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An inclusive study on the fundamentals of interval-valued fuzzy hypersoft set

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Abstract

When compared to its extension, hypersoft set, a soft set only deals with a single set of attributes, while a hypersoft set deals with several attribute-valued disjoint sets that correspond to various attributes. Several researchers have developed models based on soft sets, but the majority of these models suffer from limitations since they are inappropriate for interval-type data or uncertain data. In order to address these issues, a novel model interval-valued fuzzy hypersoft set ($IVFHS$-set) is presented in this research article. This model not only resolves the inadequacy of soft set for distinct attributes for non-overlapping attribute-valued sets, but also addresses the limitations of soft set-like models with having data in interval environment. This work modifies the current fuzzy hypersoft set concept and introduces certain fundamental ideas, such as subset, not set, whole set, and absolute relative null set, relative absolute set and aggregation operations e.g. intersection, union, extended intersection, restricted union, complement, OR, AND, difference, restricted difference are discussed under $IVFHS$-set environment with illustrated examples. Some new hybrids of fuzzy hypersoft set under interval-valued settings are also discussed. Moreover, some extensions of $IVFHS$-set are presented along with different operations.

Keywords: interval-valued fuzzy set; interval-valued fuzzy soft set; hypersoft set; set-theoretic operations.

1 introduction

Numerous intricate problems containing diverse uncertainties are addressed mathematically by the theory of fuzzy sets, the theory of probability and interval mathematics. These theories are used in a variety of mathematical areas. These theories' inherent complexities which prevent them from successfully resolving these issues. These difficulties may be caused by the parametrization tool's shortcomings. For dealing with uncertainties, a mathematical instrument that is free of these obstructions is required. To cope with uncertainties and ambiguity in data, fuzzy soft set (FS-set)[1] as a mixed variant of fuzzy set (F-set)[2] and soft set (S-set)[3] is regarded viable model. The F-set uses a membership function to give each component of the sample universe a fuzzy value between 0 and 1. In order to offer the F-set a parametrization tool, the FS-set ties an approximation function to each member of the set of parameters, allowing for the parameterization of each member with a fuzzy membership grade. Rahman et al.[4–6] revised the conventional concept of convexity and concavity in the context of the S-set, FS-set and refined intuitionistic fuzzy set ($RIFS$). Yang et al.[7] envisioned an interval-valued fuzzy soft set ($IVFS$-set) to handle the uncertain data in interval form by merging the interval-valued fuzzy set (IVF-set)[8] and S-set. Researchers[9–14] made marvelous contributions concerning real-world applications of $IVFS$-sets. The similarity measure, entropy measure and inclusion measure along with distance measure of the $IVFS$-set were devised by Xindong and Yong.[15] Intuitionistic fuzzy set IF-set[16]

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and neutrosophic set N-set[17] included non-membership and Neutral degree to extend the idea of F-set where as picture fuzzy set PFS[18] applied condition on sum of positive, negative and neutral degree to remain with in o to 1.

In S-set-like models, a single set of parameters is used to cope with the ambiguous and uncertain character of the data, however there are some circumstances in which the chosen parameters are insufficient to address decision-making issues. It is important to further categorise these parameters into the proper subclasses. The concept of the hypersoft set (HS-set) was devised by Smarandache as an enhancement of the S-set since the S-set literature is insufficient to handle such situations. In order to improve the performance of S-set, it employs a brand-new mapping approach called multi-argument approximation mapping (MAA-mapping). The domain and codomain of this mapping are assumed to be parametric tuples and the universe's power set, respectively. In this sense, it is accurate to argue that HS-set is more adaptable and trustworthy than S-set. Saeed et al.[20] and Abbas et al.[21] not only provided the fundamentals of the HS-set but also covered its characteristics and operations with the aid of several examples to assure its usage in addressing real-world problems. In the context of the HS-set, Rahman et al.[22] modified the traditional idea of convexity and concavity. The concept of rough HS-set was initiated by[23] along with some of its operational aspects in order to handle the roughness of the data. Kamacı[24] created expert systems for HS-set environments to handle circumstances requiring the multiple-decisional opinions of experts in decision-making systems. The idea was expanded by Martin & Smarandache[25] to include a plithogenic HS-set with its graphical version.

The fuzzy hypersoft set (FHS-set), a combination of the HS-set and the F-set, was developed by Yolcu and Oztürk[26] and its numerous fundamental operational properties were examined. Real-world decision-making issues were addressed by Arshad et al.[27] using distance measure of FHS-set. Rahman et al.[28] investigated the parametrization of the fuzzy hypersoft set (FHS-set) and analysed various of its practical results. Authors[29–32] developed several modifications in neutrosophic HS-set. Some fundamental concepts, including set theoretic operations, properties, laws as well as hybrids of $IVFHS$-sets are conceptualised in this work within the context of a hypersoft set in interval-value fuzzy environment. Recently Smarandache[33–35] developed new structures IndetermSoft set and IndetermHyperSoft set. He discussed the nature of product used in these structures and discussed its practical applications. Moreover, TreeSoftSet is developed as an extension of MultiSoftSet.

The remaining article is organised as follows: Section 2 provides some basic notions of HS-set and FHS-set. Some fundamental properties of $IVFHS$-set are presented in Section 3. The set theoretic operations of $IVFHS$-sets are described in Section 4. Some fundamental results, characteristics, and laws on $IVFHS$-sets are provided in Section 5. Section 6 offers several hybrid of $IVFHS$-sets, and section 7 concludes the paper.

2 Preliminaries

Let $\Bbbk$, $P(\Bbbk)$ and $C(\Bbbk)$, represent the domain of discourse, the gathering of all $\Bbbk$ subsets, and the collection of all F-sets of $\Bbbk$, respectively. E be the set of attributes. Let collection of all sub intervals of $I = [0, 1]$ be represented by $\mathcal{C}(I)$.

Definition 2.1. [2] A F-set over $\Bbbk$ is described by a membership function f_F, where $f_F : \Bbbk \to I$ is given by

$$f_F(z) = \{(z, f_F(z)) | z \in \Bbbk\},$$

this gives each $z \in \Bbbk$ a real value inside I and $f_F(z)$ is the membership rating of $z \in \Bbbk$.

Definition 2.2. [8] An IVF-set M over $\Bbbk$ is specified by a function $\Im_M : \Bbbk \to \mathcal{C}(I)$ where $\Im_M(z), z \in \Bbbk$ is an interval $[\delta, \sigma], 0 \leq \delta \leq \sigma \leq 1$, δ and σ denote lower and upper membership-rating of an individual respectively.

As an instance, assume the interval-valued fuzzy number (IVF-number) $\tau = [\delta, \sigma]$. The whole collection of IVF-numbers $\tau = [\delta, \sigma]$ over $\Bbbk$ is symbolized by $\Gamma(\Bbbk)$.

Definition 2.3. [3] A soft set S_S over $\Bbbk$ is given by $\digamma_{S_S} : E' \to P(\Bbbk)$ where $E' \subseteq E$ and

$$S_S(\zeta) = \left\{(\zeta, \digamma_{(S,E)(\zeta)}) : \zeta \in E'\right\}.$$

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Definition 2.4. [1] A fuzzy soft set (FS-set) over $\mathbb{U}$ denoted by F_S is described as $\Delta_{F_S} : E' \to C(\mathbb{U})$ and is stated as $F_S(\zeta) = \{(\zeta, \Delta_{F_S}(\zeta)) : \zeta \in E'\}, \Delta_{F_S}(\zeta) \in C(\mathbb{U})$ for $E' \subseteq E$, $\Delta_{F_S} = \varphi$ for $\zeta \notin E'$ and

$$\Delta_{F_{S_{E'}}}(\zeta) = \left\{\zeta_{\Delta_{F_{S_{E'}}}(\zeta)}(\gamma)/\gamma : \gamma \in \mathbb{U}, \zeta_{\Delta_{F_{S_{E'}}}(\zeta)}(\gamma) \in I\right\},$$

for all $\zeta \in E'$ is a F-set over $\mathbb{U}$, where the approximation function of F_S is a F-set $\Delta_{F_S}(z)$ such that, if $\Delta_{F_{S_{E'}}}(\zeta) = \varphi$, then $(\zeta, \Delta_{F_{S_{E'}}}(\zeta)) \notin F_{S_{E'}}$.

Definition 2.5. [7] The interval-valued fuzzy soft set ($IVFS$-set) over $\mathbb{U}$ denoted by $(\tilde{L}_S, E)$ and described by $\tilde{L}_S : E \to P(\mathbb{U})$.
The $IVFS$-set: a parameterized collection of IVF-subsets of $\mathbb{U}$ is stated as

$$\tilde{L}_S(\zeta) = \left\{< \tau, \kappa_{\tilde{L}_S(\zeta)}(\tau) >: \tau \in \mathbb{U}, \zeta \in E\right\},$$

such that $\tilde{L}_S(\zeta)$ denotes IVF membership-grading of $\tau \in \mathbb{U}, \zeta \in E$. $\tilde{L}_S(\zeta)$ reduces to F-set if $\kappa^-_{\tilde{L}_S(\zeta)}(\tau) = \kappa^+_{\tilde{L}_S(\zeta)}(\tau) \forall \tau \in \mathbb{U}, \zeta \in E$.

Definition 2.6. [7] The compliment $(\tilde{L}_S, E)^c = (\tilde{L}^c_S, E)$ of $IVFS$-set $(\tilde{L}_S, E)$ is defined as

$$\tilde{L}^c_S(\zeta) = [1 - \kappa^+_{\tilde{L}_S(\zeta)}(\tau), 1 - \kappa^-_{\tilde{L}_S(\zeta)}(\tau)],$$

for all $\tau \in \mathbb{U}, \zeta \in E$.

Definition 2.7. [19] The entity $(\tilde{H}_S, \eth)$ defined by $\tilde{H}_S : \eth \to P(\mathbb{U})$ is known as hypersoft set (HS-set) over $\mathbb{U}$ where $\eth_1, \eth_2, ..., \eth_n$ are disjoint attribute valued sets corresponding to attributes $\epsilon_1, \epsilon_2, ..., \epsilon_n \in E$ and $\eth = \eth_1 \times \eth_2 \times ... \times \eth_n$ be the Cartesian product of disjoint attribute valued sets.

Example 2.8. Suppose that Mr. Z is interested to purchase a laptop from market. There are five kinds of laptops which form the universal set $L = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5\}$. The attributes which are considered in the selection of laptop are discussed hereafter. $å_1$ = Company, $å_2$ =Processor, $å_3$ =RAM, $å_4$ =Hard drive, $å_5$ =Price range. The attributive valued sets corresponding to these attributes are:
$Å_1$ = { HP, Apple } = { $å_{11}$,$å_{12}$ } , corresponds to attribute $å_1$.
$Å_2$ = { 2.4 GHz, 3.4 GHz } = { $å_{21}$,$å_{22}$ } , corresponds to attribute $å_2$.
$Å_3$ = { 4 GB, 8 GB } = { $å_{31}$,$å_{32}$ } , corresponds to attribute $å_3$.
$Å_4$ = { 320 GB, 1 TB } = { $å_{41}$,$å_{42}$ } , corresponds to attribute $å_4$.
$Å_5$ = { up to 1200 US dollars } = { $å_{51}$ } corresponds to attribute $å_5$.
Å=$Å_1$×$Å_2$×$Å_3$×$Å_4$×$Å_5$
Å= { $\tau_1, \tau_2, \tau_3, ..., \tau_{16}$ } , $each \tau_i : i = 1, 2, ..., 16$ is a five tuple element.
The HS-set $(\tilde{H}_S, \eth)$ is given as

$$(\tilde{H}_S, \eth) = \left\{ \begin{array}{l} (\tau_1, \{\ell_1, \ell_2\}), (\tau_2, \{\ell_1, \ell_3\}), (\tau_3, \{\ell_2, \ell_4\}), (\tau_4, \{\ell_1, \ell_2, \ell_5\}), \\ (\tau_5, \{\ell_2, \ell_4\}), (\tau_6, \{\ell_1, \ell_2, \ell_5\}), (\tau_7, \{\ell_4, \ell_5\}), (\tau_8, \{\ell_1, \ell_2, \ell_4, \ell_5\}), \\ (\tau_9, \{\ell_1, \ell_3, \ell_5\}), (\tau_{10}, \{\ell_4, \ell_5\}), (\tau_{11}, \{\ell_2, \ell_5\}), (\tau_{12}, \{\ell_4, \ell_5\}) \\ (\tau_{13}, \{\ell_2, \ell_3, \ell_4, \ell_5\}), (\tau_{14}, \{\ell_1, \ell_5\}), (\tau_{15}, \{\ell_3, \ell_4, \ell_5\}), (\tau_{16}, \{\ell_1, \ell_2, \ell_3, \ell_4\}) \end{array} \right\}.$$

The element $(\tau_1, \{\ell_1, \ell_2\})$ of HS-set $(\tilde{H}_S, \eth)$ represents that there are two laptops ℓ_1 and ℓ_2 available in market having the following specifications;

1. Company "HP"
2. Processor of 2.4 GHz,
3. Random Access Memory (RAM) of 4 GB,
4. Hard drive of capacity 320 GB,
5. Price range upto 1200 US Dollars.

Similarly, the element $(\tau_{16}, \{\ell_1, \ell_2, \ell_3, \ell_4\})$ of HS-set $(\tilde{H}_S, \eth)$ represents that there are four laptops ℓ_1, ℓ_2, ℓ_3 and ℓ_4 available in market having the following specifications;

1. Company "Apple"
2. Processor of 3.4 GHz,
3. Random Access Memory (RAM) of 8 GB,
4. Hard drive of capacity 1 TB,
5. Price range upto 1200 US Dollars.

Definition 2.9. [19] Let $\eth_1, \eth_2, ..., \eth_n$ be attribute valued sets corresponding to attributes $\epsilon_1, \epsilon_2, ..., \epsilon_n \in \mathfrak{E}$ such that for $p, q = 1, 2, ..., n, p \neq q, \eth_p \cap \eth_q = \varphi$, then $(\tilde{L}_{FHS}, \eth)$, the pair set over $\mathbb{U}$ is said to be fuzzy hypersoft set (FHS-set) and is given by

$$(\tilde{L}_{FHS}, \eth) = \left\{(\varsigma, \tilde{L}_{FHS}(\varsigma)) : \varsigma \in \eth, \tilde{L}_{FHS}(\varsigma) \in C(\mathbb{U})\right\},$$

where $\tilde{L}_{HS} : \eth \to C(\mathbb{U})$ and $C(\mathbb{U})$ be collection of all F-sets over $\mathbb{U}$ with $\varsigma \in \eth = \eth_1 \times \eth_2 \times ... \times \eth_n$ and, $\tilde{L}(\varsigma) = \left\{\kappa_{\tilde{L}(\varsigma)}(z)/z : z \in \mathbb{U}, \kappa_{\tilde{L}(\varsigma)} \in I\right\}$ is a F-set over $\mathbb{U}$.

Example 2.10. Consider the Example 2.8. The FHS-set $(\tilde{L}_{FHS}, \eth)$ over $\mathbb{U}$ is given as $(\tilde{L}_{FHS}, \eth) =$

$$\left\{\begin{array}{l} \left(\tau_1, \left\{{}^{0.1}/_{\ell_1}, {}^{0.5}/_{\ell_2}\right\}\right), \left(\tau_2, \left\{{}^{0.2}/_{\ell_1}, {}^{0.7}/_{\ell_3}\right\}\right), \left(\tau_3, \left\{{}^{0.3}/_{\ell_2}, {}^{0.5}/_{\ell_4}\right\}\right), \left(\tau_4, \left\{{}^{0.7}/_{\ell_1}, {}^{0.3}/_{\ell_2}, {}^{0.2}/_{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{{}^{0.1}/_{\ell_2}, {}^{0.5}/_{\ell_4}\right\}\right), \left(\tau_6, \left\{{}^{0.2}/_{\ell_1}, {}^{0.6}/_{\ell_2}, {}^{0.3}/_{\ell_5}\right\}\right), \left(\tau_7, \left\{{}^{0.2}/_{\ell_4}, {}^{0.1}/_{\ell_5}\right\}\right), \\ \left(\tau_8, \left\{{}^{0.1}/_{\ell_1}, {}^{0.2}/_{\ell_2}, {}^{0.4}/_{\ell_4}, {}^{0.6}/_{\ell_5}\right\}\right), \left(\tau_9, \left\{{}^{0.3}/_{\ell_1}, {}^{0.1}/_{\ell_3}, {}^{0.3}/_{\ell_5}\right\}\right), \left(\tau_{10}, \left\{{}^{0.5}/_{\ell_4}, {}^{0.2}/_{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{{}^{0.3}/_{\ell_2}, {}^{0.6}/_{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{}^{0.4}/_{\ell_4}, {}^{0.7}/_{\ell_5}\right\}\right), \left(\tau_{13}, \left\{{}^{0.3}/_{\ell_2}, {}^{0.1}/_{\ell_3}, {}^{0.6}/_{\ell_4}, {}^{0.4}/_{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{{}^{0.4}/_{\ell_1}, {}^{0.2}/_{\ell_5}\right\}\right), \left(\tau_{15}, \left\{{}^{0.1}/_{\ell_3}, {}^{0.6}/_{\ell_4}, {}^{0.4}/_{\ell_5}\right\}\right), \left(\tau_{16}, \left\{{}^{0.5}/_{\ell_1}, {}^{0.2}/_{\ell_2}, {}^{0.3}/_{\ell_3}, {}^{0.7}/_{\ell_4}\right\}\right) \end{array}\right\}.$$

The element $\left(\tau_1, \left\{{}^{0.1}/_{\ell_1}, {}^{0.5}/_{\ell_2}\right\}\right)$ of FHS-set $(\tilde{H}_S, \eth)$ represents that there are two laptops ℓ_1 and ℓ_2 available in market having the specifications as described in Example 2.8. The availability of ℓ_1 is 0.1 or 10 % and availability of ℓ_2 is 0.5 or 50 %.

3 Interval-valued fuzzy hypersoft set ($IVFHS$-set)

In this section, $IVFHS$-set as well as some of its basic notions are illustrated with suitable examples.

Definition 3.1. [20] Let $\eth_1, \eth_2, ..., \eth_n$ are attribute valued sets corresponding to attributes $\epsilon_1, \epsilon_2, ..., \epsilon_n \in \mathfrak{E}$, then $(\tilde{L}_{IVFHS}, \eth)$, the pair set over $\mathbb{U}$ is said to be $IVFHS$-set over $\mathbb{U}$ and is defined as follow:

$$(i\tilde{L}_{IVFHS}, \eth) = \left\{(\varsigma, \tilde{L}_{IVFHS}(\varsigma)) : \varsigma \in \eth, \tilde{L}_{IVFHS}(\varsigma) \in C(\mathbb{U})\right\},$$

such that $\tilde{L}_{IVFHS} : \eth \to C(\mathbb{U})$, $C(\mathbb{U})$ denotes the family of all IVF-sets over $\mathbb{U}$, $\varsigma \in \eth = \eth_1 \times \eth_2 \times ... \times \eth_n$, and $\tilde{L}(\varsigma) = \left\{\kappa_{\tilde{L}'(\varsigma)}(z)/z : z \in \mathbb{U}, \kappa_{\tilde{L}'(\varsigma)} \in \mathcal{C}(I)\right\}$ is IVF-set over $\mathbb{U}$.

Example 3.2. Consider the Example 2.8. The $IVFHS$-set $(\tilde{L}_{IVFHS}, \eth)$ over $\mathbb{U}$ is given as

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$(\tilde{L}_{IVFHS}, \eth) =$

$$\left\{\begin{array}{l} \left(\tau_1, \left\{{[0.1,0.2]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}\right\}\right), \left(\tau_2, \left\{{[0.2,0.4]}/{\ell_1}, {[0.3,0.7]}/{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{{[0.3,0.4]}/{\ell_2}, {[0.2,0.5]}/{\ell_4}\right\}\right), \left(\tau_4, \left\{{[0.4,0.7]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}, {[0.6,0.8]}/{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{{[0.1,0.3]}/{\ell_2}, {[0.2,0.5]}/{\ell_4}\right\}\right), \left(\tau_6, \left\{{[0.2,0.6]}/{\ell_1}, {[0.4,0.6]}/{\ell_2}, {[0.5,0.6]}/{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{{[0.2,0.4]}/{\ell_4}, {[0.6,0.7]}/{\ell_5}\right\}\right), \left(\tau_8, \left\{{[0.2,0.4]}/{\ell_1}, {[0.2,0.3]}/{\ell_2}, {[0.4,0.5]}/{\ell_4}, {[0.3,0.6]}/{\ell_5}\right\}\right), \\ \left(\tau_9, \left\{{[0.2,0.3]}/{\ell_1}, {[0.1,0.5]}/{\ell_3}, {[0.3,0.7]}/{\ell_5}\right\}\right), \left(\tau_{10}, \left\{{[0.5,0.6]}/{\ell_4}, {[0.2,0.4]}/{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{{[0.3,0.5]}/{\ell_2}, {[0.4,0.6]}/{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{[0.2,0.4]}/{\ell_4}, {[0.5,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{{[0.3,0.6]}/{\ell_2}, {[0.1,0.4]}/{\ell_3}, {[0.2,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{{[0.4,0.5]}/{\ell_1}, {[0.2,0.3]}/{\ell_5}\right\}\right), \left(\tau_{15}, \left\{{[0.1,0.4]}/{\ell_3}, {[0.6,0.8]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{{[0.5,0.7]}/{\ell_1}, {[0.2,0.4]}/{\ell_2}, {[0.3,0.6]}/{\ell_3}, {[0.4,0.8]}/{\ell_4}\right\}\right) \end{array}\right\}.$$

The element $\left(\tau_1, \left\{{[0.1,0.2]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}\right\}\right)$ of $IVFHS$-set $(IV\tilde{F}HS_S, \eth)$ represents that there are two laptops ℓ_1 and ℓ_2 available in market having the specifications as described in Example 2.8. The availability of ℓ_1 is 0.1 to 0.2 i.e. 10 % to 20% which means all lower values are greater than or equal to 10 % and all upper values are less than or equal to 20 %. Similarly the availability of ℓ_2 is 0.3 to 0.5 i.e. 30 % to 50% which means all lower values are greater than or equal to 30 % and all upper values are less than or equal to 50 %.

Definition 3.3. Let $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$, are two $IVFHS$-sets over $\Bbbk$ written as $(\tilde{L'}_{H_1}, \eth_1), (\tilde{L'}_{H_2}, \eth_2) \in \c{C}_{(\tilde{L'}_H, \eth)}$(the collection of $IVFHS$-sets). $(\tilde{L'}_{H_1}, \eth_1)$ is known as $IVFHS$-subset of $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \subseteq (\tilde{L'}_{H_2}, \eth_2)$ if

1. $\eth_1 \subseteq \eth_1$
2. $\forall\, ł \in \eth_1, \tilde{L'}_{H_1}(ł) \subseteq \tilde{L'}_{H_2}(ł)$.

Example 3.4. Consider the Example 2.8.
If $(\tilde{L'}_{H_1}, \eth_1) =$

$$\left\{\left(\tau_1, \left\{{[0.1,0.2]}/{\ell_1}\right\}\right), \left(\tau_2, \left\{{[0.4,0.5]}/{\ell_3}\right\}\right), \left(\tau_4, \left\{{[0.4,0.6]}/{\ell_1}, {[0.3,0.4]}/{\ell_2}\right\}\right)\right\}$$

and $(\tilde{L'}_{H_2}, \eth_2) =$

$$\left\{\begin{array}{l} \left(\tau_1, \left\{{[0.1,0.4]}/{\ell_1}, {[0.3,o.5]}/{\ell_2}\right\}\right), \left(\tau_2, \left\{{[0.2,0.4]}/{\ell_1}, {[0.3,0.7]}/{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{{[0.3,0.4]}/{\ell_2}, {[0.2,0.5]}/{\ell_4}\right\}\right), \left(\tau_4, \left\{{[0.4,0.7]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}, {[0.1,0.2]}/{\ell_5}\right\}\right) \end{array}\right\}$$

then $(\tilde{L'}_{H_1}, \eth_1) \subseteq (\tilde{L'}_{H_2}, \eth_2)$

Definition 3.5. A set $N = N_1 \times N_2 \times ... \times N_n$ in $IVFHS$-set $(\tilde{L'}_H, \eth)$ is known as $Not\,set$ if it has a representation as

$$\propto N = \{\propto \tau_1, \propto \tau_2, ..., \propto \tau_h\}$$

where $h = \prod\limits_{p=1}^{n} |N_p|$ and each $\propto \tau_p, p \in \{1, 2, ..., h\}$ is a Not n-tuple element.

