Neutrosophic Perspective of Neutrosophic Probability Distributions and its Application

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Abstract

Neutrosophical probability is concerned with inequitable and defective topics and processes. This is a subset of Neutrosophic measures that includes a prediction of an event (as opposed to indeterminacy) as well as a prediction of some unpredictability. When there is no such thing as a non-stochastic occurrence, the Neutrosophic probability is the probability of determining a stochastic process. It is a generalisation of classical probability, which states that the probability of correctly predicting an occurrence is zero. Until now, neutrosophic probability distributions have been derived directly from conventional statistical distributions, with fewer contributions to the determination of the for statistical distribution. We introduced the Poission distribution as a limiting case of the Binomial distribution for the first time in this study, and we also proposed Neutrosophic Exponential Distribution and Uniform Distribution for the first time. With numerical examples, the validity and soundness of the proposed notions were also tested.

Keywords: Neutrosophic Statistics, Poisson,Uniform,Exponential, Probability distribution.

1.Introduction

The classical probability distributions only work with data that is given to them. The study of classical distributions with uncertain values, as well as distribution factors such as time intervals, is aided by this research. Neutrosophic probability distributions are the name given to these distributions. Prof. Florentin Smarandache proposed neutrosophic statistics for the first time in 1995. This is frequently presented as a novel area of philosophy, described as a generalisation of formal logic[1] and an extension of the intuitionistic fuzzy logic that underpins intuitionistic logic[2]. The core notions of Neutrosophic set, as described by smarandach in [3,4,5,6] and salama et al. in [7,8,9,10], provides a new framework for handling indeterminacy data concerns. The indeterminacy data can be numbers, and Neutrosophic numbers are well-defined in [11,12,13,14,15,16].

In this paper, we demonstrate how the Neutrosophic crisp sets theory [17,18] is prone to cater to difficulties that follow classical distributions while still containing data that isn't precisely described. This paves the way for dealing with issues such as whether a specific sort of data can hold all of the information it was created to hold. When classical distributions are extended to be compatible with neutrosophic logic, the parameters of the classical distribution take on indeterminate values. This aids in dealing with any situation that may emerge while working with statistical data, particularly unclear and incorrect data. In 2014, Florentin Smarandache [19,20,21] supplied both the Neutrosophic and Neutrosophic natural distributions.

In this paper, we proposed Poisson distribution as a Limiting case of Binomial distribution ,exponential distribution and uniform distribution on Neutrosophic numbers with illustrative example for the first time.

The remainder of the paper is laid out as follows. Basic definitions have been supplied in section 2 to aid comprehension. In section 3, neutrosophic Poisson distribution is proposed as a limiting case of neutrosophic Binomial distribution. In section 4, one real world problem is solved using the proposed concept. In Section 5 Neutrosophic exponential distribution proposed. In section 6 application problem of Neutrosophic exponential distribution were discussed, In section 7 Neutrosophic Uniform distribution proposed and in section 8 application of Neutrosophic uniform distribution were discussed. In section 9, we conclusion of this work is given.

2.Basics Definition

For a better understanding of the proposed notion, we offered basic concepts of neutrosophic statistical distribution in this part.

Neutrosophic Statistical Distribution:[3]

When we repeat the experiment $n \geq 1$times, the Neutrosophic binomial random variable 'x' is determined as the number of successes. The Neutrosophic binomial probability distribution is also known as the Neutrosophic probability distribution of 'x'.

Neutrosophic Binomial Random Variables: It is marked as 'x' and is determined as the number of successes when we repeat the experiment $n \geq 1$ times.

Neutrosophic Binomial Probability Distribution: Neutrosophic probability distribution is the name given to the Neutrosophic probability distribution of 'x'.

Indeterminacy: It is not constrained by the findings of experiments (Successes and Failures).

Indeterminacy Threshold: It's the amount of trials when the outcome isn't known where $th \in \{0.1.2....n\}$

Let Prob(S) =The probability of success of a given trial

Prob(F) = The probability of a given trial leads to failure, for the two S and F different from indeterminacy.

