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Article *in* Soft Computing · June 2022

DOI: 10.1007/s00500-022-06994-0

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Neutrosophic pre-*I*-open set in neutrosophic ideal bitopological space

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Abstract: In this article, we introduce the notion of neutrosophic ideal bitopological space, which is the natural generalization of neutrosophic ideal topological space. The concept of neutrosophic ideal has been studied by Albowi et.al. [1]. The concept of local function on neutrosophic bitopological space have been defined and the different properties are also studied here. Also we defined neutrosophic pre-*I*-open set in a neutrosophic ideal bitopological spaces.

Keywords: Neutrosophic topology; Neutrosophic bi-topology; Neutrosophic ideal; Neutrosophic local function; Pairwise neutrosophic open; Pairwise neutrosophic closed; Pre-*I*-open; Semi-*I*-open.

2020 AMS Classification No: 03E72; 20F10; 62C86; 90B05; 91B10.

1. Introduction: In 1965, Zadeh introduced the concept of fuzzy set, where every element had a membership value between 0 and 1. Thereafter Atanassov introduce the concept of intuitionistic fuzzy set as a generalization of fuzzy set, where every element had membership as well as non-membership value. The idea of neutrosophic set theory was introduced by Smarandache [17, 18] as a generalization of intuitionistic fuzzy set, where besides the degree of truthness, degree of falseness and degree of indeterminacy of each element. Salama and Alblowi [11] first introduced the concept of neutrosophic topological space in 2012 as generalization of fuzzy topological space. Thereafter Ozturk and Ozkan [7] introduce the concept of neutrosophic bitopologoical space. Salama and Smarandache [12] defined neutrosophic ideal theory, neutrosophic local function and generated neutrosophic topology and generalized the concept of fuzzy ideal concepts, first initiated by Sarker [15].

2. Preliminaries and Definitions

In this section, we procure some definitions, examples and some basic properties which are already established.

For more information we can refer to [7, 11, 12, 17, 18]

Definition 2.1. A neutrosophic set Y over an universal set W is a set contains triplet having the degree of truthness (T_Y), indeterminacy (I_Y) and falseness (F_Y), where $T_Y(p)$, $I_Y(p)$, $F_Y(p)\in]^-0,1^+[$, for all $p\in W$. The neutrosophic set is denoted as follows:

$Y = \{(p, T_Y(p), I_Y(p), F_Y(p)): p\in W$ and $T_Y(p), I_Y(p), F_Y(p)\in\,]^-0,1^+[\}$, where

$$^{-}0 \leq T_Y(p) + I_Y(p) + F_Y(p) \leq 3^{+}.$$

Now we may defined two neutrosophic sets 0_N and 1_N over W as follows:

1) $0_N = \{(p,0,1,1): p \in W\}$;
2) $1_N = \{(p,1,0,0): p \in W\}$.

Definition 2.2. Assume that $Y = \{(p, T_Y(p), I_Y(p), F_Y(p)): p \in W$ and $T_Y(p), I_Y(p), F_Y(p) \in\ ]^{-}0,1^{+}[\}$, $X = \{(p, T_X(p), I_X(p), F_X(p)): p \in W$ and $T_X(p), I_X(p), F_X(p) \in\ ]^{-}0,1^{+}[\}$ be two neutrosophic sets over W.

Then we may define the followings:

1) $Y^c = \{(p, 1-T_Y(p), 1-I_Y(p), 1-F_Y(p)): p \in W\}$;
2) $Y \subseteq X$ if and only if $T_Y(p) \leq T_X(p)$, $I_Y(p) \geq I_X(p)$, $F_Y(p) \geq F_X(p)$, for each $p \in W$;
3) $Y \cup X = \{(p, T_Y(p) \vee T_X(p), I_Y(p) \wedge I_X(p), F_Y(p) \wedge F_X(p)): p \in W\}$;
4) $Y \cap X = \{(p, T_Y(p) \wedge T_X(p), I_Y(p) \vee I_X(p), F_Y(p) \vee F_X(p)): p \in W\}$.

Definition 2.3. Let p, q, r be real standard and non standard subsets of $]^{-}0,1^{+}[$. A neutrosophic set $x_{p.q.r}$ is said to be a neutrosophic point (in short NP) in a non-empty set W given by $x_{p.q.r}(y) = \begin{cases} (p,q,r), if\ x = y \\ (0,0,1), if\ x \neq y \end{cases}$, for $y \in W$ is called the support of $x_{p.q.r}$; where p, q, r denotes the degree of truth, indeterminacy and falsity membership values of $x_{p.q.r}$.

