig. 5 MST of the TRIN

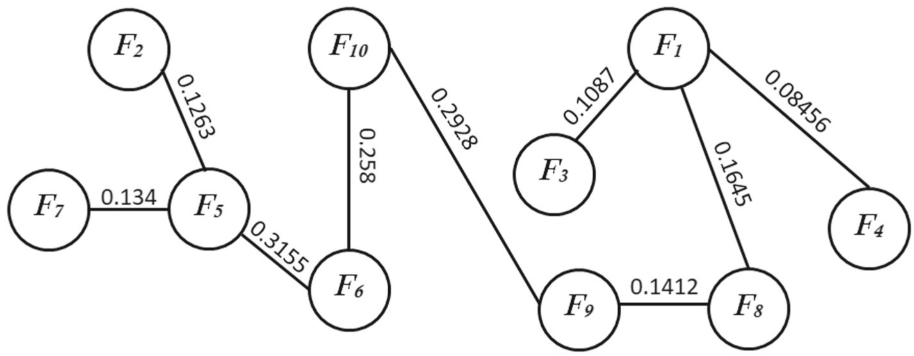


Table 4 Clustering results using TRIN-MST clustering algorithm

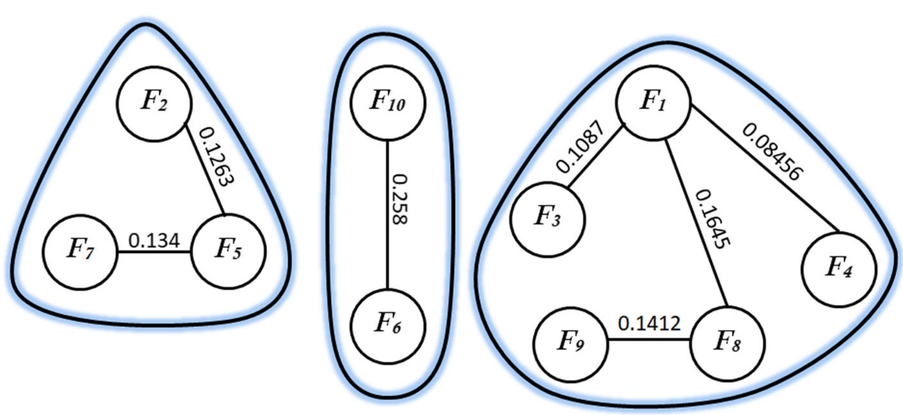
Threshold r	Corresponding clustering result
$r = d_{68} = 0.3155$	$\{F_1, F_3, F_4, F_6, F_8, F_9, F_{10}\}, \{F_2, F_5, F_7\}$
$r = d_{56} = 0.2928$	$\{F_1, F_3, F_4, F_8, F_9\}, \{F_6, F_{10}\}, \{F_2, F_5, F_7\}$

come when TRINS is used for collecting feedback from the user. It captures the feedback in a sensitive, accurate and realistic way as it deals with incomplete, imprecision, uncertain and indeterminate information. It is clearly seen that indeterminate Likert scale when compared to Likert scale gives more option to the customer to express themselves. In Likert scale, only the dominant choice is selected and vital information is lost.

6.2 Comparison with fuzzy Likert scale

Neutrosophic set is generalized as TRINS, intuitionistic fuzzy information is generalized as neutrosophic information/SVNS sets, and fuzzy information is generalized as intuitionistic fuzzy information. Thus, TRINS has the capacity to provide better precision and accuracy to represent the

Fig. 6 Clusters of customers



existing uncertain, indeterminate, vague, imperfect and unreliable information.

It has the supplementary ability to designate with more sensitivity the indeterminate and unreliable information. Whereas the SVNS can deal indeterminate and unreliable information, it cannot designate with accuracy the existing indeterminacy. It is acknowledged that neither fuzzy theory nor IFS can deal with information that is indeterminate and inconsistent in nature; however, IFS has provisions to deal and describe with incomplete information. In SVNS, truth, indeterminacy and falsity membership are characterized individualistically, and they can also be defined with respect to any of them (no restriction). This enables SVNS to be prepared to deal with indeterminate information better than IFS, whereas in TRINS, more scope is given to deal with the prevailing indeterminate and unreliable information because the indeterminacy concept is sub-classified as three distinct values. This provides more accurateness and exactness to the indeterminacy present in the data in TRINS than in SVNS.