Prob(I) = The probability of a particular trial results in an indeterminacy.

For example: for $x \in \{0,1,2....n\}, NP = \{T_x, I_x, F_x\}$

Tr_x: Chances of 'x' success, (n-x) failures, and indeterminacy, with the number of indeterminacy equal to or less than the threshold of indeterminacy.

Fal_x: Chances of 'y' success, with $y \neq x$ and $(n-y)$ failures and indeterminacy is less than the indeterminacy threshold.

Ind_x: Indeterminacy probabilities for 'z', when 'z' is strictly bigger than the indeterminacy threshold.

$$Tr_x + Ind_x + Fal_x = \left(\Pr ob(S) + \Pr ob(I) + \Pr ob(F)\right)^n$$

For complete probability, $\Pr ob(S) + \Pr ob(I) + \Pr ob(F) = 1;$

For incomplete probability, $0 \leq \Pr ob(S) + \Pr ob(I) + \Pr ob(F) < 1;$

For paraconsistent probability, $0 < \Pr ob(S) + \Pr ob(I) + \Pr ob(F) \leq 3;$

Now
$$Tr_x = \frac{n!}{x!(n-x)!}.\Pr ob(S)^x.\sum_{k=0}^{th} C_{n-x}^k \Pr ob(I)^k \Pr ob(F)^{n-x-k}$$

$$Tr_x = \frac{n!}{x!}\Pr ob(S)^x \sum_{k=0}^{th} \frac{\Pr ob(I)^k \Pr ob(F)^{n-x-k}}{k!(n-x-k)!} \quad \text{………………….(1)}$$

Equ(1) implies the Truth membership function of Neutrosophic Binomial Distribution.

$$Ind_x = \sum_{z=th+1}^{n} \frac{n!}{z!(n-z)!}.\Pr ob(I)^z.\left[\sum_{k=0}^{th} C_{n-z}^k \Pr ob(S)^z \Pr ob(F)^{n-z-k}\right]$$

$$Ind_x = \sum_{z=th+1}^{n} \frac{n!}{z!(n-z)!}.\Pr ob(I)^z.\left[\sum_{k=0}^{n-z} \frac{(n-z)!}{k!(n-z-k)!}\Pr ob(S)^k \Pr ob(F)^{n-z-k}\right]$$

$$Ind_x = \sum_{z=th+1}^{n} \frac{n!}{z!}.\Pr ob(I)^z.\left[\sum_{k=0}^{n-z} \frac{\Pr ob(S)^k \Pr ob(F)^{n-z-k}}{k!(n-z-k)!}\right]$$

…………………………(2)

Equ(2) implies the Indeterminacy of Neutrosophic Binomial Distribution.

$$Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} T_y = \sum_{\substack{y=0 \\ y\neq x}}^{n} \frac{n!}{y!}\Pr ob(S)^y \left[\sum_{k=0}^{th} \frac{\Pr ob(I)^k \Pr ob(F)^{n-y-k}}{k!(n-y-k)!}\right]$$

………………………………………………….. (3)

Equ(3) implies the Falsity of Neutrosophic Binomial Distribution.

Where C_u^v means combinations of 'u' elements taken by groups of v elements: $C_u^v = \dfrac{u!}{v!(u-v)!}$

Score Function of a single valued neutrosophic number:[11]

The score function of the single valued Neutrosophic number $b=\left(t\left(b\right),i\left(b\right),f\left(b\right)\right)$ can be communicated as follows: $S(b)=\frac{1+t\left(b\right)-f\left(b\right)}{2}$ for $s(b)\in\left[0,1\right]$

3. Neutrosophic Poisson Distribution (NPD) - Limiting case of Neutrosophic Binomial Distribution (NBD)

In this section, we proposed NPD as a limiting case of NBD under single valued neutrosophic environment.