Example 3.6. Consider the Example 2.8. Take sets N_1, N_2, N_3, N_4, N_5, we have

$$\propto N = \{\propto \tau_1, \propto \tau_2, ..., \propto \tau_{16}\}$$

where each $\propto \tau_p, p \in \{1, 2, ..., 16\}$ is a $Not\,set$ 5-tuple element.

Definition 3.7. An $IVFHS$-set $(\tilde{L'}_H, \eth_1)$ is relative null $IVFHS$-set w.r.t $\eth$, represented as $(\tilde{L'}_H, \eth_1)_\Phi$, if $\tilde{L'}_H(\tau) = \varphi \forall \tau \in \eth_1$.

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Example 3.8. Consider the Example 2.8, if

$$(\tilde{L'}_H, \eth_1)_\Phi = \{(\tau_1, \varphi), (\tau_2, \varphi), (\tau_3, \varphi)\}$$

where $\eth_1 \in \eth$.

Definition 3.9. An $IVFHS$-set $(\tilde{L'}_H, \eth_1)$ is relative whole $IVFHS$-set w.r.t $\eth_1 \subseteq E$, represented as $(\tilde{L'}_H, \eth_1)_{\mathfrak{Z}}$, if $\tilde{L'}_H(\tau) = \mathbb{U} \, \forall \tau \in \eth_1$.

Example 3.10. Consider the Example 2.8, if

$$(\tilde{L'}_H, \eth_1)_{\mathbb{U}} = \{(\tau_1, \mathbb{U}), (\tau_2, \mathbb{U}), (\tau_3, \mathbb{U})\}$$

where $\eth_1 \in \eth$.

Definition 3.11. An $IVFHS$-set $(\tilde{L'}_H, \eth)$ is absolute whole $IVFHS$-set over E, represented as $(\tilde{L'}_H, \eth)_{\mathfrak{Z}}$, if $\tilde{L'}_H(\tau) = \mathbb{U} \, \forall \tau \in \eth$.

Example 3.12. Consider the Example 2.8, if

$$(\tilde{L'}_H, \eth)_{\mathbb{U}} = \left\{ \begin{array}{l} (\tau_1, Z), (\tau_2, Z), (\tau_3, Z), (\tau_4, Z), \\ (\tau_5, Z), (\tau_6, Z), (\tau_7, Z), (\tau_8, Z), \\ (\tau_9, Z), (\tau_{10}, Z), (\tau_{11}, Z), (\tau_{12}, Z), \\ (\tau_{13}, Z), (\tau_{14}, Z), (\tau_{15}, Z), (\tau_{16}, Z) \end{array} \right\}.$$

Proposition 3.13. *$(\tilde{L'}_{H_1}, \eth_1), (\tilde{L'}_{H_2}, \eth_2), (\tilde{L'}_{H_3}, \eth_3) \in \varsigma_{(\tilde{L'}_H, \eth)}$ with $\eth_1, \eth_2, \eth_3 \subseteq \eth$, then*

1. $(\tilde{L'}_{H_1}, \eth_1) \subseteq (\tilde{L'}_{H_1}, \eth_1)_{\mathbb{U}}$
2. $(\tilde{L'}_{H_1}, \eth_1)_\Phi \subseteq (\tilde{L'}_{H_1}, \eth_1)$
3. $(\tilde{L'}_{H_1}, \eth_1) \subseteq (\tilde{L'}_{H_1}, \eth_1)$
4. $(\tilde{L'}_{H_1}, \eth_1) \subseteq (\tilde{L'}_{H_2}, \eth_2)$ *and* $(\tilde{L'}_{H_2}, \eth_2) \subseteq (\tilde{L'}_{H_3}, \eth_3)$*, then* $(\tilde{L'}_{H_1}, \eth_1) \subseteq (\tilde{L'}_{H_3}, \eth_3)$
5. $(\tilde{L'}_{H_1}, \eth_1) = (\tilde{L'}_{H_2}, \eth_2)$ *and* $(\tilde{L'}_{H_2}, \eth_2) = (\tilde{L'}_{H_3}, \eth_3)$*, then* $(\tilde{L'}_{H_1}, \eth_1) = (\tilde{L'}_{H_3}, \eth_3)$

Definition 3.14. The compliment $(\tilde{L}_{IVFHS}, \eth)^c$ of $IVFHS$-set $(\tilde{L}_{IVFHS}, \eth)$ is given by $(\tilde{L}_{IVFHS}, \eth)^c = (\tilde{L}^c_{IVFHS}, \eth)$ and is defined as

$$\tilde{L}^c_{IVFHS}(\theta) = [1 - \kappa^+_{\tilde{L}_{IVFHS(\theta)}}(\tau), 1 - \kappa^-_{\tilde{L}_{IVFHS(\theta)}}(\tau)],$$

for all $\tau \in \mathbb{U}, \theta \in \eth$.

Example 3.15. Consider the Example 2.8. The compliment $(\tilde{L}_{IVFHS}, \eth)^c$ of $IVFHS$-set $(\tilde{L}_{IVFHS}, \eth)$ over $\mathbb{U}$ is given as
$(\tilde{L}_{IVFHS}, \eth)^c =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{[0.8, 0.9]/_{\ell_1}, [0.5, 0.7]/_{\ell_2}\right\}\right), \left(\tau_2, \left\{[0.6, 0.8]/_{\ell_1}, [0.3, 0.7]/_{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{[0.6, 0.7]/_{\ell_2}, [0.5, 0.8]/_{\ell_4}\right\}\right), \left(\tau_4, \left\{[0.3, 0.6]/_{\ell_1}, [0.5, 0.7]/_{\ell_2}, [0.8, 0.9]/_{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{[0.7, 0.9]/_{\ell_2}, [0.5, 0.8]/_{\ell_4}\right\}\right), \left(\tau_6, \left\{[0.4, 0.8]/_{\ell_1}, [0.4, 0.6]/_{\ell_2}, [0.5, 0.7]/_{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{[0.6, 0.8]/_{\ell_4}, [0.3, 0.4]/_{\ell_5}\right\}\right), \left(\tau_8, \left\{[0.6, 0.9]/_{\ell_1}, [0.7, 0.8]/_{\ell_2}, [0.5, 0.6]/_{\ell_4}, [0.4, 0.7]/_{\ell_5}\right\}\right), \\ \left(\tau_9, \left\{[0.7, 0.8]/_{\ell_1}, [0.5, 0.9]/_{\ell_3}, [0.3, 0.7]/_{\ell_5}\right\}\right), \left(\tau_{10}, \left\{[0.4, 0.5]/_{\ell_4}, [0.6, 0.8]/_{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{[0.5, 0.7]/_{\ell_2}, [0.4, 0.6]/_{\ell_5}\right\}\right), \left(\tau_{12}, \left\{[0.6, 0.8]/_{\ell_4}, [0.3, 0.5]/_{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{[0.4, 0.7]/_{\ell_2}, [0.6, 0.9]/_{\ell_3}, [0.4, 0.8]/_{\ell_4}, [0.3, 0.6]/_{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{[0.5, 0.6]/_{\ell_1}, [0.7, 0.8]/_{\ell_5}\right\}\right), \left(\tau_{15}, \left\{[0.6, 0.9]/_{\ell_3}, [0.2, 0.4]/_{\ell_4}, [0.3, 0.6]/_{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{[0.3, 0.5]/_{\ell_1}, [0.6, 0.8]/_{\ell_2}, [0.4, 0.7]/_{\ell_3}, [0.3, 0.6]/_{\ell_4}\right\}\right) \end{array} \right\}.$$

Proposition 3.16. *$(\tilde{L'}_H, \eth) \in \varsigma_{(\tilde{L'}_H, \eth)}$, then*

1. $((\tilde{L}_{IVFHS}, \eth)^c)^c = (\tilde{L}_{IVFHS}, \eth)$
2. $((\tilde{L}_{IVFHS}, \eth)_\Phi)^c = (\tilde{L}_{IVFHS}, \eth)_{\mathbb{U}}$
3. $((\tilde{L}_{IVFHS}, \eth)_{\mathbb{U}})^c = (\tilde{L}_{IVFHS}, \eth)_\Phi$

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4 Set Theoretic operations on $IVFHS$-set

The purpose of this portion of the study is to describe some new $IVFHS-set$ operations.

Definition 4.1. The *union of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \sqcup (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set $(\tilde{L'}_{H_3}, \eth_3)$ with $\eth_3 = \eth_1 \cup \eth_2$ and is defined as

$$\tilde{L'}_{H_3}(\tau) = \left\{ \begin{array}{l} \tilde{L'}_{H_1}(\tau) : \tau \in \eth_1 \backslash \eth_2 \\ \tilde{L'}_{H_2}(\tau) : \tau \in \eth_2 \backslash \eth_1 \\ \tilde{L'}_{H_1}(\tau) \cup \tilde{L'}_{H_2}(\tau) : \tau \in \eth_1 \cap \eth_2 \end{array} \right\},$$

where $\tilde{L'}_{H_1}(\tau) \cup \tilde{L'}_{H_2}(\tau) =$
$\left\{ \left(\tau, \left[\max\left(\kappa_{l\tilde{L'}_{H_1}(\tau)}(z), \kappa_{l\tilde{L'}_{H_2}(\tau)}(z) \right), \max\left(\kappa_{u\tilde{L'}_{H_1}(\tau)}(z), \kappa_{u\tilde{L'}_{H_2}(\tau)}(z) \right) \right] \right) \forall z \in \mathcal{Z} \right\}.$

Example 4.2. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2) \in Ç_{(\tilde{L'}_H, \eth)}$ with $\eth_1, \eth_2 \subseteq \eth$ as in Example 2.8 such that
$(\tilde{L'}_{H_1}, \eth_1) =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{{[0.5,0.6]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}\right\}\right), \left(\tau_4, \left\{{[0.4,0.5]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}, {[0.1,0.4]}/{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{{[0.1,0.3]}/{\ell_2}, {[0.2,0.3]}/{\ell_4}\right\}\right), \left(\tau_6, \left\{{[0.2,0.6]}/{\ell_1}, {[0.4,0.8]}/{\ell_2}, {[0.3,0.5]}/{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{{[0.3,0.4]}/{\ell_4}, {[0.6,0.7]}/{\ell_5}\right\}\right), \left(\tau_9, \left\{{[0.2,0.3]}/{\ell_1}, {[0.4,0.5]}/{\ell_3}, {[0.3,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{{[0.3,0.6]}/{\ell_2}, {[0.4,0.6]}/{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{[0.2,0.4]}/{\ell_4}, {[0.6,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{{[0.3,0.6]}/{\ell_2}, {[0.3,0.4]}/{\ell_3}, {[0.2,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{{[0.4,0.5]}/{\ell_1}, {[0.2,0.5]}/{\ell_5}\right\}\right), \left(\tau_{15}, \left\{{[0.1,0.4]}/{\ell_3}, {[0.6,0.8]}/{\ell_4}, {[0.5,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{{[0.5,0.7]}/{\ell_1}, {[0.2,0.4]}/{\ell_2}, {[0.4,0.6]}/{\ell_3}, {[0.4,0.7]}/{\ell_4}\right\}\right) \end{array} \right\}$$

$(\tilde{L'}_{H_2}, \eth_2) =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{{[0.3,0.5]}/{\ell_1}, {[0.1,0.3]}/{\ell_2}\right\}\right), \left(\tau_2, \left\{{[0.6,0.7]}/{\ell_1}, {[0.2,0.4]}/{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{{[0.3,0.4]}/{\ell_2}, {[0.3,0.6]}/{\ell_4}\right\}\right), \left(\tau_4, \left\{{[0.1,0.2]}/{\ell_1}, {[0.4,0.5]}/{\ell_2}, {[0.6,0.8]}/{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{{[0.3,0.4]}/{\ell_2}, {[0.3,0.7]}/{\ell_4}\right\}\right), \left(\tau_6, \left\{{[0.4,0.7]}/{\ell_1}, {[0.2,0.3]}/{\ell_2}, {[0.2,0.6]}/{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{{[0.7,0.8]}/{\ell_4}, {[0.3,0.5]}/{\ell_5}\right\}\right), \left(\tau_9, \left\{{[0.6,0.9]}/{\ell_1}, {[0.4,0.5]}/{\ell_3}, {[0.5,0.8]}/{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{{[0.4,0.5]}/{\ell_2}, {[0.1,0.3]}/{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{[0.4,0.7]}/{\ell_4}, {[0.2,0.5]}/{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{{[0.4,0.6]}/{\ell_2}, {[0.3,0.5]}/{\ell_3}, {[0.4,0.5]}/{\ell_4}, {[0.2,0.3]}/{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{{[0.7,0.8]}/{\ell_1}, {[0.4,0.6]}/{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{{[0.1,0.3]}/{\ell_1}, {[0.2,0.4]}/{\ell_2}, {[0.4,0.7]}/{\ell_3}, {[0.3,0.7]}/{\ell_4}\right\}\right) \end{array} \right\}$$

Then $(\tilde{L'}_{H_1}, \eth_1) \sqcup (\tilde{L'}_{H_2}, \eth_2) =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{{[0.5,0.6]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}\right\}\right), \left(\tau_2, \left\{{[0.6,0.7]}/{\ell_1}, {[0.2,0.4]}/{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{{[0.3,0.4]}/{\ell_2}, {[0.3,0.6]}/{\ell_4}\right\}\right), \left(\tau_4, \left\{{[0.4,0.5]}/{\ell_1}, {[0.4,0.5]}/{\ell_2}, {[0.6,0.8]}/{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{{[0.3,0.4]}/{\ell_2}, {[0.3,0.7]}/{\ell_4}\right\}\right), \left(\tau_6, \left\{{[0.4,0.7]}/{\ell_1}, {[0.4,0.8]}/{\ell_2}, {[0.3,0.6]}/{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{{[0.7,0.8]}/{\ell_4}, {[0.6,0.7]}/{\ell_5}\right\}\right), \left(\tau_9, \left\{{[0.6,0.9]}/{\ell_1}, {[0.4,0.5]}/{\ell_3}, {[0.5,0.8]}/{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{{[0.4,0.6]}/{\ell_2}, {[0.4,0.6]}/{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{[0.4,0.7]}/{\ell_4}, {[0.6,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{{[0.4,0.6]}/{\ell_2}, {[0.3,0.5]}/{\ell_3}, {[0.4,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{{[0.7,0.8]}/{\ell_1}, {[0.4,0.6]}/{\ell_5}\right\}\right), \left(\tau_{15}, \left\{{[0.1,0.4]}/{\ell_3}, {[0.6,0.8]}/{\ell_4}, {[0.5,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{{[0.5,0.7]}/{\ell_1}, {[0.2,0.4]}/{\ell_2}, {[0.4,0.7]}/{\ell_3}, {[0.4,0.7]}/{\ell_4}\right\}\right) \end{array} \right\}.$$

Definition 4.3. The *restricted union of two $IVFHS$-sets* $(\tilde{L}'_{H_1}, ð_1)$ and $(\tilde{L}'_{H_2}, ð_2)$ denoted by $(\tilde{L}'_{H_1}, ð_1) \sqcup_R (\tilde{L}'_{H_2}, ð_2)$ is an $IVFHS$-set $(\tilde{L}'_{H_3}, ð_3)$ with $ð_3 = ð_1 \cap ð_2$ and is defined as

$$\tilde{L}'_{H_3}(\tau) = \left\{ \tilde{L}'_{H_1}(\tau) \cup \tilde{L}'_{H_2}(\tau) : \tau \in ð_1 \cap ð_2 \right\},$$

where $\tilde{L}'_{H_1}(\tau) \cup \tilde{L}'_{H_2}(\tau) =$
$\left\{ \left(\tau, \left[\max \left(\kappa_{l\tilde{L}'_{H_1}(\tau)}(z), \kappa_{l\tilde{L}'_{H_2}(\tau)}(z) \right), \max \left(\kappa_{u\tilde{L}'_{H_1}(\tau)}(z), \kappa_{u\tilde{L}'_{H_2}(\tau)}(z) \right) \right] \right) \forall z \in \mathcal{Z} \right\}.$

Example 4.4. Consider two $IVFHS$-sets $(\tilde{L}'_{H_1}, ð_1)$ and $(\tilde{L}'_{H_2}, ð_2) \in Ç_{(\tilde{L}'_H, ð)}$ with $ð_1, ð_2 \subseteq ð$ as in Example 4.2. Then $(\tilde{L}'_{H_3}, ð_3) =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{ {[0.5, 0.6]}/{\ell_1}, {[0.3, 0.5]}/{\ell_2} \right\}\right), \left(\tau_4, \left\{ {[0.4, 0.5]}/{\ell_1}, {[0.4, 0.5]}/{\ell_2}, {[0.6, 0.8]}/{\ell_5} \right\}\right), \\ \left(\tau_5, \left\{ {[0.3, 0.4]}/{\ell_2}, {[0.3, 0.7]}/{\ell_4} \right\}\right), \left(\tau_6, \left\{ {[0.4, 0.7]}/{\ell_1}, {[0.4, 0.8]}/{\ell_2}, {[0.3, 0.6]}/{\ell_5} \right\}\right), \\ \left(\tau_7, \left\{ {[0.7, 0.8]}/{\ell_4}, {[0.6, 0.7]}/{\ell_5} \right\}\right), \left(\tau_9, \left\{ {[0.6, 0.9]}/{\ell_1}, {[0.4, 0.5]}/{\ell_3}, {[0.5, 0.8]}/{\ell_5} \right\}\right), \\ \left(\tau_{11}, \left\{ {[0.4, 0.6]}/{\ell_2}, {[0.4, 0.6]}/{\ell_5} \right\}\right), \left(\tau_{12}, \left\{ {[0.4, 0.7]}/{\ell_4}, {[0.6, 0.7]}/{\ell_5} \right\}\right), \\ \left(\tau_{13}, \left\{ {[0.4, 0.6]}/{\ell_2}, {[0.3, 0.5]}/{\ell_3}, {[0.4, 0.6]}/{\ell_4}, {[0.4, 0.7]}/{\ell_5} \right\}\right), \\ \left(\tau_{14}, \left\{ {[0.7, 0.8]}/{\ell_1}, {[0.4, 0.6]}/{\ell_5} \right\}\right), \\ \left(\tau_{16}, \left\{ {[0.5, 0.7]}/{\ell_1}, {[0.2, 0.4]}/{\ell_2}, {[0.4, 0.7]}/{\ell_3}, {[0.4, 0.7]}/{\ell_4} \right\}\right) \end{array} \right\}$$

Definition 4.5. The *intersection of two $IVFHS$-sets* $(\tilde{L}'_{H_1}, ð_1)$ and $(\tilde{L}'_{H_2}, ð_2)$ denoted by $(\tilde{L}'_{H_1}, ð_1) \sqcap (\tilde{L}'_{H_2}, ð_2)$ is an $IVFHS$-set $(\tilde{L}'_{H_3}, ð_3)$ with $ð_3 = ð_1 \cap ð_2$ and is defined as

$$\tilde{L}'_{H_3}(\tau) = \left\{ \tilde{L}'_{H_1}(\tau) \cap \tilde{L}'_{H_2}(\tau) : \tau \in ð_1 \cap ð_2 \right\},$$

where $\tilde{L}'_{H_1}(\tau) \cap \tilde{L}'_{H_2}(\tau) =$
$\left\{ \left(\tau, \left[\min \left(\kappa_{lð_1(\tau)}(z), \kappa_{lð_2(\tau)}(z) \right), \min \left(\kappa_{uð_1(\tau)}(z), \kappa_{uð_2(\tau)}(z) \right) \right] \right) \forall z \in \mathcal{Z} \right\}.$

Example 4.6. Consider two $IVFHS$-sets $(\tilde{L}'_{H_1}, ð_1)$ and $(\tilde{L}'_{H_2}, ð_2) \in Ç_{(\tilde{L}'_H, ð)}$ with $ð_1, ð_2 \subseteq ð$ as in Example 4.2. Then $(\tilde{L}'_{H_1}, ð_1) \sqcap (\tilde{L}'_{H_2}, ð_2) =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{ {[0.3, 0.5]}/{\ell_1}, {[0.1, 0.3]}/{\ell_2} \right\}\right), \left(\tau_4, \left\{ {[0.1, 0.2]}/{\ell_1}, {[0.3, 0.5]}/{\ell_2}, {[0.1, 0.4]}/{\ell_5} \right\}\right), \\ \left(\tau_5, \left\{ {[0.1, 0.3]}/{\ell_2}, {[0.2, 0.3]}/{\ell_4} \right\}\right), \left(\tau_6, \left\{ {[0.2, 0.6]}/{\ell_1}, {[0.2, 0.3]}/{\ell_2}, {[0.2, 0.5]}/{\ell_5} \right\}\right), \\ \left(\tau_7, \left\{ {[0.3, 0.4]}/{\ell_4}, {[0.3, 0.5]}/{\ell_5} \right\}\right), \left(\tau_9, \left\{ {[0.2, 0.3]}/{\ell_1}, {[0.4, 0.5]}/{\ell_3}, {[0.3, 0.7]}/{\ell_5} \right\}\right), \\ \left(\tau_{11}, \left\{ {[0.3, 0.5]}/{\ell_2}, {[0.1, 0.3]}/{\ell_5} \right\}\right), \left(\tau_{12}, \left\{ {[0.2, 0.4]}/{\ell_4}, {[0.2, 0.5]}/{\ell_5} \right\}\right), \\ \left(\tau_{13}, \left\{ {[0.3, 0.6]}/{\ell_2}, {[0.3, 0.4]}/{\ell_3}, {[0.2, 0.5]}/{\ell_4}, {[0.2, 0.3]}/{\ell_5} \right\}\right), \\ \left(\tau_{14}, \left\{ {[0.4, 0.5]}/{\ell_1}, {[0.2, 0.5]}/{\ell_5} \right\}\right), \\ \left(\tau_{16}, \left\{ {[0.1, 0.3]}/{\ell_1}, {[0.2, 0.4]}/{\ell_2}, {[0.4, 0.6]}/{\ell_3}, {[0.3, 0.7]}/{\ell_4} \right\}\right) \end{array} \right\}$$

Definition 4.7. The *extended intersection of two $IVFHS$-sets* $(\tilde{L}'_{H_1}, ð_1)$ and $(\tilde{L}'_{H_2}, ð_2)$ denoted by $(\tilde{L}'_{H_1}, ð_1) \sqcap_E (\tilde{L}'_{H_2}, ð_2)$ is an $IVFHS$-set $(\tilde{L}'_{H_3}, ð_3)$ with $ð_3 = ð_1 \cup ð_1$ and is defined as

$$\tilde{L}'_{H_3}(\tau) = \left\{ \begin{array}{l} \tilde{L}'_{H_1}(\tau) : \tau \in ð_1 \backslash ð_2 \\ \tilde{L}'_{H_2}(\tau) : \tau \in ð_2 \backslash ð_1 \\ \tilde{L}'_{H_1}(\tau) \cap \tilde{L}'_{H_2}(\tau) : \tau \in ð_1 \cap ð_2 \end{array} \right\},$$

where $\tilde{L}'_{H_1}(\tau) \cap \tilde{L}'_{H_2}(\tau) =$
$\left\{ \left(\tau, \left[\min \left(\kappa_{lð_1(\tau)}(z), \kappa_{lð_2(\tau)}(z) \right), \min \left(\kappa_{uð_1(\tau)}(z), \kappa_{uð_2(\tau)}(z) \right) \right] \right) \forall z \in \mathcal{Z} \right\}.$

Example 4.8. Consider two $IVFHS$-sets $(\tilde{L}'_{H_1}, ð_1)$ and $(\tilde{L}'_{H_2}, ð_2) \in Ç_{(\tilde{L}'_H, ð)}$ with $ð_1, ð_2 \subseteq ð$ as in