TRINS deals particularly with the indeterminacy leaning towards (favouring) positive (truth), the indeterminacy leaning towards negative (false) and indeterminacy itself which other methods are incapable of doing it. It is acknowledged that when fuzzy set membership is defined with respect to truth T, the information related to indeterminacy and non-membership is missing. In IFS, memberships are defined in terms of truth and false only; here the indeterminacy is taken as what is left after the truth and false membership. The IFS cannot represent the indeterminate and inconsistent information, but it has provisions to describe and work with incomplete information. In SVNS, truth, indeterminacy and falsity membership are represented individualistically, and they can also be defined with respect to any of them (no restriction). This makes SVNS better at dealing information than IFS. TRINS when compared to SVNS/DVNS has better scope to describe and deal with the existing indeterminacy and inconsistent information because the indeterminacy concept is classified as three different values. This provides more accuracy and precision to indeterminacy in TRINS, than in SVNS. However, TRINS is better equipped to deal with indeterminacy than Fuzzy theory. Fuzzy Likert scale cannot capture indeterminate, imprecise and incomplete data. TRINS-based indeterminate Likert scale captures data in a more precise, accurate and realistic way than fuzzy Likert scale.

6.3 Further study

Multipoint indeterminate Likert scale which functions on 7 points or 10 points will be taken up for further studies. These multipoint Likert scales can be used to study a variety of sociological, economical and psychological problems. As future research, we also propose to map the middle 3 terms of TRINS to neutrosophic triplets (Vasantha et al. 2018b) and then they can be automatically mapped to neutrosophic duplets (Vasantha et al. 2018a, c) in the case that indeterminacy leaning towards false is zero.

7 Conclusions

In this paper, indeterminate Likert scaling based on TRINS was introduced; it is equipped to deal with inconsistent, uncertain, imprecise and indeterminate information which Likert scale is incapable of. Generally feedback from the customer depends on the human emotions which are mostly uncertain, inconsistent, imprecise or indeterminate in nature. Hence, indeterminate Likert scale is more apt to use for feedback than Likert scale. Indeterminate Likert scale can be easily implemented and used in mobile apps for collecting feedback. Indeterminate MST clustering algorithm was introduced to cluster the feedback obtained using distance matrices as a main measurement. Results from the clustering can be used for targeted and interactive marketing separately for each cluster.

Compliance with ethical standards

Conflict of interest All authors declare that they have no conflict of interest.

Informed consent Informed consent was obtained from all individual participants included in the study.

References

Armstrong RL (1987) The midpoint on a five-point likert-type scale. Percept Mot Skills 64(2):359–362. https://doi.org/10.2466/pms.1987.64.2.359

Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20(1):87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

Brown CN, Guskiewicz KM, Bleiberg J (2007) Athlete characteristics and outcome scores for computerized neuropsychological assessment: a preliminary analysis. J Athl Train 42(4):515

Cheng HD, Guo Y (2008) A new neutrosophic approach to image thresholding. New Math Nat Comput 4(03):291–308. https://doi.org/10.1142/S1793005708001082

Kandasamy I (2018a) Double-valued neutrosophic sets, their minimum spanning trees, and clustering algorithm. J Intell Syst 27(2):163–182. https://doi.org/10.1515/jisys-2016-0088

Kandasamy I (2018b) Indeterminate likert scale—sample dataset—customer feedback of restaurant. Mendeley Data, v1. https://doi.org/10.17632/ywjxpyw95w.1

Kandasamy I, Smarandache E (2016a) Multicriteria decision making using double refined indeterminacy neutrosophic cross entropy and indeterminacy based cross entropy. Appl Mech Mater 859:129–143. https://doi.org/10.4028/www.scientific.net/AMM.859.129

Kandasamy I, Smarandache F (2016b) Triple refined indeterminate neutrosophic sets for personality classification. In: 2016 IEEE

Symposium Series on Computational Intelligence (SSCI). IEEE, pp 1–8. https://doi.org/10.1109/SSCI.2016.7850153

Khan Q, Liu P, Mahmood T (2018) Some generalized dice measures for double-valued neutrosophic sets and their applications. Mathematics 6(7):121. https://doi.org/10.3390/math6070121

Kiedrowski PJ (2006) Quantitative assessment of a senge learning organization intervention. Learn Organ 13(4):369–383. https://doi.org/10.1108/09696470610667742