The Neutrosophic Binomial probability distribution is a law of success, indeterminacy and failure and in a series of 'n' independent trials is defined in Equation (1)-(3)

Neutrosophic Binomial Distribution under limiting case of Poisson distribution when

(i) 'n' is indefinitely large (i.e.,) $n\to\infty$
(ii) P(S),P(I),P(S) is very small when P(I),P(F),P(S) tends to zero.
(iii) $np=\lambda$ (a finite quantity)

By the definition of neutrosophic Binomial distribution truth membership of is given in Equ(1) as follows

$$T_x=\frac{n!}{x!}\Pr ob(S)^x\sum_{k=0}^{th}\frac{\Pr ob(I)^k \Pr ob(F)^{n-x-k}}{k!\left(n-x-k\right)!}$$

The truth membership function of NPD can be deterned using Eqn (1).:

$$Tr_x=\lambda(S)^x\sum_{k=0}^{th}\frac{n\left(n-1\right)\left(n-2\right)..........\left(n-k-x-1\right)}{x!k!}\left(\frac{\lambda(F)}{n}\right)^k\left(1-\frac{\lambda(I)}{n}\right)^{n-x-k}$$

$$=\lambda(S)^x\sum_{k=0}^{th}n^k\frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)..................\left(1-\frac{\left(k+x-1\right)}{n}\right)\right]}{x!k!}\left(\frac{\lambda(F)}{n}\right)^k\left(1-\frac{\lambda(I)}{n}\right)^{n-x-k}$$

$$=\lambda(S)^x\sum_{k=0}^{th}\frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)..................\left(1-\frac{\left(k+x-1\right)}{n}\right)\right]}{x!k!}\left(\lambda(F)\right)^k\left(1-\frac{\lambda(I)}{n}\right)^{n-x-k}$$

$$\text{Since}\left[n\to\infty\ \&\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-x-k}=e^{-\lambda}\right]$$

$$Tr_{\ x}=\lambda(S)^x\sum_{k=0}^{th}\frac{e^{-\lambda(I)k}\lambda(F)^k}{k!}\qquad\text{...(4)}$$

which is the truth membership of a NPD.

Using Eqn (2) indeterminacy of NPD can be determined as follows:

$$Ind_x = \lambda(I)^z \sum_{k=0}^{th} \frac{n(n-1)(n-2)..........(n-k-z-1)}{x!k!} \left(\frac{\lambda(S)}{n}\right)^k \left(1-\frac{\lambda(F)}{n}\right)^{n-z-k}$$

Since $\left[n \to \infty \,\&\, \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-z-k} = e^{-\lambda} \right]$

$$Ind_x = \sum_{z=th+1}^{n} \lambda(I)^z \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k}\lambda(S)^k}{k!} \quad \text{...(5)}$$

The falsehood membership function of NPD can be defined as follows using Eqn (3).

$$Fal_x = \lambda(S)^y \sum_{k=0}^{th} n^k \frac{\left[\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right).................\left(1-\frac{(k+y-1)}{n}\right)\right]}{x!k!} \left(\frac{\lambda(F)}{n}\right)^k \left(1-\frac{\lambda(I)}{n}\right)^{n-y-k}$$

Since $\left[n \to \infty \,\&\, \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n-y-k} = e^{-\lambda} \right]$

$$Fal_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-\lambda(I)k}\lambda(F)^k}{k!} \quad \text{...(6)}$$

Therefore the neutrosophic poisson distribution can be determined as

$$Tr_x = \lambda(S)^x \sum_{k=0}^{th} \frac{e^{-\lambda(I)k}\lambda(F)^k}{k!}, \quad I_x = \sum_{z=th+1}^{n} \lambda(I)^z \sum_{k=0}^{n-z} \frac{e^{-\lambda(F)k}\lambda(S)^k}{k!}, \quad F_x = \sum_{\substack{y=0 \\ y\neq x}}^{n} \lambda(S)^y \sum_{k=0}^{th} \frac{e^{-\lambda(I)k}\lambda(F)^k}{k!}$$

4. Application of the neutrosophic poisson distribution:

In this section, we used [23] to apply the proposed notion to a real-world problem.