Example 4.2. Then $(\tilde{L'}_{H_1}, \eth_1) \sqcap_E (\tilde{L'}_{H_2}, \eth_2) =$

$$\left\{\begin{array}{l} \left(\tau_1, \left\{{[0.3,0.5]}/{\ell_1}, {[0.1,0.3]}/{\ell_2}\right\}\right), \left(\tau_2, \left\{{[0.6,0.7]}/{\ell_1}, {[0.2,0.4]}/{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{{[0.3,0.4]}/{\ell_2}, {[0.3,0.6]}/{\ell_4}\right\}\right), \left(\tau_4, \left\{{[0.1,0.2]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}, {[0.1,0.4]}/{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{{[0.1,0.3]}/{\ell_2}, {[0.2,0.3]}/{\ell_4}\right\}\right), \left(\tau_6, \left\{{[0.2,0.6]}/{\ell_1}, {[0.2,0.3]}/{\ell_2}, {[0.2,0.5]}/{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{{[0.3,0.4]}/{\ell_4}, {[0.3,0.5]}/{\ell_5}\right\}\right), \left(\tau_9, \left\{{[0.2,0.3]}/{\ell_1}, {[0.4,0.5]}/{\ell_3}, {[0.3,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{{[0.3,0.5]}/{\ell_2}, {[0.1,0.3]}/{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{[0.2,0.4]}/{\ell_4}, {[0.2,0.5]}/{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{{[0.3,0.6]}/{\ell_2}, {[0.3,0.4]}/{\ell_3}, {[0.2,0.5]}/{\ell_4}, {[0.2,0.3]}/{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{{[0.4,0.5]}/{\ell_1}, {[0.2,0.5]}/{\ell_5}\right\}\right), \left(\tau_{15}, \left\{{[0.1,0.4]}/{\ell_3}, {[0.6,0.8]}/{\ell_4}, {[0.5,0.7]}/{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{{[0.1,0.3]}/{\ell_1}, {[0.2,0.4]}/{\ell_2}, {[0.4,0.6]}/{\ell_3}, {[0.3,0.7]}/{\ell_4}\right\}\right) \end{array}\right\}.$$

Definition 4.9. The *addition of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2) \in Ç_{(\tilde{L'}_H, \eth)}$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \boxplus (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set with $\eth_3 = \eth_1 \cup \eth_2$ and is defined as

$$\tilde{L'}_{H_3}(\tau) = \left\{\begin{array}{l} \tilde{L'}_{H_1}(\tau) : \tau \in \eth_1 \backslash \eth_2 \\ \tilde{L'}_{H_2}(\tau) : \tau \in \eth_2 \backslash \eth_1 \\ \tilde{L'}_{H_1}(\tau) \oplus \tilde{L'}_{H_2}(\tau) : \tau \in \eth_1 \cap \eth_2 \end{array}\right\},$$

where $\tilde{L'}_{H_1}(\tau) \oplus \tilde{L'}_{H_2}(\tau) =$
$$\left\{\left(\tau, \left[\begin{array}{l} \kappa_{l\tilde{L'}_{H_1}(\tau)}(z) + \kappa_{l\tilde{L'}_{H_2}(\tau)}(z) - \kappa_{l\tilde{L'}_{H_1}(\tau)}(z)\kappa_{l\tilde{L'}_{H_2}(\tau)}(z), \\ \kappa_{u\tilde{L'}_{H_1}(\tau)}(z) + \kappa_{u\tilde{L'}_{H_2}(\tau)}(z) - \kappa_{u\tilde{L'}_{H_1}(\tau)}(z)\kappa_{u\tilde{L'}_{H_2}(\tau)}(z) \end{array}\right]\right) \forall z \in \mathcal{Z}\right\}.$$

Example 4.10. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2) \in Ç_{(\tilde{L'}_H, \eth)}$ with $\eth_1, \eth_2 \subseteq \eth$ as in Example 4.2. Then $(\tilde{L'}_{H_1}, \eth_1) \boxplus (\tilde{L'}_{H_2}, \eth_2) =$

$$\left\{\begin{array}{l} \left(\tau_1, \left\{{[0.65,0.80]}/{\ell_1}, {[0.37,0.65]}/{\ell_2}\right\}\right), \left(\tau_2, \left\{{[0.60,0.70]}/{\ell_1}, {[0.20,0.40]}/{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{{[0.30,0.40]}/{\ell_2}, {[0.30,0.60]}/{\ell_4}\right\}\right), \left(\tau_4, \left\{{[0.46,0.60]}/{\ell_1}, {[0.58,0.75]}/{\ell_2}, {[0.64,0.88]}/{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{{[0.37,0.58]}/{\ell_2}, {[0.44,0.79]}/{\ell_4}\right\}\right), \left(\tau_6, \left\{{[0.52,0.88]}/{\ell_1}, {[0.52,0.86]}/{\ell_2}, {[0.44,0.80]}/{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{{[0.79,0.88]}/{\ell_4}, {[0.72,0.85]}/{\ell_5}\right\}\right), \left(\tau_9, \left\{{[0.68,0.93]}/{\ell_1}, {[0.64,0.75]}/{\ell_3}, {[0.65,0.94]}/{\ell_5}\right\}\right) \\ \left(\tau_{11}, \left\{{[0.58,0.80]}/{\ell_2}, {[0.46,0.72]}/{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{[0.52,0.82]}/{\ell_4}, {[0.68,0.85]}/{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{{[0.58,0.84]}/{\ell_2}, {[0.51,0.70]}/{\ell_3}, {[0.52,0.80]}/{\ell_4}, {[0.52,0.79]}/{\ell_5}\right\}\right), \\ \left(\tau_{14}, \left\{{[0.82,0.90]}/{\ell_1}, {[0.52,0.80]}/{\ell_5}\right\}\right), \left(\tau_{15}, \left\{{[0.10,0.40]}/{\ell_3}, {[0.60,0.80]}/{\ell_4}, {[0.50,0.70]}/{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{{[0.55,0.79]}/{\ell_1}, {[0.36,0.64]}/{\ell_2}, {[0.64,0.88]}/{\ell_3}, {[0.58,0.91]}/{\ell_4}\right\}\right) \end{array}\right\}.$$

Definition 4.11. The *multiplication of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \boxtimes (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set with $\eth_3 = \eth_1 \cup \eth_1$ and is defined as

$$\tilde{L'}_{H_3}(\tau) = \left\{\begin{array}{l} \tilde{L'}_{H_1}(\tau) : \tau \in \eth_1 \backslash \eth_2 \\ \tilde{L'}_{H_2}(\tau) : \tau \in \eth_2 \backslash \eth_1 \\ \tilde{L'}_{H_1}(\tau) \otimes \tilde{L'}_{H_2}(\tau) : \tau \in \eth_1 \cap \eth_2 \end{array}\right\},$$

where $\tilde{L'}_{H_1}(\tau) \otimes \tilde{L'}_{H_2}(\tau) =$
$$\left\{\left(\tau, \left[\kappa_{l\tilde{L'}_{H_1}(\tau)}(z)\kappa_{l\tilde{L'}_{H_2}(\tau)}(z), \kappa_{u\tilde{L'}_{H_1}(\tau)}(z)\kappa_{u\tilde{L'}_{H_1}(\tau)}(z)\right]\right) \forall z \in \mathcal{Z}\right\}.$$

Example 4.12. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2) \in Ç_{(\tilde{L'}_H, \eth)}$ with $\eth_1, \eth_2 \subseteq \eth$ as in

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Example 4.2. Then $(\tilde{L}'_{H_1}, \eth_1) \boxtimes (\tilde{L}'_{H_2}, \eth_2) =$

$$\left\{\begin{array}{l}
\left(\tau_1, \left\{{}^{[0.15, 0.30]}/_{\ell_1}, {}^{[0.03, 0.15]}/_{\ell_2}\right\}\right), \left(\tau_2, \left\{{}^{[0.60, 0.70]}/_{\ell_1}, {}^{[0.20, 0.40]}/_{\ell_3}\right\}\right), \\
\left(\tau_3, \left\{{}^{[0.30, 0.40]}/_{\ell_2}, {}^{[0.30, 0.60]}/_{\ell_4}\right\}\right), \left(\tau_4, \left\{{}^{[0.04, 0.10]}/_{\ell_1}, {}^{[0.12, 0.25]}/_{\ell_2}, {}^{[0.06, 0.32]}/_{\ell_5}\right\}\right), \\
\left(\tau_5, \left\{{}^{[0.03, 0.12]}/_{\ell_2}, {}^{[0.06, 0.21]}/_{\ell_4}\right\}\right), \left(\tau_6, \left\{{}^{[0.08, 0.42]}/_{\ell_1}, {}^{[0.08, 0.24]}/_{\ell_2}, {}^{[0.06, 0.30]}/_{\ell_5}\right\}\right), \\
\left(\tau_7, \left\{{}^{[0.21, 0.32]}/_{\ell_4}, {}^{[0.18, 0.35]}/_{\ell_5}\right\}\right), \left(\tau_9, \left\{{}^{[0.12, 0.27]}/_{\ell_1}, {}^{[0.16, 0.25]}/_{\ell_3}, {}^{[0.15, 0.56]}/_{\ell_5}\right\}\right), \\
\left(\tau_{11}, \left\{{}^{[0.12, 0.30]}/_{\ell_2}, {}^{[0.04, 0.18]}/_{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{}^{[0.08, 0.28]}/_{\ell_4}, {}^{[0.12, 0.35]}/_{\ell_5}\right\}\right), \\
\left(\tau_{13}, \left\{{}^{[0.12, 0.36]}/_{\ell_2}, {}^{[0.09, 0.20]}/_{\ell_3}, {}^{[0.08, 0.30]}/_{\ell_4}, {}^{[0.08, 0.21]}/_{\ell_5}\right\}\right), \\
\left(\tau_{14}, \left\{{}^{[0.28, 0.40]}/_{\ell_1}, {}^{[0.08, 0.30]}/_{\ell_5}\right\}\right), \left(\tau_{15}, \left\{{}^{[0.10, 0.40]}/_{\ell_3}, {}^{[0.60, 0.80]}/_{\ell_4}, {}^{[0.50, 0.70]}/_{\ell_5}\right\}\right), \\
\left(\tau_{16}, \left\{{}^{[0.05, 0.21]}/_{\ell_1}, {}^{[0.04, 0.16]}/_{\ell_2}, {}^{[0.16, 0.42]}/_{\ell_3}, {}^{[0.12, 0.49]}/_{\ell_4}\right\}\right)
\end{array}\right\}$$

Definition 4.13. *Partial membership of $IVFHS$-set* $(\tilde{L}'_{H_1}, \eth_1)$ denoted by $\boxdot(\tilde{L}'_{H_1}, \eth_1)$ is an $IVFHS$-set and is defined as

$$\boxdot(\tilde{L}'_{H_1}, \eth_1) = \left\{\odot\tilde{L}'_{H_1}(\tau) : \tau \in \eth_1\right\},$$

where $\odot\tilde{L}'_{H_1}(\tau) = \left\{\left(\tau, \left[\kappa_{l\tilde{L}'_{H_1}(\tau)}(z), 1 - \kappa_{l\tilde{L}'_{H_1}(\tau)}(z)\right]\right) \forall z \in \mathscr{Z}\right\}.$

Example 4.14. Consider $IVFHS$-set $(\tilde{L}'_{H_1}, \eth_1) \in \text{Ç}_{(\tilde{L}'_H, \eth)}$ with $\eth_1 \subseteq \eth$ as in Example 4.2 given by, $(\tilde{L}'_{H_1}, \eth_1) =$

$$\left\{\begin{array}{l}
\left(\tau_1, \left\{{}^{[0.1, 0.2]}/_{\ell_1}, {}^{[0.3, 0.4]}/_{\ell_2}\right\}\right), \left(\tau_2, \left\{{}^{[0.2, 0.4]}/_{\ell_1}, {}^{[0.3, 0.7]}/_{\ell_3}\right\}\right), \\
\left(\tau_3, \left\{{}^{[0.3, 0.4]}/_{\ell_2}, {}^{[0.2, 0.6]}/_{\ell_4}\right\}\right), \left(\tau_4, \left\{{}^{[0.4, 0.7]}/_{\ell_1}, {}^{[0.3, 0.8]}/_{\ell_2}, {}^{[0.1, 0.2]}/_{\ell_5}\right\}\right), \\
\left(\tau_5, \left\{{}^{[0.1, 0.3]}/_{\ell_2}, {}^{[0.2, 0.4]}/_{\ell_4}\right\}\right), \left(\tau_6, \left\{{}^{[0.2, 0.6]}/_{\ell_1}, {}^{[0.4, 0.6]}/_{\ell_2}, {}^{[0.3, 0.6]}/_{\ell_5}\right\}\right), \\
\left(\tau_7, \left\{{}^{[0.2, 0.4]}/_{\ell_4}, {}^{[0.6, 0.7]}/_{\ell_5}\right\}\right), \left(\tau_8, \left\{{}^{[0.1, 0.4]}/_{\ell_1}, {}^{[0.2, 0.3]}/_{\ell_2}, {}^{[0.4, 0.6]}/_{\ell_4}, {}^{[0.3, 0.6]}/_{\ell_5}\right\}\right), \\
\left(\tau_9, \left\{{}^{[0.2, 0.3]}/_{\ell_1}, {}^{[0.1, 0.2]}/_{\ell_3}, {}^{[0.3, 0.7]}/_{\ell_5}\right\}\right), \left(\tau_{10}, \left\{{}^{[0.4, 0.6]}/_{\ell_4}, {}^{[0.2, 0.4]}/_{\ell_5}\right\}\right), \\
\left(\tau_{11}, \left\{{}^{[0.3, 0.9]}/_{\ell_2}, {}^{[0.4, 0.6]}/_{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{}^{[0.2, 0.4]}/_{\ell_4}, {}^{[0.6, 0.7]}/_{\ell_5}\right\}\right), \\
\left(\tau_{13}, \left\{{}^{[0.3, 0.6]}/_{\ell_2}, {}^{[0.1, 0.4]}/_{\ell_3}, {}^{[0.2, 0.6]}/_{\ell_4}, {}^{[0.4, 0.7]}/_{\ell_5}\right\}\right), \left(\tau_{14}, \left\{{}^{[0.4, 0.6]}/_{\ell_1}, {}^{[0.2, 0.3]}/_{\ell_5}\right\}\right), \\
\left(\tau_{15}, \left\{{}^{[0.1, 0.4]}/_{\ell_3}, {}^{[0.6, 0.8]}/_{\ell_4}, {}^{[0.4, 0.7]}/_{\ell_5}\right\}\right), \\
\left(\tau_{16}, \left\{{}^{[0.6, 0.7]}/_{\ell_1}, {}^{[0.2, 0.4]}/_{\ell_2}, {}^{[0.3, 0.6]}/_{\ell_3}, {}^{[0.4, 0.7]}/_{\ell_4}\right\}\right)
\end{array}\right\}.$$

Then partial membership is given as follow:
$\boxdot(\tilde{L}'_{H_1}, \eth_1) =$

$$\left\{\begin{array}{l}
\left(\tau_1, \left\{{}^{[0.1, 0.9]}/_{\ell_1}, {}^{[0.3, 0.7]}/_{\ell_2}\right\}\right), \left(\tau_2, \left\{{}^{[0.2, 0.8]}/_{\ell_1}, {}^{[0.3, 0.7]}/_{\ell_3}\right\}\right), \\
\left(\tau_3, \left\{{}^{[0.3, 0.7]}/_{\ell_2}, {}^{[0.2, 0.8]}/_{\ell_4}\right\}\right), \left(\tau_4, \left\{{}^{[0.4, 0.6]}/_{\ell_1}, {}^{[0.3, 0.7]}/_{\ell_2}, {}^{[0.1, 0.9]}/_{\ell_5}\right\}\right), \\
\left(\tau_5, \left\{{}^{[0.1, 0.9]}/_{\ell_2}, {}^{[0.2, 0.8]}/_{\ell_4}\right\}\right), \left(\tau_6, \left\{{}^{[0.2, 0.8]}/_{\ell_1}, {}^{[0.4, 0.6]}/_{\ell_2}, {}^{[0.3, 0.7]}/_{\ell_5}\right\}\right), \\
\left(\tau_7, \left\{{}^{[0.2, 0.8]}/_{\ell_4}, {}^{[0.4, 0.6]}/_{\ell_5}\right\}\right), \left(\tau_8, \left\{{}^{[0.1, 0.9]}/_{\ell_1}, {}^{[0.2, 0.8]}/_{\ell_2}, {}^{[0.4, 0.6]}/_{\ell_4}, {}^{[0.3, 0.7]}/_{\ell_5}\right\}\right), \\
\left(\tau_9, \left\{{}^{[0.2, 0.8]}/_{\ell_1}, {}^{[0.1, 0.9]}/_{\ell_3}, {}^{[0.3, 0.7]}/_{\ell_5}\right\}\right), \left(\tau_{10}, \left\{{}^{[0.4, 0.6]}/_{\ell_4}, {}^{[0.2, 0.8]}/_{\ell_5}\right\}\right), \\
\left(\tau_{11}, \left\{{}^{[0.3, 0.7]}/_{\ell_2}, {}^{[0.4, 0.6]}/_{\ell_5}\right\}\right), \left(\tau_{12}, \left\{{}^{[0.2, 0.8]}/_{\ell_4}, {}^{[0.4, 0.6]}/_{\ell_5}\right\}\right), \\
\left(\tau_{13}, \left\{{}^{[0.3, 0.7]}/_{\ell_2}, {}^{[0.1, 0.9]}/_{\ell_3}, {}^{[0.2, 0.8]}/_{\ell_4}, {}^{[0.4, 0.6]}/_{\ell_5}\right\}\right), \left(\tau_{14}, \left\{{}^{[0.4, 0.6]}/_{\ell_1}, {}^{[0.2, 0.8]}/_{\ell_5}\right\}\right), \\
\left(\tau_{15}, \left\{{}^{[0.1, 0.9]}/_{\ell_3}, {}^{[0.4, 0.6]}/_{\ell_4}, {}^{[0.4, 0.6]}/_{\ell_5}\right\}\right), \\
\left(\tau_{16}, \left\{{}^{[0.4, 0.6]}/_{\ell_1}, {}^{[0.2, 0.8]}/_{\ell_2}, {}^{[0.3, 0.7]}/_{\ell_3}, {}^{[0.4, 0.6]}/_{\ell_4}\right\}\right)
\end{array}\right\}.$$

Definition 4.15. The *partial non-membership of $IVFHS$-set* $(\tilde{L}'_{H_1}, \eth_1)$ denoted by $\circledcirc(\tilde{L}'_{H_1}, \eth_1)$ is an $IVFHS$-set defined as

$$\circledcirc\tilde{L}'_{H_1}(\tau) = \left\{\circ\tilde{L}'_{H_1}(\tau) : \tau \in \eth_1\right\},$$

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where $\circ\tilde{L'}_{H_1}(\tau) = \left\{\left(\tau, \left[1-\kappa_{u\tilde{L'}_{H_1}(\tau)}(z), \kappa_{u\tilde{L'}_{H_1}(\tau)}(z)\right]\right) \forall z \in \mathcal{Z}\right\}$.

Example 4.16. Consider $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1) \in \c{C}_{(\tilde{L'}_H,\eth)}$ with $\eth_1 \subseteq \eth$ as in Example 4.14. Then partial non-membership is given as follow:
$\circledcirc(\tilde{L'}_{H_1}, \eth_1) =$

$$\left\{\begin{array}{l} \left(\tau_1, \left\{[0.2,0.8]/_{\ell_1}, [0.4,0.6]/_{\ell_2}\right\}\right), \left(\tau_2, \left\{[0.4,0.6]/_{\ell_1}, [0.3,0.7]/_{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{[0.4,0.6]/_{\ell_2}, [0.4,0.6]/_{\ell_4}\right\}\right), \left(\tau_4, \left\{[0.3,0.7]/_{\ell_1}, [0.2,0.8]/_{\ell_2}, [0.2,0.8]/_{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{[0.3,0.7]/_{\ell_2}, [0.4,0.6]/_{\ell_4}\right\}\right), \left(\tau_6, \left\{[0.4,0.6]/_{\ell_1}, [0.4,0.6]/_{\ell_2}, [0.4,0.6]/_{\ell_5}\right\}\right), \\ \left(\tau_7, \left\{[0.4,0.6]/_{\ell_4}, [0.3,0.7]/_{\ell_5}\right\}\right), \left(\tau_8, \left\{[0.4,0.6]/_{\ell_1}, [0.3,0.7]/_{\ell_2}, [0.4,0.6]/_{\ell_4}, [0.4,0.6]/_{\ell_5}\right\}\right), \\ \left(\tau_9, \left\{[0.3,0.7]/_{\ell_1}, [0.2,0.8]/_{\ell_3}, [0.3,0.7]/_{\ell_5}\right\}\right), \left(\tau_{10}, \left\{[0.4,0.6]/_{\ell_4}, [0.4,0.6]/_{\ell_5}\right\}\right), \\ \left(\tau_{11}, \left\{[0.1,0.9]/_{\ell_2}, [0.4,0.6]/_{\ell_5}\right\}\right), \left(\tau_{12}, \left\{[0.4,0.6]/_{\ell_4}, [0.3,0.7]/_{\ell_5}\right\}\right), \\ \left(\tau_{13}, \left\{[0.4,0.6]/_{\ell_2}, [0.4,0.6]/_{\ell_3}, [0.4,0.6]/_{\ell_4}, [0.3,0.7]/_{\ell_5}\right\}\right), \left(\tau_{14}, \left\{[0.4,0.6]/_{\ell_1}, [0.3,0.7]/_{\ell_5}\right\}\right), \\ \left(\tau_{15}, \left\{[0.4,0.6]/_{\ell_3}, [0.2,0.8]/_{\ell_4}, [0.3,0.7]/_{\ell_5}\right\}\right), \\ \left(\tau_{16}, \left\{[0.3,0.7]/_{\ell_1}, [0.4,0.6]/_{\ell_2}, [0.4,0.6]/_{\ell_3}, [0.3,0.7]/_{\ell_4}\right\}\right) \end{array}\right\}.$$

Definition 4.17. The *AND operation of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \bigwedge (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set $(\tilde{L'}_{H_3}, \eth_3)$ with $\eth_3 = \eth_1 \times \eth_2$ and for $(\tau_i, \tau_j) \in \eth_3, \tau_i \in \eth_1, \tau_j \in \eth_2$ is defined as

$$\tilde{L'}_{H_3}(\tau_1, \tau_2) = \tilde{L'}_{H_1}(\tau_1) \sqcap \tilde{L'}_{H_2}(\tau_2),$$

where $\tilde{L'}_{H_1}(\tau_1) \sqcap \tilde{L'}_{H_2}(\tau_2)$ as defined in definition 4.5.

Example 4.18. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ as in Example 2.8 given by
$(\tilde{L'}_{H_1}, \eth_1) =$

$$\left\{\begin{array}{l} \left(\tau_1, \left\{[0.4,0.5]/_{\ell_1}, [0.2,0.5]/_{\ell_2}\right\}\right), \left(\tau_2, \left\{[0.1,0.4]/_{\ell_1}, [0.6,0.8]/_{\ell_2}, [0.5,0.7]/_{\ell_3}\right\}\right), \\ \left(\tau_3, \left\{[0.5,0.7]/_{\ell_2}, [0.2,0.4]/_{\ell_3}, [0.4,0.6]/_{\ell_4}, [0.4,0.7]/_{\ell_5}\right\}\right) \end{array}\right\}$$

and
$(\tilde{L'}_{H_2}, \eth_2) =$

$$\left\{\begin{array}{l} \left(\tau_3, \left\{[0.7,0.8]/_{\ell_1}, [0.3,0.5]/_{\ell_2}\right\}\right), \left(\tau_4, \left\{[0.4,0.7]/_{\ell_3}, [0.2,0.3]/_{\ell_4}, [0.2,0.6]/_{\ell_5}\right\}\right), \\ \left(\tau_5, \left\{[0.4,0.6]/_{\ell_1}, [0.3,0.5]/_{\ell_2}, [0.4,0.5]/_{\ell_4}, [0.2,0.3]/_{\ell_5}\right\}\right), \end{array}\right\}$$

$\eth_1 \times \eth_2 =$

$$\left\{\begin{array}{l} (\lambda_1 = (\tau_1, \tau_3), \lambda_2 = (\tau_1, \tau_4), \lambda_3 = (\tau_1, \tau_5), \lambda_4 = (\tau_2, \tau_3), \lambda_5 = (\tau_2, \tau_4),), \\ (\lambda_6 = (\tau_2, \tau_5), \lambda_7 = (\tau_3, \tau_3), \lambda_8 = (\tau_3, \tau_4), \lambda_9 = (\tau_3, \tau_5)) \end{array}\right\}$$

then
$(\tilde{L'}_{H_3}, \eth_3) =$

$$\left\{\begin{array}{l} \left(\lambda_1, \left\{[0.4,0.5]/_{\ell_1}, [0.2,0.5]/_{\ell_2}\right\}\right), (\lambda_2, \{\}), \left(\lambda_3, \left\{[0.4,0.5]/_{\ell_1}, [0.2,0.5]/_{\ell_2},\right\}\right), \\ \left(\lambda_4, \left\{[0.1,0.4]/_{\ell_1}, [0.3,0.5]/_{\ell_2}\right\}\right), \left(\lambda_5, \left\{[0.4,0.7]/_{\ell_3},\right\}\right), \left(\lambda_6, \left\{[0.1,0.4]/_{\ell_1}, [0.3,0.5]/_{\ell_2}\right\}\right), \\ \left(\lambda_7, \left\{[0.5,0.7]/_{\ell_1}, [0.2,0.4]/_{\ell_2},\right\}\right), \left(\lambda_8, \left\{[0.2,0.4]/_{\ell_3}, [0.2,0.3]/_{\ell_4}, [0.2,0.6]/_{\ell_5},\right\}\right), \\ \left(\lambda_9, \left\{[0.3,0.5]/_{\ell_2}, [0.4,0.5]/_{\ell_4}, [0.2,0.3]/_{\ell_5}\right\}\right) \end{array}\right\}.$$

Definition 4.19. The *OR operation of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \bigvee (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set $(\tilde{L'}_{H_3}, \eth_3)$ with $\eth_3 = \eth_1 \times \eth_2$ and for $(\tau_i, \tau_j) \in \eth_3, \tau_i \in \eth_1, \tau_j \in \eth_2$ is defined as

$$\tilde{L'}_{H_3}(\tau_1, \tau_2) = \tilde{L'}_{H_1}(\tau_1) \sqcup \tilde{L'}_{H_2}(\tau_2),$$

where $\tilde{L'}_{H_1}(\tau_1) \sqcup \tilde{L'}_{H_2}(\tau_2)$ as defined in definition 4.1.