Definition 2.4. Let τ be a family of some neutrosophic sets defined over a non-empty set W. Then τ is said to be a neutrosophic topology (in short NT) on W if the following axioms holds:

1) $0_N, 1_N \in \tau$
2) $U_1, U_2 \in \tau \Rightarrow U_1 \cap U_2 \in \tau$
3) $\cup\, U_i \in \tau$, for every $\{U_i : i \in \Delta\} \subseteq \tau$.

In that case the pair (W,τ) is called a neutrosophic topological space (in short NTS). The elements of τ are called neutrosophic open set (in short NOS) and the complement of the members of τ are said to be neutrosophic closed set (in short NCS).

Example 2.1. Let $W = \{k_1, k_2, k_3\}$ and
$T_1 = \{(k_1,0.3,0.5,0.7), (k_2,0.5,0.6,0.4), (k_3,0.6,0.8,0.6): k_1, k_2, k_3 \in W\}$,
$T_2 = \{(k_1,0.2,0.5,0.9), (k_2,0.4,0.8,0.8), (k_3,0.5,0.9,0.8): k_1, k_2, k_3 \in W\}$,
$T_3 = \{(k_1,0.8,0.3,0.2), (k_2,0.7,0.4,0.3), (k_3,0.8,0.7,0.4): k_1, k_2, k_3 \in W\}$
are three neutrosophic set over W.
Then $\tau = \{0_N, 1_N, T_1, T_2, T_3\}$ is a neutrosophic topology on W.

Definition 2.5. Let W be an universal set and τ_1, τ_2 be two different neutrosophic topology on W. Then (W, τ_1, τ_2) is called a neutrosophic bitopological space.

Example 2.2. Let $W = \{k_1, k_2\}$ and

T_1={(k_1, 0.8, 0.4, 0.2), (k_2, 0.7, 0.4, 0.2)},

T_2={(k_1, 0.2, 0.5, 0.6), (k_2, 0.3, 0.8, 0.7)},

T_3={(k_1, 0.6, 0.1, 0.3), (k_2, 0.7, 0.3, 0.6)},

T_4={(k_1, 0.4, 0.5, 0.7), (k_2, 0.5, 0.9, 0.7)} are four neutrosophic sets over W.

Then clearly τ_1={$0_N, 1_N$, T_1, T_2} and τ_2={$0_N, 1_N$, T_3, T_4} are two different neutrosophic topology on W. So (W, τ_1, τ_2) is a neutrosophic bitopological space.

Remark 2.1. Throughout this paper we use the following notations:

$N_{\tau_i-int}\,(K)$ = The neutrosophic interior of K with respect to the neutrosophic topology τ_i , i=1,2.

$N_{\delta_i-int}\,(K)$ = The neutrosophic interior of K with respect to the neutrosophic topology δ_i , i=1,2.

Definition 2.6. A collection I of neutrosophic subsets of a neutrosophic topological space (W, τ) is said to be a neutrosophic ideal if it satisfies the following properties:

1) $Y \in I$ and $X \in I \Longrightarrow Y \cup X \in I$
2) $Y \in I$ and $X \subseteq Y \Longrightarrow X \in I$

Remark 2.2.

1) $0_N \in I$.
2) If $1_N \in I$, then I is called neutrosophic improper ideal.
3) If $1_N \notin I$, then I is called neutrosophic proper ideal.

Definition 2.7. Let (W, τ) be a neutrosophic topological space and I be an neutrosophic ideal on W. Let A be any neutrosophic subset of W. Then the neutrosophic local function $NA^*(\tau,I)$ of A is the union of all neutrosophic points $x_{p,q,r}$ such that $NA^*(\tau,I) = \cup\{x_{p,q,r} \in W$: $A \cap U \notin I$ for every U nbd of $x_{p,q,r}$ } , $NA^*(\tau,I)$ or simply NA^* is called a neutrosophic local function of A with respect to τ and I.

Theorem[12] 2.1. Assume that (W, τ) be a neutrosophic topological space and I, J be any two neutrosophic ideals on W. Then for any two arbitrary neutrosophic subsets K and L, the followings holds:

1) $K \subseteq L \Longrightarrow NK^*(\tau,I) \subseteq NL^*(\tau,I)$;
2) $I \subseteq J \Longrightarrow NK^*(\tau,I) \subseteq NK^*(\tau,J)$;
3) $N(K \cup L)^*(\tau,I) = NK^*(\tau,I) \cup NL^*(\tau,I)$;
4) $N(K \cap L)^*(\tau,I) \leq NK^*(\tau,I) \cap NL^*(\tau,I)$;
5) For any neutrosophic subset K of W, $NK^*(\tau,I)$ is a neutrosophic closed set.