The atoms of a radioactive element disintegrate at random. Every gramme of this element emits between [0.4-0.9] alpha particles per second on average. When the indeterminacy threshold is reached during the next second, the probability of the number of alpha particles emitted from one gramme is exactly two, as shown below using the proposed NPD.

Solution:

Given data: $\lambda(S)=0.4$; $\lambda(I)=0.6$; $\lambda(F)=0.9$

$$x=2; k=2$$

From Equ(4) the formula for Neutrosophic truth membership function is given by

$$Tr_x=\lambda(S)^x\sum_{k=0}^{th}\frac{e^{-\lambda(I)k}\lambda(F)^k}{k!}$$

$$=(0.4)^2\sum_{k=0}^{2}\frac{e^{-0.6k}(0.9)^k}{k!}$$

$$Tr_x=0.75$$

From Equ(5) the formula for Neutrosophic truth membership function is given by

$$Fal_x=\sum_{\substack{y=0\\ y\neq x}}^{n}\lambda(S)^y\sum_{k=0}^{th}\frac{e^{-\lambda(I)k}\lambda(F)^k}{k!}$$

$$=(0.4)^2\sum_{k=0}^{2}\frac{e^{-0.6k}(0.9)^k}{k!}$$

$$Fal_x=0.52$$

From Equ(4) the formula for Neutrosophic truth membership function is given by

$$Ind_x=\sum_{z=th+1}^{n}\lambda(I)^z\sum_{k=0}^{n-z}\frac{e^{-\lambda(F)k}\lambda(S)^k}{k!}$$

$$=\sum_{z=th+1}^{n}(0.6)^2\sum_{k=0}^{z}\frac{e^{-0.9k}(0.4)^k}{k!}$$

$$Ind_x=0.25$$

$$\left(Tr_x, Fal_x, Ind_x\right)=\left(0.75, 0.52, 0.25\right) \quad \ldots\ldots\ldots\ldots\ldots(7)$$

Therefore from Eqn.(7) we applied all the values in [11] we get,

$$S(b) = \frac{1+0.75-0.52}{2}$$
$$S(b) = 0.615$$

As a result, the probability that 1 gm will produce 0.615 alpha particles in the following second is 0.615.

5. The Neutrosophic Exponential Distribution (NED):

In this section, we proposed NED's density function for Truth membership function, Indeteminacy and also for Falsity. Also proposed mean and variance of NED, and their Distribution function.

Definition:5.1

The Neutrosophic Exponential Distribution (NED) is a generalisation of the traditional exponential distribution. It can process any type of data, including non-specific data. The density function of NED is written as follows:

Table 1:

Density function for Neutrosophic Exponential Distribution(NED)		
Truth Membership	Indeteminacy	Falsity
$f_{Neu}\left(T_x(x)\right) = \lambda_{Neu}\left(T_x(x)\right)e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}$	$f_{Neu}\left(I_x(x)\right) = \lambda_{Neu}\left(I_x(x)\right)e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}$	$f_{Neu}\left(F_x(x)\right) = \lambda_{Neu}\left(F_x(x)\right)e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}$

Where 'x' is a Neutrosophic random variable

$\lambda_{Neu}\left(T_x(x)\right)$ - Truth membership function's distribution parameter.

$\lambda_{Neu}\left(I_x(x)\right)$ - Indeterminacy membership function's distribution parameter.

$\lambda_{Neu}\left(F_x(x)\right)$ - Falsity membership function's distribution parameter.