Example 4.20. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ as in Example 4.18, then $(\tilde{L'}_{H_3}, \eth_3)$ can e given by

$(\tilde{L'}_{H_3}, \eth_3) =$

$$\left\{\begin{array}{l} \left(\lambda_1, \left\{{[0.7,0.8]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}\right\}\right), \left(\lambda_4, \left\{{[0.7,0.8]}/{\ell_1}, {[0.6,0.8]}/{\ell_2}, {[0.5,0.7]}/{\ell_3}\right\}\right) \\ \left(\lambda_2, \left\{{[0.4,0.5]}/{\ell_1}, {[0.2,0.5]}/{\ell_2}, {[0.4,0.7]}/{\ell_3}, {[0.2,0.3]}/{\ell_4}, {[0.2,0.6]}/{\ell_5}\right\}\right), \\ \left(\lambda_3, \left\{{[0.4,0.6]}/{\ell_1}, {[0.3,0.5]}/{\ell_2}, {[0.4,0.5]}/{\ell_4}, {[0.2,0.3]}/{\ell_5}\right\}\right), \\ \left(\lambda_5, \left\{{[0.1,0.4]}/{\ell_1}, {[0.6,0.8]}/{\ell_2}, {[0.5,0.7]}/{\ell_3}, {[0.2,0.3]}/{\ell_4}, {[0.2,0.6]}/{\ell_5},\right\}\right), \\ \left(\lambda_6, \left\{{[0.4,0.6]}/{\ell_1}, {[0.6,0.8]}/{\ell_2}, {[0.5,0.7]}/{\ell_3}, {[0.4,0.5]}/{\ell_4}, {[0.2,0.3]}/{\ell_5}\right\}\right), \\ \left(\lambda_7, \left\{{[0.7,0.8]}/{\ell_1}, {[0.5,0.7]}/{\ell_2}, {[0.2,0.4]}/{\ell_3}, {[0.4,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right), \\ \left(\lambda_8, \left\{{[0.5,0.7]}/{\ell_2}, {[0.4,0.7]}/{\ell_3}, {[0.4,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right), \\ \left(\lambda_9, \left\{{[0.4,0.6]}/{\ell_1}, {[0.5,0.7]}/{\ell_2}, {[0.2,0.4]}/{\ell_3}, {[0.4,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right) \end{array}\right\}.$$

Definition 4.21. The *difference of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \setminus (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set $(\tilde{L'}_{H_3}, \eth_3)$ with $\eth_3 = \eth_1 \setminus \eth_2$ and is defined as

$$\tilde{L'}_{H_3}(\tau) = \tilde{L'}_{H_1}(\tau_1) - \tilde{L'}_{H_2}(\tau_2).$$

Example 4.22. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ as in Example 4.18, then $(\tilde{L'}_{H_3}, \eth_3)$ can be given by
$(\tilde{L'}_{H_3}, \eth_3) = \left\{ \left(\tau_1, \left\{{[0.4,0.5]}/{\ell_1}, {[0.2,0.5]}/{\ell_2}\right\}\right), \left(\tau_2, \left\{{[0.1,0.4]}/{\ell_1}, {[0.6,0.8]}/{\ell_2}, {[0.5,0.7]}/{\ell_3}\right\}\right) \right\}$

Definition 4.23. The *restricted difference of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \setminus_R (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set $(\tilde{L'}_{H_3}, \eth_3)$ with $\eth_3 = \eth_1 \cap \eth_2$ and for $(\tau_i, \tau_j) \in \eth_3, \tau_i \in \eth_1, \tau_j \in \eth_2$ is defined as

$$\tilde{L'}_{H_3}(\tau) = \tilde{L'}_{H_1}(\tau) - \tilde{L'}_{H_2}(\tau_2).$$

Example 4.24. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ as in Example 4.18, then $(\tilde{L'}_{H_3}, \eth_3)$ can be given by
$(\tilde{L'}_{H_3}, \eth_3) = \left\{\left(\tau_3, \left\{{[0.2,0.4]}/{\ell_3}, {[0.4,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right)\right\}$.

Definition 4.25. The *restricted symmetric difference of two $IVFHS$-sets* $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ denoted by $(\tilde{L'}_{H_1}, \eth_1) \setminus_{RS} (\tilde{L'}_{H_2}, \eth_2)$ is an $IVFHS$-set $(\tilde{L'}_{H_3}, \eth_3)$ with $\eth_3 = \eth_1 \cap \eth_2$ and for $(\tau_i, \tau_j) \in \eth_3, \tau_i \in \eth_1, \tau_j \in \eth_2$ is defined as

$$\tilde{L'}_{H_3} = \left\{\left((\tilde{L'}_{H_1}, \eth_1) \sqcup_R (\tilde{L'}_{H_2}, \eth_2)\right) \setminus_R \left((\tilde{L'}_{H_1}, \eth_1) \cap (\tilde{L'}_{H_2}, \eth_2)\right)\right\}$$

or

$$\tilde{L'}_{H_3} = \left\{\left((\tilde{L'}_{H_1}, \eth_1) \setminus_R (\tilde{L'}_{H_2}, \eth_2)\right) \sqcup_R \left((\tilde{L'}_{H_2}, \eth_2) \setminus_R (\tilde{L'}_{H_1}, \eth_1)\right)\right\}.$$

Example 4.26. Consider two $IVFHS$-sets $(\tilde{L'}_{H_1}, \eth_1)$ and $(\tilde{L'}_{H_2}, \eth_2)$ as in Example 4.18, then we have

$\left((\tilde{L'}_{H_1}, \eth_1) \setminus_R (\tilde{L'}_{H_2}, \eth_2)\right) = \left\{\left(\tau_3, \left\{{[0.2,0.4]}/{\ell_3}, {[0.4,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right)\right\}$.

$\left((\tilde{L'}_{H_2}, \eth_2) \setminus_R (\tilde{L'}_{H_1}, \eth_1)\right) = \left\{\left(\tau_3, \left\{{[0.7,0.8]}/{\ell_1}\right\}\right)\right\}$.
then,
$(\tilde{L'}_{H_3}, \eth_3) = \left\{\left(\tau_3, \left\{{[0.7,0.8]}/{\ell_1}, {[0.2,0.4]}/{\ell_3}, {[0.4,0.6]}/{\ell_4}, {[0.4,0.7]}/{\ell_5}\right\}\right)\right\}$.

5 Basic Properties and Laws of $IVFHS$-set Operations

Some basic properties, laws and results of $IVFHS$-set are discussed in this section. All $IVFHS$-sets in $Ç_{(\tilde{L'}_H, \eth)}$ satisfy following properties, laws and results.

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1. **Idempotent Laws**
   (a) $(\tilde{L'}_H, \eth) \sqcap (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth) \sqcap_E (\tilde{L'}_H, \eth)$.
   (b) $(\tilde{L'}_H, \eth) \sqcup (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth) \sqcup_R (\tilde{L'}_H, \eth)$.
2. **Identity Laws**
   (a) $(\tilde{L'}_H, \eth) \sqcap (\tilde{L'}_H, \eth)_{\Bbbk} = (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth) \sqcap_E (\tilde{L'}_H, \eth)_{\Bbbk}$.
   (b) $(\tilde{L'}_H, \eth) \sqcup (\tilde{L'}_H, \eth)_{\Phi} = (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth) \sqcup_R (\tilde{L'}_H, \eth)_{\Phi}$.
   (c) $(\tilde{L'}_H, \eth) \setminus_R (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth)_{\Phi} = (\tilde{L'}_H, \eth) \sqcup_{RS} (\tilde{L'}_H, \eth)$.
   (d) $(\tilde{L'}_H, \eth) \setminus_R (\tilde{L'}_H, \eth)_{\Phi} = (\tilde{L'}_H, \eth) = (\tilde{L'}_H, \eth) \sqcup_{RS} (\tilde{L'}_H, \eth)_{\Phi}$.
3. **Domination Laws**
   (a) $(\tilde{L'}_H, \eth) \sqcap (\tilde{L'}_H, \eth)_{\Phi} = (\tilde{L'}_H, \eth)_{\Phi} = (\tilde{L'}_H, \eth) \sqcap_E (\tilde{L'}_H, \eth)_{\Phi}$.
   (b) $(\tilde{L'}_H, \eth) \sqcup (\tilde{L'}_H, \eth)_{\Bbbk} = (\tilde{L'}_H, \eth)_{\Bbbk} = (\tilde{L'}_H, \eth) \sqcup_R (\tilde{L'}_H, \eth)_{\Bbbk}$.
4. **Property of Contradiction**
   (a) $(\tilde{L'}_H, \eth) \sqcap (\tilde{L'}_H, \eth)^c = (\tilde{L'}_H, \eth)_{\Phi} = (\tilde{L'}_H, \eth) \sqcap_E (\tilde{L'}_H, \eth)^c$.
5. **Property of Exclusion**
   (a) $(\tilde{L'}_H, \eth) \sqcup (\tilde{L'}_H, \eth)^c = (\tilde{L'}_H, \eth)_{\Bbbk} = (\tilde{L'}_H, \eth) \sqcup_R (\tilde{L'}_H, \eth)^c$.
6. **Absorption Laws**
   (a) $(\tilde{L'}_{H_1}, \eth_1) \sqcap \Big((\tilde{L'}_{H_1}, \eth_1) \sqcup (\tilde{L'}_{H_2}, \eth_2)\Big) = (\tilde{L'}_{H_1}, \eth_1)$.
   (b) $(\tilde{L'}_{H_1}, \eth_1) \sqcup \Big((\tilde{L'}_{H_1}, \eth_1) \sqcap (\tilde{L'}_{H_2}, \eth_2)\Big) = (\tilde{L'}_{H_1}, \eth_1)$.
   (c) $(\tilde{L'}_{H_1}, \eth_1) \sqcap_E \Big((\tilde{L'}_{H_1}, \eth_1) \sqcup_R (\tilde{L'}_{H_2}, \eth_2)\Big) = (\tilde{L'}_{H_1}, \eth_1)$.
   (d) $(\tilde{L'}_{H_1}, \eth_1) \sqcup_R \Big((\tilde{L'}_{H_1}, \eth_1) \sqcap_E (\tilde{L'}_{H_2}, \eth_2)\Big) = (\tilde{L'}_{H_1}, \eth_1)$.
7. **Commutative Laws**
   (a) $(\tilde{L'}_{H_1}, \eth_1) \sqcap (\tilde{L'}_{H_2}, \eth_2) = (\tilde{L'}_{H_2}, \eth_2) \sqcap (\tilde{L'}_{H_1}, \eth_1)$.
   (b) $(\tilde{L'}_{H_1}, \eth_1) \sqcap_E (\tilde{L'}_{H_2}, \eth_2) = (\tilde{L'}_{H_2}, \eth_2) \sqcap_E (\tilde{L'}_{H_1}, \eth_1)$.
   (c) $(\tilde{L'}_{H_1}, \eth_1) \sqcup (\tilde{L'}_{H_2}, \eth_2) = (\tilde{L'}_{H_2}, \eth_2) \sqcup (\tilde{L'}_{H_1}, \eth_1)$.
   (d) $(\tilde{L'}_{H_1}, \eth_1) \sqcup_R (\tilde{L'}_{H_2}, \eth_2) = (\tilde{L'}_{H_2}, \eth_2) \sqcup_R (\tilde{L'}_{H_1}, \eth_1)$.
   (e) $(\tilde{L'}_{H_1}, \eth_1) \setminus_{RS} (\tilde{L'}_{H_2}, \eth_2) = (\tilde{L'}_{H_2}, \eth_2) \setminus_{RS} (\tilde{L'}_{H_1}, \eth_1)$.
8. **Associative Laws**
   (a) $(\tilde{L'}_{H_1}, \eth_1) \sqcap \Big((\tilde{L'}_{H_2}, \eth_2) \sqcap (\tilde{L'}_{H_3}, \eth_3)\Big) = \Big((\tilde{L'}_{H_1}, \eth_1) \sqcap (\tilde{L'}_{H_2}, \eth_2)\Big) \sqcap (\tilde{L'}_{H_3}, \eth_3)$.
   (b) $(\tilde{L'}_{H_1}, \eth_1) \sqcap_E \Big((\tilde{L'}_{H_2}, \eth_2) \sqcap_E (\tilde{L'}_{H_3}, \eth_3)\Big) = \Big((\tilde{L'}_{H_1}, \eth_1) \sqcap_E (\tilde{L'}_{H_2}, \eth_2)\Big) \sqcap_E (\tilde{L'}_{H_3}, \eth_3)$.
   (c) $(\tilde{L'}_{H_1}, \eth_1) \sqcup \Big((\tilde{L'}_{H_2}, \eth_2) \sqcup (\tilde{L'}_{H_3}, \eth_3)\Big) = \Big((\tilde{L'}_{H_1}, \eth_1) \sqcup (\tilde{L'}_{H_2}, \eth_2)\Big) \sqcup (\tilde{L'}_{H_3}, \eth_3)$.
   (d) $(\tilde{L'}_{H_1}, \eth_1) \sqcup_R \Big((\tilde{L'}_{H_2}, \eth_2) \sqcup_R (\tilde{L'}_{H_3}, \eth_3)\Big) = \Big((\tilde{L'}_{H_1}, \eth_1) \sqcup_R (\tilde{L'}_{H_2}, \eth_2)\Big) \sqcup_R (\tilde{L'}_{H_3}, \eth_3)$.
   (e) $(\tilde{L'}_{H_1}, \eth_1) \bigwedge \Big((\tilde{L'}_{H_2}, \eth_2) \bigwedge (\tilde{L'}_{H_3}, \eth_3)\Big) = \Big((\tilde{L'}_{H_1}, \eth_1) \bigwedge (\tilde{L'}_{H_2}, \eth_2)\Big) \bigwedge (\tilde{L'}_{H_3}, \eth_3)$.
   (f) $(\tilde{L'}_{H_1}, \eth_1) \bigvee \Big((\tilde{L'}_{H_2}, \eth_2) \bigvee (\tilde{L'}_{H_3}, \eth_3)\Big) = \Big((\tilde{L'}_{H_1}, \eth_1) \bigvee (\tilde{L'}_{H_2}, \eth_2)\Big) \bigvee (\tilde{L'}_{H_3}, \eth_3)$.
9. **De Morgans Laws**
   (a) $\Big((\tilde{L'}_{H_1}, \eth_1) \sqcap_E (\tilde{L'}_{H_2}, \eth_2)\Big)^c = \Big(\tilde{L'}_{H_1}, \eth_1\Big)^c \sqcup \Big(\tilde{L'}_{H_2}, \eth_2\Big)^c$.

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(b) $\left((\tilde{L}'_{H_1}, \eth_1) \sqcup (\tilde{L}'_{H_2}, \eth_2)\right)^c = \left(\tilde{L}'_{H_1}, \eth_1\right)^c \sqcap_E \left(\tilde{L}'_{H_2}, \eth_2\right)^c$.

(c) $\left((\tilde{L}'_{H_1}, \eth_1) \bigwedge (\tilde{L}'_{H_2}, \eth_2)\right)^c = \left(\tilde{L}'_{H_1}, \eth_1\right)^c \bigvee \left(\tilde{L}'_{H_2}, \eth_2\right)^c$.

(d) $\left((\tilde{L}'_{H_1}, \eth_1) \bigvee (\tilde{L}'_{H_2}, \eth_2)\right)^c = \left(\tilde{L}'_{H_1}, \eth_1\right)^c \bigwedge \left(\tilde{L}'_{H_2}, \eth_2\right)^c$.

10. **Distributive Laws**

(a) $(\tilde{L}'_{H_1}, \eth_1) \sqcap \left((\tilde{L}'_{H_2}, \eth_2) \sqcup (\tilde{L}'_{H_3}, \eth_3)\right) =$
$\left((\tilde{L}'_{H_1}, \eth_1) \sqcap (\tilde{L}'_{H_2}, \eth_2)\right) \sqcup \left((\tilde{L}'_{H_1}, \eth_1) \sqcap (\tilde{L}'_{H_3}, \eth_3)\right)$.

(b) $(\tilde{L}'_{H_1}, \eth_1) \sqcup \left((\tilde{L}'_{H_2}, \eth_2) \sqcap (\tilde{L}'_{H_3}, \eth_3)\right) =$
$\left((\tilde{L}'_{H_1}, \eth_1) \sqcup (\tilde{L}'_{H_2}, \eth_2)\right) \sqcap \left((\tilde{L}'_{H_1}, \eth_1) \sqcup (\tilde{L}'_{H_3}, \eth_3)\right)$.

(c) $(\tilde{L}'_{H_1}, \eth_1) \sqcap_E \left((\tilde{L}'_{H_2}, \eth_2) \sqcup_R (\tilde{L}'_{H_3}, \eth_3)\right) =$
$\left((\tilde{L}'_{H_1}, \eth_1) \sqcap_E (\tilde{L}'_{H_2}, \eth_2)\right) \sqcup_R \left((\tilde{L}'_{H_1}, \eth_1) \sqcap_E (\tilde{L}'_{H_3}, \eth_3)\right)$.

(d) $(\tilde{L}'_{H_1}, \eth_1) \sqcup_R \left((\tilde{L}'_{H_2}, \eth_2) \sqcap_E (\tilde{L}'_{H_3}, \eth_3)\right) =$
$\left((\tilde{L}'_{H_1}, \eth_1) \sqcup_R (\tilde{L}'_{H_2}, \eth_2)\right) \sqcap_E \left((\tilde{L}'_{H_1}, \eth_1) \sqcup_R (\tilde{L}'_{H_3}, \eth_3)\right)$.

(e) $(\tilde{L}'_{H_1}, \eth_1) \sqcap \left((\tilde{L}'_{H_2}, \eth_2) \sqcup_R (\tilde{L}'_{H_3}, \eth_3)\right) =$
$\left((\tilde{L}'_{H_1}, \eth_1) \sqcap (\tilde{L}'_{H_2}, \eth_2)\right) \sqcup_R \left((\tilde{L}'_{H_1}, \eth_1) \sqcap (\tilde{L}'_{H_3}, \eth_3)\right)$.

(f) $(\tilde{L}'_{H_1}, \eth_1) \sqcup_R \left((\tilde{L}'_{H_2}, \eth_2) \sqcap (\tilde{L}'_{H_3}, \eth_3)\right) =$
$\left((\tilde{L}'_{H_1}, \eth_1) \sqcup_R (\tilde{L}'_{H_2}, \eth_2)\right) \sqcap \left((\tilde{L}'_{H_1}, \eth_1) \sqcup_R (\tilde{L}'_{H_3}, \eth_3)\right)$.

6 Hybrids of $IVFHS$-set

In this section some hybrids of $IVFHS$-sets are discussed.

Definition 6.1. Let $\tilde{L}$, $\tilde{G}$ and $\tilde{H}$ are three $IVFHS$-sets over $\mathbb{U}$. A mapping $F(\tilde{L}, \tilde{G})$ given by $F : ivfs(\mathbb{U}) \times ivfs(\mathbb{U}) \rightarrow IVFN$ is called similarity measure if it satisfies the following conditions:

1. $F(\tilde{L}, \tilde{G})$ is IVF-number.
2. $F(\tilde{L}, \tilde{G}) = [1, 1]$ if and only if $\tilde{L} = \tilde{G}$.
3. $F(\tilde{L}, \tilde{G}) = F(\tilde{G}, \tilde{L})$.
4. If $\tilde{L} \subseteq \tilde{G} \subseteq \tilde{H}$, then $F(\tilde{L}, \tilde{H}) \subseteq F(\tilde{L}, \tilde{G})$ and $F(\tilde{L}, \tilde{H}) \subseteq F(\tilde{G}, \tilde{H})$.

Definition 6.2. Let $\tilde{L}$, $\tilde{G}$ and $\tilde{H}$ are three $IVFHS$-sets over $\mathbb{U}$. A mapping $\Delta(\tilde{L}, \tilde{G})$ given by $\Delta : ivfs(\mathbb{U}) \times ivfs(\mathbb{U}) \rightarrow IVFN$ is called distance measure if it satisfies the following conditions:

1. $\Delta(\tilde{L}, \tilde{G})$ is an IVF-number.
2. $\Delta(\tilde{L}, \tilde{G}) = [0, 0]$ if and only if $\tilde{L} = \tilde{G}$.
3. $\Delta(\tilde{L}, \tilde{G}) = \Delta(\tilde{G}, \tilde{L})$.
4. If $\tilde{L} \subseteq \tilde{G} \subseteq \tilde{H}$, then $\Delta(\tilde{L}, \tilde{G}) \subseteq \Delta(\tilde{L}, \tilde{H})$ and $\Delta(\tilde{G}, \tilde{H}) \subseteq \Delta(\tilde{L}, \tilde{H})$.

Proposition 6.3. *Let $\tilde{L}$ and $\tilde{G}$ and are two $IVFHS$-sets over $\mathbb{U}$, then the following results hold:*

1. $F(\tilde{L}, \tilde{L} \cap \tilde{G}) = F(\tilde{G}, \tilde{L} \cup \tilde{G})$.

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2. $\digamma(\tilde{L}, \tilde{L} \cup \tilde{G}) = \digamma(\tilde{G}, \tilde{L} \cap \tilde{G})$.
3. $\digamma(\tilde{L}, \tilde{G}) = \digamma(\tilde{L} \cap \tilde{G}, \tilde{L} \cup \tilde{G})$.
4. $\Delta(\tilde{L}, \tilde{L} \cap \tilde{G}) = \Delta(\tilde{G}, \tilde{L} \cup \tilde{G})$.
5. $\Delta(\tilde{L}, \tilde{L} \cup \tilde{G}) = \Delta(\tilde{G}, \tilde{L} \cap \tilde{G})$.
6. $\Delta(\tilde{L}, \tilde{G}) = \Delta(\tilde{L} \cap \tilde{G}, \tilde{L} \cup \tilde{G})$.
7. $(\digamma(\tilde{L}, \tilde{G}))^c = \Delta(\tilde{L}, \tilde{G})$.

7 Some extensions of $IVFHS$-set

Atiqe et al.[20] and Smarandache[19] discussed some hybrids of $IVFHS$-sets. Some more hybrids of $IVFHS$-sets are discussed in this section. Let $\eth_1, \eth_2, ..., \eth_n$ are attribute valued sets, each corresponding to a unique attribute $\epsilon_1, \epsilon_2, ..., \epsilon_n \in E$ such that for $p, q = 1, 2, ..., n, p \neq q, \eth_p \cap \eth_q = \varphi$. Let collection of all sub intervals of $I = [0, 1]$ be represented by $\mathcal{C}(I)$.

Definition 7.1. Let $C_{ivf}(\Bbbk)$ be the collection of interval-valued fuzzy sets. Then $(\Upsilon, \eth)$, the interval-valued intuitionistic fuzzy hypersoft set ($\mathcal{IVIFHS}$-set) over $\Bbbk$ is defined as

$$(\Upsilon, \eth) = \left\{ \begin{array}{l} (< \Upsilon_{\tilde{\mathcal{T}}}(\varsigma), \Upsilon_{\tilde{\mathcal{L}}}(\varsigma) > /\varsigma, \Upsilon) : \varsigma \in \eth, \\ \Upsilon(\varsigma) \in C_{ivf}(\Bbbk), \Upsilon_{\tilde{\mathcal{T}}}(\varsigma), \Upsilon_{\tilde{\mathcal{L}}}(\varsigma) \in \mathcal{C}(I) \end{array} \right\}$$

where $\Upsilon : \eth \to C(\Bbbk)$ is approximate function and $\varsigma \in \eth = \eth_1 \times \eth_2 \times ... \times \eth_n$ with $\Upsilon_{\tilde{\mathcal{T}}}(\varsigma), \Upsilon_{\tilde{\mathcal{L}}}(\varsigma) : \eth \to I$ are truth membership and false membership functions of $\mathcal{IVIFHS}$-set such that the sum of upper bonds of $\Upsilon_{\tilde{\mathcal{T}}}$ and $\Upsilon_{\tilde{\mathcal{L}}}$ is less than or equal to 1.