Theorem[12] 2.2. Let τ_1, τ_2 be two neutrosophic topolgies on *W*. Then for any neutrosophic ideal *I* on *W*, $\tau_1 \subseteq \tau_2 \Longrightarrow N\tau_1^* \subseteq N\tau_2^*$.

Definition 2.8. Let (W, τ) be a neutrosophic topological space and *A* be a neutrosophic subset of *W*. A function $N_{Cl}^*: P(W) \rightarrow P(W)$ is said to be neutrosophic closure operator on *A*, defined by $N_{Cl}^*(A) = A \cup NA^*(\tau, I)$. Where, $NA^*(\tau, I)$ is the neutrosophic local function of *A*.

3. Neutrosophic ideal bitopological spaces

Definition 3.1. Assume that (W, τ_1, τ_2) is a neutrosophic bitopological space and *I* be an neutrosophic ideal on *W*. Then (W, τ_1, τ_2, I) is called a neutrosophic ideal bitopological space.

Definition 3.2. Let (W, τ_1, τ_2) be a neutrosophic bitopological space and *I* be an neutrosophic ideal on *W*. Let *A* be any neutrosophic subset of *W*. Then the neutrosophic local function on bitopological space $NA^{**}(\tau_1, \tau_2, I)$ of *A* is the union of all neutrosophic points $x_{p,q,r}$ such that $NA^{**}(\tau_1, \tau_2, I) = \cup\{x_{p,q,r} \in W: A \cap U \notin I$ for every *U* nbd of $x_{p,q,r}$ either in τ_1, or $\tau_2\}$, $NA^{**}(\tau_1, \tau_2, I)$ simply NA^{**} is called a neutrosophic local function of *A* with respect to τ_1, τ_2 and *I*.

Theorem 3.1. Assume that (W, τ_1, τ_2) be a neutrosophic bitopological space and *I*, *J* be any two neutrosophic ideals on *W*. Then for any two neutrosophioc subsets *K*, *L* of (W, τ_1, τ_2), the following holds:

1) $K \subseteq L \Rightarrow NK^{**} \subseteq NL^{**}$;
2) $I \subseteq J \Longrightarrow NK^{**}(\tau, I) \subseteq NK^{**}(\tau, J)$;
3) $N(K \cup L)^{**}(\tau, I) = NK^{**}(\tau, I) \cup NL^{**}(\tau, I)$;
4) $N(K \cap L)^{**}(\tau, I) \subseteq NK^{**}(\tau, I) \cap NL^{**}(\tau, I)$.

Proof.

1) Let us assume that $K \subseteq L$, where *K* and *L* are any two neutrosophic subsets of (W, τ_1, τ_2). Let $x_{p,q,r}$ be a neutrosophic point in *W* such that $x_{p,q,r} \in NK^{**}$. Then for every neutrosophic neighbourhood *U* of $x_{p,q,r}$ either in τ_1 or τ_2 such that $K \cap U \notin I$. Since $K \subseteq L$, so $L \cap U \notin I$. This implies $x_{p,q,r} \in NL^{**}$. Therefore $NK^{**} \subseteq NL^{**}$. Hence $K \subseteq L \Rightarrow NK^{**} \subseteq NL^{**}$.
2) Let *K* be any neutrosophic subsets of (W, τ_1, τ_2) and *I*, *J* be any two neutrosophic ideals on *W*. Again let $I \subseteq J$. Let $x_{p,q,r}$ be a neutrosophic point in *W* such that $x_{p,q,r} \in NK^{**}(\tau, I)$. Then for every neutrosophic neighbourhood *U* of $x_{p,q,r}$ either in τ_1 or τ_2 , we have $K \cap U \notin I$. Since $I \subseteq J$ and $K \cap U \notin I$, so $K \cap U \notin J$. Therefore $x_{p,q,r} \in NK^{**}(\tau, J)$.
3) Let us assume that *K* and *L* are any two neutrosophic subsets of (W, τ_1, τ_2). Clearly $K \subseteq K \cup L$ and $L \subseteq K \cup L$. From (1) it is clear that $NK^{**} \subseteq N(K \cup L)^{**}$ and $NL^{**} \subseteq N(K \cup L)^{**}$. This implies $NK^{**} \cup NL^{**} \subseteq N(K \cup L)^{**}$.

Let us assume that $x_{p,q,r} \in N(K \cup L)^{**}$. Then for any neutrosophic neighbourhood U of $x_{p,q,r}$ either in τ_1 or τ_2 such that $(K \cup L) \cap U \notin I$ that is $(K \cap U) \cup (L \cap U) \notin I$. Then there is three possible cases :

Case-1: $(K \cap U) \notin I$ and $(L \cap U) \in I$

Clearly $x_{p,q,r} \in NK^{**}$ and $x_{p,q,r}$ does not belongs to NL^{**} , thus the elements $x_{p,q,r} \in NK^{**} \cup NL^{**}$.