Properties of NED:

Table 2:

	Truth Membership Function	Indeterminacy	Falsity
Expected Value for NED: $E(x)$	$E(T_x) = \frac{1}{\lambda_{Neu}\left(T_x(x)\right)}$	$E(I_x) = \frac{1}{\lambda_{Neu}\left(I_x(x)\right)}$	$E(F_x) = \frac{1}{\lambda_{Neu}\left(F_x(x)\right)}$
Variance for NED: $Var(x)$	$Var(T_x) = \frac{1}{\left(\lambda_{Neu}\left(T_x(x)\right)\right)^2}$	$Var(I_x) = \frac{1}{\left(\lambda_{Neu}\left(I_x(x)\right)\right)^2}$	$Var(F_x) = \frac{1}{\left(\lambda_{Neu}\left(F_x(x)\right)\right)^2}$

Table 3:

	Truth Membership Function	Indeterminacy	Falsity
Distribution Function for NED: $N\left(F\left(T_x, I_x, F_x\right)\right) = NP\left(X \le x\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x}\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x}\right)$	$\left(1-e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x}\right)$

6. Application of the neutrosophic exponential distribution:

We used the proposed notion in a case study problem from [21] with different intervals in this part.

Consider what amount of time it requires for a bank to end a client assistance's, which follows a dramatic dispersion with a normal of one moment. Compose a thickness work, mean, change, and dissemination work for the time it takes to end a customer's administration, and afterward propose the likelihood of ending a customer's administration in the stretch [1,2] minute.

Assume that x: indicates the time necessary per minute to terminate the client's service.

The average $\frac{1}{\lambda\left(T_x(x)\right)} = 1$; $\frac{1}{\lambda\left(I_x(x)\right)} = \frac{1}{1.5} = 0.6667$; $\frac{1}{\lambda\left(F_x(x)\right)} = \frac{1}{2} = 0.5$

The probability density function NED:

From table 1:

$$f_{Neu}\left(T_x(x)\right) = \lambda_{Neu}\left(T_x(x)\right) e^{-\left(\lambda_{Neu}\left(T_x(x)\right)\right)x} = 0.3679$$

$$f_{Neu}\left(I_x(x)\right) = \lambda_{Neu}\left(I_x(x)\right) e^{-\left(\lambda_{Neu}\left(I_x(x)\right)\right)x} = 0.3347$$

$$f_{Neu}\left(F_x(x)\right) = \lambda_{Neu}\left(F_x(x)\right) e^{-\left(\lambda_{Neu}\left(F_x(x)\right)\right)x} = 0.2707$$

From Table 2:

$$E\left(T_x\right) = \frac{1}{\lambda_{Neu}\left(T_x(x)\right)} = 1; Var\left(T_x\right) = \frac{1}{\left(\lambda_{Neu}\left(T_x(x)\right)\right)^2} = 1$$

$$E\left(I_x\right) = \frac{1}{\lambda_{Neu}\left(I_x(x)\right)} = 0.6667 ; Var\left(I_x\right) = \frac{1}{\left(\lambda_{Neu}\left(I_x(x)\right)\right)^2} = 0.4444$$

$$E\left(F_x\right) = \frac{1}{\lambda_{Neu}\left(F_x(x)\right)} = 0.5 ; Var\left(F_x\right) = \frac{1}{\left(\lambda_{Neu}\left(F_x(x)\right)\right)^2} = 0.2500$$

There's a chance that the client's service will be terminated in less than 1,0.67,0.5 minute. :

From Table 3

$$Neu\left(F\left(T_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=Neu\left(P\left(X\le 1\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(T_x\left(x\right)\right)\right)x}\right)=0.63$$

$$Neu\left(F\left(I_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=Neu\left(P\left(X\le 1\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(I_x\left(x\right)\right)\right)x}\right)=0.4883$$

$$Neu\left(F\left(F_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=Neu\left(P\left(X\le 1\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(F_x\left(x\right)\right)\right)x}\right)=0.39$$

Probability that the client's service will be terminated in less than a minute:

$$Neu\left(F\left(T_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(T_x\left(x\right)\right)\right)x}\right)=\left(1-e^{-[0.5,1.5]1}\right)$$

That is, the likelihood of terminating a client's service in less than a minute ranges between 0.085 and 0.085 when we use the numbers in [11].