Example 7.2. Let $L = \{\ell_1, \ell_2, \ell_3, \ell_4, \ell_5\}$ be the set of elements under consideration as in Example 2.8 and $T = \{\ \tau_1, \tau_2, \tau_3, ..., \tau_{16}\ \}$, where each τ_i is p^{th}-tuple, p is the product of orders of T_i. The $\mathcal{IVIFHS}$-set $(\Upsilon, \eth)$ is given by
$(\Upsilon, \eth) =$

$$\left\{ \begin{array}{l}
\left(\tau_1, \left\{\frac{[0.1,0.2];[0.8,0.9]}{\ell_1}\right\}, \left\{\frac{[0.3,0.4];[0.6,0.7]}{\ell_3}\right\}\right), \left(\tau_2, \left\{\frac{[0.3,0.6];[0.4,0.7]}{\ell_2}\right\}\right), \\
\left(\tau_3, \left\{\frac{[0.2,0.4];[0.3,0.4]}{\ell_4}\right\}\right), \left(\tau_4, \left\{\frac{[0.3,0.5];[0.2,0.4]}{\ell_3}\right\}, \left\{\frac{[0.4,0.5];[0.2,0.3]}{\ell_4}\right\} \left\{\frac{[0.1,0.3];[0.2,0.3]}{\ell_5}\right\}\right) \\
\left(\tau_5, \left\{\frac{[0.1,0.2];[0.2,0.5]}{\ell_1}\right\}, \left\{\frac{[0.4,0.5];[0.2,0.3]}{\ell_2}\right\}\right), \left(\tau_6, \left\{\frac{[0.3,0.6];[0.1,0.3]}{\ell_4}\right\}\right) \\
\left(\tau_7, \left\{\frac{[0.3,0.5];[0.2,0.4]}{\ell_1}\right\}, \left\{\frac{[0.2,0.6];[0.1,0.3]}{\ell_3}\right\}, \left\{\frac{[0.1,0.2];[0.4,0.5]}{\ell_5}\right\}\right), \left(\tau_8, \left\{\frac{[0.5,0.6];[0.2,0.3]}{\ell_1}\right\},\right) \\
\left(\tau_9, \left\{\frac{[0.3,0.4];[0.2,0.5]}{\ell_3}\right\}\right), \left(\tau_{10}, \left\{\frac{[0.1,0.3];[0.5,0.6]}{\ell_1}\right\}\right), \left(\tau_{11}, \left\{\frac{[0.3,0.4];[0.2,0.4]}{\ell_1}\right\}, \left\{\frac{[0.4,0.5];[0.1,0.3]}{\ell_5}\right\}\right), \\
\left(\tau_{12}, \left\{\frac{[0.2,0.3];[0.5,0.7]}{\ell_2}\right\}\right), \left(\tau_{13}, \left\{\frac{[0.6,0.7];[0.1,0.2]}{\ell_3}\right\}\right), \left(\tau_{14}, \left\{\frac{[0.1,0.3];[0.2,0.6]}{\ell_1}\right\}\right), \\
\left(\tau_{15}, \left\{\frac{[0.2,0.5];[0.3,0.4]}{\ell_1}\right\}, \left\{\frac{[0.2,0.3];[0.1,0.6]}{\ell_2}\right\}\right), \left(\tau_{16}, \left\{\frac{[0.2,0.3];[0.1,0.4]}{\ell_1}\right\}, \left\{\frac{[0.1,0.5];[0.2,0.4]}{\ell_4}\right\}\right)
\end{array} \right\}.$$

Definition 7.3. Let $C_{ivn}(\Bbbk)$ be the collection of interval-valued neutrosophic sets. Then $(\Gamma, \eth)$, the interval-valued neutrosophic hypersoft set ($\mathcal{IVNHS}$-set) over $\Bbbk$ is defined as

$$(\Gamma, \eth) = \{(< \Gamma_{\tilde{\mathcal{T}}}(\varsigma), \Gamma_{\tilde{\mathcal{I}}}(\varsigma), \Gamma_{\tilde{\mathcal{L}}}(\varsigma) > /\varsigma, \Gamma) : \varsigma \in \eth, \Gamma(\varsigma) \in C_{ivn}(\Bbbk), \Gamma_{\tilde{\mathcal{T}}}(\varsigma), \Gamma_{\tilde{\mathcal{I}}}(\varsigma), \Gamma_{\tilde{\mathcal{L}}}(\varsigma) \in \mathcal{C}(I)\}$$

where $\Gamma : \eth \to C(\Bbbk)$ is approximate function and $\varsigma \in \eth$ with $\Upsilon_{\tilde{\mathcal{T}}}(\varsigma), \Upsilon_{\tilde{\mathcal{I}}}(\varsigma), \Upsilon_{\tilde{\mathcal{L}}}(\varsigma) : \eth \to I$ are truth, indeterminacy and falsity membership functions of $\mathcal{IVNHS}$-set.

Example 7.4. Consider Example 7.2. The $\mathcal{IVNHS}$-set $(\Upsilon, \eth)$ is given by

$(\Gamma, \eth) =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{\frac{[0.3,0.6];[0.1,0.4];[0.2,0.5]}{\ell_1}\right\}, \left\{\frac{[0.2,0.5];[0.2,0.3];[0.1,0.2]}{\ell_3}\right\}\right), \left(\tau_2, \left\{\frac{[0.1,0.3];[0.2,0.5];[0.2,0.3]}{\ell_2}\right\}\right), \\ \left(\tau_3, \left\{\frac{[0.2,0.3];[0.1,0.4];[0.3,0.6]}{\ell_4}\right\}\right), \left(\tau_6, \left\{\frac{[0.4,0.5];[0.4,0.6];[0.2,0.3]}{\ell_4}\right\}\right), \\ \left(\tau_4, \left\{\frac{[0.3,0.4];[0.2,0.3];[0.3,0.7]}{\ell_3}\right\}, \left\{\frac{[0.6,0.7];[0.3,0.5];[0.1,0.3]}{\ell_4}\right\}, \left\{\frac{[0.3,0.6];[0.2,0.4];[0.1,0.3]}{\ell_5}\right\}\right) \\ \left(\tau_5, \left\{\frac{[0.1,0.3];[0.3,0.4];[0.4,0.5]}{\ell_1}\right\}, \left\{\frac{[0.2,0.3];[0.1,0.4];[0.4,0.7]}{\ell_2}\right\}\right), \left(\tau_9, \left\{\frac{[0.4,0.6];[0.2,0.3];[0.4,0.5]}{\ell_3}\right\}\right), \\ \left(\tau_7, \left\{\frac{[0.3,0.4];[0.3,0.7];[0.1,0.4]}{\ell_1}\right\}, \left\{\frac{[0.3,0.5];[0.4,0.6];[0.3,0.6]}{\ell_3}\right\}, \left\{\frac{[0.3,0.5];[0.2,0.4];[0.4,0.7]}{\ell_5}\right\}\right), \\ \left(\tau_{10}, \left\{\frac{[0.3,0.5];[0.2,0.5];[0.3,0.5]}{\ell_1}\right\}\right), \left(\tau_{11}, \left\{\frac{[0.4,0.7];[0.1,0.4];[0.5,0.6]}{\ell_1}\right\}, \left\{\frac{[0.3,0.5];[0.2,0.3];[0.6,0.7]}{\ell_5}\right\}\right), \\ \left(\tau_{12}, \left\{\frac{[0.6,0.8];[0.4,0.6];[0.3,0.7]}{\ell_2}\right\}\right), \left(\tau_{13}, \left\{\frac{[0.4,0.5];[0.4,0.7];[0.1,0.3]}{\ell_3}\right\}\right), \\ \left(\tau_{14}, \left\{\frac{[0.4,0.7];[0.2,0.5];[0.4,0.6]}{\ell_1}\right\}\right), \left(\tau_{15}, \left\{\frac{[0.1,0.3];[0.4,0.5];[0.2,0.4]}{\ell_1}\right\}, \left\{\frac{[0.5,0.6];[0.2,0.5];[0.4,0.8]}{\ell_2}\right\}\right), \\ \left(\tau_8, \left\{\frac{[0.1,0.2];[0.1,0.3];[0.3,0.5]}{\ell_1}\right\}\right), \left(\tau_{16}, \left\{\frac{[0.4,0.6];[0.4,0.5];[0.3,0.6]}{\ell_1}\right\}, \left\{\frac{[0.4,0.6];[0.3,0.5];[0.2,0.4]}{\ell_4}\right\}\right) \end{array} \right\}.$$

Definition 7.5. The interval-valued picture fuzzy hypersoft set ($\mathcal{IVPFHS}$-set) $(\wp, \eth)$, over $\Cup$ is given by

$$(\wp, \eth) = \{(< \wp_{\tilde{\mathcal{T}}}(\varsigma), \wp_{\tilde{\mathcal{I}}}(\varsigma), \wp_{\tilde{\mathcal{L}}}(\varsigma) > /\varsigma, \wp) : \varsigma \in \eth, \wp(\varsigma) \in C_{ivn}(\Cup), \wp_{\tilde{\mathcal{T}}}(\varsigma), \wp_{\tilde{\mathcal{I}}}(\varsigma), \wp_{\tilde{\mathcal{L}}}(\varsigma) \in \mathcal{C}\,(I)\}$$

where $\wp : \eth \to C(\Cup)$ is approximate function and $\varsigma \in \eth$ with $\wp_{\tilde{\mathcal{T}}}(\varsigma), \wp_{\tilde{\mathcal{I}}}(\varsigma), \wp_{\tilde{\mathcal{L}}}(\varsigma) : \eth \to I$ are truth, indeterminacy and falsity membership functions of $\mathcal{IVPFHS}$-set with condition that the sum of upper bonds of $\wp_{\tilde{\mathcal{T}}}, \wp_{\tilde{\mathcal{I}}}$ and $\wp_{\tilde{\mathcal{L}}}$ remain less than or equal to 1.

Example 7.6. Consider Example 7.2. The $\mathcal{IVPFHS}$-set $(\wp, \eth)$ is given by
$(\wp, \eth) =$

$$\left\{ \begin{array}{l} \left(\tau_1, \left\{\frac{[0.3,0.4];[0.1,0.2];[0.2,0.3]}{\ell_1}\right\}, \left\{\frac{[0.2,0.4];[0.2,0.3];[0.1,0.2]}{\ell_3}\right\}\right), \left(\tau_2, \left\{\frac{[0.1,0.3];[0.2,0.3];[0.2,0.3]}{\ell_2}\right\}\right), \\ \left(\tau_3, \left\{\frac{[0.2,0.3];[0.1,0.4];[0.3,0.6]}{\ell_4}\right\}\right), \left(\tau_6, \left\{\frac{[0.1,0.2];[0.4,0.5];[0.1,0.2]}{\ell_4}\right\}\right), \\ \left(\tau_4, \left\{\frac{[0.0,0.2];[0.2,0.3];[0.3,0.4]}{\ell_3}\right\}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.0,0.1]}{\ell_4}\right\}, \left\{\frac{[0.2,0.3];[0.2,0.4];[0.1,0.3]}{\ell_5}\right\}\right) \\ \left(\tau_5, \left\{\frac{[0.0,0.1];[0.3,0.4];[0.4,0.5]}{\ell_1}\right\}, \left\{\frac{[0.2,0.3];[0.1,0.2];[0.4,0.5]}{\ell_2}\right\}\right), \left(\tau_9, \left\{\frac{[0.1,0.4];[0.1,0.2];[0.1,0.3]}{\ell_3}\right\}\right), \\ \left(\tau_7, \left\{\frac{[0.3,0.4];[0.3,0.4];[0.0,0.1]}{\ell_1}\right\}, \left\{\frac{[0.2,0.4];[0.3,0.4];[0.1,0.2]}{\ell_3}\right\}, \left\{\frac{[0.0,0.3];[0.2,0.3];[0.3,0.4]}{\ell_5}\right\}\right), \\ \left(\tau_{10}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.0,0.2]}{\ell_1}\right\}\right), \left(\tau_{11}, \left\{\frac{[0.1,0.2];[0.1,0.2];[0.5,0.6]}{\ell_1}\right\}, \left\{\frac{[0.3,0.4];[0.2,0.3];[0.2,0.3]}{\ell_5}\right\}\right), \\ \left(\tau_{12}, \left\{\frac{[0.6,0.8];[0.0,0.1];[0.0,0.1]}{\ell_2}\right\}\right), \left(\tau_{13}, \left\{\frac{[0.0,0.1];[0.4,0.5];[0.0,0.3]}{\ell_3}\right\}\right), \\ \left(\tau_{14}, \left\{\frac{[0.1,0.3];[0.2,0.3];[0.3,0.4]}{\ell_1}\right\}\right), \left(\tau_{15}, \left\{\frac{[0.1,0.3];[0.4,0.5];[0.0,0.1]}{\ell_1}\right\}, \left\{\frac{[0.5,0.6];[0.2,0.3];[0.0,0.1]}{\ell_2}\right\}\right), \\ \left(\tau_8, \left\{\frac{[0.1,0.2];[0.1,0.3];[0.3,0.5]}{\ell_1}\right\}\right), \left(\tau_{16}, \left\{\frac{[0.0,0.6];[0.0,0.1];[0.0,0.2]}{\ell_1}\right\}, \left\{\frac{[0.4,0.5];[0.3,0.4];[0.0,0.1]}{\ell_4}\right\}\right) \end{array} \right\}.$$

Definition 7.7. $(\Delta, \eth)$ is called fuzzy parameterized hypersoft set ($\mathcal{FPHS}$-set) over $\Cup$ and is defined as

$$(\Delta, \eth) = \{(\xi_F(\varsigma)/\varsigma, \pi_F(\varsigma) :), \varsigma \in \eth, \pi_F(\varsigma) \in P(\Cup), \xi_F(\varsigma) \in I\}$$

where F is fuzzy set and $\xi_F : \eth \to I$ is membership function of $\mathcal{FPHS}$-set and $\pi_F : \eth \to P(\Cup)$ is approximate function.

Example 7.8. Consider Example 7.2. The $\mathcal{FPHS}$-set $(\Delta, \eth)$ is given by
$(\Delta, \eth) =$

$$\left\{ \begin{array}{l} \left(\frac{0.1}{\tau_1}, \{\ell_1, \ell_3\}\right), \left(\frac{0.3}{\tau_2}, \{\ell_2\}\right), \left(\frac{0.4}{\tau_3}, \{\ell_4\}\right), \left(\frac{0.1}{\tau_4}, \{\ell_3, \ell_4, \ell_5\}\right), \left(\frac{0.7}{\tau_5}, \{\ell_1, \ell_2\}\right), \left(\frac{0.2}{\tau_6}, \{\ell_4\}\right), \\ \left(\frac{0.1}{\tau_7}, \{\ell_1, \ell_3, \ell_5\}\right), \left(\frac{0.2}{\tau_8}, \{\ell_1\}\right), \left(\frac{0.4}{\tau_9}, \{\ell_3\}\right), \left(\frac{0.3}{\tau_{10}}, \{\ell_1\}\right), \left(\frac{0.3}{\tau_{11}}, \{\ell_1, \ell_5\}\right), \left(\frac{0.1}{\tau_{12}}, \{\ell_2\}\right), \\ \left(\frac{0.4}{\tau_{13}}, \{\ell_3\}\right), \left(\frac{0.1}{\tau_{14}}, \{\ell_1\}\right), \left(\frac{0.2}{\tau_{15}}, \{\ell_1, \ell_2\}\right), \left(\frac{0.3}{\tau_{16}}, \{\ell_1, \ell_4\}\right) \end{array} \right\}.$$

Definition 7.9. $(\nabla, \eth)$ is called fuzzy parameterized fuzzy hypersoft set ($\mathcal{FPFHS}$-set) over $\Cup$ and is defined as

$$(\nabla, \eth) = \{(\varsigma_M(\varsigma)/\varsigma, \sigma_M(\varsigma)/z) : \varsigma \in \eth, \varsigma_M(\varsigma) \in I, \sigma_M(\varsigma) \in I\}$$

where ς is fuzzy set and $\varsigma_M : \eth \to I$ is membership function of $\mathcal{FPFHS}$-set and $\sigma_M : \eth \to P(\Cup)$ is approximate function.

Example 7.10. Consider Example 7.2. The $\mathcal{FPFHS}$-set $(\nabla, \eth)$ is given by
$(\nabla, \eth) =$

$$\left\{ \begin{array}{l} \left(\frac{0.1}{\tau_1}, \left\{\frac{0.2}{\ell_1}, \frac{0.3}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_2}, \left\{\frac{0.1}{\ell_2}\right\}\right), \left(\frac{0.4}{\tau_3}, \left\{\frac{0.5}{\ell_4}\right\}\right), \left(\frac{0.1}{\tau_4}, \left\{\frac{0.2}{\ell_3}, \frac{0.4}{\ell_4}, \frac{0.1}{\ell_5}\right\}\right), \left(\frac{0.7}{\tau_5}, \left\{\frac{0.6}{\ell_1}, \frac{0.2}{\ell_2}\right\}\right), \\ \left(\frac{0.2}{\tau_6}, \left\{\frac{0.4}{\ell_4}\right\}\right), \left(\frac{0.1}{\tau_7}, \left\{\frac{0.2}{\ell_1}, \frac{0.3}{\ell_3}, \frac{0.1}{\ell_5}\right\}\right), \left(\frac{0.2}{\tau_8}, \left\{\frac{0.2}{\ell_1}\right\}\right), \left(\frac{0.4}{\tau_9}, \left\{\frac{0.5}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_{10}}, \left\{\frac{0.1}{\ell_1}\right\}\right), \\ \left(\frac{0.3}{\tau_{11}}, \left\{\frac{0.1}{\ell_1}, \frac{0.4}{\ell_5}\right\}\right), \left(\frac{0.1}{\tau_{12}}, \left\{\frac{0.3}{\ell_2}\right\}\right), \left(\frac{0.4}{\tau_{13}}, \left\{\frac{0.2}{\ell_3}\right\}\right), \left(\frac{0.1}{\tau_{14}}, \left\{\frac{0.1}{\ell_1}\right\}\right), \left(\frac{0.2}{\tau_{15}}, \left\{\frac{0.1}{\ell_1}, \frac{0.7}{\ell_2}\right\}\right), \left(\frac{0.3}{\tau_{16}}, \left\{\frac{0.2}{\ell_1}, \frac{0.4}{\ell_4}\right\}\right) \end{array} \right\}.$$

Definition 7.11. $(\mho_{fp-ivfhs}, \eth)$ is called fuzzy parameterized interval-valued fuzzy hypersoft set ($\mathcal{FPIVFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{fp-ivfhs}, \eth) = \left\{ \left(\frac{\mathcal{A}_{\mathcal{M}}(\varsigma)}{\varsigma}, \frac{\mathbb{A}_{\mathbb{M}}(\varsigma)}{z} \right) : \varsigma \in \eth, \mathcal{A}_{\mathcal{M}}(\varsigma) \in I, \mathbb{A}_{\mathbb{M}}(\varsigma) \in \mathcal{C}\left(I\right) \right\}$$

where $\mathcal{A}_{\mathcal{M}}$ is truth-ness of membership function $\mathcal{A} : \eth \to I$ and $\mathbb{A}_{\mathbb{M}}$ is truth-ness of approximate function $\mathbb{A} : \eth \to P(\mathbb{U})$ of $\mathcal{FPIVFHS}$-set.

Example 7.12. Consider Example 7.2. The $\mathcal{FPIVFHS}$-set $(\mho_{fp-ivfhs}, \eth)$ is given by
$(\mho_{fp-ivfhs}, \eth) =$

$$\left\{ \begin{array}{l} \left(\frac{0.1}{\tau_1}, \left\{\frac{[0.1,0.2]}{\ell_1}, \frac{[0.2,0.3}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_2}, \left\{\frac{[0.1,0,4]}{\ell_2}\right\}\right), \left(\frac{0.4}{\tau_3}, \left\{\frac{[0.2,0.5]}{\ell_4}\right\}\right), \left(\frac{0.1}{\tau_4}, \left\{\frac{[0.2,0.3]}{\ell_3}, \frac{[o.1,0.4]}{\ell_4}, \frac{[0.1,0.5]}{\ell_5}\right\}\right), \\ \left(\frac{0.7}{\tau_5}, \left\{\frac{[0.3,0.6]}{\ell_1}, \frac{[0.1,0.2]}{\ell_2}\right\}\right), \left(\frac{0.2}{\tau_6}, \left\{\frac{[0.2,0.4]}{\ell_4}\right\}\right), \left(\frac{0.1}{\tau_7}, \left\{\frac{[0.2,0.3]}{\ell_1}, \frac{[0.1,0.3]}{\ell_3}, \frac{[0.1,0.5]}{\ell_5}\right\}\right), \left(\frac{0.2}{\tau_8}, \left\{\frac{[0.2,0.5]}{\ell_1}\right\}\right), \\ \left(\frac{0.4}{\tau_9}, \left\{\frac{[0.3,0.5]}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_{10}}, \left\{\frac{[0.1,0.3]}{\ell_1}\right\}\right), \left(\frac{0.3}{\tau_{11}}, \left\{\frac{[0.1,0.3]}{\ell_1}, \frac{[0.1,0.4]}{\ell_5}\right\}\right), \left(\frac{0.1}{\tau_{12}}, \left\{\frac{[0.2,0.3]}{\ell_2}\right\}\right), \\ \left(\frac{0.4}{\tau_{13}}, \left\{\frac{[0.2,0.6]}{\ell_3}\right\}\right), \left(\frac{0.1}{\tau_{14}}, \left\{\frac{[0.1,0.4]}{\ell_1}\right\}\right), \left(\frac{0.2}{\tau_{15}}, \left\{\frac{[0.1,0.5]}{\ell_1}, \frac{[0.3,0.7]}{\ell_2}\right\}\right), \left(\frac{0.3}{\tau_{16}}, \left\{\frac{[0.2,0.5]}{\ell_1}, \frac{[0.1,0.4]}{\ell_4}\right\}\right) \end{array} \right\}.$$

Definition 7.13. $(\mho_{fp-ivifhs}, \eth)$ is called fuzzy parameterized interval-valued intuitionistic fuzzy hypersoft set ($\mathcal{FPIVIFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{fp-ivifhs}, \eth) = \left\{ \begin{array}{l} \left(\frac{\mathcal{B}_{\mathcal{M}}(\varsigma)}{\varsigma}, \frac{\{\mathbb{B}_{\mathbb{M}}(\varsigma);\mathbb{B}_{\mathbb{N}}(\varsigma)\}}{z} \right) : \varsigma \in \eth, \\ \mathcal{B}_{\mathcal{M}}(\varsigma) \in I, \mathbb{B}_{\mathbb{M}}(\varsigma), \mathbb{B}_{\mathbb{N}}(\varsigma) \in \mathcal{C}\left(I\right) \end{array} \right\}$$

where $\mathcal{B}_{\mathcal{M}}$ is truth-ness of membership function $\mathcal{B} : \eth \to I$ and $\mathbb{B}_{\mathbb{M}}$ and $\mathbb{B}_{\mathbb{N}}$ are truth-ness and false-ness of approximate function $\mathbb{B} : \eth \to P(\mathbb{U})$ of $\mathcal{FPIVIFHS}$-set with condition that the sum of upper bonds of $\mathbb{B}_{\mathbb{M}}$ and $\mathbb{B}_{\mathbb{N}}$ remain less than or equal to 1.