Case-2: $(L \cap U) \notin I$ and $(K \cap U) \in I$

Clearly $x_{p,q,r} \in NL^{**}$ and $x_{p,q,r}$ does not belongs to NK^{**} , thus the elements $x_{p,q,r} \in NK^{**} \cup NL^{**}$.

Case-3: $(L \cap U) \notin I$ and $(K \cap U) \notin I$

Clearly $x_{p,q,r} \in NL^{**}$ and $x_{p,q,r} \in NK^{**}$, thus the elements $x_{p,q,r} \in NK^{**} \cup NL^{**}$.

In all the cases $x_{p,q,r} \in NK^{**} \cup NL^{**}$. Therefore $N(K \cup L)^{**} \subseteq NK^{**} \cup NL^{**}$.

Hence the theorem established.

4) Let us assume that K and L are any two neutrosophic subsets of (W, τ_1, τ_2). Clearly $K \cap L \subseteq K$ and $K \cap L \subseteq L$. From (1), it is clear that $N(K \cap L)^{**} \subseteq NK^{**}$ and $N(K \cap L)^{**} \subseteq NL^{**}$. This implies that $N(K \cap L)^{**} \subseteq NK^{**} \cap NL^{**}$.

Definition 3.3. Let (W, τ_1, τ_2) be a neutrosophic bitopological space and A be a neutrosophic subset of W. A function $N_{Cl}^{**}: P(W) \to P(W)$ is said to be neutrosophic closure operator on A, defined by $N_{Cl}^{**}(A) = A \cup NA^{**}(\tau_1, \tau_2, I)$. Where, $NA^{**}(\tau_1, \tau_2, I)$ is the neutrosophic local function of A.

Result 3.1.

1) If K and L are any two neutrosophic subset of a neutrosophic bitopological space (W, τ_1, τ_2), then $N_{Cl}^{**}(K \cup L) = N_{Cl}^{**}(K) \cup N_{Cl}^{**}(L)$.
2) If K is a neutrosophic subset of a neutrosophic bitopological space (W, τ_1, τ_2), then $N_{\tau_i - int}(K) \subseteq N_{Cl}^{**}(K)$, i=1,2.

Proof.

1) Let K and L be any two neutrosophic subset of a neutrosophic bitopological space (W, τ_1, τ_2). Then $N_{Cl}^{**}(K \cup L) = (K \cup L) \cup N(K \cup L)^{**} = (K \cup L) \cup (NK^{**} \cup NL^{**}) = (K \cup NK^{**}) \cup (L \cup NL^{**}) = N_{Cl}^{**}(K) \cup N_{Cl}^{**}(L)$.
2) Assume that K be a neutrosophic subset of a neutrosophic bitopological space (W, τ_1, τ_2) and I be an neutrosophic ideal on W. Clearly $N_{\tau_i - int}(K) \subseteq K$ and $K \subseteq K \cup NK^{**}(\tau_1, \tau_2, I)$, for i=1,2. This implies $N_{\tau_i - int}(K) \subseteq K \cup NK^{**}(\tau_1, \tau_2, I)$, i=1,2. That is $N_{\tau_i - int}(K) \subseteq N_{Cl}^{**}(K)$, for i= 1,2.

Definition 3.4. An neutrosophic subset Ω of a neutrosophic ideal bitopological space (W, τ_1, τ_2, I) is said to be a

1) τ_{12}-neutrosophic-pre-*I*-open if and only if $\Omega \subseteq N_{\tau_2-int}\, N_{Cl}^{**}(\Omega)$;
2) τ_{12}-neutrosophic-semi-*I*-open if and only if $\Omega \subseteq N_{cl}^{**} N_{\tau_2-int}(\Omega)$.
3) τ_{12}-neutrosophic-α-*I*-open if and only if $\Omega \subseteq N_{\tau_2-int} N_{cl}^{**} N_{\tau_2-int}(\Omega)$.

The family of all τ_{12}-neutrosophic-pre-*I*-open and τ_{12}-neutrosophic-semi-*I*-open sets in (W,τ_1,τ_2, *I*) is denoted by NPIO(*W*) and NSIO(*W*).

Remark 3.1. Assume that *I* and *J* are two ideals on a neutrosophic bitopological space (W,τ_1,τ_2, *I*). If $I \subseteq J$ then NPIO(*W*) ⊆ NPJO(*W*), and NSIO(*W*) ⊆ NSIO(*W*).