Probability that the client's service will be terminated in less than 1.5 minutes:

$$Neu\left(F\left(I_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(I_x\left(x\right)\right)\right)x}\right)=\left(1-e^{-[0.5,1.5]1.5}\right)$$

When we utilise the numbers in [11], the probability of terminating a client's service in less than 1.5 minutes ranges between 0.2111 and 0.2111.

Probability of terminating a client's service in under 2 minutes:

$$Neu\left(F\left(F_x\right)\right)=Neu\left(P\left(X\le x\right)\right)=\left(1-e^{-\left(\lambda_{Neu}\left(F_x\left(x\right)\right)\right)x}\right)=\left(1-e^{-[0.5,1.5]2}\right)$$

When we apply the figures in [11], the probability of terminating a client's service in less than 2 minutes ranges between 0.435 and 0.435.

7. Neutrosophic Uniform Distribution (NUD):

In this section, we proposed NUD's density function for Truth membership function, Indeteminacy and also for Falsity. Also proposed mean and variance of NUD.

Definition:7.1

The numeric value of a continuous variable is its numeric value. Although X is a conventional Uniform distribution, the distribution parameters an or b, or both, are untrustworthy. For example, 'a' or 'b,' or both,' are sets of two or more components having a< b, and we can use NUD to stand for Truth Membership, Indeterminacy, and Falsity. The following is a NUD definition that has been offered.: $f\left(T_x\right)=\begin{cases}K, & a\left(T_x\right)<b\left(T_x\right)\\ 0 & otherwise\end{cases}$

Since the total probability always unity

$$\int_{a(T_x)}^{b(T_x)} f(T_x)dx = 1$$

$$\Rightarrow \int_{a(T_x)}^{b(T_x)} K dx = 1$$

$$[K]_{a(T_x)}^{b(T_x)} = 1$$

$$K = \frac{1}{b(T_x) - a(T_x)}$$

Density function of NUD for Truth Membership function is given by

$$f_{Neu}(T_x) = \frac{1}{b(T_x) - a(T_x)} \text{ for } a(T_x) < b(T_x) \ldots\ldots\ldots\ldots\ldots\ldots(8)$$

Similiarly we propose NUD for Indeterminacy

$$f_{Neu}(I_x) = \frac{1}{b(I_x) - a(I_x)} \text{ for } a(I_x) < b(I_x) \ldots\ldots\ldots\ldots\ldots\ldots(9)$$

Also we propose NUD for Falsity function

$$f_{Neu}(F_x) = \frac{1}{b(F_x) - a(F_x)} \text{ for } a(F_x) < b(F_x) \ldots\ldots\ldots\ldots(10)$$

Mean of a NUD:

Mean for NUD for Truth Membership function is given by

$$E(T_x) = \int_{a(T_x)}^{b(T_x)} f(T_x)dx = \frac{b(T_x) - a(T_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

Mean for NUD for Indeterminacy is given by

$$E(I_x) = \int_{a(T_x)}^{b(T_x)} f(I_x)dx = \frac{b(I_x) - a(I_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots(12)$$

Mean for NUD for Falsity is given by

$$E(F_x) = \int_{a(T_x)}^{b(T_x)} f(F_x)dx = \frac{b(F_x) - a(F_x)}{2} \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

Variance of NUD:

Variance of NUD for Truth Membership function is given by

$$Var(T_x) = \frac{\left(b(T_x) - a(T_x)\right)^2}{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(14)$$

Variance of NUD for Indeterminacy is given by

$$Var(I_x) = \frac{\left(b(I_x) - a(I_x)\right)^2}{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(15)$$

Variance of NUD for Falsity function is given by

$$Var(F_x) = \frac{\left(b(F_x) - a(F_x)\right)^2}{12} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(16)$$

8. Application of the neutrosophic Uniform distribution:

We used the proposed notion in a case study problem from [21] with different intervals in this part.