Example 7.14. Consider Example 7.2. The $\mathcal{FPIVIFHS}$-set $(\mho_{fp-ivifhs}, \eth)$ is given by
$(\mho_{fp-ivifhs}, \eth) =$

$$\left\{ \begin{array}{l} \left(\frac{0.1}{\tau_1}, \left\{\frac{[0.5,0.7];[0.1,0.2]}{\ell_1}, \frac{[0.2,0.5];[0.2,0.3}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_2}, \left\{\frac{[0.1,0.4];[0.2,0,4]}{\ell_2}\right\}\right), \left(\frac{0.4}{\tau_3}, \left\{\frac{[0.2,0.5];[0.3,0.4]}{\ell_4}\right\}\right), \\ \left(\frac{0.1}{\tau_4}, \left\{\frac{[0.2,0.3];[0.5,0.6]}{\ell_3}, \frac{[o.1,0.4];[0.3,0.5]}{\ell_4}, \frac{[0.1,0.2];[0.3,0.5]}{\ell_5}\right\}\right), \left(\frac{0.7}{\tau_5}, \left\{\frac{[0.1,0.2];[0.3,0.5]}{\ell_1}, \frac{[0.1,0.2];[0.4,0.7]}{\ell_2}\right\}\right), \\ \left(\frac{0.2}{\tau_6}, \left\{\frac{[0.2,0.3];[0.5,0.6]}{\ell_4}\right\}\right), \left(\frac{0.1}{\tau_7}, \left\{\frac{[0.2,0.3];[0.3,0.4]}{\ell_1}, \frac{[0.1,0.3];[0.1,0.2]}{\ell_3}, \frac{[0.1,0.2];[0.3,0.5]}{\ell_5}\right\}\right), \\ \left(\frac{0.2}{\tau_8}, \left\{\frac{[0.1,0.2];[0.2,0.5]}{\ell_1}\right\}\right), \left(\frac{0.4}{\tau_9}, \left\{\frac{[0.1,0.4];[0.3,0.5]}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_{10}}, \left\{\frac{[0.1,0.3];[0.2,0.5]}{\ell_1}\right\}\right), \\ \left(\frac{0.3}{\tau_{11}}, \left\{\frac{[0.1,0.4];[0.3,0.5]}{\ell_1}, \frac{[0.1,0.4];[0.4,0.5]}{\ell_5}\right\}\right), \left(\frac{0.1}{\tau_{12}}, \left\{\frac{[0.2,0.3];[0.4,0.6]}{\ell_2}\right\}\right), \left(\frac{0.4}{\tau_{13}}, \left\{\frac{[0.1,0.4];[0.2,0.6]}{\ell_3}\right\}\right), \\ \left(\frac{0.1}{\tau_{14}}, \left\{\frac{[0.2,0.4];[0.3,0.4]}{\ell_1}\right\}\right), \left(\frac{0.2}{\tau_{15}}, \left\{\frac{[0.1,0.3];[0.4,0.5]}{\ell_1}, \frac{[0.1,0.2];[0.6,0.7]}{\ell_2}\right\}\right), \\ \left(\frac{0.3}{\tau_{16}}, \left\{\frac{[0.2,0.4];[0.3,0.5]}{\ell_1}, \frac{[0.1,0.2];[0.4,0.6]}{\ell_4}\right\}\right) \end{array} \right\}.$$

Definition 7.15. $(\mho_{fp-ivnhs}, \eth)$ is called fuzzy parameterized interval-valued neutrosophic hypersoft set ($\mathcal{FPIVNHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{fp-ivnhs}, \eth) = \left\{ \begin{array}{l} \left(\frac{\mathcal{C}_{\mathcal{M}}(\varsigma)}{\varsigma}, \frac{\{\mathbb{C}_{\mathbb{M}}(\varsigma);\mathbb{C}_{\mathbb{N}}(\varsigma);\mathbb{C}_{\mathbb{I}}(\varsigma)\}}{z} \right) : \varsigma \in \eth, \mathcal{C}_{\mathcal{M}}(\varsigma) \in I, \\ \mathbb{C}_{\mathbb{M}}(\varsigma), \mathbb{C}_{\mathbb{N}}(\varsigma), \mathbb{C}_{\mathbb{I}}(\varsigma) \in \mathcal{C}\left(I\right) \end{array} \right\}$$

where $\mathcal{C}_{\mathcal{M}}$ is truth-ness of membership function $\mathcal{C} : \eth \to I$ and $\mathbb{C}_{\mathbb{M}}, \mathbb{C}_{\mathbb{N}}$ and $\mathbb{C}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{C} : \eth \to P(\mathbb{U})$ of $\mathcal{FPIVNHS}$-set .

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Example 7.16. Consider Example 7.2. The $\mathcal{FPIVNHS}$-set $(\mho_{fp-ivnhs}, \eth)$ is given by
$(\mho_{fp-ivnhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{0.1}{\tau_1}, \left\{\frac{[0.4,0.6];[0.3,0.4];[0.2,0.3]}{\ell_1}, \frac{[0.3,0.5];[0.2,0.5];[0.2,0.5]}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_2}, \left\{\frac{[0.1,0.3];[0.2,0.4];[0.3,0,4]}{\ell_2}\right\}\right), \\
\left(\frac{0.1}{\tau_4}, \left\{\frac{[0.3,0.5];[0.3,0.7];[0.5,0.6]}{\ell_3}, \frac{[o.1,0.2];[0.3,0.6];[0.3,0.5]}{\ell_4}, \frac{[0.1,0.2];[0.3,0.6];[0.2,0.5]}{\ell_5}\right\}\right), \\
\left(\frac{0.4}{\tau_3}, \left\{\frac{[0.1,0.4];[0.3,0.5];[0.3,0.6]}{\ell_4}\right\}\right), \left(\frac{0.7}{\tau_5}, \left\{\frac{[0.4,0.7];[0.3,0.4];[0.1,0.5]}{\ell_1}, \frac{[0.6,0.8];[0.1,0.3];[0.5,0.7]}{\ell_2}\right\}\right), \\
\left(\frac{0.2}{\tau_6}, \left\{\frac{[0.1,0.4];[0.1,0.3];[0.5,0.7]}{\ell_4}\right\}\right), \left(\frac{0.2}{\tau_8}, \left\{\frac{[0.1,0.3];[0.1,0.4];[0.2,0.3]}{\ell_1}\right\}\right), \\
\left(\frac{0.1}{\tau_7}, \left\{\frac{[0.2,0.6];[0.5,0.7];[0.3,0.4]}{\ell_1}, \frac{[0.3,0.6];[0.1,0.3];[0.2,0.3]}{\ell_3}, \frac{[0.4,0.8];[0.2,0.4];[0.3,0.4]}{\ell_5}\right\}\right), \\
\left(\frac{0.4}{\tau_9}, \left\{\frac{[0.2,0.4];[0.3,0.4];[0.1,0.5]}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_{10}}, \left\{\frac{[0.1,0.2];[0.3,0.5];[0.4,0.5]}{\ell_1}\right\}\right), \\
\left(\frac{0.3}{\tau_{11}}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.3,0.7]}{\ell_1}, \frac{[0.5,0.7];[0.1,0.3];[0.4,0.6]}{\ell_5}\right\}\right), \left(\frac{0.1}{\tau_{12}}, \left\{\frac{[0.1,0.3];[0.3,0.6];[0.3,0.6]}{\ell_2}\right\}\right), \\
\left(\frac{0.4}{\tau_{13}}, \left\{\frac{[0.3,0.6];[0.2,0.4];[0.4,0.6]}{\ell_3}\right\}\right), \left(\frac{0.2}{\tau_{15}}, \left\{\frac{[0.3,0.6];[0.2,0.3];[0.4,0.7]}{\ell_1}, \frac{[0.1,0.3];[0.5,0.7];[0.3,0.4]}{\ell_2}\right\}\right), \\
\left(\frac{0.1}{\tau_{14}}, \left\{\frac{[0.3,0.4];[0.3,0.7];[0.2,0.4]}{\ell_1}\right\}\right), \left(\frac{0.3}{\tau_{16}}, \left\{\frac{[0.2,0.6];[0.3,0.4];[0.1,0.2]}{\ell_1}, \frac{[0.6,0.8];[0.3,0.4];[0.2,0.4]}{\ell_4}\right\}\right)
\end{array}\right\}.$$

Definition 7.17. $(\mho_{fp-ivpfhs}, \eth)$ is called fuzzy parameterized interval-valued picture fuzzy hypersoft set ($\mathcal{FPIVPFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{fp-ivpfhs}, \eth) = \left\{ \begin{array}{l} \left(\frac{\mathcal{D}_{\mathcal{M}}(\varsigma)}{\varsigma}, \frac{\{\mathbb{D}_{\mathbb{M}}(\varsigma);\mathbb{D}_{\mathbb{N}}(\varsigma);\mathbb{D}_{\mathbb{I}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \mathcal{D}_{\mathcal{M}}(\varsigma) \in I, \\ \mathbb{D}_{\mathbb{M}}(\varsigma), \mathbb{D}_{\mathbb{N}}(\varsigma), \mathbb{D}_{\mathbb{I}}(\varsigma) \in \mathcal{C}\left(I\right) \end{array} \right\}$$

where $\mathcal{D}_{\mathcal{M}}$ is truth-ness of membership function $\mathcal{D} : \eth \to I$ and $\mathbb{D}_{\mathbb{M}}, \mathbb{D}_{\mathbb{N}}$ and $\mathbb{D}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{D} : \eth \to P(\mathbb{U})$ of $\mathcal{FPIVPFHS}$-set with condition that the sum of upper bonds of $\mathbb{D}_{\mathbb{M}}, \mathbb{D}_{\mathbb{N}}$ and $\mathbb{D}_{\mathbb{I}}$ remain less than or equal to 1.

Example 7.18. Consider Example 7.2. The $\mathcal{FPIVPFHS}$-set $(\mho_{fp-ivpfhs}, \eth)$ is given by
$(\mho_{fp-ivpfhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{0.1}{\tau_1}, \left\{\frac{[0.2,0.3];[0.3,0.4];[0.1,0.3]}{\ell_1}, \frac{[0.1,0.2];[0.2,0.3];[0.2,0.3]}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_2}, \left\{\frac{[0.1,0.3];[0.2,0.3];[0.3,0,4]}{\ell_2}\right\}\right), \\
\left(\frac{0.1}{\tau_4}, \left\{\frac{[0.1,0.2];[0.1,0.2];[0.5,0.6]}{\ell_3}, \frac{[0.0,0.1];[0.3,0.6];[0.2,0.3]}{\ell_4}, \frac{[0.1,0.2];[0.3,0.4];[0.2,0.4]}{\ell_5}\right\}\right), \\
\left(\frac{0.4}{\tau_3}, \left\{\frac{[0.1,0.4];[0.3,0.5];[0.0,0.1]}{\ell_4}\right\}\right), \left(\frac{0.7}{\tau_5}, \left\{\frac{[0.0,0.1];[0.3,0.4];[0.1,0.5]}{\ell_1}, \frac{[0.6,0.7];[0.0,0.1];[0.1,0.2]}{\ell_2}\right\}\right), \\
\left(\frac{0.2}{\tau_6}, \left\{\frac{[0.1,0.4];[0.1,0.3];[0.0,0.3]}{\ell_4}\right\}\right), \left(\frac{0.2}{\tau_8}, \left\{\frac{[0.1,0.3];[0.1,0.4];[0.2,0.3]}{\ell_1}\right\}\right), \\
\left(\frac{0.1}{\tau_7}, \left\{\frac{[0.2,0.6];[0.1,0.3];[0.0,0.1]}{\ell_1}, \frac{[0.3,0.6];[0.0,0.1];[0.0,0.1]}{\ell_3}, \frac{[0.4,0.6];[0.0,0.2];[0.0,0.1]}{\ell_5}\right\}\right), \\
\left(\frac{0.4}{\tau_9}, \left\{\frac{[0.2,0.4];[0.3,0.4];[0.1,0.2]}{\ell_3}\right\}\right), \left(\frac{0.3}{\tau_{10}}, \left\{\frac{[0.1,0.2];[0.3,0.5];[0.0,0.2]}{\ell_1}\right\}\right), \\
\left(\frac{0.3}{\tau_{11}}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.0,0.1]}{\ell_1}, \frac{[0.5,0.7];[0.0,0.1];[0.1,0.2]}{\ell_5}\right\}\right), \left(\frac{0.1}{\tau_{12}}, \left\{\frac{[0.1,0.3];[0.3,0.6];[0.0,0.1]}{\ell_2}\right\}\right), \\
\left(\frac{0.4}{\tau_{13}}, \left\{\frac{[0.3,0.6];[0.2,0.3];[0.0,0.1]}{\ell_3}\right\}\right), \left(\frac{0.2}{\tau_{15}}, \left\{\frac{[0.3,0.5];[0.2,0.3];[0.1,0.2]}{\ell_1}, \frac{[0.1,0.3];[0.0,0.2];[0.3,0.4]}{\ell_2}\right\}\right), \\
\left(\frac{0.1}{\tau_{14}}, \left\{\frac{[0.3,0.4];[0.1,0.2];[0.2,0.4]}{\ell_1}\right\}\right), \left(\frac{0.3}{\tau_{16}}, \left\{\frac{[0.2,0.6];[0.0,0.2];[0.1,0.2]}{\ell_1}, \frac{[0.2,0.5];[0.2,0.3];[0.0,0.1]}{\ell_4}\right\}\right)
\end{array}\right\}.$$

Definition 7.19. $(\mho_{ifp-ivfhs}, \eth)$ is called intuitionistic fuzzy parameterized interval-valued fuzzy hypersoft set ($\mathcal{IFPIVFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{ifp-ivfhs}, \eth) = \left\{\left(\frac{\{\mathcal{E}_{\mathcal{M}}(\varsigma); \mathcal{E}_{\mathcal{N}}(\varsigma)\}}{\varsigma}, \frac{\mathbb{E}_{\mathbb{M}}(\varsigma)}{z}\right) : \varsigma \in \eth, \mathcal{E}_{\mathcal{M}}(\varsigma), \mathcal{E}_{\mathcal{N}}(\varsigma) \in I, \mathbb{E}_{\mathbb{M}}(\varsigma) \in \mathcal{C}\left(I\right)\right\}$$

where $\mathcal{E}_{\mathcal{M}}, \mathcal{E}_{\mathcal{N}}$ are truth-ness and false-ness of membership function $\mathcal{E} : \eth \to I$ such that $\mathcal{E}_{\mathcal{M}} + \mathcal{E}_{\mathcal{N}} \leq 1$ and $\mathbb{E}_{\mathbb{M}}$ is truth-ness of approximate function $\mathbb{E} : \eth \to P(\mathbb{U})$ of $\mathcal{IFPIVFHS}$-set.

Example 7.20. Consider Example 7.2. The $\mathcal{IFPIVFHS}$-set $(\mho_{ifp-ivfhs}, \eth)$ is given by

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$(\mho_{ifp-ivfhs}, \eth) =$

$$\left\{\begin{array}{l} \left(\frac{\{0.5;0.1\}}{\tau_1}, \left\{\frac{[0.1,0.2]}{\ell_1}, \frac{[0.2,0.3}{\ell_3}\right\}\right), \left(\frac{\{0.6;0.3\}}{\tau_2}, \left\{\frac{[0.1,0,4]}{\ell_2}\right\}\right), \left(\frac{\{0.5;0.4\}}{\tau_3}, \left\{\frac{[0.2,0.5]}{\ell_4}\right\}\right), \\ \left(\frac{\{0.5;0.1\}}{\tau_4}, \left\{\frac{[0.2,0.3]}{\ell_3}, \frac{[o.1,0.4]}{\ell_4}, \frac{[0.1,0.5]}{\ell_5}\right\}\right), \left(\frac{\{0.7;0.2\}}{\tau_5}, \left\{\frac{[0.3,0.6]}{\ell_1}, \frac{[0.1,0.2]}{\ell_2}\right\}\right), \\ \left(\frac{\{0.4;0.2\}}{\tau_6}, \left\{\frac{[0.2,0.4]}{\ell_4}\right\}\right), \left(\frac{\{0.4;0.1\}}{\tau_7}, \left\{\frac{[0.2,0.3]}{\ell_1}, \frac{[0.1,0.3]}{\ell_3}, \frac{[0.1,0.5]}{\ell_5}\right\}\right), \left(\frac{\{0.5;0.2\}}{\tau_8}, \left\{\frac{[0.2,0.5]}{\ell_1}\right\}\right), \\ \left(\frac{\{0.4;0.2\}}{\tau_9}, \left\{\frac{[0.3,0.5]}{\ell_3}\right\}\right), \left(\frac{\{0.5;0.3\}}{\tau_{10}}, \left\{\frac{[0.1,0.3]}{\ell_1}\right\}\right), \left(\frac{\{0.5;0.1\}}{\tau_{11}}, \left\{\frac{[0.1,0.3]}{\ell_1}, \frac{[0.1,0.4]}{\ell_5}\right\}\right), \\ \left(\frac{\{0.4,0.3\}}{\tau_{12}}, \left\{\frac{[0.2,0.3]}{\ell_2}\right\}\right), \left(\frac{\{0.4;0,5\}}{\tau_{13}}, \left\{\frac{[0.2,0.6]}{\ell_3}\right\}\right), \left(\frac{\{0.6;0.2\}}{\tau_{14}}, \left\{\frac{[0.1,0.4]}{\ell_1}\right\}\right), \\ \left(\frac{\{0.7;0.2\}}{\tau_{15}}, \left\{\frac{[0.1,0.5]}{\ell_1}, \frac{[0.3,0.7]}{\ell_2}\right\}\right), \left(\frac{\{0.5;0.3\}}{\tau_{16}}, \left\{\frac{[0.2,0.5]}{\ell_1}, \frac{[0.1,0.4]}{\ell_4}\right\}\right) \end{array}\right\}.$$

Definition 7.21. $(\mho_{ifp-ivifhs}, \eth)$ is called intuitionistic fuzzy parameterized interval-valued intuitionistic fuzzy hypersoft set ($\mathcal{IFPIVIFHS}$-set) over $\Bbbk$ and is defined as

$$(\mho_{ifp-ivifhs}, \eth) = \left\{\begin{array}{l} \left(\frac{\{\mathcal{F}_{\mathcal{M}}(\varsigma);\mathcal{F}_{\mathcal{N}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{F}_{\mathbb{M}}(\varsigma);\mathbb{F}_{\mathbb{N}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \\ \mathcal{F}_{\mathcal{M}}(\varsigma), \mathcal{F}_{\mathcal{N}}(\varsigma) \in I, \mathbb{F}_{\mathbb{M}}(\varsigma), \mathbb{F}_{\mathbb{N}}(\varsigma) \in \mathcal{C}(I) \end{array}\right\}$$

where $\mathcal{F}_{\mathcal{M}}$ and $\mathcal{F}_{\mathcal{N}}$ are truth-ness and false-ness of membership function $\mathcal{F} : \eth \to I$ such that $\mathcal{F}_{\mathcal{M}} + \mathcal{F}_{\mathcal{N}} \leq 1$ and $\mathbb{F}_{\mathbb{M}}$ and $\mathbb{F}_{\mathbb{N}}$ are truth-ness and false-ness of approximate function $\mathbb{F} : \eth \to P(\Bbbk)$ of $\mathcal{IFPIVIFHS}$-set with condition that the sum of upper bonds of $\mathbb{F}_{\mathbb{M}}$ and $\mathbb{F}_{\mathbb{N}}$ remain less than or equal to 1.

Example 7.22. Consider Example 7.2. The $\mathcal{IFPIVIFHS}$-set $(\mho_{ifp-ivifhs}, \eth)$ is given by
$(\mho_{ifp-ivifhs}, \eth) =$

$$\left\{\begin{array}{l} \left(\frac{\{0.5;0.1\}}{\tau_1}, \left\{\frac{[0.5,0.7];[0.1,0.2]}{\ell_1}, \frac{[0.2,0.5];[0.2,0.3}{\ell_3}\right\}\right), \left(\frac{\{0.6;0.3\}}{\tau_2}, \left\{\frac{[0.1,0.4];[0.2,0,4]}{\ell_2}\right\}\right), \\ \left(\frac{\{0.4;0.1\}}{\tau_3}, \left\{\frac{[0.2,0.5];[0.3,0.4]}{\ell_4}\right\}\right), \left(\frac{\{0.6;0.1\}}{\tau_4}, \left\{\frac{[0.2,0.3];[0.5,0.6]}{\ell_3}, \frac{[o.1,0.4];[0.3,0.5]}{\ell_4}, \frac{[0.1,0.2];[0.3,0.5]}{\ell_5}\right\}\right), \\ \left(\frac{\{0.7;0.2\}}{\tau_5}, \left\{\frac{[0.1,0.2];[0.3,0.5]}{\ell_1}, \frac{[0.1,0.2];[0.4,0.7]}{\ell_2}\right\}\right), \left(\frac{\{0.4;0.2\}}{\tau_6}, \left\{\frac{[0.2,0.3];[0.5,0.6]}{\ell_4}\right\}\right), \\ \left(\frac{\{0.6;0.1\}}{\tau_7}, \left\{\frac{[0.2,0.3];[0.3,0.4]}{\ell_1}, \frac{[0.1,0.3];[0.1,0.2]}{\ell_3}, \frac{[0.1,0.2];[0.3,0.5]}{\ell_5}\right\}\right), \left(\frac{\{0.3;0.2\}}{\tau_8}, \left\{\frac{[0.1,0.2];[0.2,0.5]}{\ell_1}\right\}\right), \\ \left(\frac{\{0.5;0.4\}}{\tau_9}, \left\{\frac{[0.1,0.4];[0.3,0.5]}{\ell_3}\right\}\right), \left(\frac{\{0.5;0.3\}}{\tau_{10}}, \left\{\frac{[0.1,0.3];[0.2,0.5]}{\ell_1}\right\}\right), \\ \left(\frac{\{0.6;0.3\}}{\tau_{11}}, \left\{\frac{[0.1,0.4];[0.3,0.5]}{\ell_1}, \frac{[0.1,0.4];[0.4,0.5]}{\ell_5}\right\}\right), \left(\frac{\{0.7;0.1\}}{\tau_{12}}, \left\{\frac{[0.2,0.3];[0.4,0.6]}{\ell_2}\right\}\right), \\ \left(\frac{\{0.4;0.2\}}{\tau_{13}}, \left\{\frac{[0.1,0.4];[0.2,0.6]}{\ell_3}\right\}\right), \left(\frac{\{0.6;0.2\}}{\tau_{15}}, \left\{\frac{[0.1,0.3];[0.4,0.5]}{\ell_1}, \frac{[0.1,0.2];[0.6,0.7]}{\ell_2}\right\}\right), \\ \left(\frac{\{0.5;0.1\}}{\tau_{14}}, \left\{\frac{[0.2,0.4];[0.3,0.4]}{\ell_1}\right\}\right), \left(\frac{\{0.4;0.3\}}{\tau_{16}}, \left\{\frac{[0.2,0.4];[0.3,0.5]}{\ell_1}, \frac{[0.1,0.2];[0.4,0.6]}{\ell_4}\right\}\right) \end{array}\right\}.$$

Definition 7.23. $(\mho_{ifp-ivnhs}, \eth)$ is called intuitionistic fuzzy parameterized interval-valued neutrosophic hypersoft set ($\mathcal{IFPIVNHS}$-set) over $\Bbbk$ and is defined as

$$(\mho_{ifp-ivnhs}, \eth) = \left\{\begin{array}{l} \left(\frac{\{\mathcal{G}_{\mathcal{M}}(\varsigma);\mathcal{G}_{\mathcal{N}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{G}_{\mathbb{M}}(\varsigma);\mathbb{G}_{\mathbb{N}}(\varsigma);\mathbb{G}_{\mathbb{I}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \mathcal{G}_{\mathcal{M}}(\varsigma), \mathcal{G}_{\mathcal{N}}(\varsigma) \in I, \\ \mathbb{G}_{\mathbb{M}}(\varsigma), \mathbb{G}_{\mathbb{N}}(\varsigma), \mathbb{G}_{\mathbb{I}}(\varsigma) \in \mathcal{C}(I) \end{array}\right\}$$

where $\mathcal{G}_{\mathcal{M}}$ and $\mathcal{G}_{\mathcal{N}}$ are truth-ness and false-ness of membership function $\mathcal{G} : \eth \to I$ such that $\mathcal{G}_{\mathcal{M}} + \mathcal{G}_{\mathcal{N}} \leq 1$ and $\mathbb{G}_{\mathbb{M}}, \mathbb{G}_{\mathbb{N}}$ and $\mathbb{G}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{G} : \eth \to P(\Bbbk)$ of $\mathcal{IFPIVNHS}$-set .