Li Q (2013) A novel likert scale based on fuzzy sets theory. Expert Syst Appl 40(5):1609–1618. https://doi.org/10.1016/j.eswa.2012.09.015

Likert R (1932) A technique for the measurement of attitudes. Arch Psychol 22:5–55

Liu P, Li H (2017) Multiple attribute decision-making method based on some normal neutrosophic Bonferroni mean operators. Neural Comput Appl 28:179–194. https://doi.org/10.1007/s00521-015-2048-z

Liu P, Shi L (2015) The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making. Neural Comput Appl 26(2):457–471. https://doi.org/10.1007/s00521-014-1736-4

Liu P, Teng F (2017) Multiple attribute group decision making methods based on some normal neutrosophic number Heronian mean operators. J Intell Fuzzy Syst 32(3):2375–2391. https://doi.org/10.3233/JIFS-16345

Liu P, Wang Y (2014) Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. Neural Comput Appl 25(7–8):2001–2010. https://doi.org/10.1007/s00521-014-1688-8

Orr MG, Ohlsson S (2005) Relationship between complexity and liking as a function of expertise. Music Percept Interdiscip J 22(4):583–611. https://doi.org/10.1525/mp.2005.22.4.583

Postma TC (2007) Screening tools to prioritize routine dental care in an institutional environment. Mil Med 172(12):1287–1292

Rus CL, Chirică S, Raţiu L, Băban A (2014) Learning organization and social responsibility in Romanian higher education institutions. Procedia Soc Behav Sci 142:146–153. https://doi.org/10.1016/j.sbspro.2014.07.628

Russell CJ, Bobko P (1992) Moderated regression analysis and likert scales: too coarse for comfort. J Appl Psychol 77(3):336

Salama A, Haitham A, Manie A, Lotfy M (2014) Utilizing neutrosophic set in social network analysis e-learning systems. Int J Inf Sci Intell Syst 3(2):61–72

Sengur A, Guo Y (2011) Color texture image segmentation based on neutrosophic set and wavelet transformation. Comput Vis Image Underst 115(8):1134–1144. https://doi.org/10.1016/j.cviu.2011.04.001

Smarandache F (2000) A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, probability, and statistics. American Research Press, Rehoboth. https://arxiv.org/pdf/math/0101228

Smarandache F (2013) n-valued refined neutrosophic logic and its applications in physics. Prog Phys 4:143–146

Vasantha W, Smarandache F (2003) Fuzzy cognitive maps and neutrosophic cognitive maps. Xiquan. https://arxiv.org/pdf/math/0311063

Vasantha W, Smarandache F (2004) Analysis of social aspects of migrant labourers living with HIV/AIDS using fuzzy theory and neutrosophic cognitive maps: with special reference to Rural Tamil Nadu in India. Xiquan. https://arxiv.org/pdf/math/0406304

Vasantha WB, Kandasamy I, Smarandache F (2018a) Algebraic structure of neutrosophic duplets in neutrosophic rings. Neutrsophic Sets Syst 23:85–95

Vasantha WB, Kandasamy I, Smarandache F (2018b) A classical group of neutrosophic triplet groups using Z2p. Symmetry 10(6):194. https://doi.org/10.3390/sym10060194

Vasantha WB, Kandasamy I, Smarandache F (2018c) Neutrosophic duplets of Zpn, and Zpq, and their properties. Symmetry 10(8):344. https://doi.org/10.3390/sym10080345

Wang H, Smarandache F, Zhang Y, Sunderraman R (2010) Single valued neutrosophic sets. Review p. 10. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.309.9470&rep=rep1&type=pdf

Ye J (2013) Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. Int J Gen Syst 42(4):386–394. https://doi.org/10.1080/03081079.2012.761609

Ye J (2014a) A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. J Intell Fuzzy Syst 26(5):2459–2466. https://doi.org/10.3233/IFS-130916

Ye J (2014b) Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making. J Intell Fuzzy Syst 26(1):165–172. https://doi.org/10.3233/IFS-120724

Ye J (2014c) Single valued neutrosophic cross-entropy for multicriteria decision making problems. Appl Math Model 38(3):1170–1175. https://doi.org/10.1016/j.apm.2013.07.020

Zadeh LA (1965) Fuzzy sets. Inf Control 8(3):338–353

Zhang M, Zhang L, Cheng H (2010) A neutrosophic approach to image segmentation based on watershed method. Signal Process 90(5):1510–1517. https://doi.org/10.1016/j.sigpro.2009.10.021