Definition 3.5. A neutrosophic subset Ω of a neutrosophic ideal bitopological space (W,τ_1,τ_2, *I*) is said to be a τ_{12}-neutrosophic *b*-*I*-open if and only if $\Omega \subseteq N_{\tau_2-int} N_{cl}^{**}(\Omega) \cup N_{cl}^{**} N_{\tau_2-int}(\Omega)$.

Theorem 3.2. Assume that (W,τ_1,τ_2,*I*) be a neutrosophic ideal bitopological space. Then

1) Every τ_{12}-neutrosophic-α-*I*-open set is a τ_{12}-neutrosophic-pre-*I*-open set.
2) Every τ_{12}-neutrosophic-α-*I*-open set is a τ_{12}-neutrosophic-semi-*I*-open set.

Proof.

1) Let K be an arbitrary τ_{12}-neutrosophic-α-*I*-open set in a neutrosophic ideal bitopological space (W,τ_1,τ_2,*I*). So $K \subseteq N_{\tau_2-int} N_{cl}^{**} N_{\tau_2-int}(K)$ ……………….(1).
Now, $N_{\tau_2-int}(K) \subseteq K$
$\Rightarrow N_{cl}^{**} N_{\tau_2-int}(K) \subseteq N_{cl}^{**}(K)$
$\Rightarrow N_{\tau_2-int} N_{cl}^{**} N_{\tau_2-int}(K) \subseteq N_{\tau_2-int} N_{cl}^{**}(K)$ …………..(2)

From (1) and (2), we have got, $K \subseteq N_{\tau_2-int} N_{cl}^{**}(K)$. Therefore K is a τ_{12}-neutrosophic-pre-*I*-open set. Hence every τ_{12}-neutrosophic-α-*I*-open set is a τ_{12}-neutrosophic-pre-*I*-open set.

2) Let K be an arbitrary τ_{12}-neutrosophic-α-*I*-open set in a neutrosophic ideal bitopological space (W,τ_1,τ_2,*I*). So $K \subseteq N_{\tau_2-int} N_{cl}^{**} N_{\tau_2-int}(K)$ ……………..(3)
Clearly, $N_{\tau_2-int} N_{cl}^{**} N_{\tau_2-int}(K) \subseteq N_{cl}^{**} N_{\tau_2-int}(K)$ …………..(4)

From (3) and (4), we have got, $K \subseteq N_{cl}^{**} N_{\tau_2-int}(K)$. Therefore K is a τ_{12}-neutrosophic-semi-*I*-open set. Hence every τ_{12}-neutrosophic-α-*I*-open set is a τ_{12}-neutrosophic-semi-*I*-open set.

Theorem 3.3. Assume that (W,τ_1,τ_2,*I*) be a neutrosophic ideal bitopological space and Ω be a neutrosophic subset of *W*. If Ω is τ_{12}-neutrosophic-pre-*I*-open (τ_{12}-neutrosophic-semi-*I*-open) set then it is τ_{12}-neutrosophic *b*-*I*-open.

Proof. Assume that Ω be a τ_{12}-neutrosophic-pre-I-open set in a neutrosophic ideal bitopological space (W,τ_1,τ_2,I). So $\Omega \subseteq N_{\tau_2-int}N_{cl}^{**}(\Omega)$. This implies $\Omega \subseteq N_{\tau_2-int}N_{cl}^{**}(\Omega) \cup N_{cl}^{**}N_{\tau_2-int}(\Omega)$. Hence Ω is a τ_{12}-neutrosophic-b-I-open set in (W,τ_1,τ_2, I).

Similarly it can be shown that, every τ_{12}-neutrosophic-semi-I-open set is a τ_{12}-neutrosophic-b-I-open set in (W,τ_1,τ_2, I).

Remark 3.2. In a neutrosophic ideal bitopological space (W,τ_1,τ_2,I), every τ_{12}-neutrosophic-α-I-open set is a τ_{12}-neutrosophic b-I-open set.

Proof. Assume that (W,τ_1,τ_2,I) be a neutrosophic ideal bitopological space and K be an arbitrary τ_{12}-neutrosophic-α-I-open set. From **Theorem 3.1 and 3.2**, it is clear that K is a τ_{12}-neutrosophic b-I-open set.

Result 3.2. In a neutrosophic ideal bitopological space (W,τ_1,τ_2,I),

1) The union of two τ_{12}-neutrosophic-pre-I-open set is a τ_{12}-neutrosophic-pre-I-open set.
2) The union of two τ_{12}-neutrosophic-semi-I-open set is a τ_{12}-neutrosophic-semi-I-open set.

Proof.