Example 7.24. Consider Example 7.2. The $\mathcal{IFPIVNHS}$-set $(\mho_{ifp-ivnhs}, \eth)$ is given by

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$(\mho_{ifp-ivnhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.7;0.1\}}{\tau_1}, \left\{\frac{[0.4,0.6];[0.3,0.4];[0.2,0.3]}{\ell_1}, \frac{[0.3,0.5];[0.2,0.5];[0.2,0.5]}{\ell_3}\right\}\right), \left(\frac{\{0.5;0.3\}}{\tau_2}, \left\{\frac{[0.1,0.3];[0.2,0.4];[0.3,0,4]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.4;0.1\}}{\tau_4}, \left\{\frac{[0.3,0.5];[0.3,0.7];[0.5,0.6]}{\ell_3}, \frac{[o.1,0.2];[0.3,0.6];[0.3,0.5]}{\ell_4}, \frac{[0.1,0.2];[0.3,0.6];[0.2,0.5]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.4,0.2\}}{\tau_3}, \left\{\frac{[0.1,0.4];[0.3,0.5];[0.3,0.6]}{\ell_4}\right\}\right), \left(\frac{\{0.7;0.2\}}{\tau_5}, \left\{\frac{[0.4,0.7];[0.3,0.4];[0.1,0.5]}{\ell_1}, \frac{[0.6,0.8];[0.1,0.3];[0.5,0.7]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.6;0.2\}}{\tau_6}, \left\{\frac{[0.1,0.4];[0.1,0.3];[0.5,0.7]}{\ell_4}\right\}\right), \left(\frac{\{0.3;0.2\}}{\tau_8}, \left\{\frac{[0.1,0.3];[0.1,0.4];[0.2,0.3]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.4;0.1\}}{\tau_7}, \left\{\frac{[0.2,0.6];[0.5,0.7];[0.3,0.4]}{\ell_1}, \frac{[0.3,0.6];[0.1,0.3];[0.2,0.3]}{\ell_3}, \frac{[0.4,0.8];[0.2,0.4];[0.3,0.4]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.5;0.4\}}{\tau_9}, \left\{\frac{[0.2,0.4];[0.3,0.4];[0.1,0.5]}{\ell_3}\right\}\right), \left(\frac{\{0.6;0.3\}}{\tau_{10}}, \left\{\frac{[0.1,0.2];[0.3,0.5];[0.4,0.5]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.4;0.3\}}{\tau_{11}}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.3,0.7]}{\ell_1}, \frac{[0.5,0.7];[0.1,0.3];[0.4,0.6]}{\ell_5}\right\}\right), \left(\frac{\{0.7;0.1\}}{\tau_{12}}, \left\{\frac{[0.1,0.3];[0.3,0.6];[0.3,0.6]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.4,0.1\}}{\tau_{13}}, \left\{\frac{[0.3,0.6];[0.2,0.4];[0.4,0.6]}{\ell_3}\right\}\right), \left(\frac{\{0.5;0.2\}}{\tau_{15}}, \left\{\frac{[0.3,0.6];[0.2,0.3];[0.4,0.7]}{\ell_1}, \frac{[0.1,0.3];[0.5,0.7];[0.3,0.4]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.6;0.1\}}{\tau_{14}}, \left\{\frac{[0.3,0.4];[0.3,0.7];[0.2,0.4]}{\ell_1}\right\}\right), \left(\frac{\{0.4;0.3\}}{\tau_{16}}, \left\{\frac{[0.2,0.6];[0.3,0.4];[0.1,0.2]}{\ell_1}, \frac{[0.6,0.8];[0.3,0.4];[0.2,0.4]}{\ell_4}\right\}\right)
\end{array}\right\}.$$

Definition 7.25. $(\mho_{ifp-ivpfhs}, \eth)$ is called intuitionistic fuzzy parameterized interval-valued picture fuzzy hypersoft set ($\mathcal{IFPIVPFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{ifp-ivpfhs}, \eth) = \left\{\begin{array}{l} \left(\frac{\{\mathcal{H}_{\mathcal{M}}(\varsigma);\mathcal{H}_{\mathcal{N}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{H}_{\mathbb{M}}(\varsigma);\mathbb{H}_{\mathbb{N}}(\varsigma);\mathbb{H}_{\mathbb{I}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \mathcal{H}_{\mathcal{M}}(\varsigma), \mathcal{H}_{\mathcal{N}}(\varsigma) \in I, \\ \mathbb{H}_{\mathbb{M}}(\varsigma), \mathbb{H}_{\mathbb{N}}(\varsigma), \mathbb{H}_{\mathbb{I}}(\varsigma) \in \mathcal{C}\,(I) \end{array}\right\}$$

where $\mathcal{H}_{\mathcal{M}}$ and $\mathcal{H}_{\mathcal{N}}$ such that $\mathcal{H}_{\mathcal{M}} + \mathcal{H}_{\mathcal{N}} \leq 1$are truth-ness and false-ness of membership function $\mathcal{H} : \eth \to I$ and $\mathbb{H}_{\mathbb{M}}, \mathbb{H}_{\mathbb{N}}$ and $\mathbb{H}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{H} : \eth \to P(\mathbb{U})$ of $\mathcal{IFPIVPFHS}$-set with condition that the sum of upper bonds of $\mathbb{H}_{\mathbb{M}}, \mathbb{H}_{\mathbb{N}}$ and $\mathbb{H}_{\mathbb{I}}$ remain less than or equal to 1.

Example 7.26. Consider Example 7.2. The $\mathcal{IFPIVPFHS}$-set $(\mho_{ifp-ivpfhs}, \eth)$ is given by
$(\mho_{ifp-ivpfhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.7;0.1\}}{\tau_1}, \left\{\frac{[0.4,0.5];[0.3,0.4];[0.0,0.1]}{\ell_1}, \frac{[0.3,0.5];[0.2,0.3];[0.0,0.1]}{\ell_3}\right\}\right), \left(\frac{\{0.5;0.3\}}{\tau_2}, \left\{\frac{[0.1,0.3];[0.2,0.4];[0.0,0.1]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.4;0.1\}}{\tau_4}, \left\{\frac{[0.3,0.5];[0.3,0.4];[0.0,0.1]}{\ell_3}, \frac{[o.1,0.2];[0.3,0.6];[0.1,0.2]}{\ell_4}, \frac{[0.1,0.2];[0.3,0.6];[0.0,0.1]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.4,0.2\}}{\tau_3}, \left\{\frac{[0.1,0.4];[0.3,0.5];[0.0,0.1]}{\ell_4}\right\}\right), \left(\frac{\{0.7;0.2\}}{\tau_5}, \left\{\frac{[0.4,0.5];[0.3,0.4];[0.0,0.1]}{\ell_1}, \frac{[0.6,0.8];[0.0,0.1];[0.0,0.1]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.6;0.2\}}{\tau_6}, \left\{\frac{[0.1,0.4];[0.1,0.3];[0.1,0.2]}{\ell_4}\right\}\right), \left(\frac{\{0.3;0.2\}}{\tau_8}, \left\{\frac{[0.1,0.3];[0.1,0.4];[0.1,0.2]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.4;0.1\}}{\tau_7}, \left\{\frac{[0.2,0.3];[0.5,0.6];[0.0,0.1]}{\ell_1}, \frac{[0.1,0.3];[0.1,0.3];[0.2,0.3]}{\ell_3}, \frac{[0.4,0.5];[0.2,0.3];[0.1,0.2]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.5;0.4\}}{\tau_9}, \left\{\frac{[0.2,0.4];[0.3,0.4];[0.1,0.2]}{\ell_3}\right\}\right), \left(\frac{\{0.6;0.3\}}{\tau_{10}}, \left\{\frac{[0.1,0.2];[0.3,0.5];[0.2,0.3]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.4;0.3\}}{\tau_{11}}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.1,0.2]}{\ell_1}, \frac{[0.5,0.7];[0.0,0.1];[0.1,0.2]}{\ell_5}\right\}\right), \left(\frac{\{0.7;0.1\}}{\tau_{12}}, \left\{\frac{[0.1,0.3];[0.3,0.6];[0.0,0.1]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.4,0.1\}}{\tau_{13}}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.1,0.2]}{\ell_3}\right\}\right), \left(\frac{\{0.5;0.2\}}{\tau_{15}}, \left\{\frac{[0.3,0.5];[0.2,0.3];[0.0,0.1]}{\ell_1}, \frac{[0.1,0.3];[0.1,0.2];[0.3,0.4]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.6;0.1\}}{\tau_{14}}, \left\{\frac{[0.3,0.4];[0.0,0.2];[0.2,0.4]}{\ell_1}\right\}\right), \left(\frac{\{0.4;0.3\}}{\tau_{16}}, \left\{\frac{[0.2,0.3];[0.3,0.4];[0.1,0.2]}{\ell_1}, \frac{[0.0,0.2];[0.3,0.4];[0.2,0.4]}{\ell_4}\right\}\right)
\end{array}\right\}.$$

Definition 7.27. $(\mho_{np-ivfhs}, \eth)$ is called neutrosophic parameterized interval-valued fuzzy hypersoft set ($\mathcal{NPIVFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{np-ivfhs}, \eth) = \left\{\begin{array}{l} \left(\frac{\{\mathcal{J}_{\mathcal{M}}(\varsigma);\mathcal{J}_{\mathcal{N}}(\varsigma);\mathcal{J}_{\mathcal{I}}(\varsigma)\}}{\varsigma}, \frac{\mathbb{J}_{\mathbb{M}}(\varsigma)}{z}\right) : \varsigma \in \eth, \\ \mathcal{J}_{\mathcal{M}}(\varsigma), \mathcal{J}_{\mathcal{N}}(\varsigma), \mathcal{J}_{\mathcal{I}}(\varsigma) \in I, \mathbb{J}_{\mathbb{M}}(\varsigma) \in \mathcal{C}\,(I) \end{array}\right\}$$

where $\mathcal{J}_{\mathcal{M}}, \mathcal{J}_{\mathcal{N}}$ and $\mathcal{J}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{J} : \eth \to I$ and $\mathbb{J}_{\mathbb{M}}$ is truth-ness of approximate function $\mathbb{J} : \eth \to P(\mathbb{U})$ of $\mathcal{NPIVFHS}$-set.

Example 7.28. Consider Example 7.2. The $\mathcal{NPIVFHS}$-set $(\mho_{np-ivfhs}, \eth)$ is given by

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$(\mho_{np-ivfhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.8;0.5;0.4\}}{\tau_1},\left\{\frac{[0.1,0.2]}{\ell_1},\frac{[0.2,0.3}{\ell_3}\right\}\right),\left(\frac{\{0.6;0.4;0.3\}}{\tau_2},\left\{\frac{[0.1,0,4]}{\ell_2}\right\}\right),\left(\frac{\{0.7;0.5;0.4\}}{\tau_3},\left\{\frac{[0.2,0.5]}{\ell_4}\right\}\right),\\
\left(\frac{\{0.5;0.3;0.1\}}{\tau_4},\left\{\frac{[0.2,0.3]}{\ell_3},\frac{[o.1,0.4]}{\ell_4},\frac{[0.1,0.5]}{\ell_5}\right\}\right),\left(\frac{\{0.7;0.5;0.1\}}{\tau_5},\left\{\frac{[0.3,0.6]}{\ell_1},\frac{[0.1,0.2]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.7;0.4;0.2\}}{\tau_6},\left\{\frac{[0.2,0.4]}{\ell_4}\right\}\right),\left(\frac{\{0.5;0.3;0.1\}}{\tau_7},\left\{\frac{[0.2,0.3]}{\ell_1},\frac{[0.1,0.3]}{\ell_3},\frac{[0.1,0.5]}{\ell_5}\right\}\right),\left(\frac{\{0.5;0.2;0.1\}}{\tau_8},\left\{\frac{[0.2,0.5]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.7;0.4;0.2\}}{\tau_9},\left\{\frac{[0.3,0.5]}{\ell_3}\right\}\right),\left(\frac{\{0.4;0.5;0.3\}}{\tau_{10}},\left\{\frac{[0.1,0.3]}{\ell_1}\right\}\right),\left(\frac{\{0.5;0.5;0.1\}}{\tau_{11}},\left\{\frac{[0.1,0.3]}{\ell_1},\frac{[0.1,0.4]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.7;0.4,0.3\}}{\tau_{12}},\left\{\frac{[0.2,0.3]}{\ell_2}\right\}\right),\left(\frac{\{0.4;0,5;0.2\}}{\tau_{13}},\left\{\frac{[0.2,0.6]}{\ell_3}\right\}\right),\left(\frac{\{0.8;0.6;0.2\}}{\tau_{14}},\left\{\frac{[0.1,0.4]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.5;0.7;0.2\}}{\tau_{15}},\left\{\frac{[0.1,0.5]}{\ell_1},\frac{[0.3,0.7]}{\ell_2}\right\}\right),\left(\frac{\{0.6;0.2;0.4\}}{\tau_{16}},\left\{\frac{[0.2,0.5]}{\ell_1},\frac{[0.1,0.4]}{\ell_4}\right\}\right)
\end{array}\right\}.$$

Definition 7.29. $(\mho_{np-ivifhs}, \eth)$ is called neutrosophic parameterized interval-valued intuitionistic fuzzy hypersoft set ($\mathcal{NPIVIFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{np-ivifhs}, \eth) = \left\{\begin{array}{l} \left(\frac{\{\mathcal{K}_{\mathcal{M}}(\varsigma);\mathcal{K}_{\mathcal{N}}(\varsigma);\mathcal{K}_{\mathcal{I}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{K}_{\mathbb{M}}(\varsigma),\mathcal{K}_{\mathcal{N}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \\ \mathcal{K}_{\mathcal{M}}(\varsigma), \mathcal{K}_{\mathcal{N}}(\varsigma), \mathcal{K}_{\mathcal{I}}(\varsigma) \in I, \mathbb{K}_{\mathbb{M}}(\varsigma), \mathbb{K}_{\mathbb{N}}(\varsigma) \in \mathcal{C}(I) \end{array}\right\}$$

where $\mathcal{K}_{\mathcal{M}}, \mathcal{K}_{\mathcal{N}}$ and $\mathcal{K}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{K} : \eth \to I$ and $\mathbb{K}_{\mathbb{M}}$ and $\mathbb{K}_{\mathbb{N}}$ are truth-ness and false-ness of approximate function $\mathbb{K} : \eth \to P(\mathbb{U})$ of $\mathcal{NPIVIFHS}$-set with condition that the sum of upper bonds of $\mathbb{K}_{\mathbb{M}}$ and $\mathbb{K}_{\mathbb{N}}$ remain less than or equal to 1.

Example 7.30. Consider Example 7.2. The $\mathcal{NPIVIFHS}$-set $(\mho_{ifp-ivifhs}, \eth)$ is given by
$(\mho_{ifp-ivifhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.8;0.5;0.1\}}{\tau_1},\left\{\frac{[0.5,0.7];[0.1,0.2]}{\ell_1},\frac{[0.2,0.5];[0.2,0.3}{\ell_3}\right\}\right),\left(\frac{\{0.6;0.3;0.1\}}{\tau_2},\left\{\frac{[0.1,0.4];[0.2,0,4]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.6;0.4;0.1\}}{\tau_3},\left\{\frac{[0.2,0.5];[0.3,0.4]}{\ell_4}\right\}\right),\left(\frac{\{0.6;0.3;0.1\}}{\tau_4},\left\{\frac{[0.2,0.3];[0.5,0.6]}{\ell_3},\frac{[o.1,0.4];[0.3,0.5]}{\ell_4},\frac{[0.1,0.2];[0.3,0.5]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.8;0.7;0.2\}}{\tau_5},\left\{\frac{[0.1,0.2];[0.3,0.5]}{\ell_1},\frac{[0.1,0.2];[0.4,0.7]}{\ell_2}\right\}\right),\left(\frac{\{0.6;0.3;0.2\}}{\tau_6},\left\{\frac{[0.2,0.3];[0.5,0.6]}{\ell_4}\right\}\right),\\
\left(\frac{\{0.6;0.2;0.1\}}{\tau_7},\left\{\frac{[0.2,0.3];[0.3,0.4]}{\ell_1},\frac{[0.1,0.3];[0.1,0.2]}{\ell_3},\frac{[0.1,0.2];[0.3,0.5]}{\ell_5}\right\}\right),\left(\frac{\{0.7;0.3;0.2\}}{\tau_8},\left\{\frac{[0.1,0.2];[0.2,0.5]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.5;0.4;0.5\}}{\tau_9},\left\{\frac{[0.1,0.4];[0.3,0.5]}{\ell_3}\right\}\right),\left(\frac{\{0.7;0.2;0.3\}}{\tau_{10}},\left\{\frac{[0.1,0.3];[0.2,0.5]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.4;0.6;0.3\}}{\tau_{11}},\left\{\frac{[0.1,0.4];[0.3,0.5]}{\ell_1},\frac{[0.1,0.4];[0.4,0.5]}{\ell_5}\right\}\right),\left(\frac{\{0.7;0.3;0.1\}}{\tau_{12}},\left\{\frac{[0.2,0.3];[0.4,0.6]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.5;0.4;0.2\}}{\tau_{13}},\left\{\frac{[0.1,0.4];[0.2,0.6]}{\ell_3}\right\}\right),\left(\frac{\{0.6;0.2;0.2\}}{\tau_{15}},\left\{\frac{[0.1,0.3];[0.4,0.5]}{\ell_1},\frac{[0.1,0.2];[0.6,0.7]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.4;0.5;0.1\}}{\tau_{14}},\left\{\frac{[0.2,0.4];[0.3,0.4]}{\ell_1}\right\}\right),\left(\frac{\{0.8;0.4;0.3\}}{\tau_{16}},\left\{\frac{[0.2,0.4];[0.3,0.5]}{\ell_1},\frac{[0.1,0.2];[0.4,0.6]}{\ell_4}\right\}\right)
\end{array}\right\}.$$

Definition 7.31. $(\mho_{np-ivnhs}, \eth)$ is called neutrosophic parameterized interval-valued neutrosophic hypersoft set ($\mathcal{NPIVNHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{np-ivnhs}, \eth) = \left\{\begin{array}{l} \left(\frac{\{\mathcal{L}_{\mathcal{M}}(\varsigma);\mathcal{L}_{\mathcal{N}}(\varsigma);\mathcal{L}_{\mathcal{I}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{L}_{\mathbb{M}}(\varsigma);\mathbb{L}_{\mathbb{N}}(\varsigma);\mathbb{L}_{\mathbb{I}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \\ \mathcal{L}_{\mathcal{M}}(\varsigma), \mathcal{L}_{\mathcal{N}}(\varsigma), \mathcal{L}_{\mathcal{I}}(\varsigma) \in I, \mathbb{L}_{\mathbb{M}}(\varsigma), \mathbb{L}_{\mathbb{N}}(\varsigma), \mathbb{L}_{\mathbb{I}}(\varsigma) \in \mathcal{C}(I) \end{array}\right\}$$

where $\mathcal{L}_{\mathcal{M}}, \mathcal{L}_{\mathcal{N}}$ and $\mathcal{L}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{L} : \eth \to I$ and $\mathbb{L}_{\mathbb{M}}, \mathbb{L}_{\mathbb{N}}$ and $\mathbb{L}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{L} : \eth \to P(\mathbb{U})$ of $\mathcal{NPIVNHS}$-set .

Example 7.32. Consider Example 7.2. The $\mathcal{NPIVNHS}$-set $(\mho_{np-ivnhs}, \eth)$ is given by

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$(\mho_{np-ivnhs},\eth)=$

$$\left\{\begin{array}{l}
\left(\frac{\{0.5;0.7;0.1\}}{\tau_1},\left\{\frac{[0.4,0.6];[0.3,0.4];[0.2,0.3]}{\ell_1},\frac{[0.3,0.5];[0.2,0.5];[0.2,0.5]}{\ell_3}\right\}\right),\\
\left(\frac{\{0.8;0.5;0.1\}}{\tau_2},\left\{\frac{[0.1,0.3];[0.2,0.4];[0.3,0,4]}{\ell_2}\right\}\right),\left(\frac{\{0.7;0.4,0.2\}}{\tau_3},\left\{\frac{[0.1,0.4];[0.3,0.5];[0.3,0.6]}{\ell_4}\right\}\right),\\
\left(\frac{\{0.5;0.4;0.1\}}{\tau_4},\left\{\frac{[0.3,0.5];[0.3,0.7];[0.5,0.6]}{\ell_3},\frac{[o.1,0.2];[0.3,0.6];[0.3,0.5]}{\ell_4},\frac{[0.1,0.2];[0.3,0.6];[0.2,0.5]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.7;0.3;0.1\}}{\tau_5},\left\{\frac{[0.4,0.7];[0.3,0.4];[0.1,0.5]}{\ell_1},\frac{[0.6,0.8];[0.1,0.3];[0.5,0.7]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.8;0.6;0.2\}}{\tau_6},\left\{\frac{[0.1,0.4];[0.1,0.3];[0.5,0.7]}{\ell_4}\right\}\right),\left(\frac{\{0.5;0.3;0.2\}}{\tau_8},\left\{\frac{[0.1,0.3];[0.1,0.4];[0.2,0.3]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.5;0.4;0.1\}}{\tau_7},\left\{\frac{[0.2,0.6];[0.5,0.7];[0.3,0.4]}{\ell_1},\frac{[0.3,0.6];[0.1,0.3];[0.2,0.3]}{\ell_3},\frac{[0.4,0.8];[0.2,0.4];[0.3,0.4]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.4;0.5;0.4\}}{\tau_9},\left\{\frac{[0.2,0.4];[0.3,0.4];[0.1,0.5]}{\ell_3}\right\}\right),\left(\frac{\{0.7;0.4;0.3\}}{\tau_{10}},\left\{\frac{[0.1,0.2];[0.3,0.5];[0.4,0.5]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.6;0.4;0.3\}}{\tau_{11}},\left\{\frac{[0.3,0.4];[0.2,0.4];[0.3,0.7]}{\ell_1},\frac{[0.5,0.7];[0.1,0.3];[0.4,0.6]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.7;0.3;0.2\}}{\tau_{12}},\left\{\frac{[0.1,0.3];[0.3,0.6];[0.3,0.6]}{\ell_2}\right\}\right),\left(\frac{\{0.8;0.4,0.3\}}{\tau_{13}},\left\{\frac{[0.3,0.6];[0.2,0.4];[0.4,0.6]}{\ell_3}\right\}\right),\\
\left(\frac{\{0.5;0.7;0.2\}}{\tau_{15}},\left\{\frac{[0.3,0.6];[0.2,0.3];[0.4,0.7]}{\ell_1},\frac{[0.1,0.3];[0.5,0.7];[0.3,0.4]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.7;0.4;0.3\}}{\tau_{16}},\left\{\frac{[0.2,0.6];[0.3,0.4];[0.1,0.2]}{\ell_1},\frac{[0.6,0.8];[0.3,0.4];[0.2,0.4]}{\ell_4}\right\}\right),\\
\left(\frac{\{0.4;0.3;0.1\}}{\tau_{14}},\left\{\frac{[0.3,0.4];[0.3,0.7];[0.2,0.4]}{\ell_1}\right\}\right)
\end{array}\right\}.$$

Definition 7.33. $(\mho_{np-ivpfhs},\eth)$ is called neutrosophic parameterized interval-valued picture fuzzy hypersoft set ($\mathcal{NPIVPFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{np-ivpfhs},\eth)=\left\{\begin{array}{l}\left(\frac{\{\mathcal{M}_{\mathcal{M}}(\varsigma);\mathcal{M}_{\mathcal{N}}(\varsigma);\mathcal{M}_{\mathcal{I}}(\varsigma)\}}{\varsigma},\frac{\{\mathbb{M}_{\mathbb{M}}(\varsigma);\mathbb{M}_{\mathbb{N}}(\varsigma);\mathbb{M}_{\mathbb{I}}(\varsigma)\}}{z}\right):\varsigma\in\eth,\\ \mathcal{M}_{\mathcal{M}}(\varsigma),\mathcal{M}_{\mathcal{N}}(\varsigma),\mathcal{M}_{\mathcal{I}}(\varsigma)\in I,\mathbb{M}_{\mathbb{M}}(\varsigma),\mathbb{M}_{\mathbb{N}}(\varsigma),\mathbb{M}_{\mathbb{I}}(\varsigma)\in\mathcal{C}(I)\end{array}\right\}$$

where $\mathcal{M}_{\mathcal{M}},\mathcal{M}_{\mathcal{N}}$ and $\mathcal{M}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{M}:\eth\to I$ and $\mathbb{M}_{\mathbb{M}},\mathbb{M}_{\mathbb{N}}$ and $\mathbb{M}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{M}:\eth\to P(\mathbb{U})$ of $\mathcal{NPIVPFHS}$-set with condition that the sum of upper bonds of $\mathbb{M}_{\mathbb{M}},\mathbb{M}_{\mathbb{N}}$ and $\mathbb{M}_{\mathbb{I}}$ remain less than or equal to 1.