The station official explained that assuming 'x; is a variable that denotes a person's waiting time for a passenger's bus (in minutes), the bus arrival time is not mentioned.:

1-the bus arrival time is: either now or in 5 minutes [0,5] or in 15-20 minutes [15,20], then

2- the bus arrival time is: either now or in 5 minutes [0,5], or in 20-25 minutes [20,25], then

3- the bus will arrive in either 5 minutes [0,5] or 25-30 minutes [25,30], depending on when you arrive.

Here $a(T_x) = a(I_x) = a(F_x) = [0,5]$

$$b(T_x) = [15,20];$$
$$b(I_x) = [20,25];$$
$$b(F_x) = [25,30];$$

Then the density

Density function of NUD for Truth Membership function is given by

From Eqn(8)⇒ $f_{Neu}(T_x) = \frac{1}{b(T_x) - a(T_x)} = \frac{1}{[15,20]-[0,5]} = \frac{1}{[10,15]} = [0.067, 0.1]$

Density function of NUD for Indeterminacy is given by

From Eqn(9)⇒ $f_{Neu}(I_x) = \frac{1}{b(I_x) - a(I_x)} = \frac{1}{[20,25]-[0,5]} = \frac{1}{[15,20]} = \frac{[0.05, 0.067]}{2} = [0.025, 0.033]$

Density function of NUD for Falsity function is given by

From Eqn(10)⇒ $f_{Neu}(F_x) = \frac{1}{b(F_x) - a(F_x)} = \frac{1}{[25,30]-[0,5]} = \frac{1}{[20,25]} = [0.04, 0.05]$

Mean of the bus arrival time 15-20 minutes is given by

$$E(T_x)=\frac{b(T_x)-a(T_x)}{2}=\frac{[15,20]-[0,5]}{2}=\frac{[10,15]}{2}=\frac{[0.067,0.1]}{2}=[0.0335,0.05]$$

Mean of the bus arrival time 20-25 minutes is provided by

$$E(I_x)=\frac{b(I_x)-a(I_x)}{2}=\frac{[20,25]-[0,5]}{2}=\frac{[15,20]}{2}=\frac{[0.05,0.067]}{2}=[0.025,0.0335]$$

Mean of the bus arrival time 25-30 minutes is provided by

$$E(F_x)=\frac{b(F_x)-a(F_x)}{2}=\frac{[25,30]-[0,5]}{2}=\frac{[20,25]}{2}=\frac{[0.04,0.05]}{2}=[0.02,0.025]$$

Variance of the bus arrival time 15-20 minutes is given by

From Eqn(14)$\Rightarrow$ $Var(T_x)=\frac{(b(T_x)-a(T_x))^2}{12}=\frac{[0.0335,0.5]^2}{12}=[0.000093,0.0208]$

Variance of the bus arrival time 20-25 minutes is given by

From Eqn(15)$\Rightarrow$ $Var(I_x)=\frac{(b(I_x)-a(I_x))^2}{12}=\frac{[0.05,0.067]^2}{12}=[0.000208,0.000374]$

Variance of the bus arrival time 25-30 minutes is given by

From Eqn(16)$\Rightarrow$ $Var(F_x)=\frac{(b(F_x)-a(F_x))^2}{12}=\frac{[0.04,0.05]^2}{12}=[0.000133,0.000208]$

9. Conclusion

Classical probability solely considers determinate data, but neutrosophic probability considers indeterminate data with varying degrees of indeterminacy. Hence in this paper, we proposed many of the standard distribution called Poisson distribution as a limiting case of Binomial distribution,NED,NUD under neutrosophic environment. Also, using the proposed concept probability value has been obtained for a real world problem. In future, probability distributions may be proposed under different neutrosophic environment.

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References:

[1] Zadeh. L,Fuzzy Sets. Inform.Control 8,(1965).

[2] Atanassov.K, Intuitionistic fuzzy sets. In V.Sgurev, ed., ITKRS Session, Sofia, June 1983, Central Sci. and Techn. Library, Bulg. Academy of Sciences, (1984).