1) Assume that K and L be any two τ_{12}-neutrosophic-pre-I-open set in a neutrosophic ideal bitopological space (W,τ_1,τ_2,I). So $K \subseteq N_{\tau_2-int}N_{cl}^{**}(K)$ and $L \subseteq N_{\tau_2-int}N_{cl}^{**}(L)$.
   Now $K \cup L \subseteq N_{\tau_2-int}N_{cl}^{**}(K) \cup N_{\tau_2-int}N_{cl}^{**}(L)$
   $\subseteq N_{\tau_2-int}(N_{cl}^{**}(K) \cup N_{cl}^{**}(L))$
   $= N_{\tau_2-int}N_{cl}^{**}(K \cup L)$
   $\Rightarrow K \cup L \subseteq N_{\tau_2-int}N_{cl}^{**}(K \cup L)$. Hence $K \cup L$ is a τ_{12}-neutrosophic-pre-I-open set in (W,τ_1,τ_2,I).
   Therefore the union of two τ_{12}-neutrosophic-pre-I-open set is a τ_{12}-neutrosophic-pre-I-open set.
2) Assume that K and L be any two τ_{12}-neutrosophic-pre-I-open set in a neutrosophic ideal bitopological space (W,τ_1,τ_2,I). So $K \subseteq N_{cl}^{**}N_{\tau_2-int}(K)$ and $L \subseteq N_{cl}^{**}N_{\tau_2-int}(L)$.
   Now $K \cup L \subseteq N_{cl}^{**}N_{\tau_2-int}(K) \cup N_{cl}^{**}N_{\tau_2-int}(L)$
   $= N_{cl}^{**}(N_{\tau_2-int}(K) \cup N_{\tau_2-int}(L))$
   $\subseteq N_{cl}^{**}N_{\tau_2-int}(K \cup L)$
   $\Rightarrow K \cup L \subseteq N_{cl}^{**}N_{\tau_2-int}(K \cup L)$. Therefore $K \cup L$ is a τ_{12}-neutrosophic-semi-I-open set in (W,τ_1,τ_2,I).
   Hence the union of two τ_{12}-neutrosophic-semi-I-open set is a τ_{12}-neutrosophic-semi-I-open set.

Theorem 3.4. Assume that (W,τ_1,τ_2,I) be a neutrosophic ideal bitopological space. If $\Omega \in$ NPIO(W) and $\Omega \subseteq \Phi \subseteq N_{\tau_2-int}(N_{cl}^{**}(\Omega))$, then $\Phi \in$ NPIO(W).

Proof. Assume that $\Omega \in$ NPIO(W). So $\Omega \subseteq N_{\tau_2-int}N_{cl}^{**}(\Omega)$.

Also $\Omega \subseteq \Phi \subseteq N_{\tau_2-int}N_{cl}^{**}(\Omega)$ …………………..(1)

Now $\Omega \subseteq \Phi$

$\Rightarrow N_{cl}^{**}(\Omega) \subseteq N_{cl}^{**}(\Phi)$

$\Rightarrow N_{\tau_2-int}(N_{cl}^{**}(\Omega)) \subseteq N_{\tau_2-int}(N_{cl}^{**}(\Phi))$ …………….(2)

From (1) and (2) we have got,

$\Omega \subseteq \Phi \subseteq N_{\tau_2-int}(N_{cl}^{**}(\Omega)) \subseteq N_{\tau_2-int}(N_{cl}^{**}(\Phi))$

$\Rightarrow \Phi \subseteq N_{\tau_2-int}(N_{cl}^{**}(\Phi))$.

Therefore, $\Phi \in$ NPIO(W).

Theorem 3.5. Assume that (W,τ_1,τ_2,I) be a neutrosophic ideal bitopological space. If $\Omega \in$ NSIO(W) and $\Omega \subseteq \Phi \subseteq N_{cl}^{**}N_{\tau_2-int}(\Omega)$ then $\Phi \in$ NSIO(W).

Proof. Assume that $\Omega \in$ NSIO(W). So $\Omega \subseteq N_{cl}^{**}N_{\tau_2-int}(\Omega)$
Also $\Omega \subseteq \Phi \subseteq N_{cl}^{**}N_{\tau_2-int}(\Omega)$ …………….(3)

Now $\Omega \subseteq \Phi$

$\Rightarrow N_{\tau_2-int}(\Omega) \subseteq N_{\tau_2-int}(\Phi)$

$\Rightarrow N_{cl}^{**}(N_{\tau_2-int}(\Omega)) \subseteq N_{cl}^{**}(N_{\tau_2-int}(\Phi))$ ……………(4)

From (3) and (4), we have got

$\Omega \subseteq \Phi \subseteq N_{cl}^{**}N_{\tau_2-int}(\Omega) \subseteq N_{cl}^{**}(N_{\tau_2-int}(\Phi))$

$\Rightarrow \Phi \subseteq N_{cl}^{**}(N_{\tau_2-int}(\Phi))$

Therefore, $\Phi \in$ NSIO(W).