Example 7.34. Consider Example 7.2. The $\mathcal{NPIVPFHS}$-set $(\mho_{np-ivpfhs},\eth)$ is given by
$(\mho_{np-ivpfhs},\eth)=$

$$\left\{\begin{array}{l}
\left(\frac{\{0.5;0.7;0.1\}}{\tau_1},\left\{\frac{[0.4,0.5];[0.3,0.4];[0.0,0.1]}{\ell_1},\frac{[0.3,0.5];[0.2,0.3];[0.1,0.2]}{\ell_3}\right\}\right),\\
\left(\frac{\{0.8;0.5;0.1\}}{\tau_2},\left\{\frac{[0.1,0.3];[0.2,0.4];[0.2,0,3]}{\ell_2}\right\}\right),\left(\frac{\{0.7;0.4,0.2\}}{\tau_3},\left\{\frac{[0.1,0.4];[0.3,0.5];[0.0,0.1]}{\ell_4}\right\}\right),\\
\left(\frac{\{0.5;0.4;0.1\}}{\tau_4},\left\{\frac{[0.3,0.5];[0.3,0.4];[0.0,0.1]}{\ell_3},\frac{[o.1,0.2];[0.3,0.6];[0.1,0.2]}{\ell_4},\frac{[0.1,0.2];[0.3,0.6];[0.0,0.1]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.7;0.3;0.1\}}{\tau_5},\left\{\frac{[0.4,0.5];[0.3,0.4];[0.0,0.1]}{\ell_1},\frac{[0.6,0.7];[0.1,0.2];[0.0,0.1]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.8;0.6;0.2\}}{\tau_6},\left\{\frac{[0.1,0.4];[0.1,0.3];[0.1,0.3]}{\ell_4}\right\}\right),\left(\frac{\{0.5;0.3;0.2\}}{\tau_8},\left\{\frac{[0.1,0.3];[0.1,0.4];[0.2,0.3]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.5;0.4;0.1\}}{\tau_7},\left\{\frac{[0.2,0.3];[0.5,0.6];[0.0,0.1]}{\ell_1},\frac{[0.3,0.6];[0.1,0.3];[0.0,0.1]}{\ell_3},\frac{[0.4,0.5];[0.2,0.3];[0.1,0.2]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.4;0.5;0.4\}}{\tau_9},\left\{\frac{[0.2,0.4];[0.3,0.4];[0.1,0.2]}{\ell_3}\right\}\right),\left(\frac{\{0.7;0.4;0.3\}}{\tau_{10}},\left\{\frac{[0.1,0.2];[0.3,0.4];[0.2,0.3]}{\ell_1}\right\}\right),\\
\left(\frac{\{0.6;0.4;0.3\}}{\tau_{11}},\left\{\frac{[0.3,0.4];[0.2,0.3];[0.1,0.3]}{\ell_1},\frac{[0.5,0.6];[0.1,0.2];[0.1,0.2]}{\ell_5}\right\}\right),\\
\left(\frac{\{0.7;0.3;0.2\}}{\tau_{12}},\left\{\frac{[0.1,0.3];[0.3,0.4];[0.1,0.3]}{\ell_2}\right\}\right),\left(\frac{\{0.8;0.4,0.3\}}{\tau_{13}},\left\{\frac{[0.3,0.4];[0.2,0.3];[0.0,0.2]}{\ell_3}\right\}\right),\\
\left(\frac{\{0.5;0.7;0.2\}}{\tau_{15}},\left\{\frac{[0.3,0.6];[0.2,0.3];[0.0,0.1]}{\ell_1},\frac{[0.1,0.3];[0.5,0.6];[0.0,0.1]}{\ell_2}\right\}\right),\\
\left(\frac{\{0.7;0.4;0.3\}}{\tau_{16}},\left\{\frac{[0.2,0.3];[0.3,0.4];[0.1,0.2]}{\ell_1},\frac{[0.6,0.8];[0.0,0.1];[0.0,0.1]}{\ell_4}\right\}\right),\\
\left(\frac{\{0.4;0.3;0.1\}}{\tau_{14}},\left\{\frac{[0.3,0.4];[0.3,0.4];[0.1,0.2]}{\ell_1}\right\}\right)
\end{array}\right\}.$$

Definition 7.35. $(\mho_{pfp-ivfhs},\eth)$ is called picture fuzzy parameterized interval-valued fuzzy hypersoft set ($\mathcal{PFPIVFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{pfp-ivfhs},\eth)=\left\{\begin{array}{l}\left(\frac{\{\mathcal{N}_{\mathcal{M}}(\varsigma);\mathcal{N}_{\mathcal{N}}(\varsigma);\mathcal{N}_{\mathcal{I}}(\varsigma)\}}{\varsigma},\frac{\{\mathbb{N}_{\mathbb{M}}(\varsigma)\}}{z}\right):\varsigma\in\eth,\\ \mathcal{N}_{\mathcal{M}}(\varsigma),\mathcal{N}_{\mathcal{N}}(\varsigma),\mathcal{N}_{\mathcal{I}}(\varsigma)\in I,\mathbb{N}_{\mathbb{M}}(\varsigma)\in\mathcal{C}(I)\end{array}\right\}$$

where $\mathcal{N}_{\mathcal{M}},\mathcal{N}_{\mathcal{N}}$ and $\mathcal{N}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{N}:\eth\to I$ such that $\mathcal{N}_{\mathcal{M}}+\mathcal{N}_{\mathcal{N}}+\mathcal{N}_{\mathcal{I}}\leq 1$ and $\mathbb{N}_{\mathbb{M}}$ is truth-ness of approximate function $\mathbb{N}:\eth\to P(\mathbb{U})$ of $\mathcal{PFPIVFHS}$-set.

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Example 7.36. Consider Example 7.2. The $\mathcal{PFPIVFHS}$-set $(\mho_{pfp-ivfhs}, \eth)$ is given by
$(\mho_{pfp-ivfhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.2;0.5;0.1\}}{\tau_1}, \left\{\frac{[0.3,0.9]}{\ell_1}, \frac{[0.5,0.8]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.2;0.6;0.1\}}{\tau_2}, \left\{\frac{[0.2,0.6]}{\ell_2}\right\}\right), \left(\frac{\{0.2;0.1,0.2\}}{\tau_3}, \left\{\frac{[0.3,0.7]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.2;0.7;0.1\}}{\tau_4}, \left\{\frac{[0.3,0.9]}{\ell_3}, \frac{[0.3,0.7]}{\ell_4}, \frac{[0.2,0.6]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.2;0.6;0.1\}}{\tau_5}, \left\{\frac{[0.4,0.9]}{\ell_1}, \frac{[0.3,0.4]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.3;0.4;0.1\}}{\tau_6}, \left\{\frac{[0.2,0.6]}{\ell_4}\right\}\right), \left(\frac{\{0.7;0.1;0.1\}}{\tau_8}, \left\{\frac{[0.3,0.7]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.2;0.6;0.1\}}{\tau_7}, \left\{\frac{[0.5,0.8]}{\ell_1}, \frac{[0.4,0.6]}{\ell_3}, \frac{[0.6,0.7]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.1;0.6;0.1\}}{\tau_9}, \left\{\frac{[0.1,0.4]}{\ell_3}\right\}\right), \left(\frac{\{0.3;0.2;0.3\}}{\tau_{10}}, \left\{\frac{[0.3,0.7]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.2;0.1;0.1\}}{\tau_{11}}, \left\{\frac{[0.2,o.5]}{\ell_1}, \frac{[0.1,0.7]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.3;0.4;0.1\}}{\tau_{12}}, \left\{\frac{[0.1,0.5]}{\ell_2}\right\}\right), \left(\frac{\{0.1;0.1,0.5\}}{\tau_{13}}, \left\{\frac{[0.2,0.9]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.2;0.4;0.4\}}{\tau_{15}}, \left\{\frac{[0.3,0.6]}{\ell_1}, \frac{[0.2,0.8]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.2;0.5;0.1\}}{\tau_{16}}, \left\{\frac{[0.8,0.9]}{\ell_1}, \frac{[0.3,0.5]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.2;0.5;0.1\}}{\tau_{14}}, \left\{\frac{[0.5,0.8]}{\ell_1}\right\}\right)
\end{array}\right\}.$$

Definition 7.37. $(\mho_{pfp-ivifhs}, \eth)$ is called picture fuzzy parameterized interval-valued intuitionistic fuzzy hypersoft set ($\mathcal{PFPIVIFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{pfp-ivifhs}, \eth) = \left\{ \begin{array}{c} \left(\frac{\{\mathcal{O}_{\mathcal{M}}(\varsigma);\mathcal{O}_{\mathcal{N}}(\varsigma);\mathcal{O}_{\mathcal{I}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{O}_{\mathbb{M}}(\varsigma);\mathbb{O}_{\mathbb{N}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \\ \mathcal{O}_{\mathcal{M}}(\varsigma), \mathcal{O}_{\mathcal{N}}(\varsigma), \mathcal{O}_{\mathcal{I}}(\varsigma) \in I, \mathbb{O}_{\mathbb{M}}(\varsigma), \mathbb{O}_{\mathbb{N}}(\varsigma) \in \mathcal{C}(I) \end{array} \right\}$$

where $\mathcal{O}_{\mathcal{M}}, \mathcal{O}_{\mathcal{N}}$ and $\mathcal{O}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{O} : \eth \to I$ such that $\mathcal{O}_{\mathcal{M}}+\mathcal{O}_{\mathcal{N}}+\mathcal{O}_{\mathcal{I}} \leq 1$ and $\mathbb{O}_{\mathbb{M}}$ and $\mathbb{O}_{\mathbb{N}}$ are truth-ness and false-ness of approximate function $\mathbb{O} : \eth \to P(\mathbb{U})$ of $\mathcal{PFPIVIFHS}$-set with condition that the sum of upper bonds of $\mathbb{O}_{\mathbb{M}}$ and $\mathbb{Q}_{\mathbb{N}}$ remain less than or equal to 1.

Example 7.38. Consider Example 7.2. The $\mathcal{PFPIVIFHS}$-set $(\mho_{pfp-ivifhs}, \eth)$ is given by
$(\mho_{pfp-ivifhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.2;0.3;0.1\}}{\tau_1}, \left\{\frac{[0.4,0.5];[0.3,0.4]}{\ell_1}, \frac{[0.2,0.3];[0.5,0.6]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.4;0.3;0.1\}}{\tau_2}, \left\{\frac{[0.2,0.4];[0.2,0,3]}{\ell_2}\right\}\right), \left(\frac{\{0.3;0.3,0.2\}}{\tau_3}, \left\{\frac{[0.4,0.5];[0.3,0.4]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.3;0.4;0.2\}}{\tau_4}, \left\{\frac{[0.3,0.6];[0.1,0.3]}{\ell_3}, \frac{[o.1,0.3];[0.3,0.4]}{\ell_4}, \frac{[0.1,0.3];[0.3,0.6]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.2;0.3;0.3\}}{\tau_5}, \left\{\frac{[0.4,0.5];[0.3,0.4]}{\ell_1}, \frac{[0.3,0.5];[0.1,0.4]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.3;0.1;0.2\}}{\tau_6}, \left\{\frac{[0.2,0.3];[0.1,0.4]}{\ell_4}\right\}\right), \left(\frac{\{0.1;0.2;0.6\}}{\tau_8}, \left\{\frac{[0.3,0.4];[0.2,0.5]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.1;0.5;0.3\}}{\tau_7}, \left\{\frac{[0.5,0.6];[0.0,0.2]}{\ell_1}, \frac{[0.1,0.4];[0.2,0.4]}{\ell_3}, \frac{[0.2,0.4];[0.1,0.5]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.4;0.2;0.3\}}{\tau_9}, \left\{\frac{[0.1,0.4];[0.1,0.3]}{\ell_3}\right\}\right), \left(\frac{\{0.2;0.1;0.3\}}{\tau_{10}}, \left\{\frac{[0.3,0.5];[0.1,0.4]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.2;0.4;0.3\}}{\tau_{11}}, \left\{\frac{[0.2,0.4];[0.1,o.5]}{\ell_1}, \frac{[0.1,0.2];[0.1,0.7]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.3;0.5;0.2\}}{\tau_{12}}, \left\{\frac{[0.1,0.3];[0.3,0.4]}{\ell_2}\right\}\right), \left(\frac{\{0.3;0.4,0.2\}}{\tau_{13}}, \left\{\frac{[0.2,0.6];[0.1,0.3]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.2;0.2;0.4\}}{\tau_{15}}, \left\{\frac{[0.3,0.4];[0.2,0.4]}{\ell_1}, \frac{[0.2,0.5];[0.4,0.5]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.2;0.6;0.1\}}{\tau_{16}}, \left\{\frac{[0.2,0.3];[0.3,0.5]}{\ell_1}, \frac{[0.4,0.5];[0.3,0.4]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.2;0.1;0.1\}}{\tau_{14}}, \left\{\frac{[0.3,0.4];[0.2,0.3]}{\ell_1}\right\}\right)
\end{array}\right\}.$$

Definition 7.39. $(\mho_{pfp-ivnhs}, \eth)$ is called picture fuzzy parameterized interval-valued neutrosophic hypersoft set ($\mathcal{PFPIVNHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{pfp-ivnhs}, \eth) = \left\{ \begin{array}{c} \left(\frac{\{\mathcal{P}_{\mathcal{M}}(\varsigma);\mathcal{P}_{\mathcal{N}}(\varsigma);\mathcal{P}_{\mathcal{I}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{P}_{\mathbb{M}}(\varsigma);\mathbb{P}_{\mathbb{N}}(\varsigma);\mathbb{P}_{\mathbb{I}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \\ \mathcal{P}_{\mathcal{M}}(\varsigma), \mathcal{P}_{\mathcal{N}}(\varsigma), \mathcal{P}_{\mathcal{I}}(\varsigma) \in I, \mathbb{P}_{\mathbb{M}}(\varsigma), \mathbb{P}_{\mathbb{N}}(\varsigma), \mathbb{P}_{\mathbb{I}}(\varsigma) \in \mathcal{C}(I) \end{array} \right\}$$

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where $\mathcal{P}_{\mathcal{M}}, \mathcal{P}_{\mathcal{N}}$ and $\mathcal{P}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{P} : \eth \rightarrow I$ such that $\mathcal{P}_{\mathcal{M}} + \mathcal{P}_{\mathcal{N}} + \mathcal{P}_{\mathcal{I}} \leq 1$ and $\mathbb{P}_{\mathbb{M}}, \mathbb{P}_{\mathbb{N}}$ and $\mathbb{P}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{P} : \eth \rightarrow P(\mathbb{U})$ of $\mathcal{PFPIVNHS}$-set.

Example 7.40. Consider Example 7.2. The $\mathcal{PFPIVNHS}$-set $(\mho_{pfp-ivnhs}, \eth)$ is given by $(\mho_{pfp-ivnhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.2;0.4;0.1\}}{\tau_1}, \left\{\frac{[0.4,0.5];[0.2,0.6];[0.3,0.4]}{\ell_1}, \frac{[0.2,0.3];[0.5,0.9];[0.1,0.2]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.4;0.2;0.1\}}{\tau_2}, \left\{\frac{[0.2,0.4];[0.4,0.8];[0.2,0.3]}{\ell_2}\right\}\right), \left(\frac{\{0.3;0.2,0.1\}}{\tau_3}, \left\{\frac{[0.4,0.8];[0.6,0.7];[0.3,0.5]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.3;0.2;0.1\}}{\tau_4}, \left\{\frac{[0.3,0.6];[0.3,0.8];[0.0,0.1]}{\ell_3}, \frac{[o.1,0.3];[0.3,0.7];[0.4,0.8]}{\ell_4}, \frac{[0.1,0.7];[0.3,0.6];[0.0,0.5]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.2;0.4;0.3\}}{\tau_5}, \left\{\frac{[0.4,0.5];[0.0,0.1];[0.3,0.6]}{\ell_1}, \frac{[0.3,0.7];[0.0,0.5];[0.1,0.6]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.3;0.3;0.2\}}{\tau_6}, \left\{\frac{[0.2,0.3];[0.1,0.5];[0.6,0.9]}{\ell_4}\right\}\right), \left(\frac{\{0.1;0.1;0.6\}}{\tau_8}, \left\{\frac{[0.3,0.4];[0.1,0.4];[0.2,0.6]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.1;0.2;0.1\}}{\tau_7}, \left\{\frac{[0.5,0.7];[0.3,0.6];[0.0,0.5]}{\ell_1}, \frac{[0.1,0.4];[0.3,0.8];[0.0,0.4]}{\ell_3}, \frac{[0.2,0.4];[0.4,0.9];[0.1,0.4]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.4;0.0;0.4\}}{\tau_9}, \left\{\frac{[0.1,0.5];[0.2,0.6];[0.1,0.3]}{\ell_3}\right\}\right), \left(\frac{\{0.2;0.3;0.3\}}{\tau_{10}}, \left\{\frac{[0.3,0.5];[0.2,0.4];[0.2,0.7]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.3;0.3;0.3\}}{\tau_{11}}, \left\{\frac{[0.2,0.4];[0.2,0.6];[0.1,0.9]}{\ell_1}, \frac{[0.1,0.5];[0.2,0.4];[0.1,0.7]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.2;0.1;0.2\}}{\tau_{12}}, \left\{\frac{[0.1,0.3];[0.3,0.7];[0.7,0.8]}{\ell_2}\right\}\right), \left(\frac{\{0.3;0.1,0.3\}}{\tau_{13}}, \left\{\frac{[0.3,0.6];[0.5,0.8];[0.1,0.4]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.5;0.1;0.4\}}{\tau_{15}}, \left\{\frac{[0.3,0.7];[0.2,0.4];[0.4,0.6]}{\ell_1}, \frac{[0.5,0.8];[0.1,0.5];[0.6,0.7]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.1;0.7;0.1\}}{\tau_{16}}, \left\{\frac{[0.2,0.5];[0.4,0.5];[0.1,0.5]}{\ell_1}, \frac{[0.4,0.7];[0.3,0.5];[0.2,0.4]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.2;0.4;0.1\}}{\tau_{14}}, \left\{\frac{[0.3,0.6];[0.2,0.6];[0.1,0.4]}{\ell_1}\right\}\right)
\end{array}\right\}.$$

Definition 7.41. $(\mho_{pfp-ivpfhs}, \eth)$ is called picture fuzzy parameterized interval-valued picture fuzzy hypersoft set ($\mathcal{PFPIVPFHS}$-set) over $\mathbb{U}$ and is defined as

$$(\mho_{pfp-ivpfhs}, \eth) = \left\{ \begin{array}{l} \left(\frac{\{\mathcal{Q}_{\mathcal{M}}(\varsigma);\mathcal{Q}_{\mathcal{N}}(\varsigma);\mathcal{Q}_{\mathcal{I}}(\varsigma)\}}{\varsigma}, \frac{\{\mathbb{Q}_{\mathbb{M}}(\varsigma);\mathbb{Q}_{\mathbb{N}}(\varsigma);\mathbb{Q}_{\mathbb{I}}(\varsigma)\}}{z}\right) : \varsigma \in \eth, \\ \mathcal{Q}_{\mathcal{M}}(\varsigma), \mathcal{Q}_{\mathcal{N}}(\varsigma), \mathcal{Q}_{\mathcal{I}}(\varsigma) \in I, \mathbb{Q}_{\mathbb{M}}(\varsigma), \mathbb{Q}_{\mathbb{N}}(\varsigma), \mathbb{Q}_{\mathbb{I}}(\varsigma) \in \mathcal{C}\left(I\right) \end{array} \right\}$$

where $\mathcal{Q}_{\mathcal{M}}, \mathcal{Q}_{\mathcal{N}}$ and $\mathcal{Q}_{\mathcal{I}}$ are truth-ness, false-ness and indeterminacy of membership function $\mathcal{Q} : \eth \rightarrow I$ such that $\mathcal{Q}_{\mathcal{M}} + \mathcal{Q}_{\mathcal{N}} + \mathcal{Q}_{\mathcal{I}} \leq 1$ and $\mathbb{Q}_{\mathbb{M}}, \mathbb{Q}_{\mathbb{N}}$ and $\mathbb{Q}_{\mathbb{I}}$ are truth-ness false-ness and indeterminacy of approximate function $\mathbb{Q} : \eth \rightarrow P(\mathbb{U})$ of $\mathcal{PFPIVPFHS}$-set with condition that the sum of upper bonds of $\mathbb{Q}_{\mathbb{M}}, \mathbb{Q}_{\mathbb{N}}$ and $\mathbb{Q}_{\mathbb{I}}$ remain less than or equal to 1.

Example 7.42. Consider Example 7.2. The $\mathcal{PFPIVPFHS}$-set $(\mho_{pfp-ivpfhs}, \eth)$ is given by $(\mho_{pfp-ivpfhs}, \eth) =$

$$\left\{\begin{array}{l}
\left(\frac{\{0.2;0.7;0.1\}}{\tau_1}, \left\{\frac{[0.4,0.5];[0.0,0.1];[0.3,0.4]}{\ell_1}, \frac{[0.2,0.3];[0.3,0.5];[0.1,0.2]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.4;0.5;0.1\}}{\tau_2}, \left\{\frac{[0.2,0.4];[0.1,0.3];[0.2,0.3]}{\ell_2}\right\}\right), \left(\frac{\{0.5;0.4,0.1\}}{\tau_3}, \left\{\frac{[0.1,0.4];[0.0,0.1];[0.3,0.5]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.5;0.4;0.1\}}{\tau_4}, \left\{\frac{[0.3,0.4];[0.3,0.5];[0.0,0.1]}{\ell_3}, \frac{[o.1,0.2];[0.3,0.4];[0.1,0.2]}{\ell_4}, \frac{[0.1,0.2];[0.3,0.5];[0.0,0.1]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.2;0.3;0.1\}}{\tau_5}, \left\{\frac{[0.4,0.5];[0.0,0.1];[0.3,0.4]}{\ell_1}, \frac{[0.6,0.7];[0.0,0.1];[0.1,0.2]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.1;0.4;0.2\}}{\tau_6}, \left\{\frac{[0.1,0.3];[0.1,0.3];[0.1,0.3]}{\ell_4}\right\}\right), \left(\frac{\{0.5;0.3;0.2\}}{\tau_8}, \left\{\frac{[0.1,0.4];[0.1,0.3];[0.2,0.3]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.5;0.2;0.1\}}{\tau_7}, \left\{\frac{[0.5,0.6];[0.2,0.3];[0.0,0.1]}{\ell_1}, \frac{[0.1,0.3];[0.3,0.5];[0.0,0.1]}{\ell_3}, \frac{[0.2,0.3];[0.4,0.5];[0.1,0.2]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.4;0.1;0.4\}}{\tau_9}, \left\{\frac{[0.3,0.4];[0.2,0.4];[0.1,0.2]}{\ell_3}\right\}\right), \left(\frac{\{0.2;0.4;0.3\}}{\tau_{10}}, \left\{\frac{[0.3,0.4];[0.1,0.2];[0.2,0.3]}{\ell_1}\right\}\right), \\
\left(\frac{\{0.3;0.1;0.3\}}{\tau_{11}}, \left\{\frac{[0.3,0.4];[0.2,0.3];[0.1,0.2]}{\ell_1}, \frac{[0.1,0.2];[0.2,0.3];[0.1,0.2]}{\ell_5}\right\}\right), \\
\left(\frac{\{0.4;0.3;0.2\}}{\tau_{12}}, \left\{\frac{[0.1,0.2];[0.3,0.4];[0.1,0.3]}{\ell_2}\right\}\right), \left(\frac{\{0.3;0.3,0.3\}}{\tau_{13}}, \left\{\frac{[0.3,0.4];[0.0,0.3];[0.1,0.2]}{\ell_3}\right\}\right), \\
\left(\frac{\{0.5;0.1;0.2\}}{\tau_{15}}, \left\{\frac{[0.3,0.4];[0.2,0.3];[0.0,0.1]}{\ell_1}, \frac{[0.5,0.6];[0.1,0.3];[0.0,0.1]}{\ell_2}\right\}\right), \\
\left(\frac{\{0.1;0.2;0.5\}}{\tau_{16}}, \left\{\frac{[0.2,0.3];[0.2,0.5];[0.1,0.2]}{\ell_1}, \frac{[0.6,0.7];[0.0,0.1];[0.0,0.1]}{\ell_4}\right\}\right), \\
\left(\frac{\{0.4;0.4;0.1\}}{\tau_{14}}, \left\{\frac{[0.3,0.4];[0.2,0.3];[0.1,0.2]}{\ell_1}\right\}\right)
\end{array}\right\}.$$

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8 Conclusion

Under the $IVFHS$-set environment, basic set laws, aggregation techniques, and fundamental features are described in this paper. Furthermore, possible hybrids of the FHS-set for interval type data are examined. More hybrids with cubic set, expert set, rough set etc., as well as algebraic structures like HS-matrices, HS-relations, HS-functions, HS-functional spaces, HS-topological spaces, HS-groups, HS-rings and HS-vector spaces may be developed in the future with interval settings.

Conflicts of Interest

Authors declare no conflicts of interest.

Data Availability Statement

No data is associated with this research work.

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