[3] Smarandache, F, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set Probability, and Statistics University of New Mexico, Gallup, NM 87301,USA,(2002).

[4] Smarandache, F.A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability,American Research Press, Rehoboth, NM,1999.

[5] Smarandache, F.Neutrosophic set a generalization of the intuitionistic fuzzy sets. Inter. J.Pure Appl. Math.,24,287-297,(2005).

[6] Smarandache.F, Introduction to Neutrosophic Measure, Integral, Probability. Sitech Education Publisher, (2015).

[7] Hanafy, I.M,Salama,A.A and Mahfouz, K,Correlation of Neutrosophic Data, International Refereed Journal of Engineering and Science (IRJES),1(2),PP39-33,2012.

[8] Hanafy,I.M.,Salama,A.A, and Mahfouz, K.M Neutrosophic Classical Events and its Probability. International Journal of Mathematics and Computer Applications Research (IJMCAR), 3(1),171-178,(2013).

[9] Salama, A.A, and Alblowi,S.A,Generalized Neutrosophic Set and Generalized Neutrosophic Sapace. Journal Computer Sci. Engineering, 2(7),129-132,(2012).

[10] Salama,A.A, and Alblowi, S.A,Neutrosophic Set and Neutrosophic Topological Spaces, ISOR J. Mathematics, 3(3),31-35,(2012).

[11] Mondal, K., Pramanik, S., & Smarandache, F. NN-harmonic mean aggregation operators-based MCGDM strategy in a Neutrosophic number environment. Axoims 2018,7.12: DOI:10.3390/axioms7010012.

[12] Pramanik,S., & Dey,P.P (2018). Bi-level linear programming problem with Neutrosophic numbers. Neutrosophic Sets and Systems, 21,110-121. Https://doi.org/10.5281/zenoda.1408663.

[13] Pramanik, S., Banerjee, D.(2018). Neutrosophic number goal programming for multi-objective linear programming problem inneutrosophic number environment. MOJ Current Research & Review, 1(3),135-141. DOI:10.15406/mojcrr.2018.01.0021.

[14] Banerjee, D.Pramanik,S (2018). Single-objective linear goal programming problem with Neutrosophic numbers. International, Journal of Engineering Science & Research Technology, 7(5),454-469.

[15] Florentin Smarandache: A Unifying Field in Logic,Neutorsophy, Neutrosophic Set,Neutrosophic Probability and Statistics (Sixth Edition). InfoLearn Quest, 1998-2007,156 p.http://fs.unm.edu/eBook-Neutrosophics6.pdf.

[16] Florentin Smarandache: Neurosophic Overset, Neutrosophic Underset, and Neutrosophic Offset.Similarly for Neutrosophic Over-/Under-/Off-Logic,Probability, and Statistics. Pons Editions, Brussels, 2016, 168p.http://fs.unm.edu/Neutosophic Overset UndersetOffset.pdf.

[17] Salama.A.A, Smarandache. F, Neutrosophic Crisp Set Theory. Education Publishing, Columbus, (2015).

[18] Salama. A.A, Smarandache. F, and Kroumov. V,Neutrosophic Crisp Sets & Neutrosophic Crisp Topological Spaces. Neutrosophic Sets and Systems, Vol. 2, pp.25-30, (2014).

[19] Smarandache.F, Neutorsophical statistics Sitech & Education publishing,(2014).

[20] Patro.S.K,Smarandache. F. The Neutrosophic Statistics Distribution, More Problems, More Solutions. Neutrosophic Sets and Systems, Vol.12,(2016).

[21] Rafif Alhabib, Moustafa Mzher Ranna et.al "Some Neutrosophic Probability Distributions" Neutrosophic Sets and Systems, Vol.22,(2018).

[22] S.K. Patr,F.Smarandache "The Neutrosophic Statistical Distribution, more problems, more solutions" Neutrosophic Sets and Systems,Vol.12,(2016).

[23] Gupta.A.C.Kapoor.V.K.Mathematics Statistics, Schand publisher, India,(2009).