Proposition 3.1. Assume that (W,τ_1,τ_2, I) be a neutrosophic ideal bitopological space.

Then

1) $\Omega \in$ NPIO(W) if and only if there exists a neutrosophic open set U either in τ_1 or τ_2 such that $U \subseteq \Omega \subseteq N_{\tau_2-int}(U)$.
2) $\Omega \in$ NSIO(W) if and only if there exists a neutrosophic open set U either in τ_1 or τ_2 such that $U \subseteq \Omega \subseteq N_{cl}^{**}(U)$.

Proof.

1) Assume that $\Omega \in$ NPIO(W). So $\Omega \subseteq N_{\tau_2-int} N_{cl}^{**}(\Omega)$. Now by taking $N_{cl}^{**}(\Omega) = U$, we have $U = N_{cl}^{**}(\Omega) \subseteq \Omega \subseteq N_{\tau_2-int}(U)$.
   Conversely, let U be an neutrosophic open set either in τ_1 or τ_2 such that $U \subseteq \Omega \subseteq N_{\tau_2-int}(U)$. Now $U \subseteq \Omega$. This implies $U \subseteq N_{cl}^{**}(\Omega)$. Hence $N_{\tau_2-int}(U) \subseteq N_{\tau_2-int}(N_{cl}^{**}(\Omega))$. Therefore $\Omega \subseteq N_{\tau_2-int}(N_{cl}^{**}(\Omega))$ i.e. $\Omega \in$ NPIO(W).
2) The proof is similar to the proof of the first part of Proposition 3.1.

Definiton 3.6. A function ξ: (W,τ_1,τ_2,I)→(M,δ_1,δ_2) is called a

1) τ_{12}-neutrosophic-pre-I-continuous function if $\xi^{-1}(\Omega)$ is a τ_{12}-neutrosophic-pre-I-open set in (W,τ_1,τ_2,I) whenever Ω is a δ_2-open set in (M,δ_1,δ_2).
2) τ_{12}-neutrosophic-semi-I-continuous function if $\xi^{-1}(\Omega)$ is a τ_{12}-neutrosophic-semi-I-open set in (W,τ_1,τ_2,I) whenever Ω is a δ_2-open set in (M,δ_1,δ_2).
3) τ_{12}-neutrosophic-α-I-continuous function if $\xi^{-1}(\Omega)$ is a τ_{12}-neutrosophic-α-I-open set in (W,τ_1,τ_2,I) whenever Ω is a δ_2-open set in (M,δ_1,δ_2).
4) τ_{12}-neutrosophic-b-I continuous function if $\xi^{-1}(\Omega)$ is a τ_{12}-neutrosophic-b-I-open set in (W,τ_1,τ_2,I) whenever Ω is a δ_2-open set in (M,δ_1,δ_2,I).

Theorem 3.6.

1) Every τ_{12}-neutrosophic-α-I-continuous function is a τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function.
2) Every τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function is a τ_{12}-neutrosophic-b-I continuous function.

Proof.

1) Assume that ξ be an arbitrary τ_{12}-neutrosophic-α-I-continuous function from (W,τ_1,τ_2,I) to (M,δ_1,δ_2). Let A be a δ_2-neutrosophic-open set in M. Then by hypothesis $\xi^{-1}(A)$ is a τ_{12}-neutrosophic-α-I-open set in W. Since every τ_{12}-neutrosophic-α-I-open set is a τ_{12}-neutrosophic-pre-I-open (τ_{12}-neutrosophic-semi-I -open) set, so $\xi^{-1}(A)$ is a τ_{12}-neutrosophic-pre-I-open (τ_{12}-neutrosophic-semi-I -open) set. Therefore ξ is a τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I -continuous) function. Hence every τ_{12}-neutrosophic-α-I-continuous function from (W,τ_1,τ_2,I) to (M,δ_1,δ_2) is a τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function.

2) Let ξ be an arbitrary τ_{12}-neutrosophic-pre-I-continuous function from (W,τ_1,τ_2,I) to (M,δ_1,δ_2). Let A be a δ_2-neutrosophic-open set in M. Then by hypothesis $\xi^{-1}(A)$ is a τ_{12}-neutrosophic-pre-I-open set in W. Since every τ_{12}-neutrosophic-pre-I-open set is a τ_{12}-neutrosophic-b-I-open set, so $\xi^{-1}(A)$ is a τ_{12}-neutrosophic-b-I-open set in W. Therefore ξ is a τ_{12}-neutrosophic b-I continuous function. Hence every τ_{12}-neutrosophic-pre-I-continuous function from (W,τ_1,τ_2,I) to (M,δ_1,δ_2) is a τ_{12}-neutrosophic b-I continuous function.

   Similarly it can be shown that every τ_{12}-neutrosophic-semi-I-continuous function is a τ_{12}-neutrosophic b-I continuous function.

Theorem 3.7. A function $\xi:(W,\tau_1,\tau_2,I)\to(M,\delta_1,\delta_2)$ is τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function if and only if for each neutrosophic point $x_{p,q,r}$ in W and for each δ_2-neutrosophic-open set K in M such that $\xi(x_{p,q,r})\in K$, there exists a τ_{12}-neutrosophic-pre-I-open (τ_{12}-neutrosophic-semi-I-open) set L in W such that $x_{p,q,r}\in L$, $\xi(L)\subseteq K$.

Proof. Assume that $\xi:(W,\tau_1,\tau_2,I)\to(M,\delta_1,\delta_2)$ be a τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function. Let $x_{p,q,r}$ be a neutrosophic point in W and K be a δ_2-neutrosophic-open set in M such that $\xi(x_{p,q,r})\in K$. By hypothesis $\xi^{-1}(K)$ is a τ_{12}-neutrosophic-pre-I-open (τ_{12}-neutrosophic-semi-I-open) set in W. Take $\xi^{-1}(K)=L$. Then $x_{p,q,r}\in L$ and $\xi(L)\subseteq K$.

Conversely, let K be a δ_2-neutrosophic-open set in M and let $x_{p,q,r}\in\xi^{-1}(K)$. Then $\xi(x_{p,q,r})\in K$. Then by hypothesis, there exists a τ_{12}-neutrosophic-pre-I-open (τ_{12}-neutrosophic-semi-I-open) set L in W such that $x_{p,q,r}\in L$ and $\xi(L)\subseteq K$. This implies that $x_{p,q,r}\in L\subseteq\xi^{-1}(K)$. Therefore $\xi^{-1}(K)$ is a τ_{12}-neutrosophic-pre-I-open (τ_{12}-neutrosophic-semi-I-open) set in W. Hence ξ is a τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function from (W,τ_1,τ_2,I) to (M,δ_1,δ_2).

Theorem 3.8. If $\xi:(W,\tau_1,\tau_2,I)\to(M,\delta_1,\delta_2)$ is a τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function then $\xi^{-1}(N_{\delta_2-int}(K))\subseteq N_{cl}^{**}N_{\tau_2-int}(\xi^{-1}(K))$, for every neutrosophic subset K of M.

Proof. Assume that $\xi:(W,\tau_1,\tau_2,I)\to(M,\delta_1,\delta_2)$ is a τ_{12}-neutrosophic-pre-I-continuous (τ_{12}-neutrosophic-semi-I-continuous) function and K be an arbitrary neutrosophic subset of M. Then $N_{\delta_2-int}(K)$ is δ_2-neutrosophic-open set in M. Then by hypothesis, $\xi^{-1}(N_{\delta_2-int}(K))$ is τ_{12}-neutrosophic-pre-I-open (τ_{12}-neutrosophic-semi-I-open) set in W.

Hence $\xi^{-1}(N_{\delta_2-int}(K))\subseteq N_{cl}^{**}N_{\tau_2-int}(\xi^{-1}(N_{\delta_2-int}(K)))$. …………(1)

Now $N_{\delta_2-int}(K)\subseteq K$

$\Rightarrow \xi^{-1}(N_{\delta_2-int}(K))\subseteq\xi^{-1}(K)$

$\Rightarrow N_{\tau_2-int}(\xi^{-1}(N_{\delta_2-int}(K)))\subseteq N_{\tau_2-int}(\xi^{-1}(K))$ …………(2)

From (1) and (2), we have got

$\xi^{-1}(N_{\delta_2-int}(K)) \subseteq N_{cl}^{**}N_{\tau_2-int}(\xi^{-1}(N_{\delta_2-int}(K))) \subseteq N_{cl}^{**}N_{\tau_2-int}(\xi^{-1}(K))$

$\Rightarrow \xi^{-1}(N_{\delta_2-int}(K)) \subseteq N_{cl}^{**}N_{\tau_2-int}(\xi^{-1}(K))$, for every neutrosophic subset K of M.

4. Conclusion

In this article, we introduce neutrosophic pre-*I*-open set, neutrosophic semi-*I*-open set, neutrosophic b-*I*-open set and investigate some of their properties. By defining different types of neutrosophic *I*-open set, we prove some theorems on neutrosophic ideal bitopological space and few result are given. In the future, we hope that based on these theorems, remarks, results and notions in neutrosophic ideal bitopological space many new investigations can be done.

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